Performance of the He–Ne Gas Laser as an Interferometer for Measuring Plasma Density

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The characteristics of the He–Ne gas laser used in a new simple interferometric technique have been studied experimentally and theoretically. The interferometer has two novel features: first, the intensity of the laser itself is used to detect the fringes and second, because the intensities of the 0.63-μ (red) and 3.39-μ (infrared) laser beams are coupled, interference in the infrared can be detected by a simple photomultiplier monitoring the red beam.

The system does not respond instantaneously to changes in the optical path length; experimental measurements show that when the red beam is used to follow interference in the infrared, the maximum detectable response is limited to about 3×10⁴ fringes per second. Discussion of the frequency response and the cross-coupling between the two wavelengths leads to the conclusion that the frequency response is limited by the red channel only.

Experimental details of the interferometer are described, including the application of a multipass system which, with some loss in spatial resolution, increases the sensitivity of the interferometer by at least a factor of 20.

1. INTRODUCTION

This paper discusses the main aspects of a new interferometric technique which uses the 0.6328-μ (red) and 3.591-μ (infrared) radiation from a He–Ne gas laser to measure the electron density in a plasma.¹

Interferometry using visible light is an established method of measuring plasma density,² ³ while the increased sensitivity associated with longer wavelengths has been exploited for many years in microwave interferometers⁴; in addition, an interferometer operating in the far infrared has been developed recently.⁵ The laser interferometer,⁶ however, has unique features which introduce major simplifications both in optical adjustment and in detection of the interference fringes. The interferometer uses only one mirror in addition to those in the laser, and the intensity of the laser itself is used to observe the fringes. Both laser wavelengths, i.e., 0.63 and 3.4 μ, are available for interferometry but interference in the infrared can be detected by a photomultiplier tube measuring the intensity of the 0.63-μ radiation alone; no infrared detector is necessary. The interferometer is suitable for measuring the density of plasmas which produce one or more fringes; with the infrared radiation the condition for this is \( n_e \geq 3.3 \times 10^{16}/\text{cm}^3 \), where \( n_e \) is the average number density of electrons along the line of sight and \( L \) is the length of the plasma in cm. Because the interferometer does not respond instantaneously to changes in optical path length, the system is not suitable for use with plasmas in which the electron density changes very rapidly; a practical upper limit is about 3 fringes per microsecond.

² J. Dyson, R. V. Williams, and K. M. Young, Nature 195, 1291 (1962).

2. PRINCIPLES OF THE METHOD

Plasma Equations

The refractive index \( \mu \) of a plasma for electromagnetic radiation of frequency \( \omega/2\pi \), when \( \omega \) is much greater than both the plasma frequency \( \omega_p \) and the electron cyclotron frequency, is given by

\[
\mu = 1 - \frac{1}{2} \left( \frac{\omega_p}{\omega} \right)^2,
\]

where

\[
\omega_p^2 = 4 \pi n_e e^2/m_e.
\]

Thus the electron number density \( n_e \) of a plasma can be found by measuring its refractive index with an interferometer using radiation of a suitable frequency. The number \( N \) of interference fringes produced by a plasma of length \( L \) is given by

\[
N = (\mu - 1)2L/\lambda,
\]

where \( \lambda \) is the wavelength and the interferometer beam has a total path length of \( 2L \) within the plasma.

From Eqs. (1) and (2),

\[
N = (\omega_p/\omega)^2 L/\lambda
\]

\[
= 8.9 \times 10^{-14} n_e LA.
\]

Note that the number of fringes is proportional to the wavelength used.

Description of the Laser Interferometer

When the output beam from one end of a laser is reflected back into the optical cavity by means of an external mirror (see Fig. 1) interference occurs between the reflected beam and the cavity oscillation; the whole laser intensity is then strongly dependent on the phase of the reflected radiation. The laser intensity undergoes one cycle of modulation for each complete wavelength change in the optical path from the laser to the external mirror and back, thus changes in the path length can
be measured simply by counting maxima and minima in the laser intensity. The cycles of modulation correspond to the fringes of a conventional interferometer. If the optical path length changes very slowly, the fringes, i.e., modulations in laser intensity, can easily be seen by eye. At higher frequencies the fringes can be detected by a simple photomultiplier monitoring the light output from the other end of the laser. This effect was observed by King and Steward who showed that it enabled long path length interferometry to be carried out with a continuously operating He–Ne gas laser and one external mirror.

