Combinatorics, 2020 Fall, USTC
Homework 7

- The due is on Tuesday, Nov. 17, at beginning of the class.

1. Prove that for any intersecting family $\mathcal{F} \subset 2^{[n]}$, there exists an intersecting family $\mathcal{F}' \subset 2^{[n]}$ satisfying that $|\mathcal{F}'| = 2^{n-1}$ and $\mathcal{F} \subset \mathcal{F}'$.

2. Let G be an n-vertex d-regular graph and let $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$ be the eigenvalues of G. Prove that $\lambda_1 = d$.

3. Given the same conditions as in the previous problem, prove that if G is bipartite, then $\lambda_n = -d$.

4. Find the maximum number of line segments that a Hasse diagram of a poset $(X, \prec)$ with $|X| = n$ can have. Then define a poset which achieves this maximum number and draw its Hasse diagram.

5. For two natural numbers a, b, the symbol $a|b$ means that "a divides b". In other words, there exists a natural number c such that $b = ac$. First verify that the relation "$|$" is a partial ordering on the set $\mathbf{N}$ of natural numbers. Then prove that every finite poset can be embedded into $(\mathbf{N}, |)$.