

## Impact resistance and energy absorption of functionally graded cellular structures

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**Abstract.** The dynamic response of functionally graded cellular structures subjected to impact of a finite mass was investigated in this paper. Compared to a cellular structure with a uniform cell size, the one with gradually changing cell sizes may improve many properties. Based on the two-dimensional random Voronoi technique, a two-dimensional topological configuration of cellular structures with a linear density-gradient in one direction was constructed by changing the cell sizes. The finite element method using ABAQUS/Explicit code was employed to investigate the energy absorption and the influence of gradient on stress wave propagation. Results show that functionally graded cellular structures studied are superior in energy absorption to the equivalent uniform cellular structures under low initial kinetic energy impacts, and the performance of such structures can be significantly improved when the density difference is enlarged. The stress levels at the impact and support ends may be reduced by introducing a gradual change of density in cellular structures when the initial impact velocity is low.

### Introduction

Functionally graded materials (FGMs) are a novel class of materials with a gradual change in volume fractions or mechanical properties of its constituent phases in one or two directions for specific performance requirements [1, 2]. They are ideal for applications as heat-resistant, energy absorption and load-bearing structures in advanced aircraft and aerospace engines [2-4]. The reasons are that these materials can reduce interfacial stress, increase the bonding strength, improve the surface properties and provide protection against adverse thermal and chemical environment [5-7].

A gradual distribution of pores from interior to surface may improve many mechanical properties, e.g., impact resistance, heat insulation and bending resistance. Examples of these functionally graded cellular structures (FGCSs) are widespread in nature, such as bamboo, teeth and bird beaks [8, 9]. Recently, functionally graded cellular structures have become a hot research topic because the structural components may bear many superior features in their application [10-12]. One type of FGCSs with varying the thickness of the regular honeycomb walls to generate gradients was reported by Ali et al. [10, 11]. Their graded honeycomb structure has a stepwise increment in the plateau stress level. It was found that such structure exhibited better energy absorbing characteristics for a wide range of impact velocities than regular honeycombs. Another type of FGCSs has been developed by Ajdari et al. [12]. Their cellular structures were divided into five equal-size regions in the crushing direction and different cell wall thicknesses were assigned to each region. They found that introducing a density gradient could significantly change the energy absorption property and deformation modes under low/high velocity impact.

In this paper, we construct a new type of FGCSs with a linear density-gradient in one direction and use a finite element method to study their dynamic response to impact loading. Different magnitude of gradient is used to investigate its influence on the energy absorption and the stresses on the impact and support surfaces.

### Materials and methods

**FGCS-generating method.** A mathematical method is developed to construct FGCSs with a linear density-gradient in the  $y$ -direction by changing the cell size but keeping the cell-wall thickness identical. The density in  $y$ -direction can be determined as

$$\rho(y) = \rho_0 \left[ 1 + \lambda \left( \frac{y}{L_y} - 1/2 \right) \right], \quad (1)$$

where  $\lambda$  is a dimensionless parameter characterizing the density gradient with  $d\rho/dy = \lambda\rho_0/L_y$ ,  $\rho_0$  the average density and  $L_y$  the length in the  $y$ -direction of the FGCS.

First,  $N$  nuclei are generated in a square area  $A_0$  by the principle that the distance between any two adjacent nuclei  $i$  and  $j$  is not less than  $\delta_{ij}$ , and the distance  $\delta_{ij}$  can be defined as

$$\delta_{ij} = \frac{\rho_0 \delta_0}{2} \left( \frac{1}{\rho(y_i)} + \frac{1}{\rho(y_j)} \right), \quad (2)$$

where  $\delta_0$  is the distance between any two adjacent nuclei of regular hexagonal honeycomb, which can be given by [13]

$$\delta_0 = \sqrt{2A_0/\sqrt{3}N}. \quad (3)$$

Then, cells are generated based on 2D random Voronoi technique [13, 14].

In our studies, specimens with different gradients are constructed in an area  $A_0 = 100 \times 100 \text{ mm}^2$  with 400 nuclei. FGCS specimens with uniform density ( $\lambda = 0$ ) and linear decreasing density from the upper end to the lower end ( $\lambda = 0.8$  and  $1.6$ ) are shown in Fig. 1. By transposing the upper and lower surfaces of specimens with  $\lambda = 0.8$  and  $1.6$ , we can obtain the other two specimens with linear increasing density from the upper end to the lower end,  $\lambda = -0.8$  and  $-1.6$ , respectively.