The He–Ne laser is particularly useful in this application because it can generate simultaneously both visible light at 0.63-μ wavelength and infrared at 3.4 μ. The presence of the visible beam simplifies optical alignment, and both wavelengths are available for interferometry. Furthermore, because the two wavelengths arise from a common upper energy level (3s2) in the neon atom there is a coupling between the radiation intensities at the two wavelengths; any modulation in the intensity of laser action at 3.4 μ produces a complementary modulation in the intensity of the 0.63-μ radiation. Thus interference in the infrared can be detected by an ordinary photomultiplier which responds only to visible light. We can therefore take advantage of the increased sensitivity associated with the longer wavelength [Eq. (3)] and at the same time retain the convenience of working with visible light.

The reciprocal effect, i.e., modulation of the infrared intensity caused by interference in the red, is not observed so the coupling in this direction is assumed to be weak.

3. FREQUENCY RESPONSE AND CROSS COUPLING

Experiments

The maximum rate of change of optical path length which the interferometer can detect was measured by using a rotating mirror to reflect the output beam back into the laser. The beam was directed to one side of the axis of rotation so that at one instant in each revolution the beam was reflected by a part of the mirror moving towards the laser. Modulation at the Doppler frequency corresponding to the mirror velocity was observed, the depth of modulation decreasing as the frequency increased. Figure 2 shows how the depth of modulation of the red output from the laser varies with frequency when interference takes place in first the infrared and then the red; in both cases the modulation is expressed as a fraction of that observed with low-frequency infrared fringes. With interference in the infrared the depth of modulation has fallen to 1/e at 150 kc/sec and the upper frequency limit of detection is about 3 Mc/sec. This limit implies that the maximum rate of change of n which can be measured when the red output is used to detect infrared fringes is

\[
d\tilde{n}/dt = 10^{10}/L \text{ cm}^{-4} \text{ sec}^{-1}.
\]

The upper limit here is imposed not by the detector but by the response of the laser itself. If, on the other hand, the infrared output is used to detect infrared fringes, then the upper frequency limit (≈100 kc/sec) is determined by the time response of commercially available detectors.

The complementarity of modulation in the red and infrared radiation is shown in Fig. 3. These oscillograms were obtained when interference occurred in the infrared radiation as a result of random vibrations of the external mirror which reflected the beam back into one end of the laser. The intensity of the red beam from the other end of the laser was measured by a photo-

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Fig. 1. Diagram of the interferometer measuring electron density in a gas discharge.

Fig. 2. Depth of modulation of red output against frequency, for interference in the red and infrared.

Fig. 3. Complementarity of the red and infrared radiation. Upper trace, 0.63-μ intensity; Lower trace, 3.4-μ intensity.

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multiplier and at the same time the infrared intensity was measured by an indium antimonide detector; both beams were interrupted at 800 cps by a rotating slotted disk so that the depth of modulation of each could be seen. The upper trace is the output from the photomultiplier (signal increasing downwards) and the lower trace is that from the infrared detector (signal increasing upwards). By adjusting the gains of the oscilloscope amplifiers the two traces can be made to match like two pieces of a jigsaw puzzle, showing the high degree of complementarity between the intensities of the two beams.

