Applied Mechanics and Materials Vols. 496-500 (2014) pp 609-612

doi:10.4028/www.scientific.net/AMM.496-500.609

Dynamic Response of Elastic Beam to a Moving Pulse: A Theoretical Study on Critical Velocity

Chunqiang Qian, Zhijun Zheng [a], Jilin Yu

CAS Key Laboratory of Mechanical Behavior and Design of Materials,
University of Science and Technology of China, Hefei, Anhui 230026, PR China

[a] Corresponding author. zjzheng@ustc.edu.cn

Keywords: moving pulse; Bernoulli-Euler beam; critical velocity; Fourier transform

Abstract. Dynamic behavior of a semi-infinite elastic beam to a moving single sinusoidal pulse was theoretical investigated. An analytical model was developed based on the Bernoulli-Euler beam theory. The solutions of the deflection and stress of beam were obtained by using the superposition principle and applying the techniques of Fourier transform. It is found that when the moving pulse reaches a critical velocity for a given moving pulse duration, the maximal absolute value of stress in beam attains its maximum value.

Introduction

Aerodynamic problems of high-speed trains have become more and more important as trains speed up [1]. One of them is the air pressure pulse acting on the carriage structure occurring as trains pass each other, which may cause the instability of carriage structure [2]. However, the mechanism of interaction between moving pulse and stress wave in structure is still not clear.

Raghunathan et al. [1] summarized the typical features of the air pressure pulse. One feature is that the air pressure pulse has alternating positive/negative amplitude values, similar to that of a single sinusoidal pulse. Thus, we focus on understanding the mechanism of dynamic response of a beam to a moving single sinusoidal pulse in this paper. The problems about moving loads extensively investigated are mainly in the fields of transportation, such as tunnel, rail and launcher. A beam to different kinds of moving loads, such as moving constant concentrated force [3, 4], moving harmonic concentrated force [5] and moving line load [6], have been investigated. An important phenomenon was extensively found, i.e. the moving load traveling at a critical velocity will cause significant vibration in a structure. The critical velocity is dependent on the properties of material and structure.

A theoretical model of an elastic beam subjected to a moving single sinusoidal pulse is developed in this paper. The techniques of the cosine Fourier transform and its inverse transform are applied to obtain the solutions. The response of the deflection and stress of beam are studied.

Theoretical model

Consider an elastic beam with density ρ, Young's modulus E, cross-section area A and moment of inertia I subjected to a moving single sinusoidal pulse with velocity v, as shown in Fig. 1.

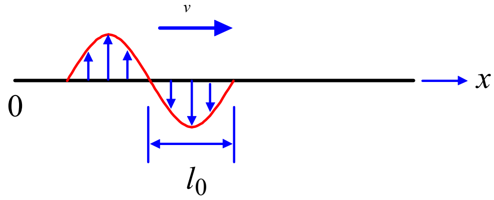

Fig. 1. Semi-infinite elastic beam subjected to a moving single sinusoidal pulse.

Based on the Bernoulli-Euler beam theory, the beam deflection, y, is governed by

$$\frac{\partial^4 y}{\partial x^4}+\frac{1}{a^2}\frac{\partial^2 y}{\partial t^2}=\frac{1}{EI}F(x,t),\quad x\geq 0\,, \tag{1}$$

where x is the position coordinate, t the time, $F(x, t)$ moving pulse and $a^2=EI/(\rho A)$. The single sinusoidal pulse can be expressed as

$$F(x,t)=-F_0\mathrm{H}(x-vt-2l_0)\mathrm{H}(vt-x)\sin\left[\alpha(x-vt)\right],\quad x\geq 0\,, \tag{2}$$

where F_0 is the peak amplitude of the moving pulse, l_0 the half length of moving pulse, α wavenumber (equals to π/l_0), and $\mathrm{H}(\cdot)$ the Heaviside step function. The boundary conditions are given by $y_x(0, t) = 0$, $y_{xxx}(0, t) = 0$, $y(\infty, t) = 0$, $y_t(\infty, t) = 0$, and the initial conditions are written as $y(x, 0) = 0$, $y_t(x, 0) = 0$, where the subscript represents the derivative of the variable with respect to the subscript.

