Key Engineering Materials
ISSN: 1662-9795, Vol. 803, pp 134-139
doi:10.4028/www.scientific.net/KEM.803.134

Submitted: 2019-01-22
Accepted: 2019-02-13
Online: 2019-05-20

Revisiting the Deshpande-Fleck Foam Model

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Keywords: Metal foam; Finite element analysis; Constitutive relation; Ellipticity

Abstract. The self-similar isotropic hardening model developed by Deshpande and Fleck has been widely used. An important issue in this model is to determine the value of ellipticity. The ellipticity was treated as a constant in the subsequent yield, but different values were suggested in the literature. In this paper a cell-based finite element model based on the 3D Voronoi technique is used to verify the Deshpande-Fleck foam model. It is found that the ellipticity determined from uniaxial and hydrostatic compressions varies with the equivalent plastic strain.

Introduction

Cellular materials, such as corks, honeycombs and metal foams, are a kind of materials with low relative density (< 0.3) and can absorb massive impact energy with large deformation [1]. It is necessary to understand the constitutive behavior of metal foams under complex loadings to improve their engineering design. Metal foams can yield under a hydrostatic pressure. This makes their constitutive model very different from the traditional plasticity theory of dense metals, which does not consider the effect of hydrostatic pressure on yielding. Several forms of the yield functions, based on the von Mises effective stress σ_e and the mean stress σ_m, have been proposed to describe the constitutive relation of metal foams [1-4].

An elliptic yield function was proposed by Deshpande and Fleck [4]

$$\sqrt{\sigma_e^2+\alpha^2\sigma_m^2}-\sqrt{1+(\alpha/3)^2}Y=0\,, \tag{1}$$

where α is the ellipticity of the ellipse representing the shape of the yield surface in the σ_m–σ_e plane, and Y the uniaxial yield strength. It was assumed that this elliptic yield surface evolves in a geometrically self-similar manner with the equivalent plastic strain. This model, known as the Deshpande-Fleck foam model (or short as the D-F model), attracts a lot of attention due to its simplicity with only two parameters, namely the ellipticity α and the uniaxial yield strength Y. The ellipticity α was considered to be a constant in subsequent yield surfaces. The initial elliptical yield surface of this model has been verified by more and more experiments when suitable plastic Poisson's ratios were employed [5-7]. The Deshpande-Fleck foam model has also been successfully applied to the finite element softwares LS-DYNA and ABAQUS [8,9].

A hardening law is also essential to describe the constitutive relation. The uniaxial stress–strain relation is most commonly used to characterize the hardening law. Various phenomenological models have been proposed to characterize the stress–strain relation of foams under uniaxial compression [10-12]. However, those models either had too many parameters or can not characterize the stress–strain relation very well. Recently, Zheng et al. [13] proposed a rate-independent, rigid–plastic hardening (R-PH) idealization to describe the uniaxial compression behavior of metal foams. The nominal stress–strain relation can be expressed as

$$\sigma_{\mathrm{n}} = \sigma_{\mathrm{n0}} + C\varepsilon_{\mathrm{n}} \big/ \left(1-\varepsilon_{\mathrm{n}}\right)^2 , \tag{2}$$

where σ_{n0} is the nominal initial crushing stress and C the strain hardening parameter. This model is very simple, with only two parameters, but is accurate enough and has been verified by many numerical simulations and experiments [14-17]. Due to its accuracy and simplicity, the R-PH model has been widely applied. For example, the design criteria of cellular sacrificial cladding based on the R-PH model has been verified by a cell-based finite element model [14]. An analytical model based on the R-PH model has been used to predict the blast response of density-graded cellular rods and a good agreement was achieved between the theoretical predictions and the numerical results [15]. Based on the R-PH model, a nonlinear plastic shock model was employed to guide the design strategy to determine the relative density distribution of graded cellular materials for desirable crashworthiness requirements and it was well verified by a cell-based finite element method [16].

The present study aims to revisit the Deshpande-Fleck foam model. A cell-based finite element model of 3D Voronoi structure is used to verify the prediction and the R-PH idealization is employed to describe the stress–strain relation of the cellular material under uniaxial compression.