**Theory of Frequency Response**

The variation in the intensities $\varepsilon_1$ and $\varepsilon_2$ of radiation at the two wavelengths can be represented by the pair of coupled nonlinear differential equations

$$d\varepsilon_1/dt = a_1\varepsilon_1 + b_1\varepsilon_2 + c_1\varepsilon_1\varepsilon_2,$$

$$d\varepsilon_2/dt = a_2\varepsilon_2 + b_2\varepsilon_2 + c_2\varepsilon_1\varepsilon_2,$$

where the 'a' terms represent the combined effect of stimulated emission, absorption and mirror losses, the 'b' terms represent the reduction in the population inversion as the radiation intensity increases, and the 'c' terms take account of the cross coupling, i.e., the depopulation of the upper level by laser action at the other wavelength. For laser action to occur it is necessary that $a>0$. The equilibrium intensities are given by

$$\varepsilon_{10} = (a_1+c_1b_2)/(b_1b_2-c_1c_2),$$

$$\varepsilon_{20} = (a_2+c_2b_1)/(b_1b_2-c_1c_2).$$

If, now, interference occurs in the red light (subscript 1, say) and the optical path is changing such that fringes appear at the frequency $\omega/2\pi$, then the intensity modulation in the red $\varepsilon_1$ is obtained by assuming $a_1$ to have a small oscillating part $a_1e^{i\omega t}$ and carrying out a linearized perturbation analysis to give

$$\varepsilon_1 = a_1e^{i\omega t}+\frac{c_1\varepsilon_{10}\varepsilon_{20}}{(i\omega-b_1\varepsilon_{10})} = a_1 e^{i\omega t}.$$  

If interference occurs in the infrared (subscript 2) and the intensity of the red light is measured, then we have

$$\varepsilon_2 = \frac{c_2\varepsilon_{10}\varepsilon_{20}}{(i\omega-b_2\varepsilon_{20})} = \frac{a_2\varepsilon_{20}}{b_2}.$$  

Since the observed frequency responses in the two cases are the same, we conclude that over the experimental range

$$\omega \ll b_1\varepsilon_{10},$$

which leads to a common frequency response given by the simple relation

$$|\varepsilon_{10}|/|\varepsilon_{20}| = (1+i\omega/\omega_b)^{-1},$$

i.e.,

$$|\varepsilon_{10}|/|\varepsilon_{20}| = [1+(\omega/\omega_b)^{2}]^{-1},$$

where

$$\omega_b = (a_1b_2-a_2b_1)/b_2.$$  

A good fit is obtained to both sets of experimental points in Fig. 1 with $\omega_b/2\pi = 45$ kc/sec.

If we apply the condition $\omega << \omega_b$ and calculate the fractional depths of modulation $P_1$ and $P_2$ of the red and infrared intensities when interference is taking place in the infrared, we obtain

$$(P_1/P_2)_r = c_1\varepsilon_{20}/b_1\varepsilon_{10}. $$  

For the other case, when interference occurs in the red,

$$(P_1/P_2)_r = b_2\varepsilon_{20}/c_2\varepsilon_{10}. $$

Experimentally, the first ratio was approximately unity and the second ratio much greater than one, so we have

$$c_1\varepsilon_{20}/b_1\varepsilon_{10} = 1,$$  

$$b_2\varepsilon_{20}/c_2\varepsilon_{10} >> 1.$$  

Referring to the original Eqs. (4) and (5), these inequalities simply mean that the cross-coupling term is relatively unimportant as far as the infrared intensity is concerned while for the red it is of comparable importance with the direct nonlinear term.

If interference in the infrared is detected in the infrared we have

$$\varepsilon_2 = \frac{i\omega-b_2\varepsilon_{20}}{c_2\varepsilon_{10}} = -\frac{c_2\varepsilon_{10}\varepsilon_{20}}{(i\omega-b_2\varepsilon_{20})} = \frac{a_2\varepsilon_{20}}{b_2}.$$  

Applying the conditions (10), (15), and (16) to Eq. (17) we obtain

$$|\varepsilon_{10}|/|\varepsilon_{20}| = 1.$$  

The over-all conclusion is that the frequency response of the interferometer is at present limited by the red light channel only; a substantial increase in the maximum observable fringe rate could be expected if a fast infrared detector were available to measure the modulation of the infrared beam directly.

**Note added in proof**: This measurement has now been done by J. M. P. Quinn of this laboratory. He observes a flat frequency response extending beyond 1 Mc/sec.

