Electric probes in plasmas

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This paper provides a background for the use of Langmuir and gridded energy analyzer probes in diagnosing plasmas with varied characteristics. Theory is illustrated which governs the analysis of data from, and the design of these probes. Several probe analysis techniques and some of their typical problems are presented.

I. INTRODUCTION

Electric and magnetic probes are among the earliest and most basic plasma diagnostics. They are used in a wide variety of plasmas ranging from the low-density, low-magnetic-field space plasmas to those at the edge of fusion research devices. Interest in probes has increased and waned over the years since Irving Langmuir first explored their usefulness. They have lately enjoyed a revival in fusion research because of the recently recognized importance conferred upon the plasma edge. Although electric probes are fairly straightforward to design, build and operate, theoretical models used to analyze the resultant data can be quite complicated. Generally, the degree of difficulty encountered in applying the selected model depends on what accuracy is desired. Choosing a theory depends on two criteria: (1) whether a correct theory exists for the situation in hand, and (2) whether the accuracy and difficulty of performing the theoretical analysis is justified by the accuracy of the experimental data.

This paper presents a brief theoretical background of the theory for the operation of, and analysis of data from, Langmuir electric and gridded-energy-analyzer (GEA) probes. Further in-depth theory is available in a number of references.1-4 Design criteria and further examples of the use of a variety of probes can be found in the literature.5-7 Perhaps of greater interest to an experimentalist reading this paper is the discussion of various data analysis methods.

II. SIMPLE PROBE THEORY

A. Qualitative description of the Langmuir trace

The general form of a Langmuir probe current versus bias voltage characteristic is shown in Fig. 1 (a). In the following discussion, current being drawn by the probe is designated as positive. \( \Phi_p \) is the plasma potential with respect to the probe ground. When \( \Phi \), the probe bias, is very negative with respect to \( \Phi_p \), the electric field around the probe will prevent all but the highest energy electrons from reaching the probe, effectively reducing the electron current to zero. The current collected by the probe \( I_{si} \) is then due entirely to positive ions, which encounter only an attracting electric field; thus it is termed the “ion-saturation current.”

As \( \Phi \) is increased, the number of electrons which are able to overcome the repelling electric field and contribute a negative current increases exponentially, reducing \( I \) from the value \( I_{si} \). Eventually, the electron current collected is equal to \( -I_{si} \) at \( \Phi = \Phi_F \). \( \Phi_F \) is less than \( \Phi_p \) because the electron thermal velocity is \( (M_e/m_e)^{1/2} \) greater than that of the ions.

When the probe is allowed to “float,” independent of a bias, it quickly develops the potential \( \Phi_f \) to repel electrons.

Further increase of the probe bias to \( \Phi_p \) allows the electron current to completely dominate \( I \). At this probe bias, electrons are unrestricted from being collected by the probe. Any further increase in \( \Phi \) will simply add energy to the electrons, not increase the current drawn. Thus the term “electron-saturation current” is used in this limit.

B. Sheath analysis for nonmagnetized plasmas

When a probe is immersed in the plasma it strongly perturbs the potential over a small region designated the sheath. This perturbation is limited by electron shielding to several Debye lengths.
\( \lambda_{\text{Debye}} = [\varepsilon_0 T_e / n_e e^2]^{1/2} \)
in distance from the probe.\(^8\) The geometry that will serve as a basis for the following discussion is illustrated in Fig. 1(b). The probe surface is designated by \( x = s \) and the unperturbed plasma by \( x = \infty \). The sheath thickness \( x_s - x_p \) is assumed to be much less than \( x_s \) allowing use of a planar approximation, independent of probe geometry. The plasma potential at infinity \( \Phi(\infty) \) is defined to be zero.

The theory of the flow of charged particles to an electrical probe can be extremely complex. In this section we will introduce Langmuir probe theory by way of the simplest case\(^9\); a probe immersed in a zero-magnetic-field plasma with the temperature of the ions much less than that of the electrons. The following additional assumptions are made about the plasma in which the probe is immersed: (1) electron and ion densities are equal; (2) Debye length \( \ll \) probe dimensions \( \ll \); electron and ion mean free paths; (3) Maxwellian velocity distributions far away from the probe; (4) no bulk motion of the unperturbed plasma \( \left( v_{\text{diss}} < v_{\text{thermal}} \right) \); (5) no secondary electron emission from the probe surface; (6) fully ionized, \( Z = 1 \) plasma.

