Robust Surface Reconstruction via Dictionary Learning

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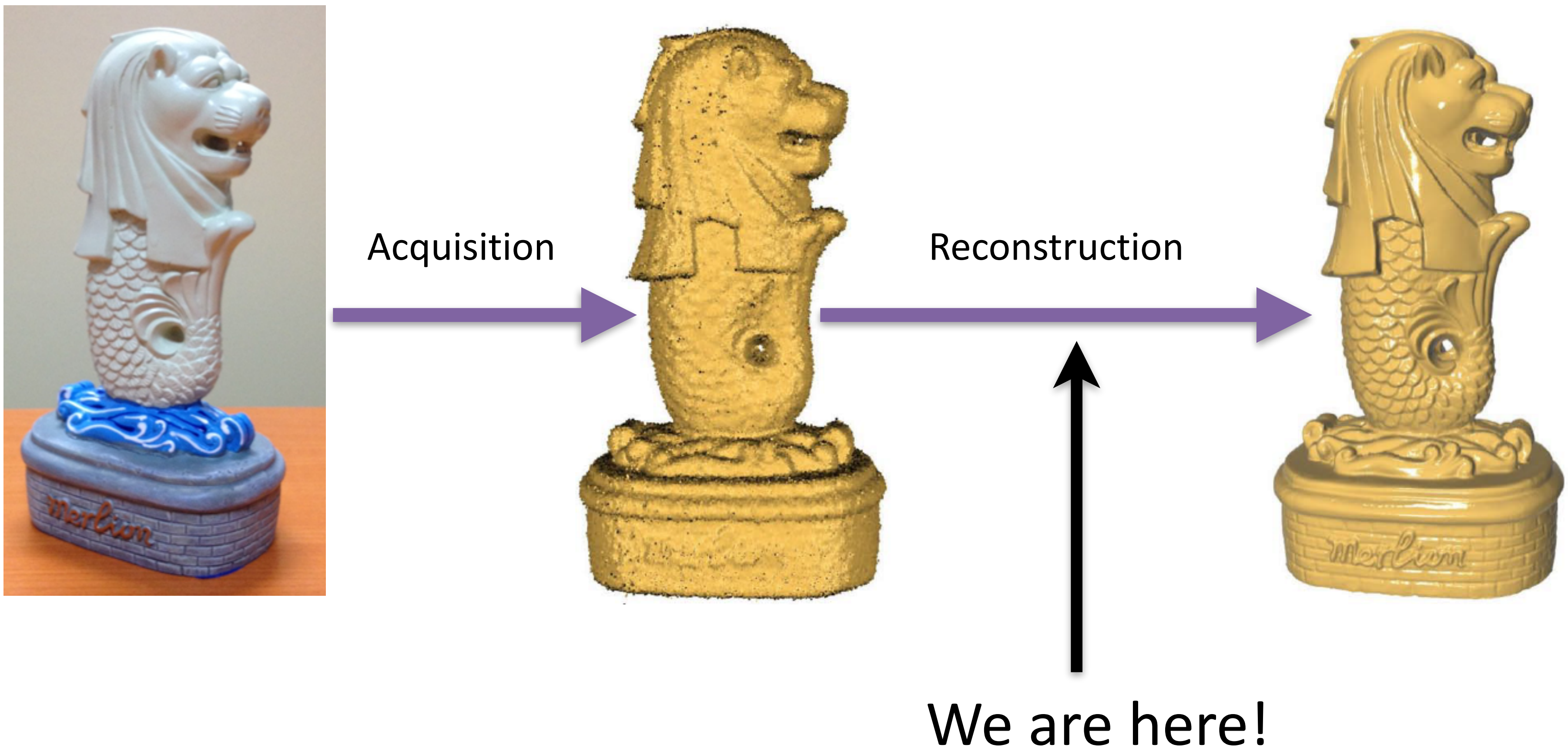
Motivation

3D scanners are everywhere:

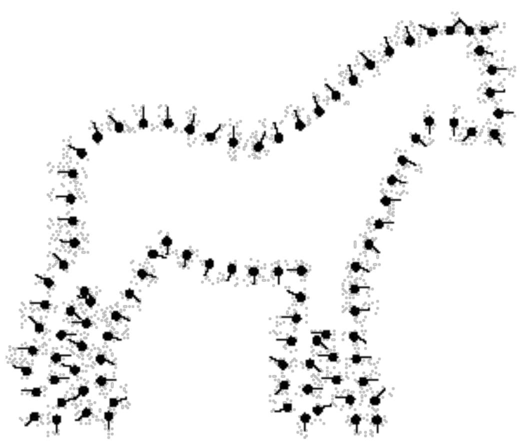
Modeling Pipeline



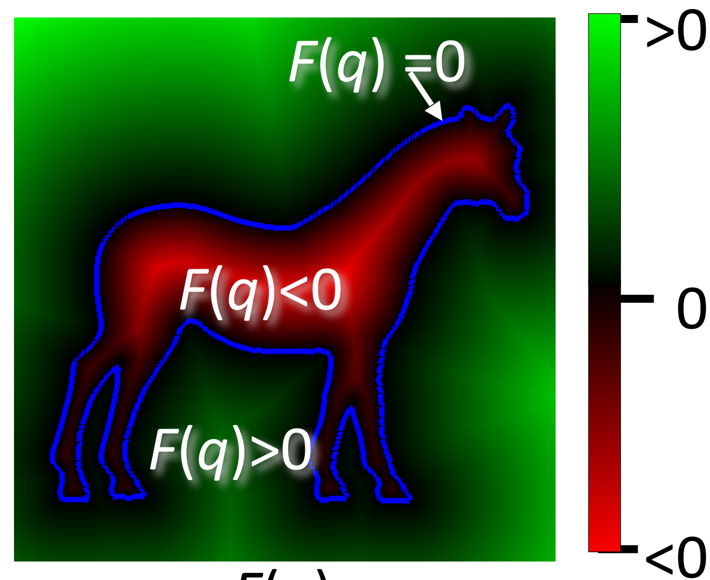
Implicit Function Fitting

Given point samples:

- Define a function with value zero at the points.
- Extract the zero isosurface.

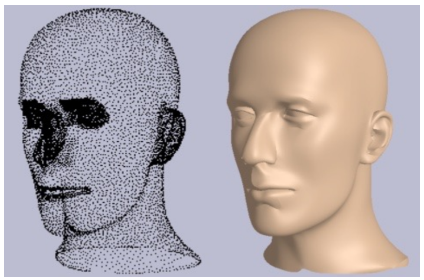


Sample points



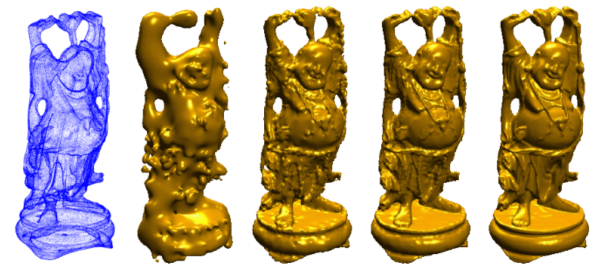
$F(q)$

Related work



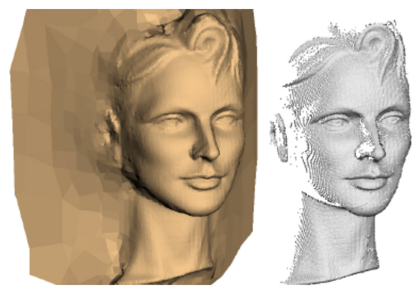
[Hoppe *et al*. 1992]

[Curless and Levoy 1996]

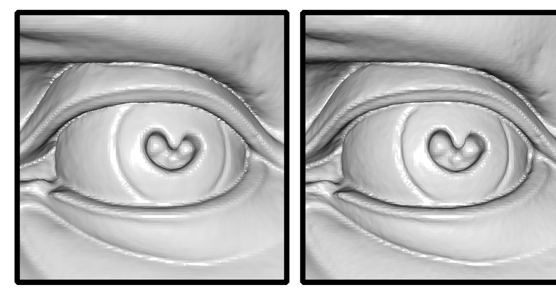


[Carr *et al*. 2001]

[Kazhdan *et al*. 2006]



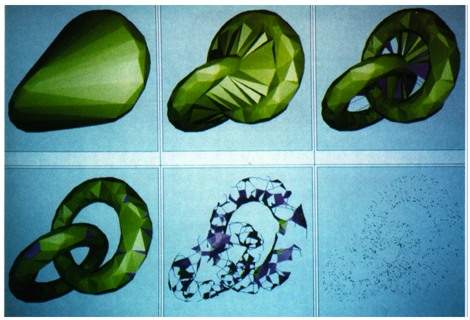
[Calakli and Taubin 2011]



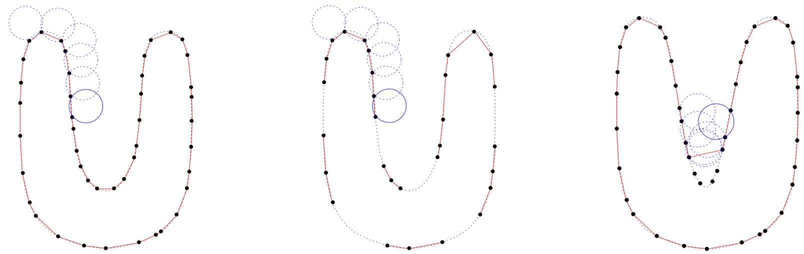
[Kazhdan *et al*. 2013]

……

Combinatorial Algorithms

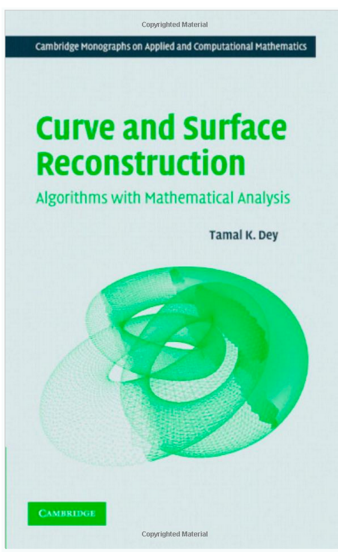


[Bolitho *et al*. 1994]



[Bernardini *et al*. 1999]

[Amenta *et al*. 2001]



[Dey 2007]