Note also that, because Eq. (8) is complex, some distortion due to phase shift may be expected when the bandwidth of the fringe signal includes the critical frequency $\omega_b$. This may introduce a maximum error of $\frac{1}{2}$ of a fringe into the simple fringe counting technique.

**4. EXPERIMENTAL MEASUREMENT OF PLASMA DENSITY**

The laser interferometer has been used to measure the electron density in a pulsed gas discharge; a short account of this work has already been given. Figure 1
tive interference completely stops the infrared laser action. The nonsinusoidal shape of the fringes makes the interferometer unsuitable for use with plasmas which produce less than one fringe.

In the arrangement of Fig. 1 the laser beam is divergent and the area of the reflected beam at the laser is about 100 times that of the output beam, thus only a small fraction of the output energy is injected back into the laser. Because of the beam divergence, the additional optical cavity formed by the external mirror can be regarded as having a low $Q$ value. A modification of the system has been reported, in which the additional cavity has a high $Q$ giving increased sensitivity.

5. PRACTICAL DETAILS

The laser used in this work (see Fig. 6) was made at the Services Electronics Research Laboratory, Baldock. The quartz discharge tube, 0.7-cm bore and 70 cm long, contains 86% helium and 14% neon at a pressure of 1.2 Torr. An rf discharge is excited by a 36-Mc/sec oscillator which can supply about 80 W to the discharge tube. The optical cavity is formed by two mirrors, one plane and one concave (150-cm focal length) mounted at the ends of an aluminum tube 80 cm long. Each mirror has a multilayer dielectric coating designed

is a diagram of the experiment. The discharge tube is in the optical path between the laser and the external mirror and as the electron density changes during the discharge pulse the laser intensity goes through successive maxima and minima which correspond to interference fringes. The lower traces in Figs. 4(a) and 4(b) are infrared fringes detected by a photomultiplier which measures the red light intensity, while the upper traces show the gas discharge current; oscillograms 4(a) and 4(b) have different sweep speeds. The electron density at any time in the discharge is determined simply by counting fringes and using Eq. (3). Figure 5 shows red fringes obtained on the same discharge when the infrared beam was suppressed by a plain glass filter.

There is no difficulty in counting the fringes in Figs. 4 and 5 because the electron density changes in a regular manner. In plasmas where large irregular fluctuations in density occur the fringe pattern may not be readily interpretable and care is needed in counting the fringes; in these cases comparison of measurements at the two wavelengths may provide a useful check.

The nonlinear relation between the laser intensity and the phase of the reflected radiation is shown by the pointed character of the fringes in Fig. 4. Fringes more sinusoidal in shape can be obtained by reducing the intensity of the reflected radiation; on the other hand, if the intensity of the reflected beam is increased, the fringes eventually develop flat bottoms when destruc-

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Fig. 4 (a) Upper trace, discharge current. Lower trace, infrared fringes. Sweep speed, 50 $\mu$sec per large division. (b) Upper trace, discharge current. Lower trace, infrared fringes. Sweep speed, 500 $\mu$sec per large division.

Fig. 5 (a) Red fringes. Sweep speed, 50 $\mu$sec per large division. (b) Red fringes. Sweep speed, 500 $\mu$sec per large division.

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beam passes through the system and is returned along its path by a fourth concave mirror D; the beam is refocused on each reflection and its diameter remains less than 3 mm throughout its path. The number of passes is increased by tilting the mirrors A and C; Figure 7(b) shows the ray diagram for 8 double passes. The interferometer has been operated satisfactorily at 3.4 μ with 20 double passes between mirrors of 200-cm radius of curvature. In any particular application the number of useful passes may be limited by reflection losses at the mirror surfaces, by absorption losses in intervening windows, or by spurious fringes produced by vibration of the mirrors. In an ordinary laboratory with no special precautions taken to isolate the optical components from floor or air-borne vibrations, spurious infrared fringes occurred at a maximum rate of roughly 100 n/sec, where is the number of double passes. A further limitation to the number of passes is imposed by the unavoidable decrease in spatial resolution as the number of passes increases; for example, the cross-sectional area sampled by the beam increased from a circle of 0.3 cm in diameter with one double pass to a rectangle 2×0.5 cm with the 20 double passes mentioned above.