The task of any probe analysis model is to determine the unperturbed values (in absence of probe) of density and temperature from the measured variation of probe current with changing bias potential. Here, we will concentrate on the range of probe bias where electrons are repelled, i.e., bias potential less than the plasma potential.

1. Density

First, let us direct our attention to the electrons. Their density \( n_e(x_1) \) can be obtained by integration over the local distribution function. The relationship of the distribution function at an arbitrary \( x_1 \) to that at \( \infty \) can be determined by several factors: first, particles are conserved so that along a particle's trajectory in phase space \((x_1,v_1) \rightarrow (x_2,v_2)\);

\[
f(x_1,v_1)dx_1dv_1 = f(x_2,v_2)dx_2dv_2.
\]

Second, Liouville's theorem states that the phase volume is a constant\(^10\). In other words, \( dx_1dv_1 \) is a constant allowing us to equate \( f(x_1,v_1) \) and \( f(x_2,v_2) \). Third, conservation of energy enables us to relate the potential and kinetic energies at the two locations \((x_1,v_1)\) and \((x_2,v_2)\) by

\[
\frac{1}{2} mv^2(x) + e\Phi(x) = \frac{1}{2} mv^2(\infty) + e\Phi(\infty).
\]

We can now write down the local distribution function in terms of value at \( x_2 = \infty \) and furthermore integrate for \( n_e(x_1) \)

\[
n_e(x_1) = \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dv = \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dv \left[ \frac{m_e}{2\pi kT} \right]^{1/2} \times \exp \left[ -\frac{m_ev^2}{2} + e\Phi(x)/kT \right].
\]

For the moment, \( T \) stands for only the electron temperature. The integration limits are determined by the existence of two classes of electrons: (1) electrons traveling towards the probe \((v < 0)\), and (2) electrons that have been repelled before reaching the probe and are traveling away \((0 < v < v_c)\).

\[ v_c = \sqrt{2e[\Phi(x) - \Phi(x_p)]/m_e} \]
is the minimum velocity which electrons need to overcome the electric repulsion of the probe and be collected there. Completing the integration we have

\[
n_e(x_1) = \frac{n_e}{2} \exp [e\Phi(x_1)/kT] \times \{ 1 + \text{erf}[\sqrt{e\Phi(x_1) - \Phi(x_p)}/kT] \}.\]

At the \( x \) of interest, near the sheath, \( \Phi(x_1) - \Phi(x_p) \gg kT/e \) and the Boltzmann relation is retrieved,

\[
n_e(x_1) = n_e \exp [e\Phi(x_1)/kT].\]

For \( x_1 > x \), the plasma is quasineutral. Therefore, Eqs. (4) and (5) apply equally well to ions in that region.

2. Fluxes

Next, let us turn our attention to the ion and electron fluxes, and thus the current they carry to the probe. The ion velocity outside the sheath is determined by Eq. (2), where both the unperturbed potential and the ion velocity at \( x = -\infty \) are defined to be zero. Combining this knowledge with Eq. (5) provides a description of the ion flux outside the sheath region,

\[
\Gamma_i(x) = n_i(x) v_i(x) = n_e \exp [e\Phi(x)/kT] \times [ -2e\Phi(x)/M_i ]^{1/2} \quad (x > x_s),
\]

where we have dropped the subscript on \( x \). Since there are no particle sources within the sheath, the current flowing to the probe is constant in \( x \). The electron flux to the probe is just the random flux reduced by the Boltzmann factor evaluated at the probe potential, so that the total current collected by a probe at bias \( \Phi(x_p) \) is

\[
I = e \left[ \frac{1}{4} n_e \bar{C}_e A_p \exp \left( \frac{e\Phi(x_p)}{kT} \right) \right.
+ A_i n_e \exp \left( \frac{e\Phi(x)}{kT} \right) \left( -2e\Phi(x) / M_i \right)^{1/2} \left],
\]

where \( A_i \) and \( A_p \) are the ion and electron probe areas, \( x_s \) the sheath position, and \( C_e \) the electron thermal speed. The potential at the sheath edge, \( \Phi_s = \Phi(x = x_s) \) as yet undetermined. \( \Phi_s \) can be determined by solving Poisson's equation in the vicinity of the sheath.