……

Our Approach: A Combinatorial One

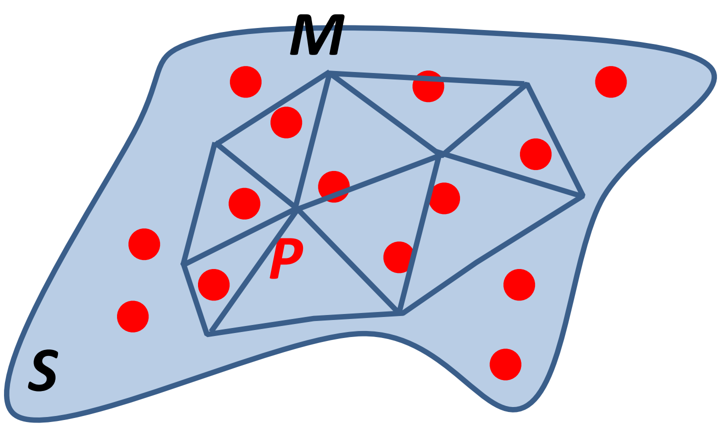
Problem Statement

- **Input**: A point set sampled from a piecewise smooth surface (with features) ***S***

$$\mathbb{P} = \{\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_n\}$$

- **Output**: A triangular mesh

$$M = \{\mathbb{V}, \mathbb{F}\}$$



to approximate ***S*** with minimal approx. error

Assumptions

- Points ***P*** might have artifacts due to

sampling density, noise, outliers, misalignment, and missing data

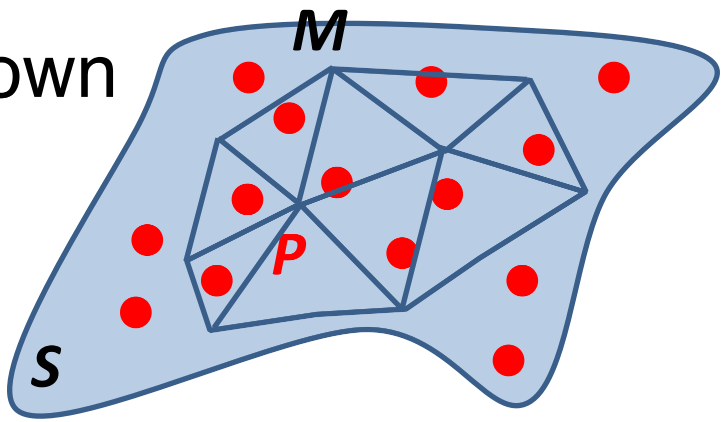
Input: *P*
Output: *M*

- The underlying surface S is unknown

Approx. error d(*S*, *M*)

↓

Approx. error d(*P*, *M*)



Approximation Error

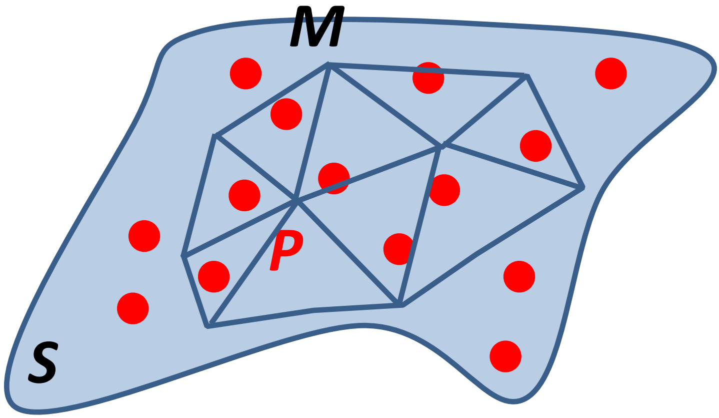
- How to measure the distance between

    the point set *P*

  and

    the mesh *M*?

    d(*P*, *M*)

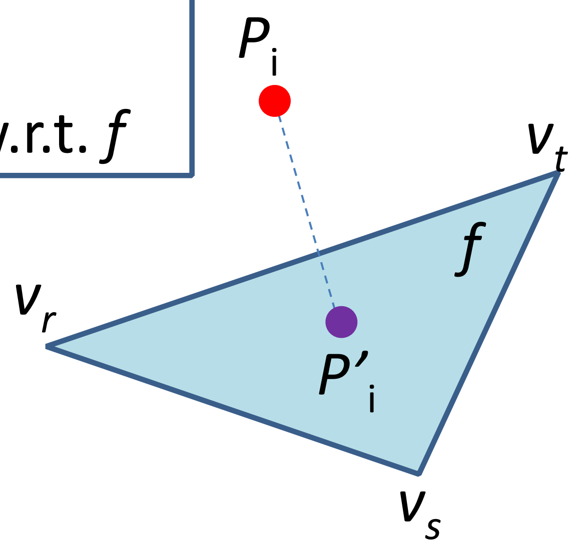


Approximation Error

- Distance from one point P_i to a triangle f

$$\mathbf{p}_i' = \alpha^* \mathbf{v}_r + \beta^* \mathbf{v}_s + \gamma^* \mathbf{v}_t$$

$(\alpha^*, \beta^*, \gamma^*)$: barycentric coordinates of P'_i w.r.t. f



$$d(\mathbf{p}_i, f) = \|\mathbf{p}_i - \mathbf{p}_i'\|$$

$$= \min_{\substack{\alpha+\beta+\gamma=1 \\ \alpha,\beta,\gamma \geq 0}} \|\mathbf{p}_i - (\alpha \mathbf{v}_r + \beta \mathbf{v}_s + \gamma \mathbf{v}_t)\|$$

Matrix Representation

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m] \in \mathbb{R}^{3\times m}$$

: vertex matrix of M

$$\mathbf{p}'_i = \alpha^*\mathbf{v}_r + \beta^*\mathbf{v}_s + \gamma^*\mathbf{v}_t$$

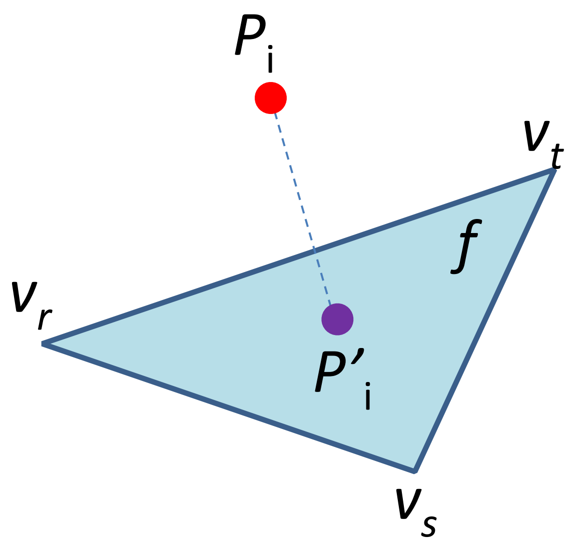
$$= (\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m)\begin{pmatrix} b_{i1} \\ b_{i2} \\ \vdots \\ b_{im} \end{pmatrix}$$

$$= (\cdots, \mathbf{v}_r, \cdots, \mathbf{v}_s, \ldots, \mathbf{v}_t, \cdots)\begin{pmatrix} \vdots \\ \alpha^* \\ \vdots \\ \beta^* \\ \vdots \\ \gamma^* \\ \vdots \end{pmatrix}$$

$$= \mathbf{V}\mathbf{b}_i$$

at most three nonzero elements

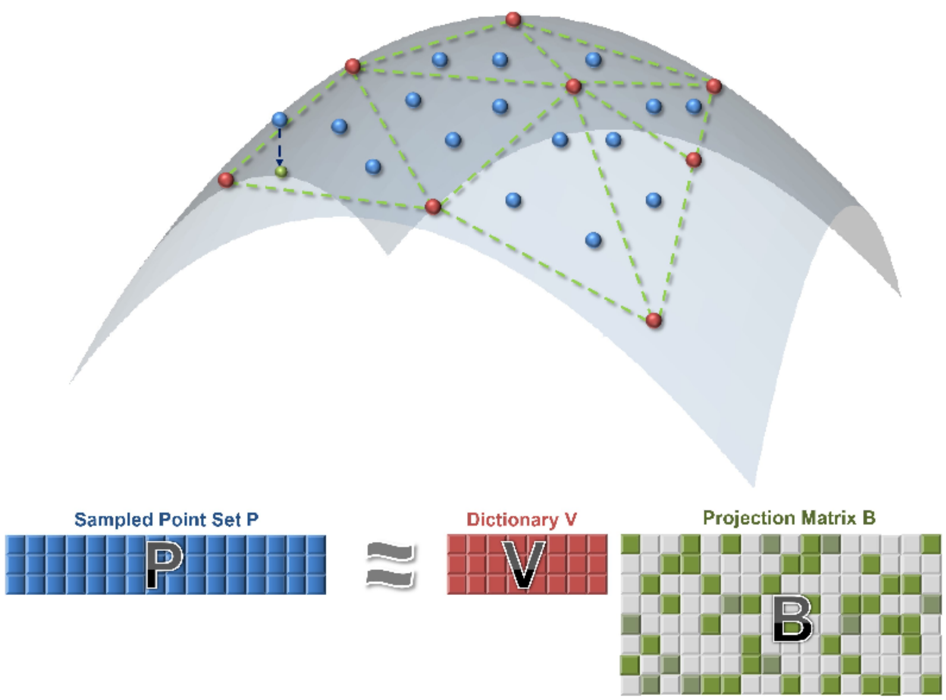
$$\|\mathbf{b}_i\|_0 \leq 3$$



Approximation Error

Denote

$$\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_n]$$
$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m]$$
$$\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_n]$$
$$\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_n]$$



then

$$\mathbf{P} = \mathbf{V}\mathbf{B} + \mathbf{Z}$$

From the **dictionary learning** perspective:

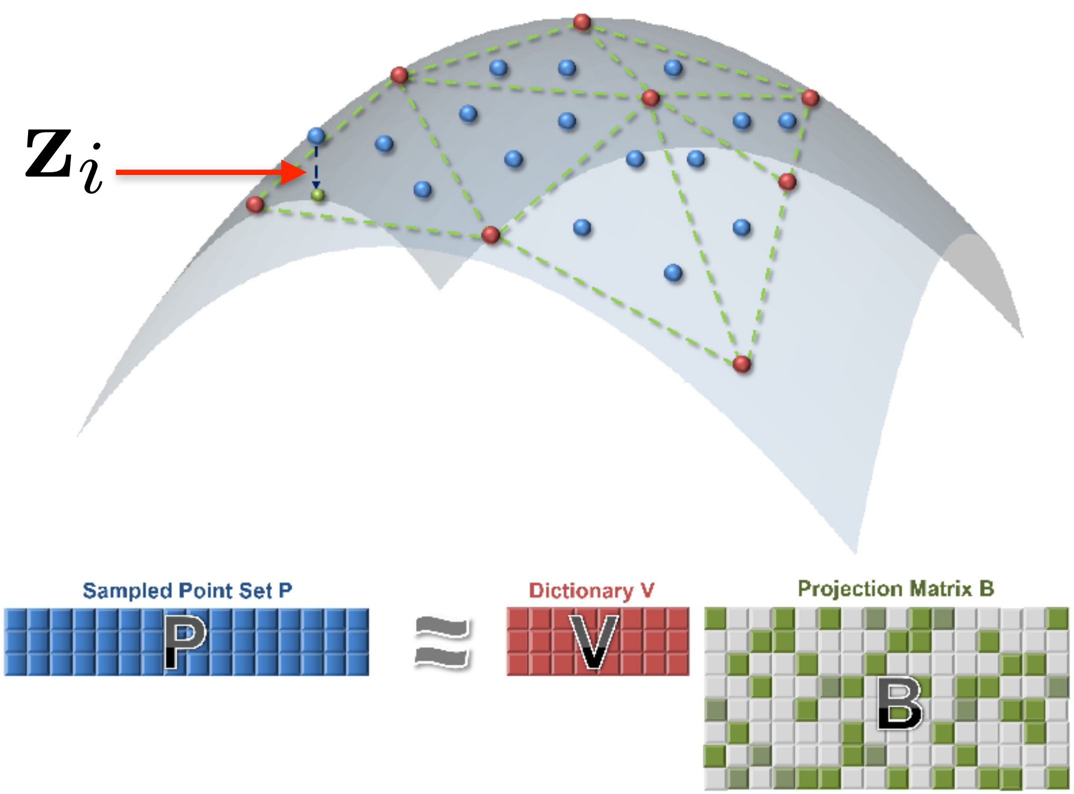
P: the given signal (sample points)

V: the dictionary elements (mesh vertices)

B: the sparse coding matrix (mesh connectivity)

