Mass Impact of Density-graded Cellular Metals in a Temperature Field

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Abstract: We consider the problem of a density-graded cellular rod in a temperature gradient field axially subjected to a mass impact. Two-dimensional cell-based finite element models and one-dimensional shock models are employed to explore the mechanisms of deformation and wave propagation. The yield stress distribution in a cellular specimen depends on both the density gradient field and the temperature gradient field. The stress distribution and the yield stress distribution are analyzed. For the increasing yield stress along the impact direction, one shock front propagates from the proximal end to the distal end of the specimen. For the decreasing yield stress along the impact direction, two shock fronts propagate in opposite directions and the one close to the proximal end ceases at a particular time. The predicted stresses of the extended shock models are compared well with the numerical results.

Introduction

Cellular metals exhibit excellent properties, such as light weight, high stiffness, good compressibility and heat resistance, and thus have been applied in many engineering fields and even in some extreme environmental conditions, e.g. high temperature and high velocity impact. Introducing a density gradient to cellular metals may further improve their high designability and multifunction application. To get some design principle of graded cellular metals (GCMs), we consider the problem of a density-graded cellular rod in a temperature gradient field axially subjected to a mass impact in this paper.

Some types of cell-based finite element (FE) models and shock models have been developed for GCMs. A varying cell-size distribution method was developed to construct cell-based FE models of GCMs and study their energy absorption ability [1]. The shock model based on the rate-independent, rigid–perfectly plastic–locking (R-PP-L) idealization [2] well explains the uniform cellular metals under dynamic loading. Shen et al. [3] extended the R-PP-L shock model for GCMs with a linear strength distribution. Wang et al. [4] studied the case with a linear density distribution. In this study, a temperature gradient field is introduced to further extend the work.

Cell-based finite element models

Consider a type of GCMs with a linear density distribution along the \(X\)-axis, i.e.

\[ \rho_t(X) = \rho_0 \cdot \left[1 + \gamma \left(\frac{X}{L} - 1/2\right)\right], \tag{1} \]

where \(\gamma\) is the density-gradient parameter, \(\rho_0\) the average density and \(L\) the length of the cellular specimens. In numerical simulations, the cell-based FE models of GCMs are constructed by the 2D Voronoi technique with a new principle of seeding nuclei [1]. The principle is that the distance between any two nuclei \(i\) and \(j\) should be larger than the minimum allowable distance, given by

\[ \delta_{ij}^{\min} = (1 - k) \cdot 2 \rho_h / \rho_t(X_j), \quad X_j = (X_i + X_j) / 2, \tag{2} \]

\(\rho_h\) is the characteristic density of the cellular specimen, \(k\) the distance fraction for nuclei, \(\delta_{ij}^{\min}\) the minimum distance, and \(X_j\) the distance in the \(X\)-direction.
where \( h \) is the cell-wall thickness, \( \rho_s \) the density of cell-wall material, \( k \) the cell irregularity, and \((X_i, Y_i)\) the location of nucleus \( i \). In this study, \( \rho_0/\rho_s = 0.1, k = 0.2, h = 0.26 \text{ mm} \) and \( \rho_s = 2700 \text{ kg/m}^3 \). Three cellular samples \((\gamma = -1, 0 \text{ and } 1)\) with length \( L = 400 \text{ mm} \), width \( W = 100 \text{ mm} \) and thickness \( H = 1 \) mm are shown in Fig. 1.