Models

Deshpande-fleck foam model. The Deshpande-Fleck foam model [4] uses an elliptic yield surface in the mean stress vs. von Mises effective stress plane and assumes similar behaviors in compression and tension, as depicted in Fig. 1. The elliptic yield surface centers at the origin of the σ_{m}–σ_{e} plane and evolves in a self-similar manner governed by the equivalent plastic strain. The equivalent plastic strain rate is defined according to the equivalent plastic work rate, i.e. $\sigma_{\mathrm{c}}\dot{\bar{\varepsilon}}^{\mathrm{p}} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}$, where σ_{c} is the uniaxial compression stress associated to current yield surface, $\boldsymbol{\sigma}$ the stress tensor and $\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}$ the plastic strain rate tensor. It is assumed that the elastic strain is negligibly small so that the total strain equals the plastic strain.

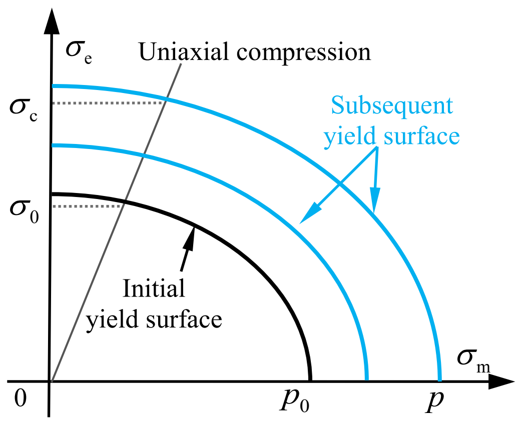

Fig. 1 Yield surfaces for the Deshpande-Fleck foam model.

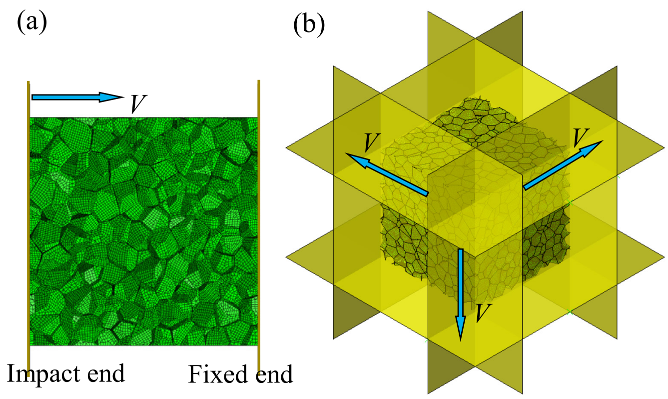

Fig. 2 Cell-based finite element models of (a) uniaxial compression and (b) hydrostatic compression.

Cell-based finite element model. The 3D Voronoi technique is widely used to simulate random foams because their foaming process is much like the principle of Voronoi diagram. This technique was employed to construct closed-cell foam models, as done in Ref. [13].

In this study, the foam specimen is a cube with a side length of 30 mm containing 1200 nuclei. The irregularity is set to 0.4. Cell walls are meshed with hybrid S3R and S4R elements and the characteristic size of shell elements is set to about 0.3 mm after the mesh sensitivity analysis [13]. Shell elements having short edges are re-meshed to save computational cost. The relative density ρ of the foam specimen is 0.1. The base material is aluminum, which is assumed to be rate-independent and elastic–linear plastic hardening. The material parameters are density 2700 kg/m^3, Young's modulus 70 GPa, Poisson's ratio 0.33, yield stress 80 MPa and Tangent modulus 30 MPa, as used in Ref. [18].