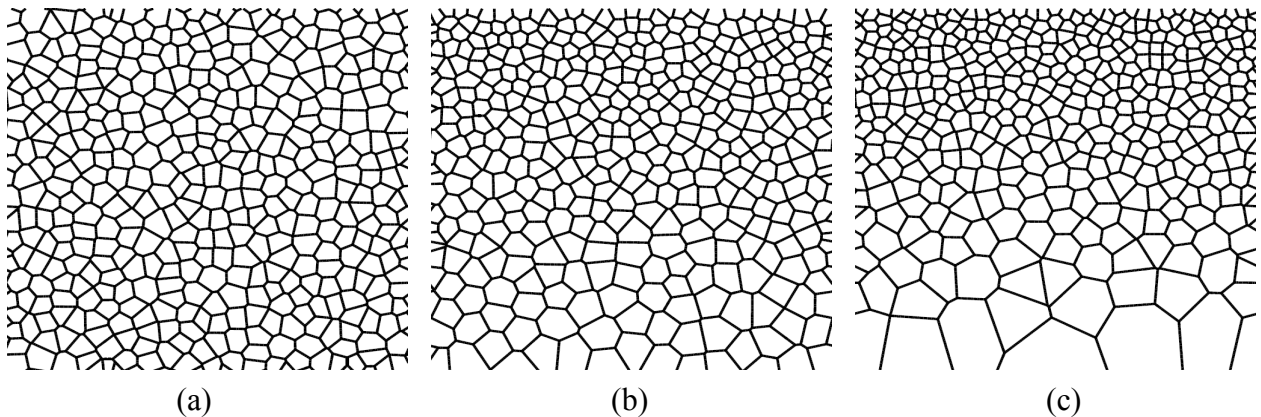


Fig. 1. Schematic diagrams of the FGCS specimens with (a)  $\lambda = 0$ , (b)  $\lambda = 0.8$  and (c)  $\lambda = 1.6$ .

**Finite element models.** To explore the energy absorbing behavior of FGCSs and the stresses on the impact and support surfaces, a series of numerical impact tests on FGCSs with different initial impact energy are performed using the ABAQUS/Explicit finite element code. In the finite element models, a mass,  $M$ , with an initial impact velocity  $V_0$  is attached to a rigid plate on the upper surface,

as schematically shown in Fig. 2a. The cell wall material is taken to be elastic-perfectly plastic. Its Young's modulus, yield stress, Poisson's ratio and mass density are 69 GPa, 175 MPa, 0.3 and 2700 kg/m<sup>3</sup>, respectively, the same as those used by Zheng et al. [14]. The cell thickness is given by

$$t = \frac{\rho_0 A_0}{\rho_s \sum l_i}, \quad (4)$$

where  $l_i$  is the  $i$ -th cell wall length, and  $\rho_s$  the density of cell walls. The relative density  $\rho_0/\rho_s$  of FGCSs is taken to be 0.05 in this paper. The cell-wall thicknesses of FGCSs with  $\lambda = 0, 0.8, 1.6, -0.8$  and  $-1.6$  are 0.12855, 0.13603, 0.14591, 0.13603 and 0.14591 mm, respectively. Each edge of the cell wall is divided into several shell elements of type S4R (a 4-node quadrilateral shell element with reduced integration) with five integration points through the shell thickness. The number of shell elements depends on the cell-wall length, with an average of six and the mean element length of about 0.7 mm. Each cell is defined as a single self-contact surface, and the lower surface is on a rigid support plate. The nominal stresses on the impact and support surfaces are calculated. An example is shown in Fig. 2b.