Impact tests of the cellular specimens in a temperature gradient field are simulated by using the FE code ABAQUS/Explicit. Two steps are implemented in the numerical analysis: Step 1, the thermal analysis, and Step 2, the mechanical analysis. In Step 1, the steady-state heat conduction in the cellular specimens is considered to determine the local temperature of every element. For a cellular specimen, its two ends are restricted with isothermal boundary conditions and the other sides are with adiabatic boundary conditions. The thermal conduction of cell-wall material, \( k_s \), is taken as 230 W/(m·K) and the temperatures on the left and right ends of a specimen are \( T_1 = 300^\circ\text{C} \) and \( T_2 = 20^\circ\text{C} \), respectively. In Step 2, we consider the cellular specimens placed before a stationary rigid wall and axially impacted by a mass \( M \) traveling with an initial velocity \( V_0 \), see Fig. 1a. The well-known Johnson-Cook model describes the constitutive behavior of metals subjected to large strains, high strain rates and high temperatures. During the deformation process of cellular metals, plastic hinge and bending are dominant, but large strains and high strain rates do not play decisive roles. Thus, we consider the cell-wall material to be rate-independent, elastic-perfectly plastic with its mechanical properties dependent on the local temperature. By introducing a dimensionless temperature \( \theta = (T-T_{\text{room}})/(T_{\text{melt}}-T_{\text{room}}) \), in which \( T \) is the temperature, \( T_{\text{melt}} \) the melting temperature of the cell-wall material (\( T_{\text{melt}} = 660^\circ\text{C} \)), \( T_{\text{room}} \) the room temperature (\( T_{\text{room}} = 20^\circ\text{C} \)), the Young’s modulus [5] and the yield stress of cell-wall material are given by

\[
\begin{align*}
E(T) &= E_{\text{RT}} \cdot \left[ 1 - \alpha_m \theta / \left( 1 - (1 - \alpha_m) \theta_0 \right) \right], \\
\sigma_y(T) &= \sigma_{y,\text{RT}} \cdot (1 - \theta)
\end{align*}
\]

where \( E_{\text{RT}} \) and \( \sigma_{y,\text{RT}} \) are the properties at room temperature (\( E_{\text{RT}} = 69 \text{ GPa} \), \( \sigma_{y,\text{RT}} = 170 \text{ MPa} \)), \( \alpha_m = 0.5 \) and \( \theta_0 = \alpha T = -273^\circ\text{C} = -0.4578 \). Material properties of every element in the cell-based FE models are assigned by Eq. (3) according to its local temperature. Additionally, the Possion’s ratio is always taken to be 0.3.

**Numerical results**

Deformation patterns for the three cellular specimens subjected to mass impact with \( M = 4 \text{ g} \) and \( V_0 = 120 \text{ m/s} \) in a temperature gradient field are shown in Fig. 1. The deformation processes of \( \gamma = 0 \) and 1 are very similar to those in a uniform temperature field, i.e. cells close to the proximal end of a specimen are crushed progressively and a deformation band like a structural shock wave front propagates towards the distal end. For \( \gamma = -1 \), one shock front appears close to the proximal end and another shock front appears near the distal end. The two shock fronts propagate in opposite directions. The shock front close to the proximal end stops at a particular time, which will be predicted from the extended R-PP-L model. The difference of deformation patterns is due to the strength distributions in the cellular specimens, as discussed later.

The nominal stress-time curves at the proximal and distal ends of the cellular specimens with \( \gamma = -1, 0 \text{ and } 1 \) are shown in Fig. 2. Hereinafter, the impact/support stress refers to the stress at the proximal/distal end of a specimen. The numerical results of stresses have oscillations, which are ignored in the following descriptions. The impact stress of \( \gamma = -1 \) drops rapidly to zero and then increases gradually with the increasing time. Similar features of the impact stresses at the initial stage are also found for \( \gamma = 0 \) and 1, but the impact stresses decrease gradually with the increasing time in the later stage. The support stresses of \( \gamma = -1 \text{ and } 1 \) increase gradually with the increasing time, but that of \( \gamma = 0 \) is much stable.
Fig. 1. Deformation patterns of graded cellular metals with (a) $\gamma = -1$, (b) $\gamma = 0$ and (c) $\gamma = 1$. ($M = 4$ g, $V_0 = 120$ m/s, $T_1 = 300 ^\circ$C and $T_2 = 20 ^\circ$C)

Fig. 2. Nominal stress-time curves of graded cellular metals with (a) $\gamma = -1$, (b) $\gamma = 0$ and (c) $\gamma = 1$ under mass impact in a temperature field.