Z: the residuals

Approximation Error



$$E_{\text{appr}} = \frac{1}{n}\sum_{i=1}^{n} \|\mathbf{z}_i\|_2^2 = \frac{1}{n}\sum_{i=1}^{n} \|\mathbf{p}_i - \mathbf{V}\mathbf{b}_i\|_2^q$$

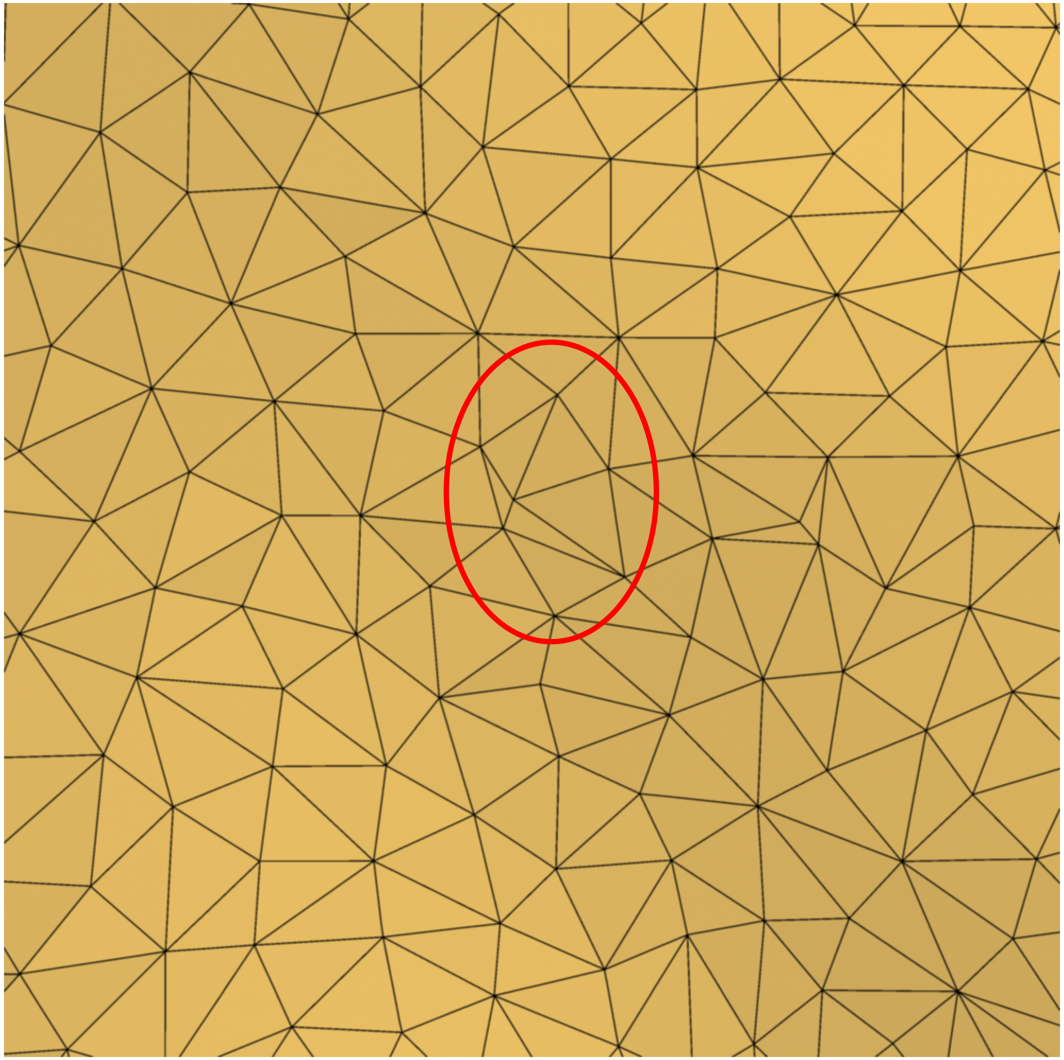
s.t. $\|\mathbf{b}_i\|_0 \leq 3, \;\; \|\mathbf{b}_i\|_1 = 1, \;\; \mathbf{b}_i \geq 0, \; \forall i$

$L_{2,q}$ Norm

$$E_{\mathrm{appr}} = \frac{1}{n}\|\mathbf{P} - \mathbf{VB}\|_{2,q} = \frac{1}{n}\sum_{i=1}^{n}\|\mathbf{p}_i - \mathbf{Vb}_i\|_2^q$$

- 0 < q <= 1
- Group Sparsity
- Resisting noise and outlier

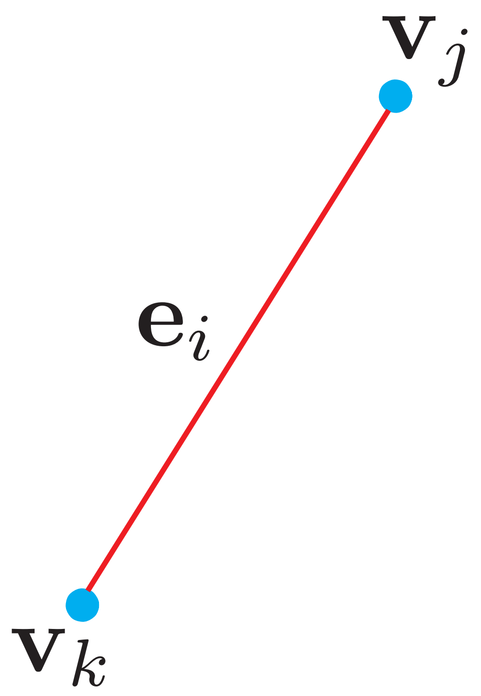
Result of $\min E_{\text{appr}}$



Regularization Term

- High quality mesh

$$
\begin{aligned}
E_{\text{edge}} &= \frac{1}{l}\sum_{i=1}^{l}\|\mathbf{e}_i\|_2^2 \\
&= \frac{1}{l}\sum_{i=1}^{l}\|\mathbf{v}_j - \mathbf{v}_k\|_2^2 \\
&= \frac{1}{l}\|\nabla \mathbf{V}\|_2^2
\end{aligned}
$$



Formulation

- With some regularization:

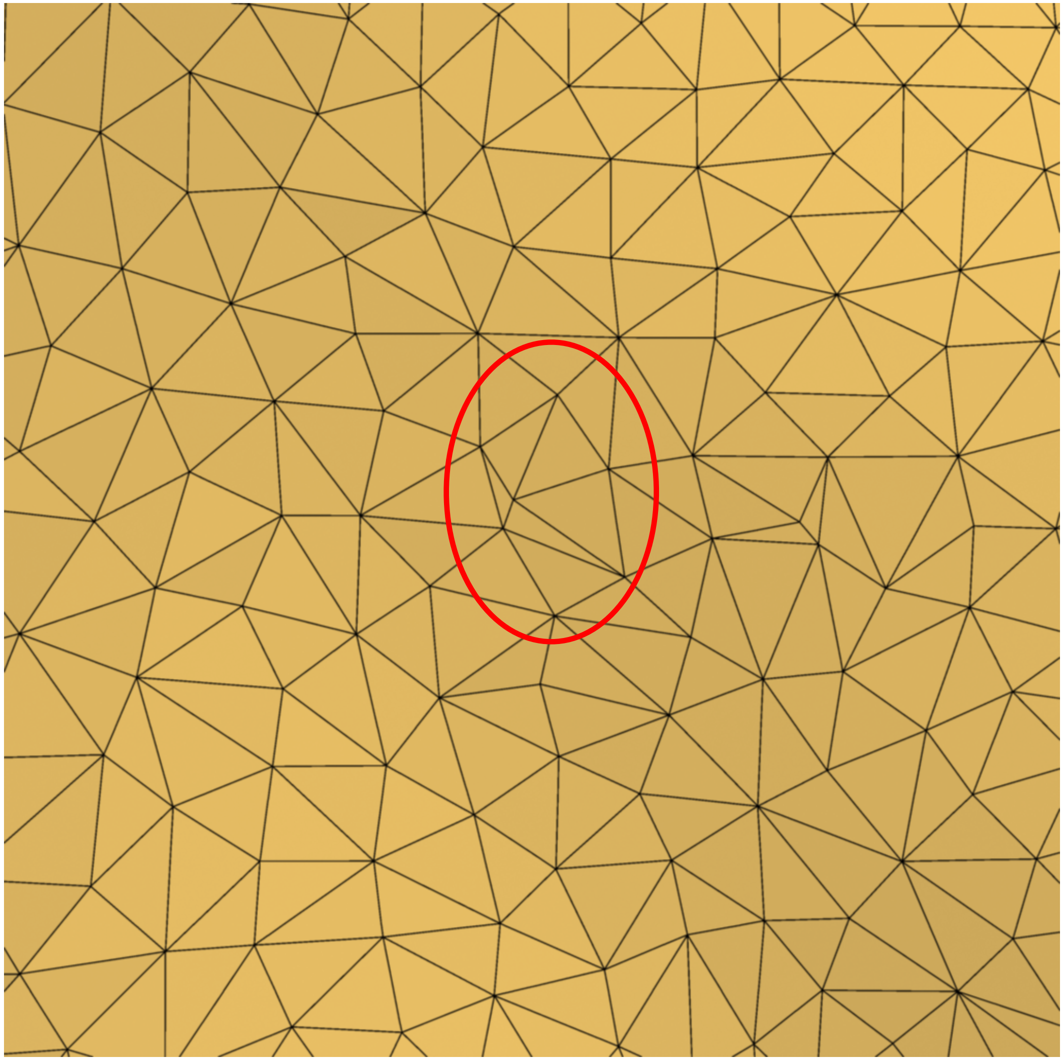
$$\min_{\mathbf{V},\mathbf{B}} \quad E = E_{\text{appr}} + E_{\text{reg}}$$

$$\text{s.t.} \quad \|\mathbf{b}_i\|_0 \leq 3, \quad \|\mathbf{b}_i\|_1 = 1, \quad \mathbf{b}_i \geq 0, \qquad \forall i$$