3. Potential at the sheath edge

We now turn our attention to just inside the sheath, where ion and electron densities are no longer equal. To determine the ion density for \( x < x_s \) we note that in planar geometry, the ion flux must be constant across the sheath and so

\[
n_i(x) = n_i [v_i/v(x)] = n_e \sqrt{\Phi} / \Phi(x),
\]

\[(x \leq x_s),\]

where \( n_e \) and \( \Phi \) are the values of ion density and potential at the sheath edge. Then Poisson's equation inside the sheath becomes

\[
\nabla^2 \Phi = - \left( n_e / \varepsilon_0 \right) \left[ (\Phi / \Phi(x))^{1/2} - 2 \exp [e(\Phi(x) - \Phi_s)/kT] \right],
\]


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which, in the region near the sheath, can be expanded\textsuperscript{9} as
\[ \nabla^2 \Phi = - \left( \frac{e n_s}{e_0} \right) \left[ - \frac{1}{2 \Phi_s} - \frac{e}{kT} \left( \Phi(x) - \Phi_s \right) \right]. \]

Nonoscillatory solutions of this equation are possible for \( \Phi_s < -kT/2e \). The value of \( d\Phi/dx \) for \( x \gg x_s \) can be obtained from Poisson's equation in the quasineutral region outside the sheath.\textsuperscript{3} \( \Phi(x) \) has infinite slope for \( \Phi_s > -kT/2 \). Therefore, we determine that
\[ \Phi_s = -kT/2e. \]
There are no solutions of Poisson's equation (no sheath) for \( \Phi(x_s) > -kT/2e \).

4. Total current to probe

Substituting the value for the sheath potential, Eq. (11), into Eq. (7), the current collected by the probe is then
\[ I = I_{se} \exp \left[ e\Phi(x_s)/kT \right] + I_u = -eA_x \left\{ \frac{1}{4\pi n_0 \overline{C}_e} \exp \left[ e\Phi(x_s)/kT \right] \right. \]
\[ \left. -n_0 \left( kT_e/M_j \right)^{1/2} \exp \left( -\frac{x}{r} \right) \right\} \]
\[ I_u = -eA_p \left( \frac{1}{4\pi n_0 \overline{C}_e} \right), \]
\[ I_{se} = 0.6 \ln C_s A_p e. \]
The sheath area has been taken to be approximately equal to that of probe; \( C_s \) is the plasma sound speed. The floating potential can be solved for, by setting Eq. (12) to zero,
\[ e\Phi/kT_e = 0.5 \left[ \ln \left( 2\pi m_e/M_j \right) - 1 \right]. \]

\( T_e \) can be obtained by examining the slope of the curve described by Eq. (12) in the exponential region between \( I_{se} \) and \( I_e \). Then, the density can finally be deduced from Eq. (14).

Throughout this discussion we have effectively assumed the ion temperature should be low enough such that 0.5\( M_j \overline{v}^2 (\infty) < -e\Phi(x_s) \). If finite ion temperature is allowed at \( x = \infty \), then the specific ion orbits and probe geometry must be included in the model. No simple formulas exist for replacing Eq. (14) because of the numerical integrations involved, but some results indicate a weak dependence of the constant 0.61 (\( = e^{-1/2} \)) in that formula. Results for monoenergetic ions with \( T_e = 0.01 \times T_e \) and 0.5 \( \times T_e \) incident on a spherical probe\textsuperscript{4} yield coefficients of 0.57 and 0.54, respectively. Therefore, in practice, Eq. (14) is widely used.

III. REFINEMENTS TO LANGMUIR PROBE THEORY

A. Practical approach to probe theory with nonzero magnetic field

The above theory is even further complicated by the inclusion of magnetic field effects. Ions and electrons spiral around the magnetic field lines with a radius, in the plane perpendicular to the field line, of \( r_i = m_i eB \). If this Larmor radius for both ions and electrons is much greater than the probe dimension \( a \), then the previous zero-magnetic-field results are recovered.