To detect fringes occurring at frequencies up to about 500 kc/sec almost any photomultiplier tube can be used; a stop of the same diameter as the laser beam and a red filter (Wratten 25) are necessary to reduce background light from the rf discharge in the laser tube. As the frequency of the fringes increases, their amplitude decreases (see Fig. 2) and the upper frequency limit of detection is determined by the noise level of the photomultiplier. Different photomultiplier tubes were tried; the best signal-to-noise ratio was obtained with an E.M.I. 9637 TA which is a 'venetian blind' tube with a tri-alkali photocathode and 5 dynodes.

In some applications trouble may arise from un-
wanted photomultiplier signals caused by background light from the plasma itself; this can be reduced simply by moving the laser and photomultiplier further away from the plasma. The interferometer operates satisfactorily over lengths of 20 m or more and there is no particular virtue in working close to the experimental plasma; indeed, this should be avoided if large stray magnetic fields are present because they may cause unwanted movement of the optical components and complicate the laser operation by the Zeeman splitting of energy levels.

6. CONCLUSION

Among the generally accepted techniques for measuring electron density in plasmas, interferometry has the advantage of providing readily interpretable experimental data, independent of the electron temperature. In spite of this advantage only microwave interferometers have been widely used in experimental plasma physics and these are limited to low-density plasmas \((n < 10^{14} \text{ cm}^{-3})\) by the frequency of available microwave generators \((<1.5 \times 10^{9} \text{ cps})\). Optical interferometers, which are capable of examining much denser plasmas, have had limited use as practical diagnostic instruments. This is partly because plasmas of current interest are not sufficiently refractive to visible light and partly because of the difficulty of setting up and adjusting the components of a conventional interferometer around the complex and inaccessible apparatus commonly encountered in modern plasma physics.

The interferometer which has been described largely overcomes both of these difficulties. The use of infrared radiation, together with the ease of using a multipass system, readily increases the number of fringes and makes possible interferometry on plasmas which are not strongly refractive to visible light. But the main advantage of the system is its simplicity; the laser itself acts as both light source and fringe shift detector and the only optical adjustment required is the alignment of a single mirror.

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Excitation of Hypersonic Vibrations by Means of Photoelastic Coupling of High-Intensity Light Waves to Elastic Waves*

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A theory of the excitation of elastic waves arising from photoelastic coupling with light is presented. It is similar in character to the theory of optical parametric amplification in spatially extended media. Special attention is given to the self-excited transient case. A novel form for the space-time development of the instability appears for the case in which the spatial dimensions of the interaction region are large compared to the distance traveled by an elastic wave during the illumination time. The predicted effects should be readily observable with existing giant pulse lasers.

1. INTRODUCTION

PHOTOELASTIC instability of quartz and sapphire under illumination by an intense optical-maser-produced light pulse has recently been observed.\(^1\) The process can be described as stimulated Brillouin scattering and is an example of Raman maser action in which the Raman excitation is a lattice vibration in the acoustic branch. The excitation of the elastic vibration is, of course, accompanied by the appearance of the Raman scattered light with its frequency shifted downward by that of the elastic wave. The relations between frequency and direction of the scattered light and associated elastic wave are the same as those pertaining to the Stokes component of Brillouin scattering.

We present here a classical theory of the process based upon an elastic continuum description of the lattice. By way of introduction we recall that the process may be classically described as follows: An electromagnetic field in a body produces an electrostrictive force density linearly dependent upon the quantities \((\partial / \partial x_2)E_xE_x\). In the case of a single plane traveling wave the disturbance produced by this force can be resolved into a static component and a disturbance propagating with the velocity of light. The elastic deformation produced by this disturbance is strongly inhibited by the discrepancy

\(^1\) The principal results of this paper were presented at the Symposium on Ultrasonics in Solids, 23 March, APS Meeting, Philadelphia, Bull. Am. Phys. Soc. 9, 222 (1964).