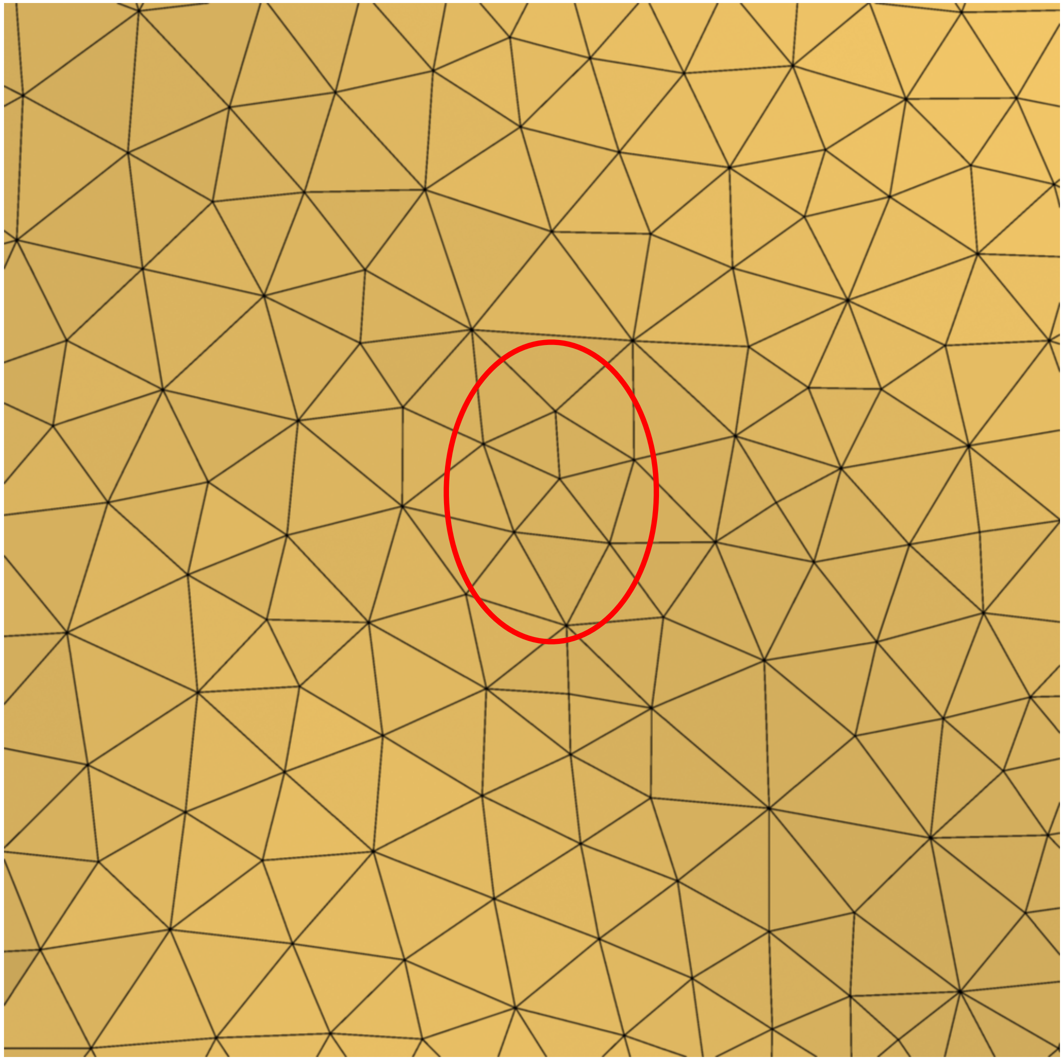
$$\mathbf{B} \in \mathbb{MT}$$

Manifold mesh space

Without Edge Regularization



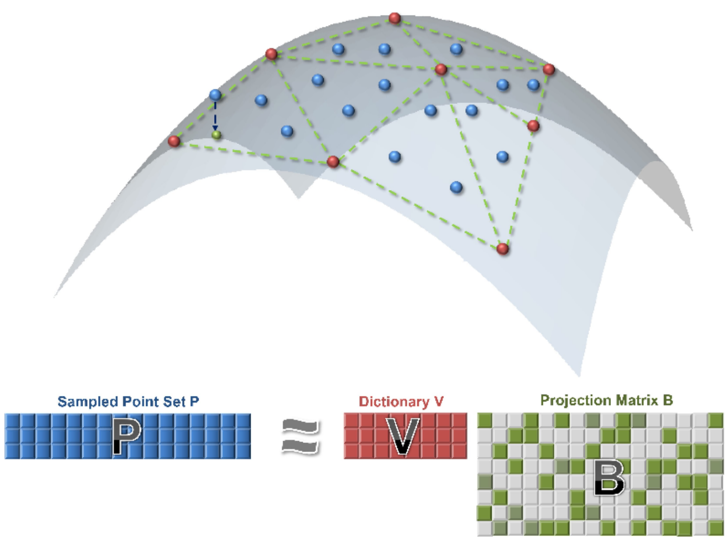
With Edge Regularization



Recap: Proposed Model

$$\min_{\mathbf{V},\mathbf{B}} \quad E = E_{\text{appr}} + E_{\text{reg}}$$
$$\text{s.t.} \quad \|\mathbf{b}_i\|_0 \leq 3, \quad \|\mathbf{b}_i\|_1 = 1, \quad \mathbf{b}_i \geq 0, \qquad \forall i$$
$$\mathbf{B} \in \mathbb{MT}$$

V: the vertices (geometry)
B: encodes the connectivity of V (topology)



Optimization

$$\min_{\mathbf{V},\mathbf{B}} \quad E = E_{\text{appr}} + E_{\text{reg}}$$
$$\text{s.t.} \quad \|\mathbf{b}_i\|_0 \leq 3, \quad \|\mathbf{b}_i\|_1 = 1, \quad \mathbf{b}_i \geq 0, \qquad \forall i$$
$$\mathbf{B} \in \mathbb{MT}$$

V: the vertices (geometry)
B: encodes the connectivity of V (topology)

- Alternative optimization:
  - Sparse coding: connectivity optimization
  - Dictionary update: Vertices position optimization

Pipeline

Input:

Point cloud: $\mathbf{P} = [\mathbf{p}_1, \cdots, \mathbf{p}_n] \in \mathbb{R}^{3\times n}$;

Initialize dictionary $\mathbf{V}$ and sparse coding matrix $\mathbf{B}$ from $\mathbf{P}$;

repeat

Update matrix $\mathbf{B}$ (Sparse Coding);

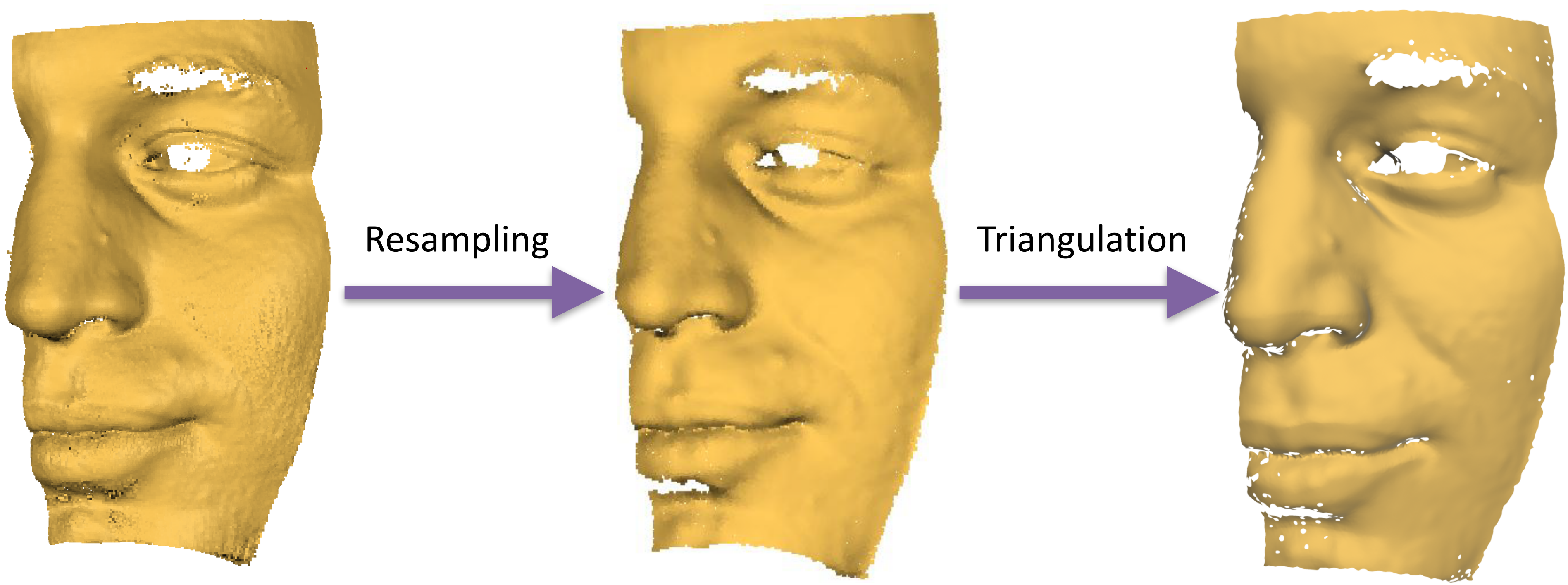
Update $\mathbf{V}$ (Dictionary Update);

until convergence

return Mesh $M(\mathbb{V}, \mathbb{F})$;

Initialization

- Resampling the point cloud
- Locally find the best connectivities respect to the proposed objective energy



Sparse Coding

- Connectivity Optimization: $\text{update matrix } \mathbf{B}$
- Only edge swap, add triangle, remove triangle are supported to preserve the manifold constraint
- A recursive strategy

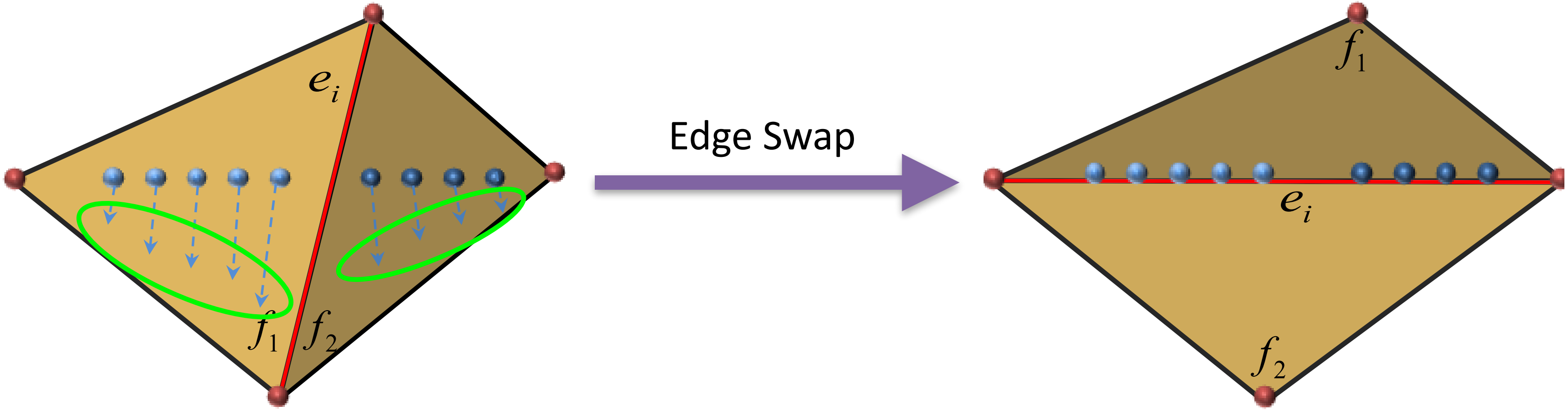
Linear in practice!

Sparse Coding

- How to maintain manifoldness?

