Combinatorics, 2016 Fall, USTC
Homework 7

- The due is on Tuesday, Nov. 8, at beginning of the class.

- Solve all problems.

1. Let $1 \leq d_1 \leq \ldots \leq d_n$ be integers. Prove that there exists a tree with degrees $d_1, \ldots, d_n$ if and only if $\sum_{i=1}^{n} d_i = 2n - 2$.

2. Given a tree T and two vertices x, y of T, let $d(x, y)$ be the length of the unique path in T between x and y. Determine those trees T on n vertices for which

$$\sum_{x,y \in V(T)} d(x, y)$$

is maximal and minimal, respectively.

3. Show that any tree has more leaves than vertices of degree at least 3.

4. Let $n = 2k$. Characterize all interesting families $\mathcal{F} \subseteq \binom{[n]}{k}$ with $|\mathcal{F}| = \binom{n-1}{k-1}$.