To obtain the solution of the above equations, we first consider the problem of the beam subjected to a moving continuous sinusoidal pulse and then apply the superposition principle. The single sinusoidal pulse in Eq. (2) can be rewritten as

$$F\left(x,t\right)=\left[F_{\mathrm{c}}(x,t)-F_{\mathrm{c}}(x+2l_0,t)\right]\mathrm{H}(x)=\begin{cases}F_{\mathrm{c}}(x,t), & 0\leq t\leq T\\ F_{\mathrm{c}}(x,t)-F_{\mathrm{c}}(x+2l_0,t)\,\mathrm{H}(x), & t>T\end{cases}, \tag{3}$$

with the moving continuous sinusoidal pulse defined as

$$F_{\mathrm{c}}(x,t)=-F_0\mathrm{H}(vt-x)\sin[\alpha(x-vt)],\quad x\geq 0\,, \tag{4}$$

where T is the pulse width (equals to $2l_0/v$). Applying the cosine Fourier transform and inverse cosine Fourier transform, we can obtain the solution of the deflection of beam subjected to the moving pulse $F_{\mathrm{c}}(x,t)$,

$$y_{\mathrm{c}}\left(x,t\right)=\frac{2\alpha F_0}{\pi EI}\int_0^{\infty}\frac{1}{k^2-\alpha^2}\left[\frac{\cos\alpha vt-\cos ak^2t}{k^4-\alpha^2v^2/a^2}-\frac{\cos kvt-\cos ak^2t}{k^2(k^2-v^2/a^2)}\right]\cos kx\,dk\,. \tag{5}$$

Thus, the solution of the deflection of beam subjected to the moving pulse in Eq. (3) is

$$y(x,t)=\left[y_{\mathrm{c}}(x,t)-y_{\mathrm{c}}(x+2l_0,t)\right]\mathrm{H}(x)\,. \tag{6}$$

We can further obtain the maximum bending stress in the beam subjected to the pulse in Eq. (3)

$$\sigma(x,t)=\left[\sigma_{\mathrm{c}}(x,t)-\sigma_{\mathrm{c}}(x+2l_0,t)\right]\mathrm{H}(x)\,, \tag{7}$$

with

$$\sigma_{\mathrm{c}}(x,t)=-\frac{Eh}{2}\frac{\partial^2 y_{\mathrm{c}}}{\partial x^2}=\frac{\alpha hF_0}{\pi I}\int_0^{\infty}\frac{k^2}{k^2-\alpha^2}\left[\frac{\cos\alpha vt-\cos ak^2t}{k^4-\alpha^2v^2/a^2}-\frac{\cos kvt-\cos ak^2t}{k^2(k^2-v^2/a^2)}\right]\cos kx\,\mathrm{d}k\,, \tag{8}$$

where h is the beam thickness.

Eq. (5) and (8) can be evaluated by using complex function techniques. Here, we directly evaluate the two expressions with the adaptive Gauss-Kronrod quadrature (function name: quadgk) in Matlab.

Results and discussion

To illustrate some features of the deflection and stress of beam, we take the material properties of the beam $\rho = 2700$ kg/m^3 and $E = 70$ GPa. The cross-section of the beam is square with the side length $c = 0.6$ m. The pulse duration, T, is set to be 0.1 s.

The typical profiles of deflection and absolute value of stress, σ_a, are shown in Fig. 2. It is observed that the vibration of beam is concentrated in front of the moving pulse. The stress in the region the moving pulse has passed is almost equal to zero.

The maximal absolute values of stress, denoted as $\sigma_{a,max}$, varying with time for different velocities of moving pulse are shown in Fig. 3a. For a particular velocity of moving pulse, $\sigma_{a,max}$ increases with time at first (stage I) and then tends to be constant (stage II). For different velocities of moving pulse, the distance between the maximal bending stress and the moving pulse front, D, is shown in Fig. 3b. It is found that the distance D remains a small value at the initial period, and then increases almost linearly with time. The transition time corresponds to the time when the stress $\sigma_{a,max}$ transits from stage I to stage II. So, the position corresponding to $\sigma_{a,max}$ stays in the moving pulse front before it attains the stress level of stage II, and then moves away from the moving pulse front.

As shown in Fig. 3a, $\sigma_{a,max}$ fluctuates with time in stage II. To evaluate the level of maximal absolute value of stress in stage II, we calculate the average value of maximal absolute value of stress in this stage, denoted as $\bar{\sigma}_{a,max}$. As the velocity of moving pulse increases, $\bar{\sigma}_{a,max}$ increases first before reaching its maximum value and then decreases, as shown in Fig. 4. $\bar{\sigma}_{a,max}$ reaches its maximum value when the moving pulse moves at a certain velocity, which is termed as the critical velocity. In the case $T = 0.1$ s, the critical velocity is 221 m/s.