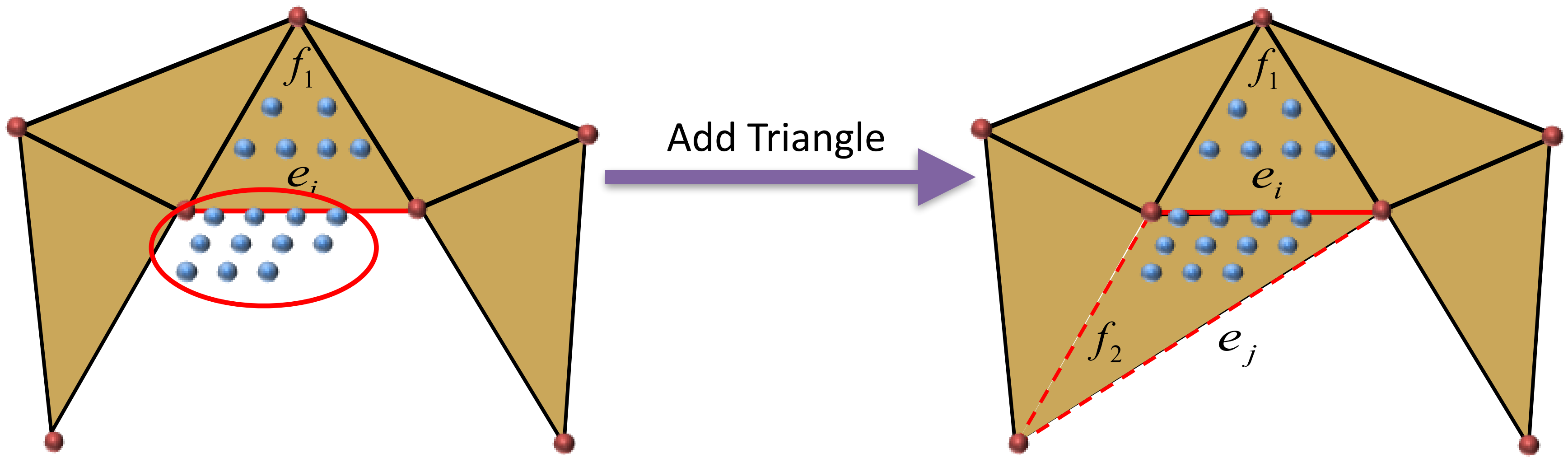
- How to fast update the connectivities?

Sparse Coding: Edge Swap

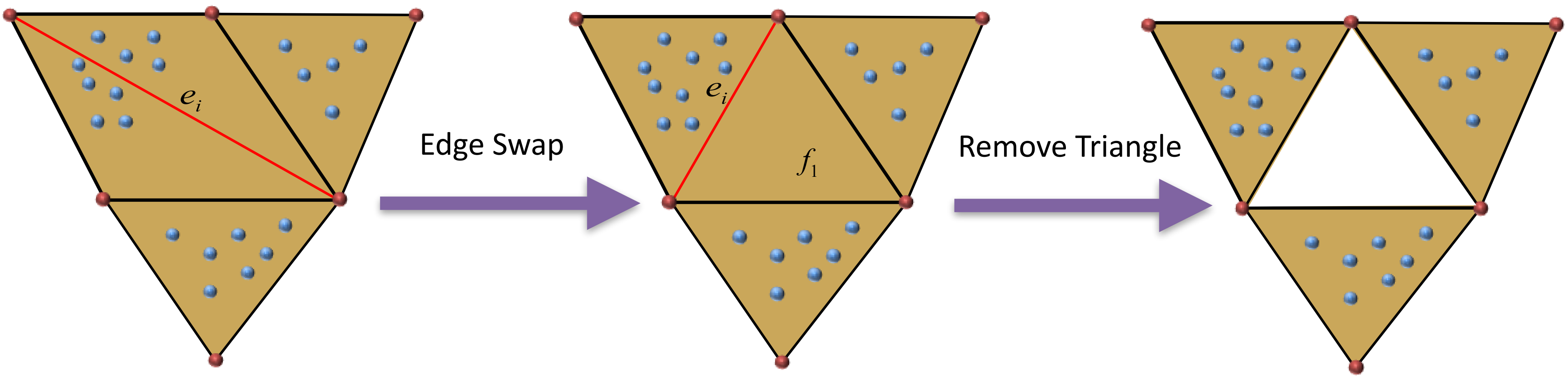


$$d(\mathbf{P}, \mathbf{M}^k) > d(\mathbf{P}, \mathbf{M}^{k+1})$$

Sparse Coding: Add Triangle



Sparse Coding: Remove Triangle

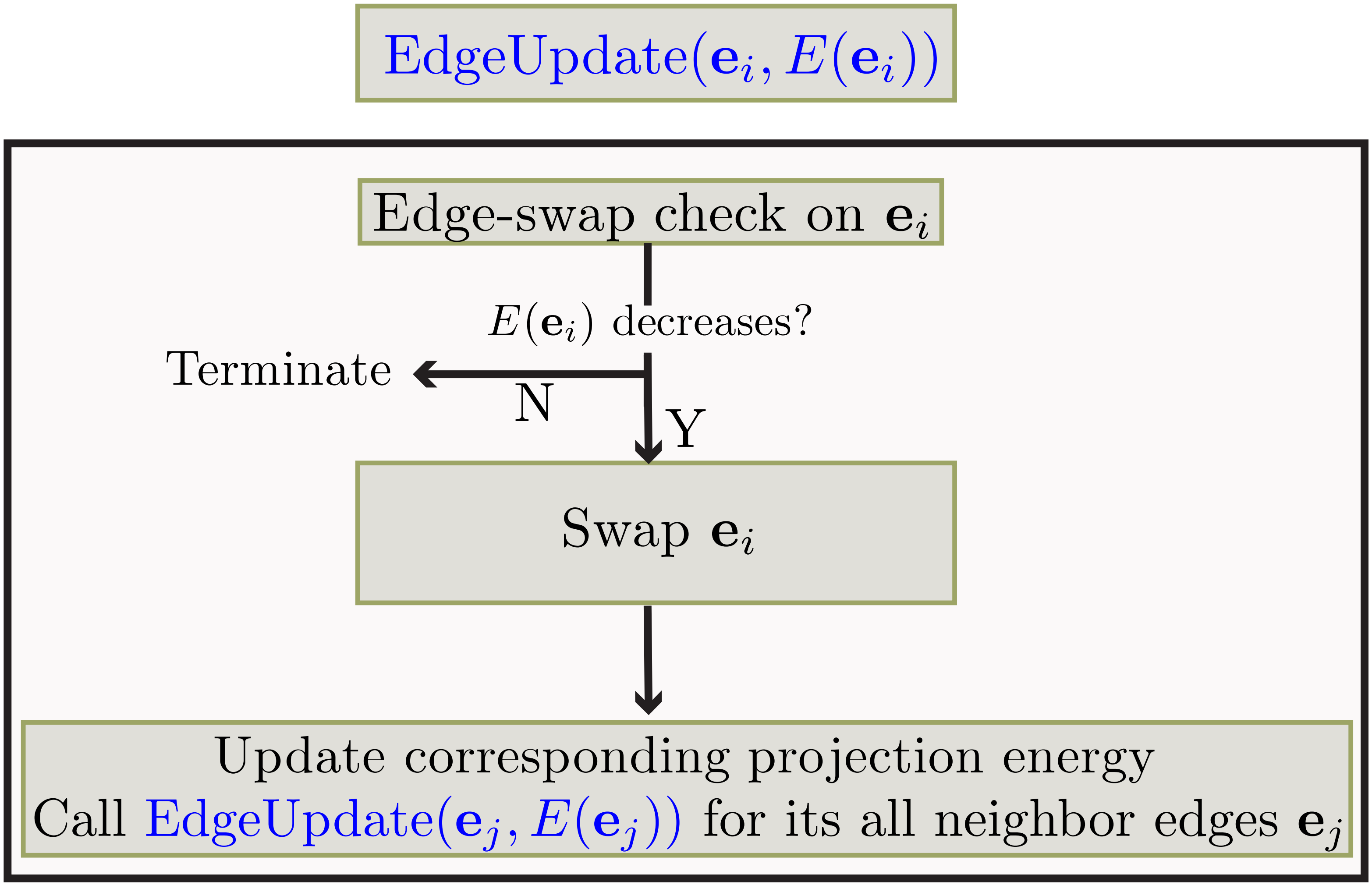


Sparse Coding

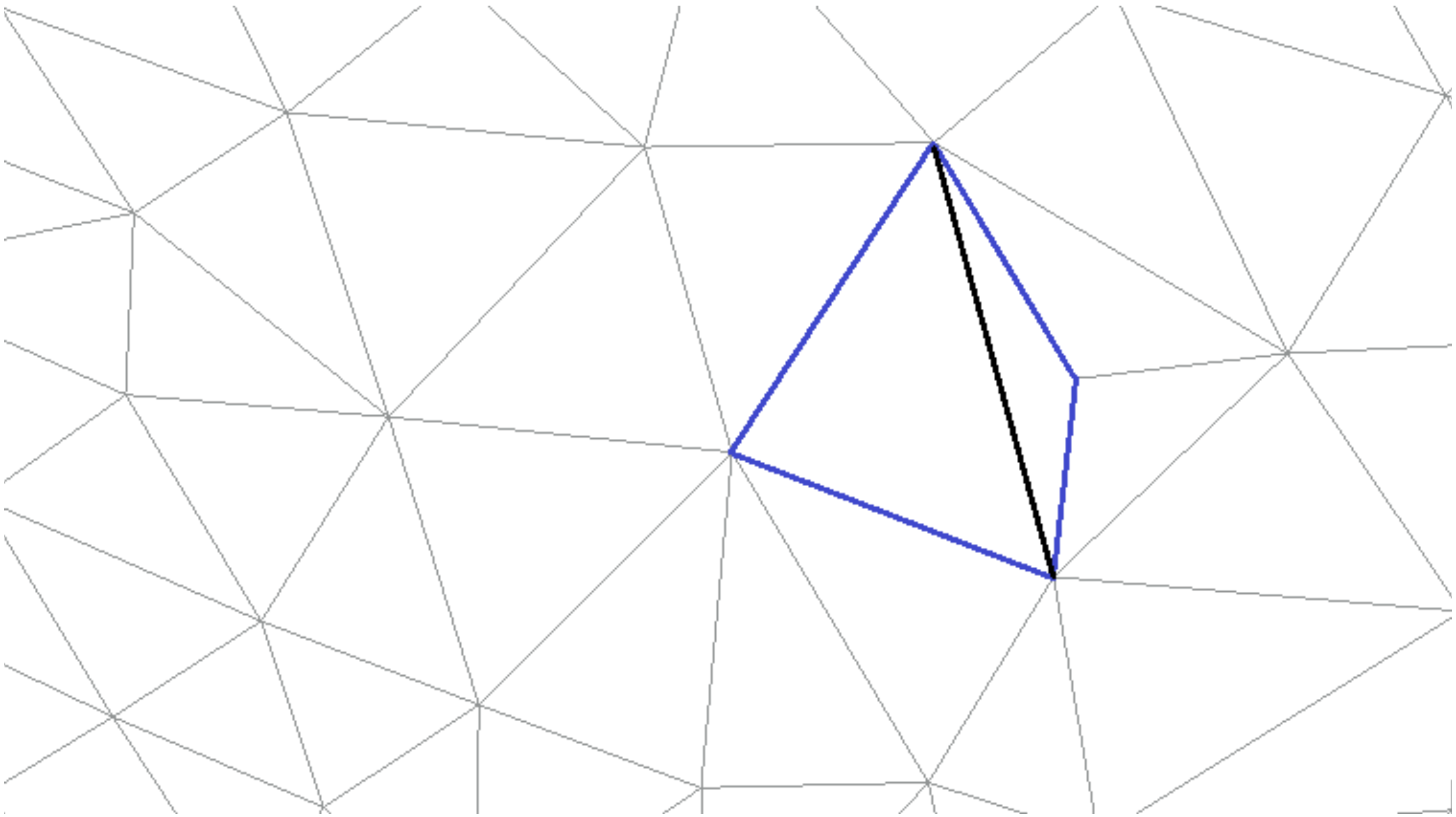
- How to maintain manifoldness?

- How to fast update the connectivities?