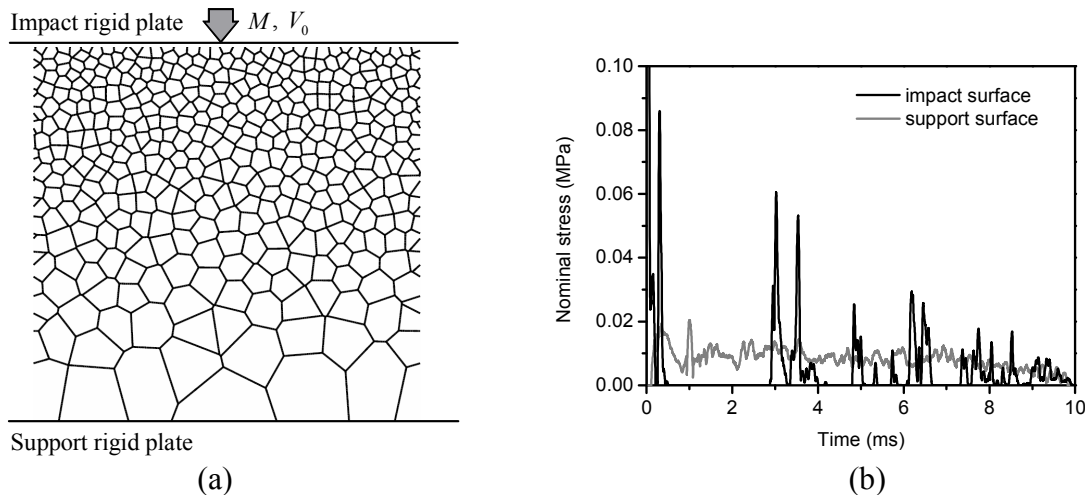


Fig. 2. (a) Schematic diagram of the finite element model. (b) Nominal stresses on the two end surfaces for an FGCS with  $\lambda = 1.6$  ( $V_0 = 5$  m/s and  $M = 1.35$  g).

## Results and discussion

**Energy absorption efficiency.** Energy absorption capacity is an important feature of cellular structures. During an impact event, the total internal energy absorbed by the structure consists of elastic strain energy, plastic dissipation energy and artificial strain energy which is used to control zero energy modes of elements. The energy-time curves of an FGCS ( $\lambda = 1.6$ ) with an initial kinetic energy of 132.3 mJ are shown in Fig. 3a. It can be observed that the initial kinetic energy has almost transformed into plastic dissipation energy at the end of impact.

To characterize the energy absorption capacity of cellular structures, we define an energy absorption ratio as  $U/E_k$ , where  $U$  is the total internal energy absorbed by the cellular structure at the end of impact and  $E_k$  (equals to  $MV_0^2/2$ ) is the initial kinetic energy of the impact plate. The energy absorption ratios of FGCSs under impact with four types of initial kinetic energy, i.e., 16.875, 33.075, 67.5 and 132.3 mJ, are shown in Fig. 3b. For a low kinetic energy impact (e.g.,  $E_k = 16.875$  and 33.075 mJ), the graded cellular structures perform a better energy absorption capacity than the uniform ones ( $\lambda = 0$ ). The energy absorption capacity of FGCSs with  $\lambda = 0.8$  and  $-0.8$ ,  $\lambda = 1.6$  and  $-1.6$  is at the same level, respectively. It transpires that FGCSs demonstrate a superior energy absorption capacity than the uniform cellular structure when the impact kinetic energy is low. However, with the

increase of initial kinetic energy, the advantages in energy absorption of FGCSs are not significant (e.g.,  $E_k = 67.5$  and  $132.3$  mJ). This may be due to the difference of deformation modes under low/high impact velocity.

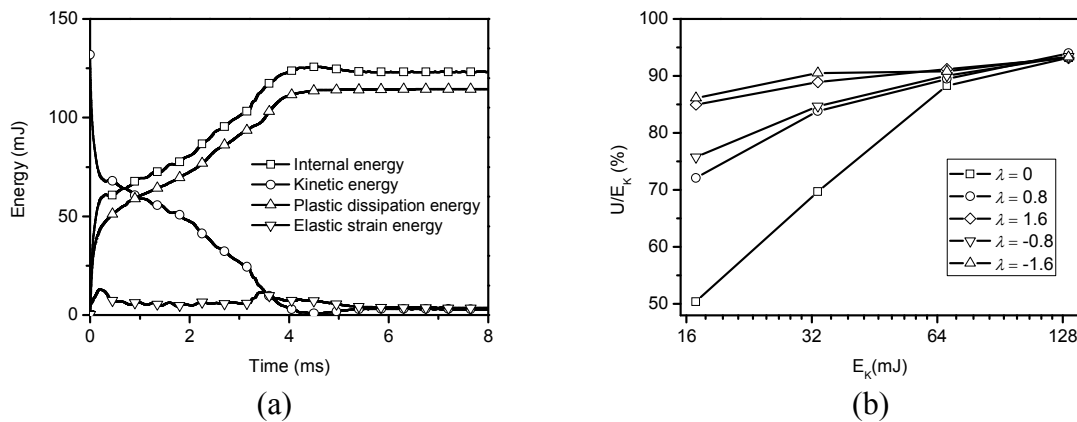


Fig. 3. (a) Energy-time curves of an FGCS with  $\lambda = 1.6$  under mass impact with an initial kinetic energy of  $132.3$  mJ. (b) Variation of energy absorption ratio of FGCSs with initial kinetic energy, where  $E_k$ -axis is plotted in the  $\log_2$  scale.