Shock models and predictions
In this section, we extend the R-PP-L shock model to explain the deformation and stress features of the above cases. Similar work has been carried out in Refs. [3, 4], but their predictions are not suitable for all the present cases. In this study, both the density gradient field and the temperature gradient field influence the strength distribution in cellular specimens. Unfortunately, the strength distribution is often not linear. It is assumed that the local yield stress for the R-PP-L idealization depends on the local relative density $\rho = \rho_f/\rho_s$ and the local temperature $T$, but the locking strain only depends on the local relative density, written as

$$
\begin{align*}
\sigma_T(\rho, T) &= c\rho^\gamma \sigma_y(T) = c\rho^\gamma (1-\theta) \sigma_{y,RT}, \\
\epsilon_L(\rho) &= \epsilon_{0L}(\rho) = r(1-\rho)
\end{align*}
$$

where $\epsilon_D$ is the densification strain, and $c$ and $r$ two fitting constants. Here, $c = 0.417$ and $r = 0.725$. The local temperature in a cellular specimen can be determined by solving the 1D steady-state heat transfer equation

$$
\frac{d}{dX} \left( k_e(\rho) \frac{dT}{dX} \right) = 0 ,
$$

where $k_e(\rho)$ is the local apparent thermal conductivity, which can be defined as $k_e(\rho) = \zeta \rho k_s$ with $\zeta$ being the tortuous shape of cell walls [6]. Considering the density distribution in Eq. (1) and the temperatures at the two ends of the specimen, we obtain
\[ T(X) = \begin{cases} 
\frac{T_1 + (T_2 - T_1) \cdot \ln (1 + \alpha X / L)}{\ln (1 + \alpha)}, & \gamma \neq 0 \\
T_1 + (T_2 - T_1) \cdot X / L, & \gamma = 0 
\end{cases} \]  

(6)

where \( \alpha = \gamma (1 - \gamma^2) \). Substituting Eqs. (1) and (6) into Eq. (4), we can obtain the yield stress distribution. The temperature distributions and the yield stress distributions for the three specimens are shown in Fig. 3. The yield stress distribution for \( \gamma = 0 \) is linear, thus the predictions in Ref. [3] can be used for this case. However, the yield stresses for \( \gamma = -1 \) and 1 are nonlinear. Thus, we need to extend the R-PP-L shock model for these two cases.

For the increasing yield stress along the impact direction (e.g., \( \gamma = 0 \) and 1), only one shock front propagates from the proximal end to the distal end. This is similar to that of uniform cellular metals with a uniform temperature field. Thus, the R-PP-L shock model can be easily extended for this case. The main difference is due to that the yield stress in a cellular specimen depends on both the local relative density and the local temperature. By using the kinematic and kinetic compatibility conditions across the shock front, the shock speed and the shock-enhanced stress can be given by

\[
\begin{align*}
\frac{d \Phi(t)}{dt} &= v \left( \frac{\sigma(t)}{\rho(t)} \right) \\
\sigma(t) &= \sigma(t) \left( \frac{\rho(t)}{\rho_s}, T(t) \right) + \rho(t) \left( \frac{\mu}{\rho_s} \right) v^2
\end{align*}
\]  

(7)

where \( \Phi(t) \) is the Lagrangian location of the shock front at time \( t \), and \( v \) the velocity of mass together with the portion of the cellular specimen behind the shock front, which is governed by the inertial law

\[
\frac{dv}{dt} = - \sigma(t) / (m + \int_{\Phi} \rho(X) dX)
\]  

(8)

where \( m = M / A_0 \) with \( A_0 \) being the cross-sectional area of the cellular specimen (\( m = 40 \text{ kg/m}^2 \) for the above cases). A numerical method (e.g. the Runge-Kutta algorithm) is needed to solve Eqs. (7) and (8) with the initial conditions \( \Phi(0) = 0 \) and \( v(0) = V_0 \). The impact stress can be determined from \( \sigma(t) = -m \cdot dv/dt \), while the support stress \( \sigma_s = \sigma(t) \rho(t) \). The stress distributions in a cellular specimen with \( \gamma = 1 \) are shown in Fig. 4a.