The determination of \( T_e \) can be divided into three regimes: \( r_i > a \); \( r_i < a \); \( r_i \sim a \). The first, as described above, is equivalent to that of the unmagnetized plasma. In the second regime, one must assume that the electron term of Eq. (12) is reduced due to the limiting rate of cross-field diffusion.

Bohm has calculated a reduction factor \( R \) for the case of all positive ions repelled from the probe\textsuperscript{4} (electron-saturation). \( R \) decreases as some monotonic function of the ratio of perpendicular to parallel diffusion coefficients. The simplest approach to determining \( T_e \) in this regime is to assume \( R \sim \text{const} \) for all probe biases. Then one can again use Eq. (12) to determine \( T_e \). This results in higher values of \( T_e \) than allowing \( R \) to be a function of probe bias.\textsuperscript{11}

In the remaining regime, \( r_i \sim a \), \( T_e \) should be determined by fitting Eq. (12) to the data only over that portion where \( r_i < a \).

Once \( T_e \) is determined from the above recipes, then the unperturbed density \( n_0 \) follows from Eq. (14) assuming, for \( r_i < a \), that the probe area is reduced to its projection along the magnetic field.

The determination of the plasma potential in the case of a magnetized plasma is difficult. This is because a "knee" in the Langmuir trace as shown in Fig. 1(a) is no longer simply related to the plasma potential. One must then use Eq. (15) which relates the plasma potential to the potential of a floating probe. More elaborate theories which include effects such as finite ion temperature can also be used.\textsuperscript{11,12}

B. Analytic result for nonzero magnetic field, \( T_j \sim 0 \)

An analytic treatment of probes applicable to the case of a high magnetic field plasma does exist.\textsuperscript{13} Although not derived explicitly for the case of a plasma with magnetic field, this theory allows collisions sufficient to provide the minimum particle source required to make the one-dimensional problem soluble. Too many collisions cause an ion to lose its memory of the potential where it was borne along the potential gradient. The values of the sheath potential, ion flux and floating potential are modified as follows:
\[ \Phi_s = -0.854-ke/T_e, \]
\[ \Gamma_i = A_i n_0 \left( 2kT_e/M_i \right)^{1/2} \left( 1/\pi \right) \phi_s^{-1/2} \]
\[ = 0.49n_0 C_i A_i, \]
\[ \Phi_i = \left( kT_e/2e \right) \ln \left( 4m_e/0.854\pi M_i \right). \]

\( A_i \) is again the projected area of the sheath along the field line. The most significant differences between this 1-D model which allows particle sources and the model of Sec. II B is found in the value of the sheath potential. Given the accuracy of typical probe data, a coefficient of 0.5 in Eqs. (14) and (17) is adequate for most cases, except perhaps when the ion flux is reduced by limits on perpendicular diffusion into the probe flux tube.

C. Nonzero magnetic field, \( T_j > 0 \)

A kinetic treatment of the general case of one-dimensional plasma flows is given by Emmett.\textsuperscript{14} Unfortunately, when applied to the analysis of probes in strong magnetic fields it leads to unphysical results. In the limit of \( T_j \gg T_e \), the predicted ion flux to the surface becomes twice the random value. This effect is due to a source model which assumes ions are borne with the local \( T_i \). It is hard to ascertain the effect of such a source model at smaller ratios of \( T_j/T_e \).

Possibly the most general theory that can be applied to the variety of situations outlined above is that of Stangeby.\textsuperscript{11,12}
This approach, in contrast to others discussed, is a fluid treatment which is not rigorously correct in a nearly collisionless regime. Nevertheless, most results obtained by kinetic treatments can be reproduced by this model in the appropriate limits. The primary attraction of this model is from an application point of view. It purports to cover the complete range of nonmagnetized to magnetized plasmas.

IV. GRIDDED ENERGY ANALYZER ANALYSIS

The Langmuir probe is a small but rugged diagnostic for determining electron temperatures, through the electron distribution function Eq. (12), and ion densities. Unfortunately, it cannot be used to determine the ion temperature nor the existence of nonthermal components in either species.