The quasi-static uniaxial and hydrostatic compressions of foam specimens are simulated by using the finite element (FE) code ABAQUS/Explicit. Two rigid plates are employed on the two parallel planes of a foam specimen to apply quasi-static uniaxial compression. One of the rigid plate is fixed, while the other travels at a constant velocity of V = 10 m/s towards the fixed plate, as shown in Fig. 2(a). Three pairs of rigid planes are employed on a cube foam specimen to apply hydrostatic compression, as depicted in Fig. 2(b). Similarly, in each direction, one of the rigid plates is fixed, while the other moves with a constant velocity of V = 10 m/s towards the fixed plate. General contacts were applied to all possible contacts with a friction coefficient of 0.2 between the shell elements, and no friction between the rigid plates and the specimen.

Results and Discussion

Calculation methods of the ellipticity. An important issue in the Deshpande-Fleck foam model is to determine the value of ellipticity. Deshpande and Fleck [4] assumed the associated flow and gave the ellipticity

$$\alpha = \sqrt{9\left(1-2\nu^{\mathrm{p}}\right)\Big/\left[2\left(1+\nu^{\mathrm{p}}\right)\right]}. \tag{3}$$

This formula was widely used in the literature, but in fact it is contradictory for a fully compressible material having a plastic Poisson's ratio of $\nu^{\mathrm{p}} = 0$. When the plastic Poisson's ratio is close to zero, deformations in three orthogonal directions are almost independent, which leads to the initial yield strength under hydrostatic compression is equal to the initial yield strength under uniaxial compression. In other words, the ellipticity α is approximately equal to 1 when the plastic Poisson's ratio is close to zero, but Eq. (3) gives $\alpha \approx 2.12$. A similar definition was defined using the elastic Poisson's ratio ν instead of the plastic Poisson's ratio ν^{p} [19], i.e.

$$\alpha = \sqrt{9\left(1-2\nu\right)\Big/\left[2\left(1+\nu\right)\right]}. \tag{4}$$

It is doubtful to use elastic parameters to characterize the plastic behavior.

Another way to compute the shape parameter ellipticity α was given by [4,9]

$$\alpha = 3\sigma_0\Big/\sqrt{9p_0^2-\sigma_0^2}, \tag{5}$$

where σ_0 is the initial yield stress under uniaxial compression and p_0 the initial yield stress under hydrostatic compression. This method has been adopted in ABAQUS. All of the methods mentioned above for determining α assume it remains constant in the subsequent yield.

Eq. (3) and Eq. (4) give $\alpha \approx 2.12$ and 1.64 with the plastic Poisson's ratio $\nu^{\mathrm{p}} = 0$ and the Poisson's ratio $\nu = 0.15$, respectively. When $\nu^{\mathrm{p}} = 0$, deformations in three orthogonal directions are almost independent, which leads to the initial yield strength under hydrostatic compression equals the initial yield strength under uniaxial compression. Thus, Eq. (5) gives $\alpha \approx 1.06$.

Fitting parameters of the R-PH model. The quasi-static uniaxial stress–strain data are obtained from the cell-based FE model with a relative density of 0.1, as shown in Fig. 3. The nominal initial crushing stress σ_{n0} in the R-PH model is obtained by averaging the stress in a small plastic strain range (0–0.2) in the initial crushing stage. The strain hardening parameter C is determined by fitting using the least square method. Fitting the numerical results gives σ_{n0} = 3.66 MPa and C = 0.452 MPa. The fitting curve of the R-PH model is also shown in Fig. 3. It appears that the R-PH idealization can characteristic the stress–strain behavior well.

Under uniaxial compression, the true stress almost equals the nominal stress because the plastic Poisson's ratio of low-density foams is negligibly small [2,5,10]. The relation between the true (logarithmic) plastic axial strain and the nominal strain can be expressed as $\varepsilon = -\ln(1-\varepsilon_n)$, where compression strain is positive. Then the R-PH model can be re-written approximately as

$$\sigma_c = \sigma_{n0} + C\left(e^{2\varepsilon} - e^{\varepsilon}\right). \tag{6}$$

This formula is used in the following predictions.