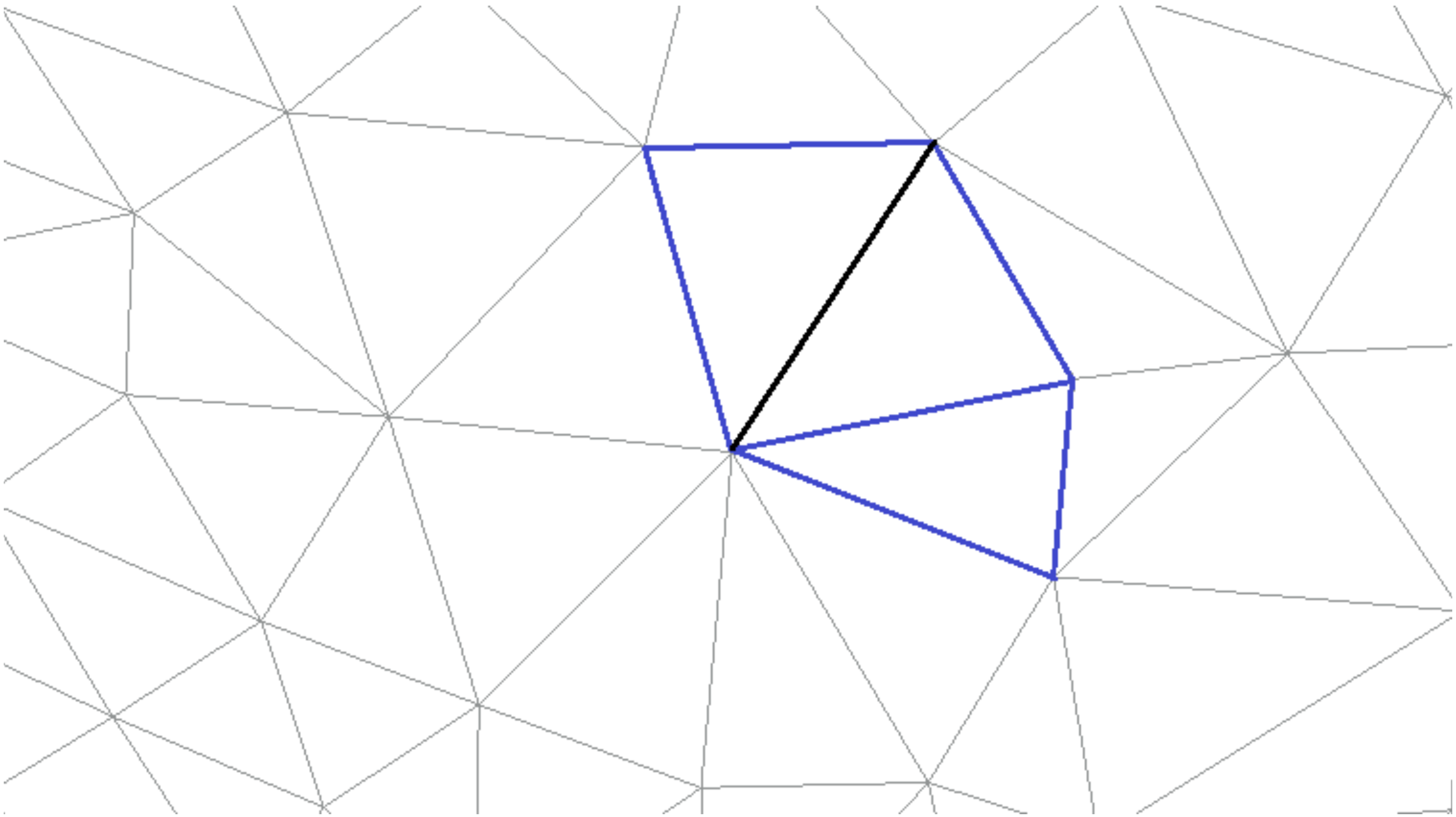
A Recursive Strategy



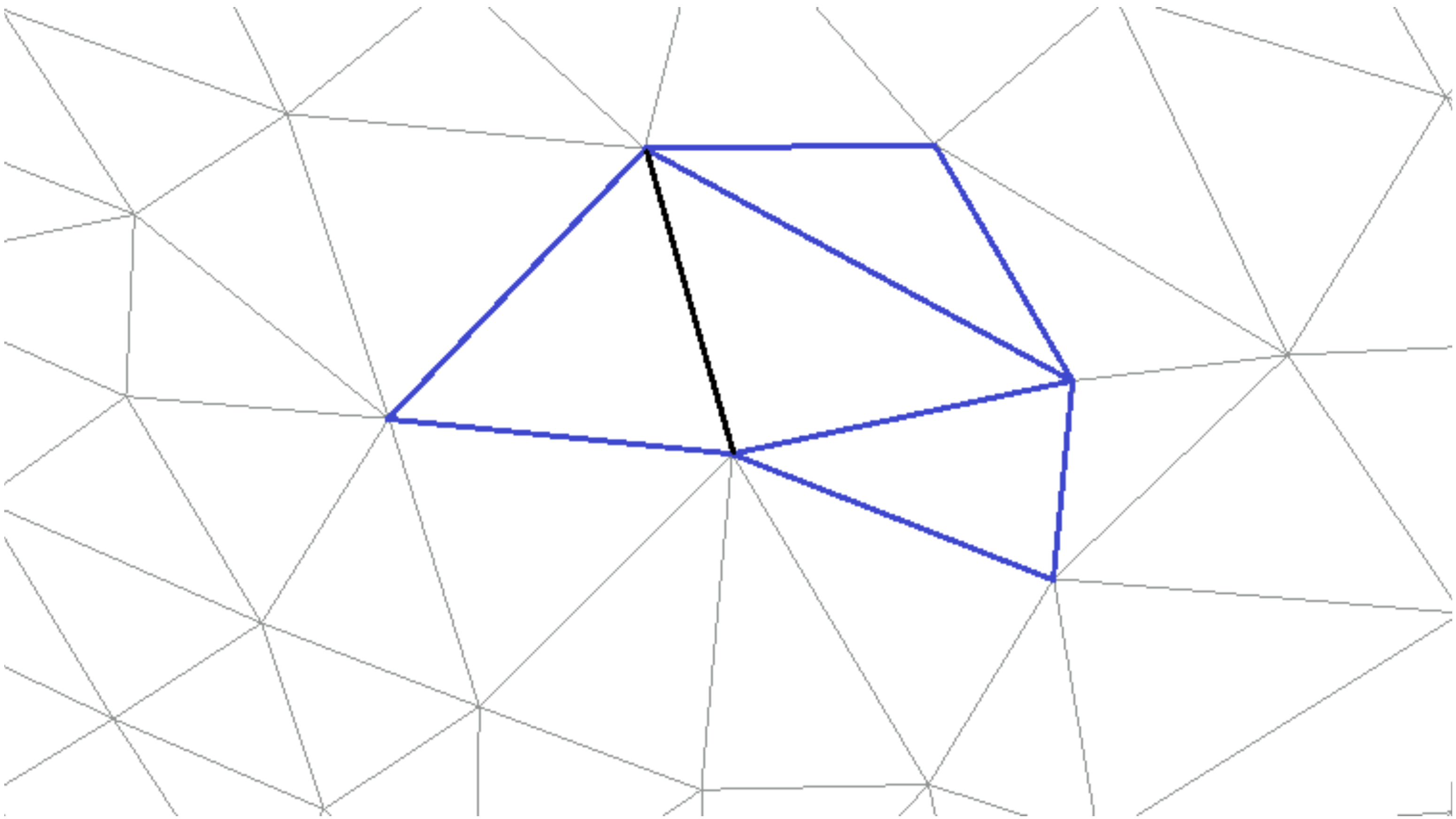
Edge Update Propagation



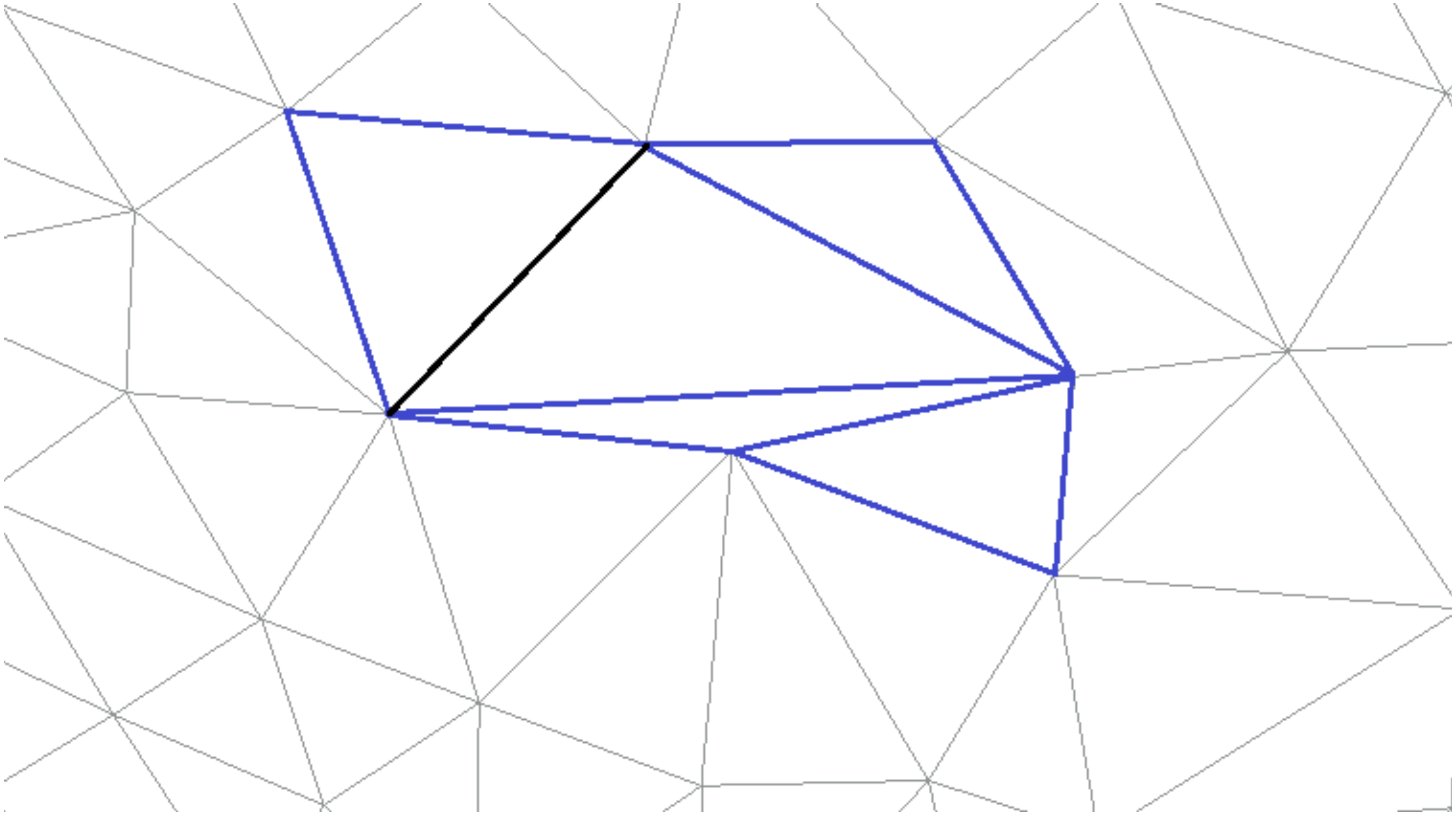