**Influence of density gradient on stress wave.** The stress can be amplified or diminished following its propagation through the functionally graded materials, as reported by Kiernan et al. [15]. When cellular materials are used for protection, e.g., packaging and helmet, the level of impact stress and support stress is very vital to minimise the peak acceleration of the impact body and reduce the reaction force of the support one. The impact stress and support stress of an FGCS specimen with  $\lambda = 1.6$ ,  $V_0 = 5$  m/s and  $M = 1.35$  g are shown in Fig. 2b. For  $\lambda > 0$ , the oscillation of the stress on the impact surface is rapid and intense, but that on the support surface is much smooth. However, for  $\lambda < 0$ , the result is reversed. A smooth stress level is ideal for protecting applications. So, we only consider the stress on the support surface for FGCSs with  $\lambda > 0$  and the stress on the impact surface for FGCSs with  $\lambda \leq 0$  under initial impact velocities of 5 and 50 m/s are shown in Figs. 4 and 5, respectively. Under low velocity impact (e.g., 5 m/s), the stress amplitudes in the case of  $\lambda = 0.8, 1.6, -0.8$  and  $-1.6$  is much longer than that of a uniform structure ( $\lambda = 0$ ), as seen in Figs. 4a and 5a. Under high velocity impact (e.g., 50 m/s), the stress amplitudes in the case of  $\lambda = 0.8, 1.6, -0.8$  and  $-1.6$  are larger than that of a uniform structure ( $\lambda = 0$ ), but the time duration in the case of  $\lambda = 0, 0.8$  and  $1.6$  is nearly equivalent to that of a uniform structure ( $\lambda = 0$ ) and that in the case of  $\lambda = -0.8$  and  $-1.6$  is shorter than that of a uniform structure ( $\lambda = 0$ ), as seen in Figs. 4b and 5b.

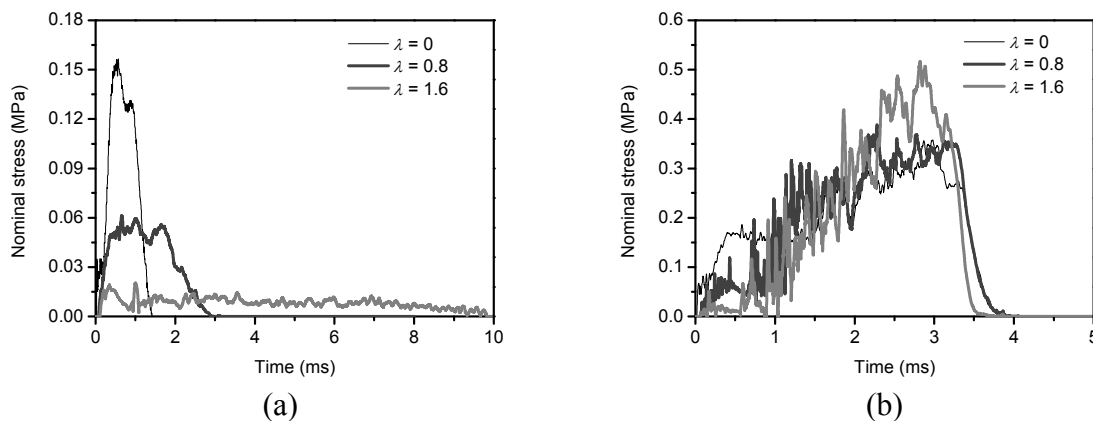


Fig. 4. Evolution of the stress on the support surface of FGCSs with  $\lambda \geq 0$  with  $M = 1.35$  g and an initial impact velocity of (a) 5 m/s and (b) 50 m/s.