The GEA complements the Langmuir probe because of its ability to measure those different parameters as well as \( T_e \). What is sacrificed in going to this type of probe is small probe size and a straightforward density measurement. Examples of its use are found in the work of Matthews\(^{15}\) and others.\(^{16-18}\)

A typical GEA geometry is shown in Fig. 2(a). The entrance to the GEA can either be a grid or a more rugged knife-edge slit depending on particle and heat flux considerations. The purpose of the entrance aperture is to produce a sheath which simultaneously reduces the electron heat flux to the interior of the analyzer and retains the Maxwellian nature of the ion distribution function.\(^ {15}\) In other words, the aperture should be small enough to allow the sheath potential perturbation due to the presence of the detector to extend uniformly across the slit, but large enough to allow a detectable flux to enter. The entering current would then be that of Eq. (12) with the same caveats (that explicit references to the GEA analysis will be limited to some examples in Sec. V C).

The first grid \( G_1 \) is used to repel the unwanted species, in this case electrons. The bias \( \Phi_2 \) of the second grid \( G_2 \) is then varied to limit the collection of ions to those with unperturbed velocity \( (x = \infty) \) less than

\[
v_e = \sqrt{2e(\Phi_2 - \Phi_{\text{plasma}})/M_i}.
\]

The last grid is biased to suppress secondary electron emission from the collector. For measurement of the electron temperature, most ions must be repelled. \( G_i \) should be biased such that \( \Phi_i > \Phi_{\text{plasma}} + \delta_x \times T_e, \delta_x > 1. \) \( \Phi_2 \) is varied negative with respect to \( \Phi_{\text{slit}} \) to “sweep out” the electron distribution function.

To determine the temperature of either species one examines the variation of current drawn at the collector with varying \( G_2 \) bias similar to the determination of \( T_e \) from a Langmuir probe trace. Specifically, in determining \( T_e \), when \( \Phi_2 \) is negative with respect to \( \Phi_{\text{slit}} \), the electron current collected will just be the standard Langmuir electron current:

\[
I = -e \cdot TF(\text{electrons}) A_{\text{slit}} (1/4n_{\infty} C_e) \times \exp(\Phi_2 - \Phi_{\text{plasma}}), \tag{19}
\]

where \( TF \) is the transmission factor of the grids. The exponential term is just the Boltzmann factor for the reduction in the random electron flux. For the case of a magnetized plasma, an additional factor \( R \) must be included in Eq. (19) as described in Section III A. For the purposes of obtaining \( T_e \), Eq. (19) can be rewritten

\[
I = I_0 \exp\{e(\Phi_2 - \Phi_{\text{slit}})\}; \quad \Phi_2 < \Phi_{\text{slit}}. \tag{20}
\]

The ions are only slightly more complicated:

\[
I = TF(\text{ions}) e A_{\text{slit}} (0.5n_{\infty} C_i) \times \exp\{-e(\Phi_2 - \Phi_{\text{plasma}})/kT_e\}; \quad \Phi_2 > \Phi_{\text{plasma}}
= TF(\text{ions}) e A_{\text{slit}} (0.5n_{\infty} C_i); \quad \Phi_2 < \Phi_{\text{plasma}}. \tag{21}
\]

Again we have assumed \( |\Phi_{\text{plasma}} - \Phi_{\text{slit}}| > |\text{sheath potential}| \) and that the ion flux entering the aperture is 0.5\( n_{\infty} \) C\(_i\) with a minimum parallel velocity of

\[
\sqrt{2e(\Phi_{\text{plasma}} - \Phi_{\text{slit}})/M_i}.
\]

The primary drawback of the GEA probe is its size which necessarily creates a larger perturbation than a Langmuir probe. The distance between the entrance slit and the leading edge of the GEA housing limits the amount of plasma that can be sampled. Estimation of the unperturbed density is much more complicated than for a Langmuir probe. Transmission factors, which are energy and species dependent, must be calculated with the use of Monte Carlo methods to follow individual particle orbits.\(^ {15,16}\) The perturbing effect of such a large structure must also be taken into account.\(^ {11,12}\)