Level 2



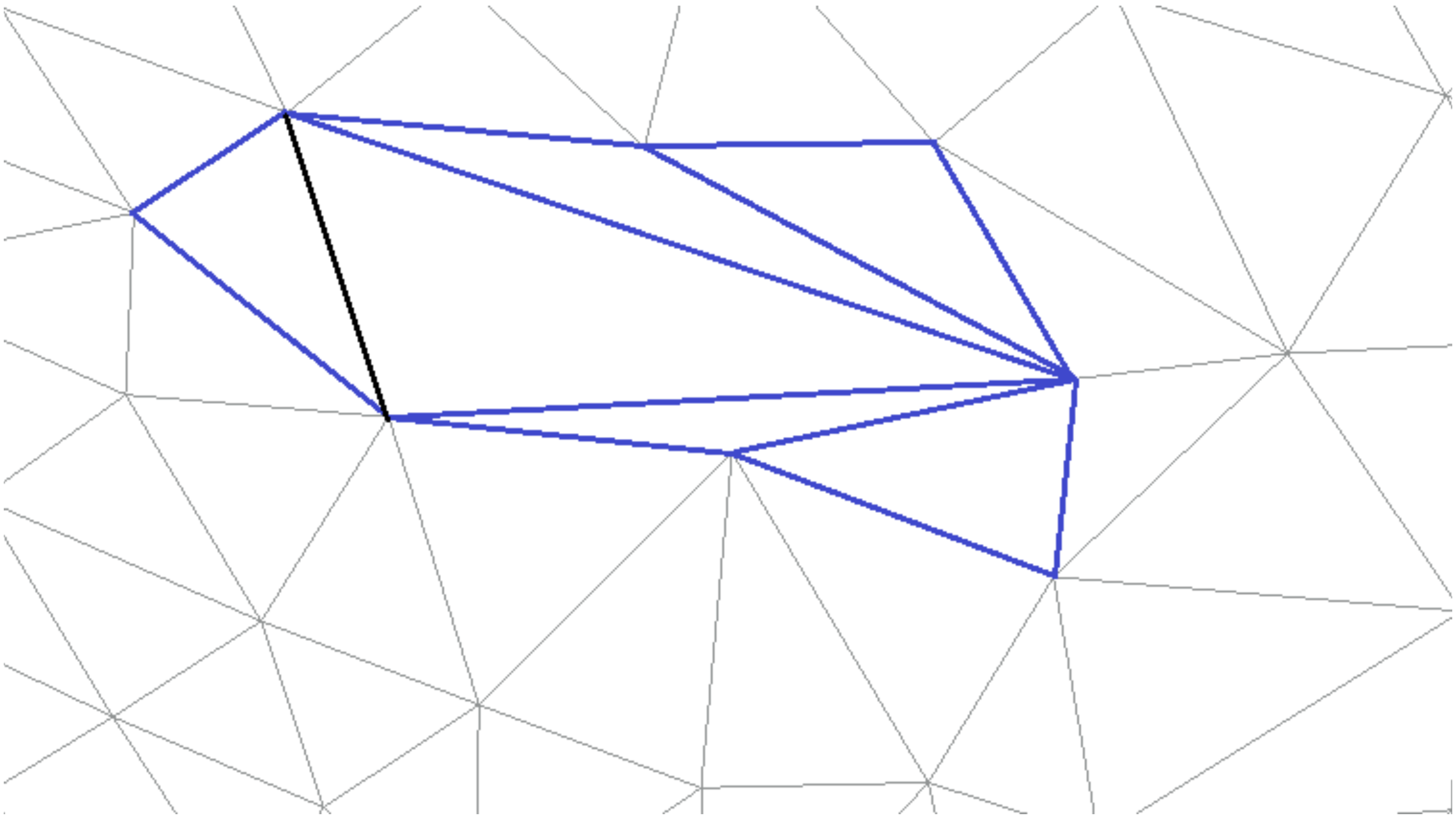
Level 3



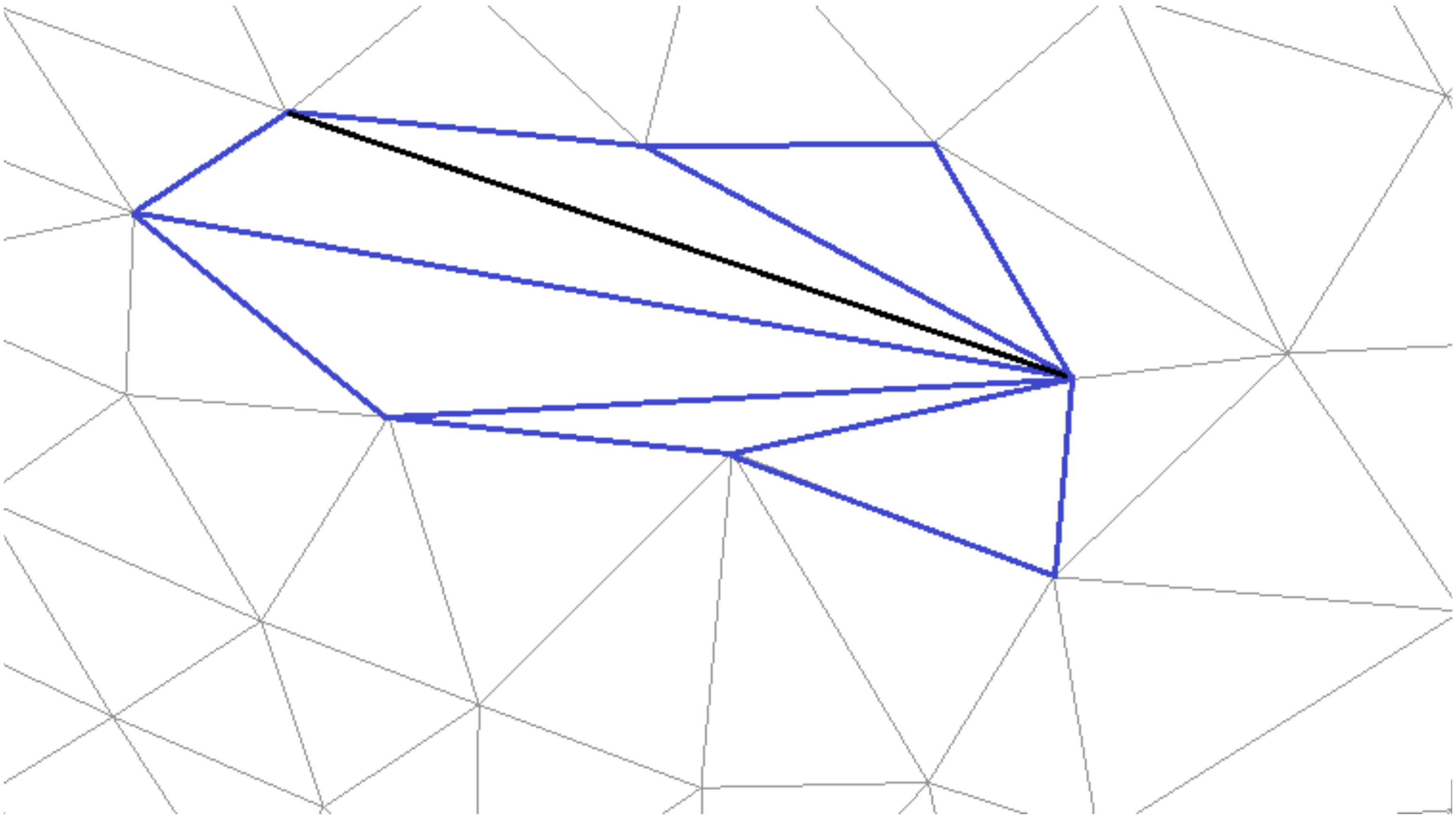
Level 4



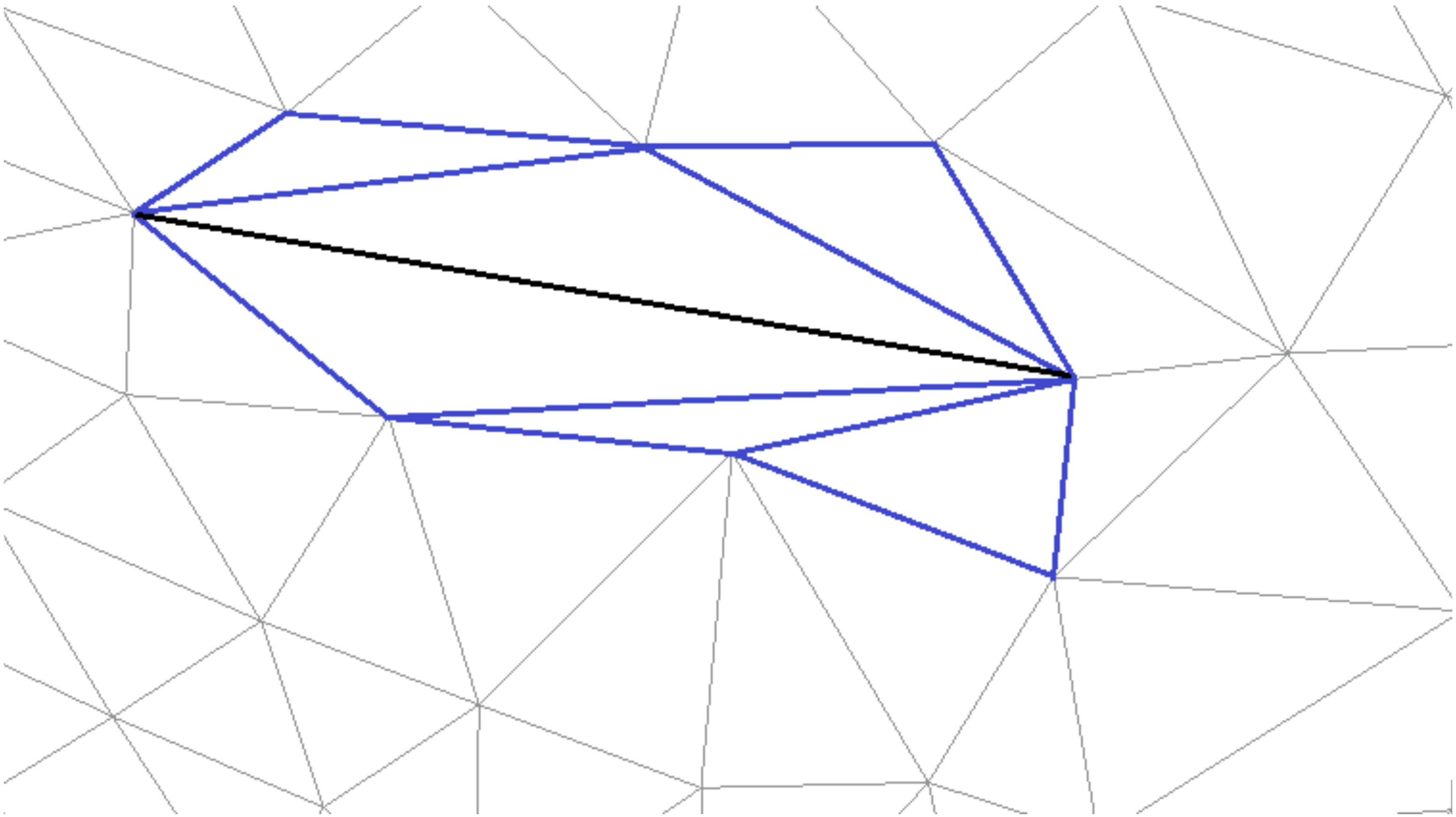
Level 5