Predictions of a typical loading. The mean stress and the von Mises stress under specific loads can be expressed by the D-F model and the uniaxial stress. The stress triaxiality parameter, $\eta = \sigma_m/\sigma_e$ [4], is widely used to characterize the direction of loading. By combining the D-F model with η, the mean stress σ_m can be expressed as

$$\sigma_m = \eta\sigma_c\sqrt{\left(1+\alpha^2/9\right)\Big/\left(1+\eta^2\alpha^2\right)}. \tag{7}$$

Under hydrostatic compression, i.e. when $\eta \to \infty$, the von Mises stress σ_e is zero and the mean stress σ_m can be written as

$$\sigma_m = p = \sigma_c\sqrt{1/\alpha^2 + 1/9}, \tag{8}$$

where p is the hydrostatic pressure.

Hydrostatic pressure obtained from cell-based finite element model under hydrostatic compression and predictions using the R-PH model and the D-F model with different α are illustrated in Fig. 4. The results show that the prediction of the D-F model using α = 1.06 determined by Eq. (5) has a better agreement with the numerical results than those of Eq. (3) and Eq. (4). Nevertheless, the prediction using α = 1.06 is deteriorated when the strain is larger than 0.3, but it is improved when strain is close to the theoretical compaction strain defined as $-\ln\rho$. So, all the results do not provide satisfactory predictions and thus considering α as a constant may be improper.

A variable ellipticity. According to the definition of isotropic ellipse yield surface, the ellipticity can be determined by two points on the ellipse, i.e.

$$\alpha = 3\sigma_c\Big/\sqrt{9p^2 - \sigma_c^2}, \tag{9}$$

where σ_c is the uniaxial stress and p the hydrostatic pressure. In this study, three random samples were used to validate the dispersion of samples. The variations of α with true equivalent plastic strain obtained from the three samples are depicted in Fig. 5. The results show obviously that α is not a constant.

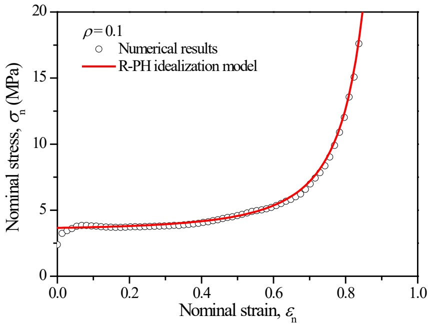

Fig. 3 Quasi-static uniaxial compression stress–strain relation and the fitting by R-PH model.

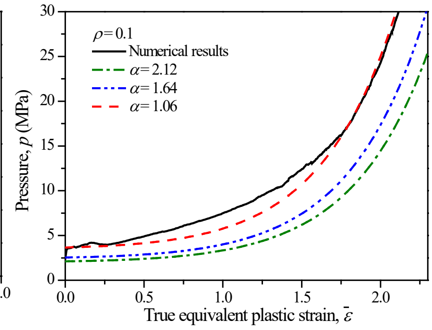

Fig. 4 Predictions of hydrostatic pressure using different value of α.

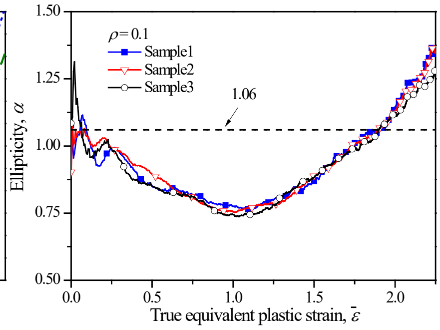

Fig. 5 Ellipticity α calculated by Eq. (9).

Conclusions

The self-similar isotropic hardening model developed by Deshpande and Fleck [4] has been verified by a cell-based finite element model based on the 3D Voronoi technique. Hydrostatic pressure is predicted by the uniaxial stress–strain relation and a constant ellipticity α, based on the Deshpande-Fleck foam model. The uniaxial stress–strain relation is fitted well by the R-PH model. However, all the results do not provide a perfect prediction, hence considering α as a constant may be incorrect. Thus, inversely, uniaxial compression and hydrostatic compression are used to determine the value of ellipticity α. The results show that the ellipticity α varies with the equivalent plastic strain.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (Projects Nos. 11772330 and 11372308) and the Fundamental Research Funds for the Central Universities (Grant No. WK2480000003).

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