V. FITTING TECHNIQUES AND PRACTICAL CONSIDERATIONS

In this section the subject of fitting the above models to the data is addressed. For most cases, the comments pertaining to Langmuir probes are equally applicable to the GEA so that explicit references to the GEA analysis will be limited to some examples in Sec. V C.
A. Logarithmic determination of $T_e$

The advantage of this technique is its simplicity. An estimate of $I_{\text{sat}}$ (ion - saturation current, $I_a$) is made utilizing the knowledge that it is equal to the total current collected at large negative biases. The estimated $I_{\text{sat}}$ is then subtracted from the current measurements at probe voltages, $\Phi < \Phi_{\text{plasma}}$. Fig. 1(a), leaving only the exponential portion of Eq. (12). $T_e$ is determined by fitting a straight line to the logarithm of that difference. Appropriate transformation of the data weighting must be included$^{19}$ in the least-squares fit. Once $T_e$ is determined, the ion density is calculated using the estimated value of $I_{\text{sat}}$ and either of Eqs. (14) or (17).

There are several obstacles implicit in obtaining $T_e$ in this fashion from a Langmuir trace. The ion-saturation current is estimated by averaging current measurements in some appropriate voltage region. Similarly, the fit to the exponential part of the trace must also be limited to a range of points below electron-saturation and above the voltage where $(I - I_{\text{sat}})$ is on the order of the uncertainty in $I_{\text{sat}}$. These limits must be determined at the initiation of the fitting procedure. In practice, the upper-voltage limit to ion-saturation $V_a$ is usually predetermined by external input. $V_a$, the voltage at the onset of electron-saturation, which can be set equal to the upper-voltage limit to the exponential fit, can be determined visually by the appearance of a “knee” in the collected current above $V_a$.

A numerical algorithm can also be used to determine the knee in the $\ln(I)$ plot [Fig. 3(a)]. A first value of $V_a$ should be guessed either from preanalysis input or from a previous time-step fit. Then, straight lines are fitted to $\ln(I - I_{\text{sat}})$ on either side of it. The intersection of those two lines provides a new guess for $V_a$. The iteration is continued until the change in $V_a$ from one iteration to the next is less than some predetermined parameter.

Let us consider how accurate this determination of $T_e$ is. We have estimated $I_{\text{sat}}$ by an average over $k$ data points where the electron current is approximately zero. The error in determining $I_{\text{sat}}$ is then $\epsilon/\sqrt{k}$, where $\epsilon$ is the average signal uncertainty. If those points where $(I - I_{\text{sat}})$ is of order $\epsilon/\sqrt{k}$ are included in any linear fit to the logarithmically transformed data, the slope will be flatter than $1/T_e$, thus overestimating the value of $T_e$.

The error in $I_{\text{sat}}$ can be reduced by increasing $k$, the number of data points with current in ion saturation. If the digitization rate is held constant, the result of increasing $k$ is that either the time resolution is degraded or the number of data points used to determine $T_e$ is reduced.

There is one extra step that can be undertaken in pursuing better accuracy in $I_{\text{sat}}$ and $T_e$ by this technique. After determining the limits in voltage over which the logarithmic fit should be applied, the value of $I_{\text{sat}}$ and thus $T_e$ can be iterated, based on the deviation of the data from the fitted line. The deviation of the data will change in sign depending on the sign of the difference between “real” and guessed $I_{\text{sat}}$. The merits of performing these extra iterations are dubious in light of the minimal gain in accuracy and the advantages of the exponential fit which will be described next. An example of the logarithmic fit to one Langmuir trace, without this last iteration, is shown in Fig. 3(b).

B. Exponential fit

A more accurate, but minimally more complicated technique is to fit $I_{\text{sat}}$ and $T_e$ simultaneously. After finding the knee in the data at $V_a$, as outlined in Sec. V A, the data for lower bias voltages can be analyzed by a least-squares fit to a version of Eq. (12),

$$I = I_{\text{sat}} + b \exp(-V/kT_e),$$

(22)

which is linear in $b$ and $I_{\text{sat}}$, but nonlinear in $T_e$. If we define a new variable

$$Z(V, T_e) = \exp(-V/kT_e),$$

(23)

then $I_{\text{sat}}$ and $b$ can be solved for in terms of $T_e$ through the usual linear least-squares fit. Therefore, in practice, the nonlinear least squares fit to all three parameters can be found by iteration on one ($T_e$). This technique fits the data with uniform weighting and negates the need for a large number of data points at large negative bias. Results for an exponential fit are also shown in Fig. 3(b).

