Combinatorics, 2020 Fall, USTC
Homework 12

- The due is on Tuesday, Jan. 12, 2021, at beginning of the class.

1. Prove that among any $2^{k-1}+1$ vectors in $\mathbb{F}_2^n$, there are k vectors which are linearly independent.

2. Let $L \subseteq \{0,1,...,n\}$ be a set. Let $\{A_1, A_2, ..., A_m\}$ be an L-intersecting family of subsets of $[n]$, where each A_i is of a constant size, say k. Prove that $m \leq \binom{n}{|L|}$.

3. Suppose $R_1, ..., R_m \subseteq [n]$ satisfy that $|R_i| \neq 0 \mod 6$ for every i, and $|R_i \cap R_j| = 0 \mod 6$ for every $i \neq j$. Prove that $m \leq 2n$.

4. Derive the following result from Bollobás's theorem. Let $A_1, ..., A_m$ be subsets of size a and $B_1, ..., B_m$ be subsets of size of b such that $|A_i \cap B_i| = t$ for all i and $|A_i \cap B_j| > t$ for all $i \neq j$. Then $m \leq \binom{a+b-t}{a-t}$.

5. Let $A_1, ..., A_m$ and $B_1, ..., B_m$ be finite subsets such that $A_i \cap B_i = \emptyset$ for all i and $A_i \cap B_j \neq \emptyset$ for all $i < j$. Prove that if $|A_i| \leq a$ and $|B_i| \leq b$ for all i, then $m \leq \binom{a+b}{a}$.