For the decreasing yield stress along the impact direction (e.g., \( \gamma = -1 \)), two shock fronts are formed at the two ends of the cellular specimen and propagate towards each other. At time \( t \), the stress distribution is denoted as \( \sigma(X) \), and the shock fronts close to the proximal and distal ends locate at \( \Phi_1 \) and \( \Phi_2 \) in the Lagrangian coordinate, respectively. The uncruhked region between the two shock fronts \( (\Phi_1 < X < \Phi_2) \) travels with a velocity \( v_{\text{in}} \), which can be determined by using the inertial law as...
\[
\frac{dv_u(t)}{dt} = -\frac{1}{\rho_i(X)} \frac{d\bar{\sigma}(X)}{dX} = \sigma_i(\rho_i(\Phi_i)/\rho_s,T(\Phi_i)) - \int_{\phi_1}^{\phi_2} \rho_i(X) dX. \tag{9}
\]

It can be found that the stress in the uncrushed region is less than the local yield stress. In Refs. [3, 4], the stress in the uncrushed region is exactly equal to the local yield stress. This difference should be noted. The motion of the mass together with the portion behind shock front 1 is the same as Eq. (8) with \( \Phi \) being replaced by \( \Phi_1 \). From the kinematic and kinetic compatibility conditions across the shock fronts, we obtain the shock speeds

\[
\begin{align*}
\frac{d\Phi_1}{dt} &= (v - v_u) / \varepsilon_0(\rho_i(\Phi_i)/\rho_s) \\
\frac{d\Phi_2}{dt} &= -v_u / \varepsilon_0(\rho_i(\Phi_2)/\rho_s)
\end{align*}
\tag{10}
\]

and the shock-enhanced stresses

\[
\begin{align*}
\sigma_B &= \sigma_i(\rho_i(\Phi_1)/\rho_s,T(\Phi_1)) + \rho_i(\Phi_1)(v - v_u)^2 / \varepsilon_0(\rho_i(\Phi_1)/\rho_s) \\
\sigma_B &= \sigma_i(\rho_i(\Phi_2)/\rho_s,T(\Phi_2)) + \rho_i(\Phi_2)v_u^2 / \varepsilon_0(\rho_i(\Phi_2)/\rho_s)
\end{align*}
\tag{11}
\]

A numerical method is employed to solve these equations with initial conditions \( \Phi_1(0) = 0, \Phi_2(0) = L, v(0) = V_0 \) and \( v_u(0) = 0 \). At time \( t_1 \) corresponding to \( v(t_1) = v_u(t_1) \), shock front 1 ceases. After time \( t_1 \), \( v_u(t) = v(t) \) and the governing equations can be similarly obtained. The impact stress is determined by \( \sigma_i = -m \cdot dv/dt \) and the support stress \( \sigma_s = \sigma_B \). The stress distributions in a cellular specimen with \( \gamma = -1 \) are shown in Fig. 4b.

Fig. 4. Stress distributions in cellular specimens with (a) \( \gamma = 1 \) and (b) \( \gamma = -1 \). \( M = 4 \text{ g}, V_0 = 120 \text{ m/s}, T_1 = 300^\circ \text{C}, T_2 = 20^\circ \text{C} \)

The predicted stresses at the two ends of cellular specimens with \( \gamma = -1, 0 \) and 1 under mass impact in the temperature field are also shown in Fig. 2. For \( \gamma = -1 \), the time \( t_1 \) is predicted as 3.672 ms, which is close to the time corresponding to the numerical result, as shown in Fig. 2a. The predictions of the shock models are compared well with the numerical results.

Summary

Two-dimensional GCM models are constructed by using a varying cell-size distribution method. The thermal analysis and the mechanical analysis are carried out to examine the response of GCM specimens subjected to a mass impact in a temperature field. The R-PP-L shock model is extended and used to explain the mechanism of shock wave propagations. The predicted stresses of the extended shock model are compared well with the numerical results.

The strength distribution in a GCM specimen depends on both the density gradient field and the temperature gradient field. Thus, density and temperature distributions significantly affect the
mechanisms of deformation and wave propagation. The results in this study can provide an insight into crashworthiness of GCMs applied in a temperature field.

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