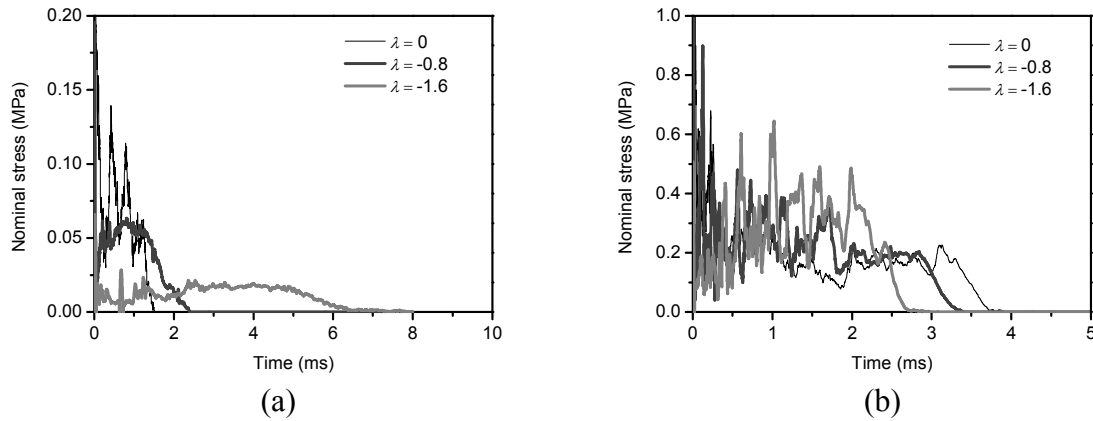


Fig. 5. Evolution of the stress on the impact surface of FGCSs with  $\lambda \leq 0$  with  $M = 1.35$  g and an initial impact velocity of (a) 5 m/s and (b) 50 m/s.

The nominal stresses on the two end surfaces are averaged from the beginning to the end of impact. The sensitivity of the average nominal stresses on the support surface and impact surface to the impact velocity is shown in Fig. 6. The nominal stresses of FGCSs with different gradients are different at the same impact velocity. For low impact velocities, the average stress on the support surface decreases with the increasing gradient, but less difference is found for high impact velocities, as seen in Fig. 6a. The average stress on the impact surface decreases with the decreasing gradient under low impact velocities (e.g., 5, 10 and 20 m/s), but it trends to the opposite when  $V_0 > 30$  m/s, as seen in Fig. 6b. This difference in the average nominal stress may be due to the difference of deformation modes under low/high impact velocity. It seems that the density gradient can influence the propagation of stress wave and the deformation modes of cellular structures. The mechanism will be explored in detail in our further research.

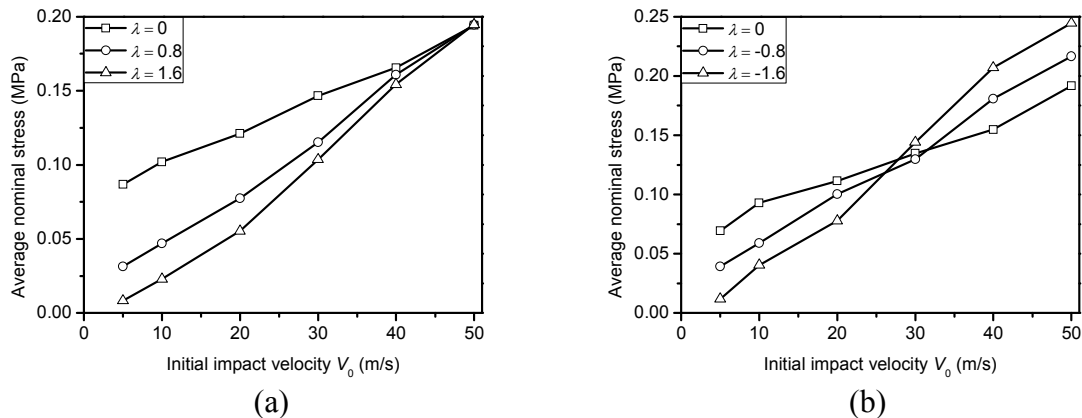


Fig. 6. Sensitivity of (a) average nominal stress on the support surface and (b) that on the impact surface to the initial impact velocity.

## Conclusions

A mathematical method was developed to construct functionally graded cellular structures with continuously changing densities in this paper. The finite element method was employed to study the response of FGCSs with a constant density-gradient under dynamic loading. It is found that FGCSs exhibit superior energy absorption characteristics than the equivalent uniform structures under low impact energy but this superiority diminish with the increase of impact energy.

The density gradient can influence the propagation of stress wave and the stress level on the two end surfaces. The stress on the support surface is much smooth for  $\lambda > 0$ , and that on the impact

surface is much smooth for  $\lambda < 0$ . This is ideal for protecting applications. So, we can choose FGCSs with  $\lambda > 0$  to protect the object attached on the support end and those with  $\lambda < 0$  to protect the object impacted in the case of low velocity impact.

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