On the Inelastic Failure Criterion for Structures Subjected to Large Dynamic Loads

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Summary

This article is concerned with the response of structural members which are subjected to impact loads causing large inelastic deformations and material failure. The literature published on the dynamic inelastic failure of structural members is reviewed from which a need for additional theoretical and experimental studies emerges clearly.

Some experimental results for clamped mild steel beams under impact loading are simulated numerically using the finite-element code ABAQUS with plane stress elements. The critical conditions of the beams during the response are revealed and various possible failure criteria are examined and discussed by comparing the numerical predictions with the experimental results.

Keywords: dynamic inelastic failure, structural impact, beams, finite-element calculations.

1 Introduction

This article is concerned with the response of structural members which are subjected to impact loads causing large inelastic deformations and failure. Engineers and designers in many industries require information on the maximum amount of energy which can be absorbed for a specified deformation. For example, the deformations of passenger cars are designed in a range of accident scenarios to maintain a survivable volume with acceleration levels which avoid human injury [1]. In other practical situations, the maximum amount of energy which can be absorbed prior to a material failure is required. This information would be necessary in order to assess the integrity of LNG tankers involved in a collision, for example. Many other safety calculations and hazard assessments involving impact, dynamic pressure or explosive loads would require information on structural impact.

Many authors [1-7] have examined the behaviour of structures subjected to impact loads which produce large ductile deformations. These methods of analysis may be used to predict the impact energy absorbed by a structure and the associated magnitude of the permanent deformations when assuming that the material has an unlimited ductility. No information is obtained, therefore, on the structural integrity. However, this aspect is important for many engineering problems since designers need to assess the integrity of critical components which could be breached in various accident scenarios involving dynamic events.

A survey on the dynamic inelastic failure of beams was presented in Reference [8]. The beams were made from ductile materials, which could be modelled as rigid, perfectly plastic, and they were subjected either to a uniformly distributed impulsive velocity, as an idealisation of an explosion, or to a mass impact to idealise a dropped object loading. Thus, large plastic strains could be produced and the possibility of material rupture was studied for sufficiently large dynamic loads.

It was observed that the different dynamic loadings examined in Reference [8] may cause the development of different failure modes. The simplest failure modes were associated with a uniformly distributed impulsive velocity loading. A rigid-plastic method of analysis for this particular problem [6] shows that membrane forces as well as bending moments must be retained in the basic equations for the response of axially restrained beams subjected to large dynamic loads which cause transverse displacements exceeding the beam thickness, approximately. This is known as a Mode I response where the dynamic energy is absorbed plastically without material failure.

If the external impulse is severe enough then the large strains which are developed at the supports of an axially restrained beam, would cause rupture of the material which is known as a Mode II failure. At still higher impulsive velocities, the influence of transverse shear forces dominates the response, and failure is more localised and occurs due to excessive transverse shear displacements (Mode III).

It is important to note that a Mode III transverse shear failure is more likely to occur in dynamically loaded beams than in similar statically loaded beams. Thus, even a beam with a solid rectangular cross-section and a large length-to-thickness ratio can suffer a transverse shear failure under a dynamic loading (Mode III response), as observed by Menkes and Opat [9] and analysed in Reference [10]. This effect is even more pronounced for beams with open cross-sections.

Paradoxically, it is observed in Reference [8] that despite the lower impact velocities of the mass impact case, the failure behaviour is much more complex than the impulsive loading case. The Mode II and III failure modes discussed above for an impulsive loading also occur for a mass impact loading. However, it transpires that other more complex failure modes may develop. For example, a striker may cause an indentation on the struck surface of a beam, which, if sufficiently severe, may lead to failure. The impact velocity of a strike near to the support of a beam might not be sufficiently severe to cause a transverse shear failure (Mode III) but could distort severely a beam and cause rupture due to the combined effect of transverse shear force, membrane force and bending moment.

It is evident [8] that the dynamic inelastic rupture of beams and other structures is an extremely complex phenomenon and that there is a pressing need for the development of a reliable criterion which can be used in theoretical methods, numerical schemes and computer codes in order to predict the onset of structural failure due to material rupture for hazard assessments and safety calculations throughout the field of engineering.

In an attempt to obtain a universal failure criterion, which could be used for a large class of dynamic structural problems, an energy density failure criterion was introduced in Reference [11] and discussed in Reference [12]. It is assumed that rupture occurs in a structure when the absorption of plastic work (per unit volume) reaches a critical value which contains the plastic work contribution related to all of the stress components.