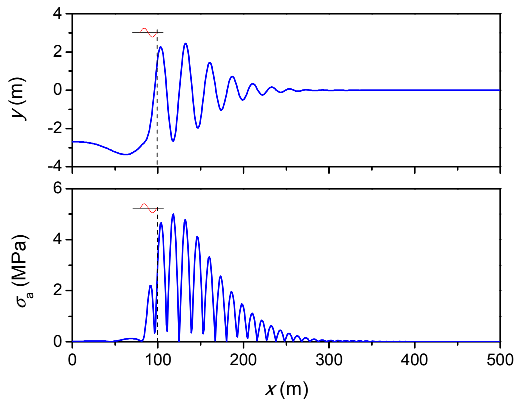

Fig. 2 The deflection and absolute value of stress, σ_a, of beam at time 0.5 s for the case of $v = 200$ m/s and $T = 0.1$ s.

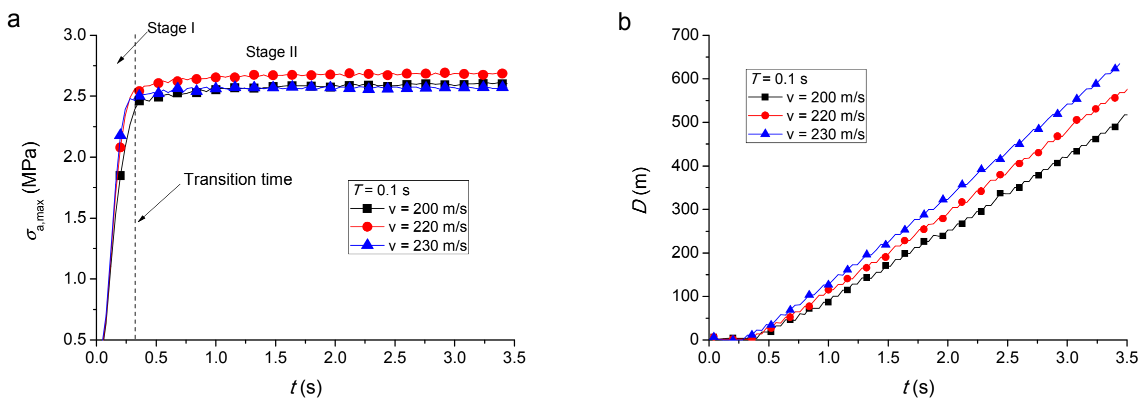

Fig. 3 (a) Maximal absolute values of stress, $\sigma_{a,max}$, vs. time and (b) distance between the maximal absolute values of stress and the moving pulse front, D, vs. time for different velocities of moving pulse.

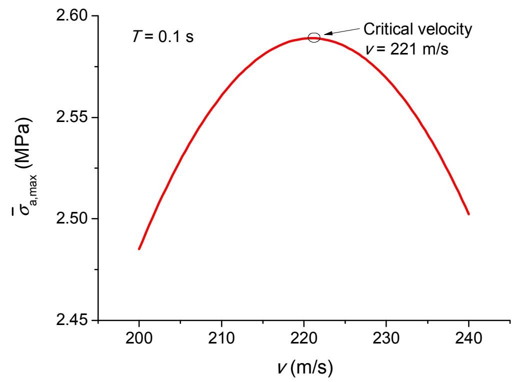

Fig. 4 The average value of maximal absolute values of stress, $\bar{\sigma}_{a,max}$, for different velocities of moving pulse.

Summary

In this paper, the techniques of Fourier transform and its inverse transform are applied to solve the problem of transient response of an elastic beam subjected to a moving single sinusoidal pulse. The maximal absolute value of the stress in the beam increases linearly with time at an initial period and then tends to be constant. For a particular pulse duration, there is a critical velocity at which the average of maximal absolute value of stress reaches its maximum.

Acknowledgements

The financial supports from the National Basic Research Program of China (973 program, Grant No. 2011CB711100), the National Natural Science Foundation of China (Project No. 11372307) and the Chinese Academy of Sciences (Grant No. KJCX2-EW-L03) are acknowledged.

References

[1] R.S. Raghunathan, H.D. Kim and T. Setoguchi, Aerodynamics of high-speed railway train, Prog. Aerospace Sci. 38 (2002) 469-514.

[2] H.Q. Tian, S. Yao and S.G. Yao, Influence of the air pressure pulse on car-body and side-windows of two meeting trains, China Railway Sci. 21 (4) (2000) 6-12.

[3] S.P. Timoshenko, Method of analysis of static and dynamic stresses in rails, Proc. Second International Congress for Applied Mechanics, Zurich, Switzer-land, (1927) 1–12.

[4] L. Florence, Traveling force on a Timoshenko beam, J. Appl. Mech. Trans. 32 (1965) 351 –359.

[5] Y.Y. Chen and Y.H. Huang, Dynamic characteristics of an infinite and finite railways to moving loads, J. Eng. Mech. 129 (9) (1997) 987-995.

[6] L. Sun, Dynamic displacement response of beam-type structures to moving loads, Int. J. Solids Struct. 38 (2001) 8869–8878.