A version of the above exponential fit can be used to apply the Stangeby probe model$^{11,12}$ to Langmuir probe data.$^{20}$ Unfortunately, the process is significantly more complicated. In addition, the benefits gained by choosing said model are questionable.$^{20}$

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**Fig. 3.** (a) Determination of $V_a$, the onset of electron saturation. Circles are the data, lines are fits to $(I - I_{\text{sat}})$ above and below $V_a$. (b) Langmuir trace: circles are the data, solid line is the logarithmic fit, broken line the exponential fit.

C. GEA fit

As discussed in the beginning of Sec. V, the fit to GEA data is very similar to that for Langmuir probes. First, let us discuss the case of determining the ion temperature. A plot of ion current versus positive $G_2$ bias is shown in Fig. 4(a). There is a knee in this data, corresponding to $\Phi_2 = \Phi_{\text{plasma}}$ [Eq. (21)], below which the current is unaffected by bias. This is because the ions are "borne" with the plasma potential far away from the probe. After determining this knee voltage in the fashion described earlier, the ion temperature can again be fit either by logarithmic or exponential methods.

A plot of electron current versus $G_2$ bias is shown in Fig. 4(b). There is no need to find a knee in this curve as per the Langmuir probe analysis. All of the data shown should be useful in determining $T_e$. A direct fit by either logarithmic or exponential methods is appropriate. In fact, the logarithmic fit should be simpler for the GEA than for Langmuir probes, if for large negative $G_2$ bias, the collected current reduces to zero as it should.

In the case of a nonthermal component or two-temperature population, the GEA analysis becomes significantly more complicated. Care must be taken to increase the magnitude of the $G_2$ bias swing to appropriately reduce the collected current to zero. Then an iterative technique, assuming knowledge of the temperature of one component to fit the other, is repeated until convergence.

D. Bias voltage waveforms

To maximize the time resolution of the analyzed probe data, it is useful to examine the relative merits of different waveforms for biasing the Langmuir probe or GEA. Short period voltage sweeps increase the time resolution of the analyzed data, but also increase the uncertainty of the result due to fewer data points. The error signals caused by stray-capacitance-induced displacement currents are also increased. In practice, most of the induced error signal can be subtracted at the initiation of data analysis.

The number of points sampled with the probe in ion-saturation, exponential or electron-saturation during a single sweep is transformed by the bias voltage waveform. A sinusoidal waveform has the advantage of only one frequency component, but the disadvantage of sampling the greatest number of points in electron and ion saturation. A triangular or sawtooth sweep is a better choice from this point of view. It is also easily generated by analog waveform generators, and a greater percentage of the data is taken in the exponential part of the Langmuir trace. An even greater fraction of the data can be taken in the exponential part of the Langmuir trace if the experimenter has the ability to digitally program more "exotic" bias waveforms. $V(t) = \text{arc sin}(t)$, for $-1 < t < 1$, is such an example. It is particularly well suited for the determination of $T_e$ by the exponential fitting technique of Sec. V B.

Such flexibility in generating waveforms can also be applied to GEA grid biases as well. For example, $G_1$ and $G_2$ can alternate from one sweep to the next, between bias potentials needed to measure ion and electron temperatures.16

VI. SUMMARY

Theory that provides an exact description of the operation of a given probe can be quite complicated. However, in practice, several simple formulas reviewed in this paper can be applied with reasonable accuracy. Numerical algorithms are also outlined which can provide efficient computer analysis of Langmuir probe and GEA data.

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Fig. 4. GEA data: for measurement of ion (a) and electron (b) temperatures. Circles are the data, solid line the exponential fit.
Fluids 23, 803 (1980).
20B. LaBombard, Ph.D. thesis (soon to be finished).