The numerical predictions of the energy density failure criterion in Reference [11] for the dimensionless impulses at the transitions between a Mode I and Mode II failure and between a Mode II and Mode III failure compare favourably with the corresponding experimental results of Menkes and Opat [9] and the theoretical rigid, perfectly plastic predictions in Reference [10]. Many other theoretical predictions for the various parameters in an impulsively loaded beam are presented in References [11] and [12] to which an interested reader is referred. The energy density failure criterion predicted good agreement with the available experimental results on impulsively loaded beams and confirmed broadly the failure characteristics which were first observed using the elementary analysis developed in Reference [10].

The critical energy density failure criterion was also used in Reference [13] to examine the failure of fully clamped beams subjected to mass impact loads.

Some recent experimental and theoretical studies have been reported on the dynamic inelastic failure of fully clamped circular plates [14,15] and square plates [16] subjected to uniformly distributed impulsive loads. The influence of the boundary conditions on the magnitude of the critical impulse is shown to be very important, which, in fact, has already been observed for beams [17,18]. This complicates further the predictions of an energy density failure criterion but shows the importance of specifying carefully the exact details of the boundary conditions in both experimental tests and theoretical studies.

The entire area of structural failure due to material rupture is a very important one in engineering, but the present state of knowledge is incomplete from both experimental and theoretical viewpoints. The simple rigid-plastic methods of analysis have, in fact, been quite successful for predicting the failure of several structural problems which have been examined and they are particularly useful for preliminary design. The energy density failure criterion is also promising for predicting the failure of a broader range of structural problems but it is more difficult to use and is a function of the strain rate [19]. Eventually, it could be incorporated into any theoretical method or numerical scheme, but insufficient information is available currently to do this with confidence except for beams and frames. It is often assumed that rupture occurs when the equivalent strain in a structural member reaches the rupture strain recorded in a static uniaxial test. It is important to emphasise that, generally speaking, this assumption is incorrect [20] and that, moreover, the rupture strain is a complex function of the strain rate [21,22].

Finite-element methods have been used by several authors to simulate the dynamic inelastic response of structures [18,23]. For example, Clift et al. [23] observed that only the criterion based on the critical generalised plastic work density successfully predicted sites found experimentally in several metal forming operations. A numerical simulation of clamped aluminium alloy beams impacted transversely by a mass at different locations on the span, reported in Reference [18], showed that the mode II failure of the beams could be predicted by a criterion based on the maximum tensile strain, or equivalently, the overall rotation angle. These studies have shown the potential of combining numerical investigations with experimental tests to obtain the criterion which controls a structural failure.

This present study reports on part [24] of a systematic research programme on the dynamic inelastic failure of structures, and is a continuation of the previous careful experimental investigation reported in Reference [25] on clamped beams struck by a solid mass. The mechanical properties of the mild steel and aluminium alloy specimen materials were obtained from both quasi-static and dynamic uniaxial tensile tests. The mild steel beams were loaded at the mid-span and the one-quarter-span positions.

A numerical simulation of the experimental tests on the mild steel beams in Reference [25] is presented in Reference [24] and discussed briefly here. The actual experimentally determined material properties are used in the finite-element models and the global response and strain history are compared with the experimental data. Detailed information on the dynamic inelastic response of the beams related to structural failure is obtained. Through the combined study of the experimental investigation in Reference [25] and the present numerical simulation using ABAQUS, the reliability of both numerical and experimental results is confirmed and the critical conditions of the beams during the response are revealed. Various possible failure criteria are then examined by comparing the experimental and numerical results.

2 Numerical Results

Table 1 contains a comparison of the transverse displacements for dynamically loaded steel beams obtained numerically with the corresponding experimental data from Reference [25]. The agreement between the numerical results and the experimental data is quite reasonable, particularly when considering that the values of W_f were obtained with a limited temporal and spatial resolution of the high-speed photographs. The calculated deflection-time history for specimen SB07, from Reference [25], which was loaded dynamically at the one-quarter span position and was observed to be cracked and severely necked after a test, is compared with the experimental curves in Figure 1. Excellent agreement is observed

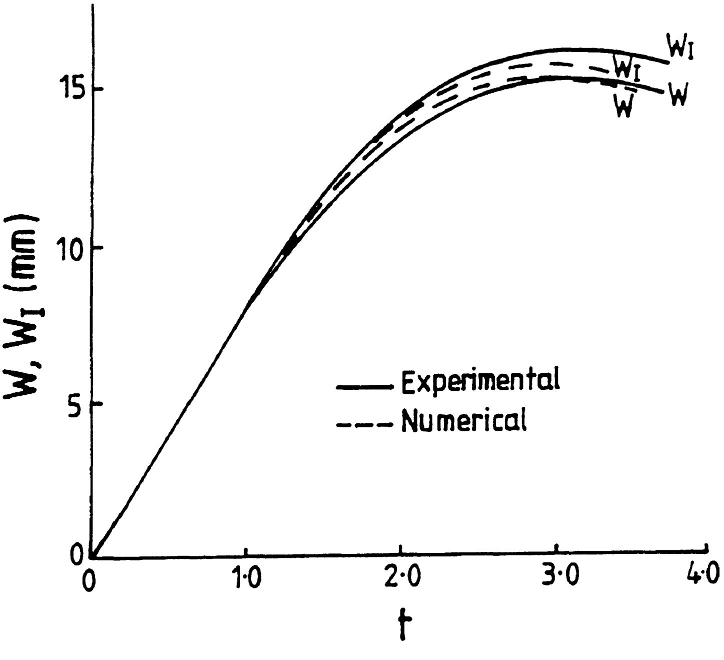

Figure 1: Deflection-time history for specimen SB07 from Reference [25]

The comparisons in Table 1 and Figure 1 together with others made in Reference [24] for strain-time histories from strain gauges and indentations underneath the striker, as well as quasi-static test results, confirm the accuracy and reliability of the numerical scheme. Thus, it is possible to explore many other parameters which might control failure with a degree of confidence. In particular, it is possible to examine various quantities in the numerical calculations at the critical impact energy input which is observed to cause failure in the experimental test beams in Reference [25]. These comparisons can be made without introducing any material failure criteria into the numerical scheme, and, therefore, observations are made without any prior prejudice towards any particular failure criterion.

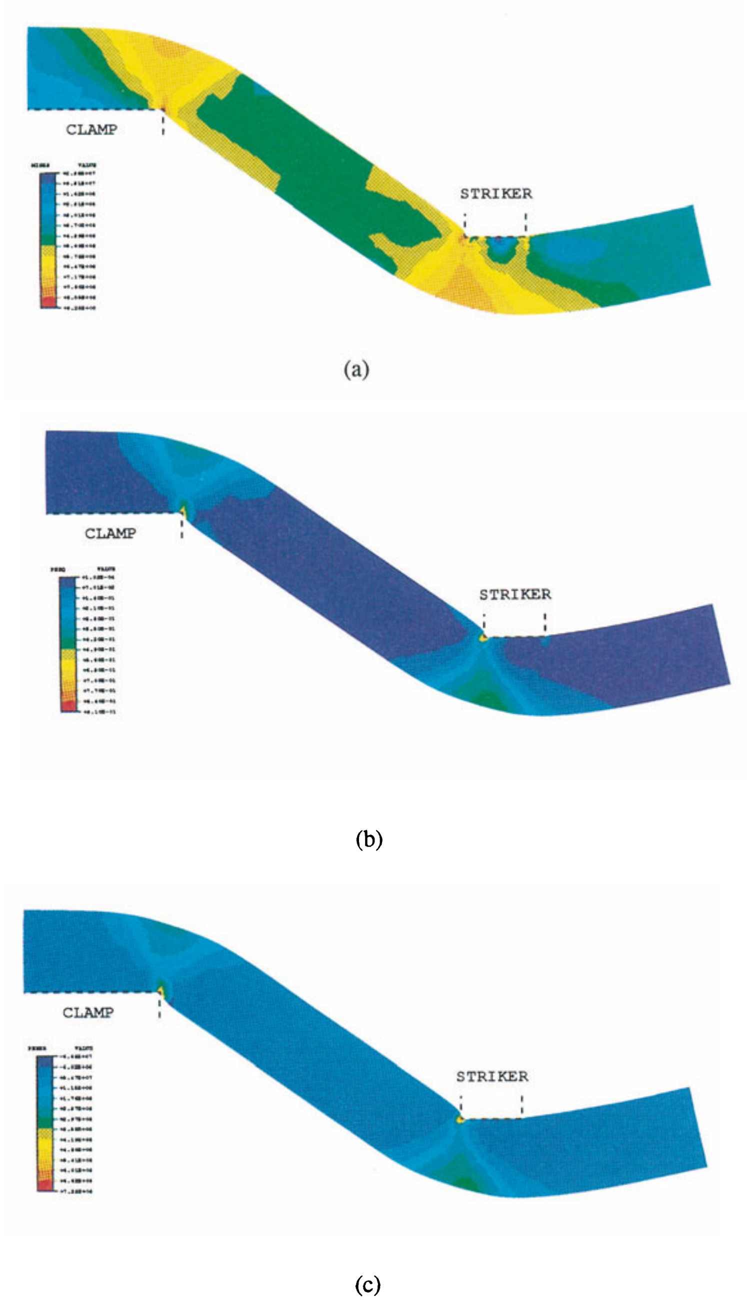

Figure 2: Distribution of (a) Mises stress, (b) equivalent plastic strain and (c) plastic strain energy density for specimen SBO7 at t = 2.86 ms

Table 1: Numerical and experimental results for the transverse displacements of dynamically loaded mild steel beams

Specimen	Numerical	Experimental	Comments on Tests
SBO9	W_p = 20.56 mm	W_p = 20.9 mm	
SBO8	W_{max} = 21.26 mm	W_f = 21.8 mm	just broken
SBO6	W_{max} = 22.80 mm	W_f = 22.8 mm	broken
SBO7	W_p = 14.97 mm	W_p = 14.5 mm	crack and severe necking
SBO5	W_{max} = 16.03 mm	W_f = 15.0 mm	broken
SBO4	W_{max} = 17.34 mm	W_f = 16.0 mm	broken

3 Discussion

The maximum values of the tensile stress and strain are located on the lower surface underneath the striker, at the symmetric axis for the specimens impacted at the mid-span, or slightly towards the support for those impacted at the one-quarter span position, where necking or a tensile tearing failure occurs. On the other hand, the maximum values of the Mises stress, Tresca stress, shear strain, equivalent plastic strain and the plastic strain energy density occur on the upper surface, at, or near, the impact point where a shear failure occurs. The distributions of the Mises stress, equivalent plastic strain and plastic strain energy density for specimen SBO7 at the cessation of material inelastic flow are shown in Figure 2. It transpires that the equivalent plastic strain and the plastic strain energy density are concentrated near the impact point, in contrast with the smooth change along the axial direction found in the neck region of a tensile specimen, as shown in Reference [24], for example.

The distribution of the Tresca stress along the ridge of the Tresca stress contour across the beam thickness tends to be uniform at the later stage of the response except in a region very near to the impact point as shown in Figure 3. The angle of the ridge of the Tresca stress contour is coincident with the fracture or sliding surface observed in the dynamic tests reported in Reference [25], indicating that failure develops along the direction of the maximum shear stress.

The results of the numerical simulation are in excellent agreement with the experimental data reported in Reference [25] on the impact behaviour of mild steel beams, especially when considering the fact that there are no adjustable parameters in the specification of the material properties, which were obtained experimentally for the same material. Hence, the numerical results here and in

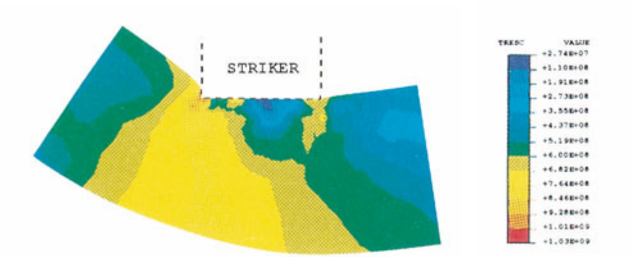

Figure 3: Contour of Tresca Stress for specimen SB07 at t = 2.86 ms

Reference [24] together with the experimental data in Reference [25] provide a reliable set of information for further analysis.

In comparison with the quasi-static loading cases, the stress wave effect is important during the early stage of the response for beams subjected to an impact loading even though the maximum impact velocity is only 8 m/s. The stress also increases in the dynamic cases for materials having a strain-rate hardening effect. Complex failure modes with the combined effects of tensile tearing and shearing, and possibly, geometrical instability, were obtained experimentally. However, the geometrical instability is delayed by the inertia effect in the dynamic cases.

Among the possible failure criteria discussed in Reference [24], the maximum membrane force in a beam cross-section appears to be the most promising for a tensile failure, while the maximum Tresca stress or Mises stress and the maximum plastic strain energy density are worthy of further study for a shear failure. The maximum Tresca stress, or shear strain at the centroidal axis, and the maximum yield index, or failure index based on an interaction yield surface, are promising parameters for a global failure criterion. Further investigations are required to confirm the reliability of these criteria, especially for predicting a shear failure since only one set of test results, i.e., the beams loaded at one quarter-span position, are related to shear failure in the experimental study reported in Reference [25].

4 Conclusions

The approach used in the present numerical study provides a critical way to evaluate the reliability and generality of a criterion for predicting the failure of structures subjected the dynamic loads which produce large inelastic strains. The key requirement is a complete set of reliable experimental data including both the structural response and the dynamic properties of the material. Unfortunately, there is a paucity of such data, especially those related to the rupture behaviour of materials under dynamic loading, including the influence of the strain-rate and the hydrostatic pressure. It is clear that further experimental studies and numerical investigations are necessary to achieve a better understanding of the phenomena of structural failure.

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