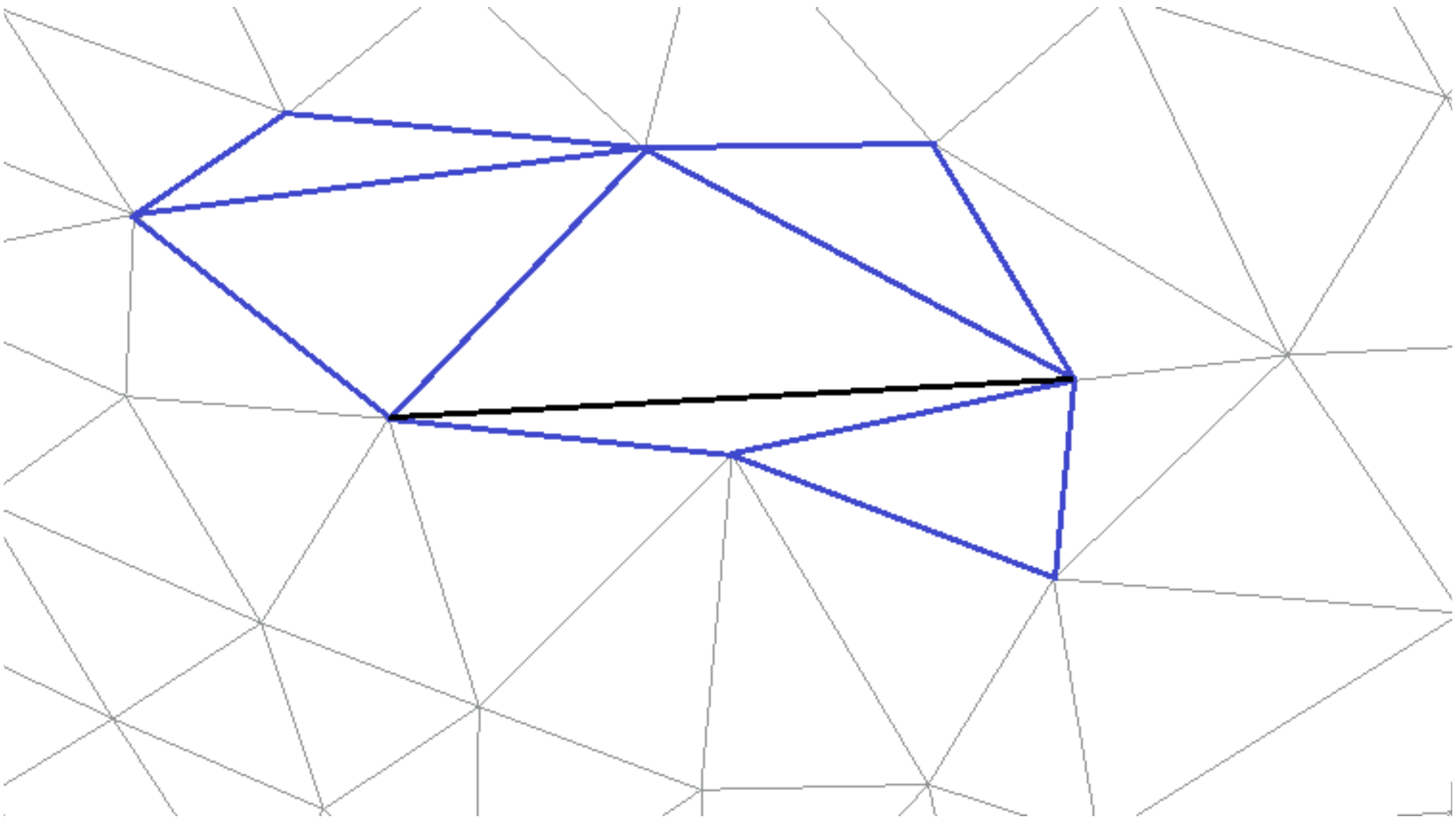
Level 6



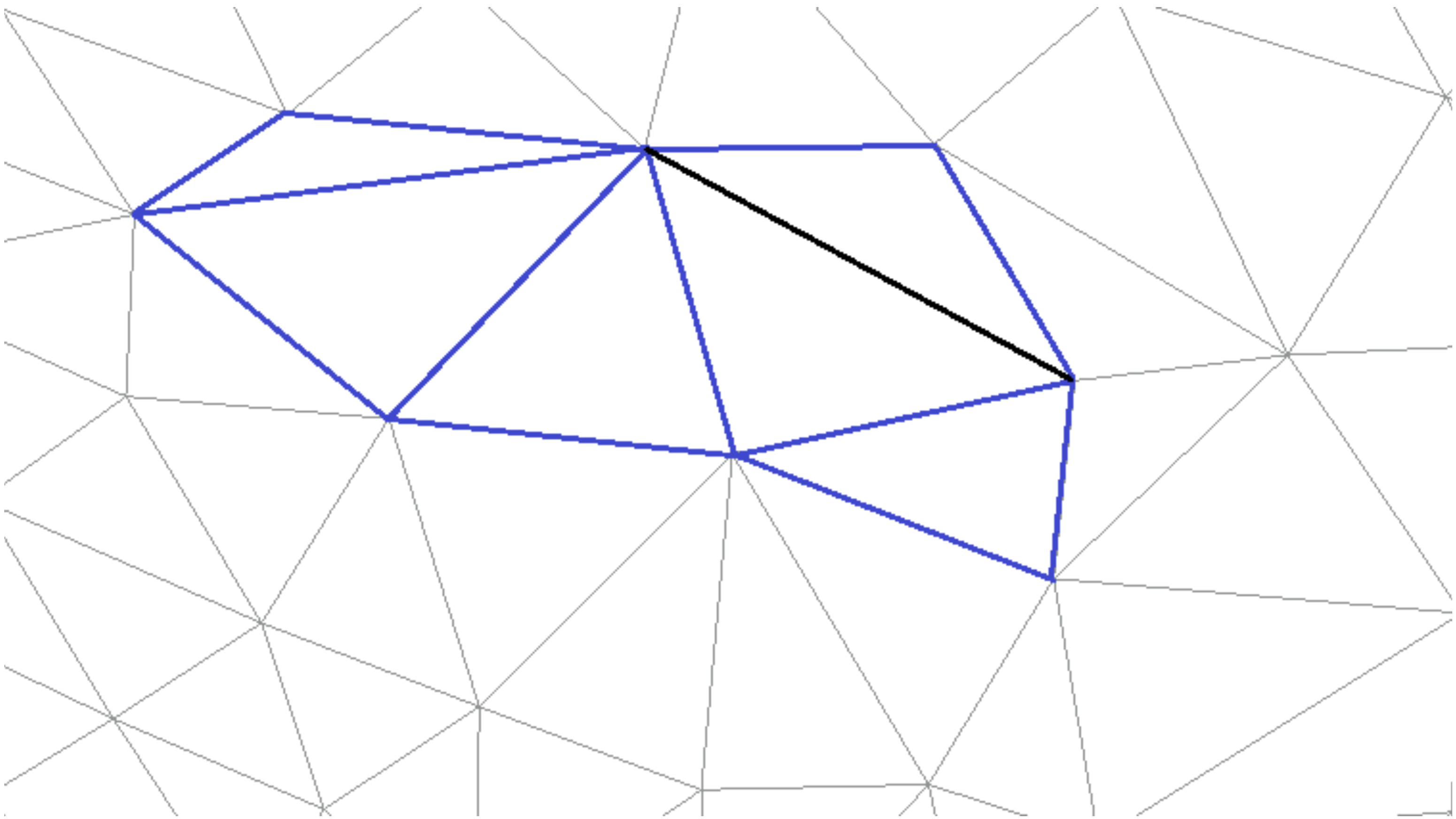
Level 7



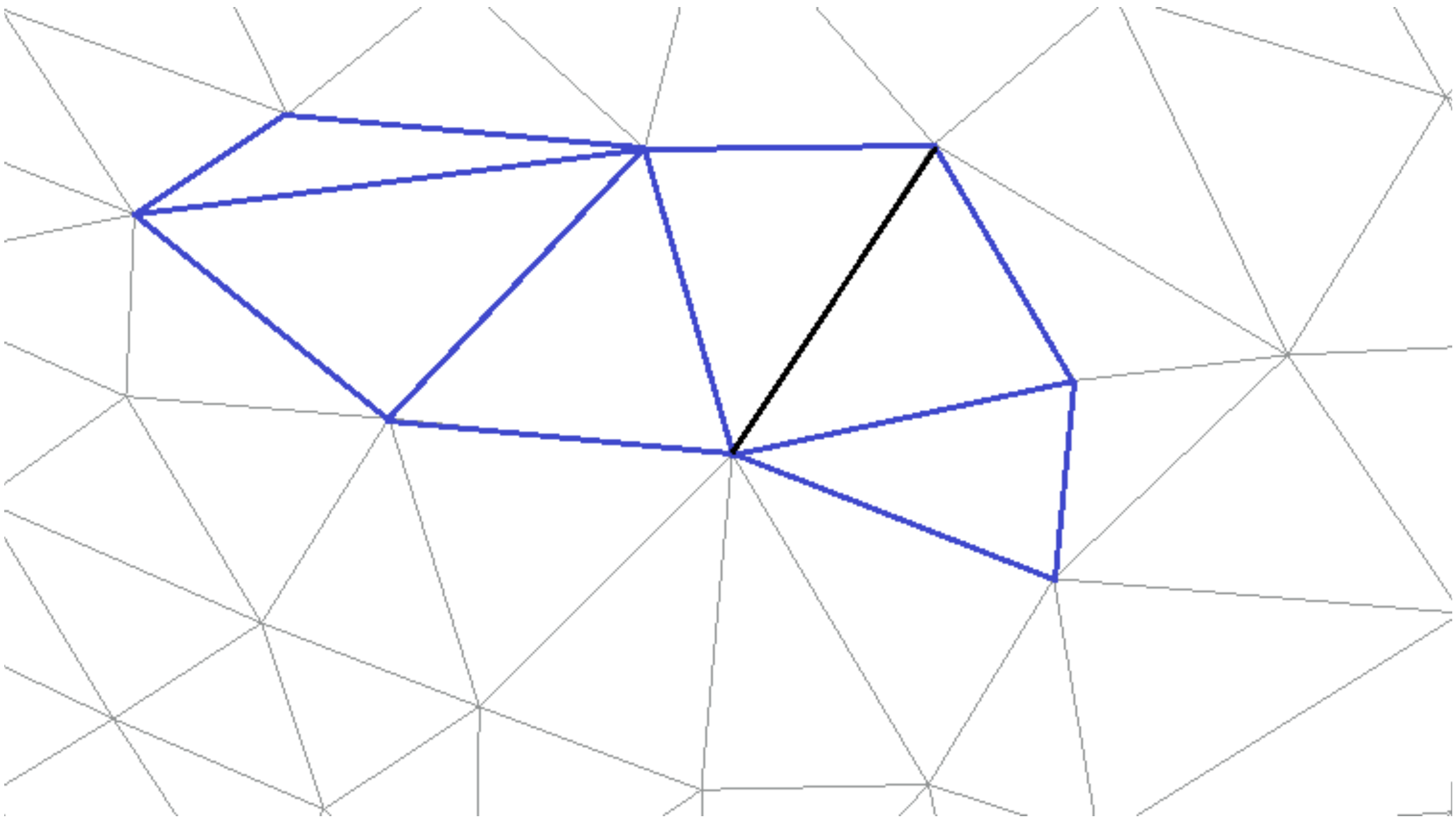
Level 8



Level 9



Level 10



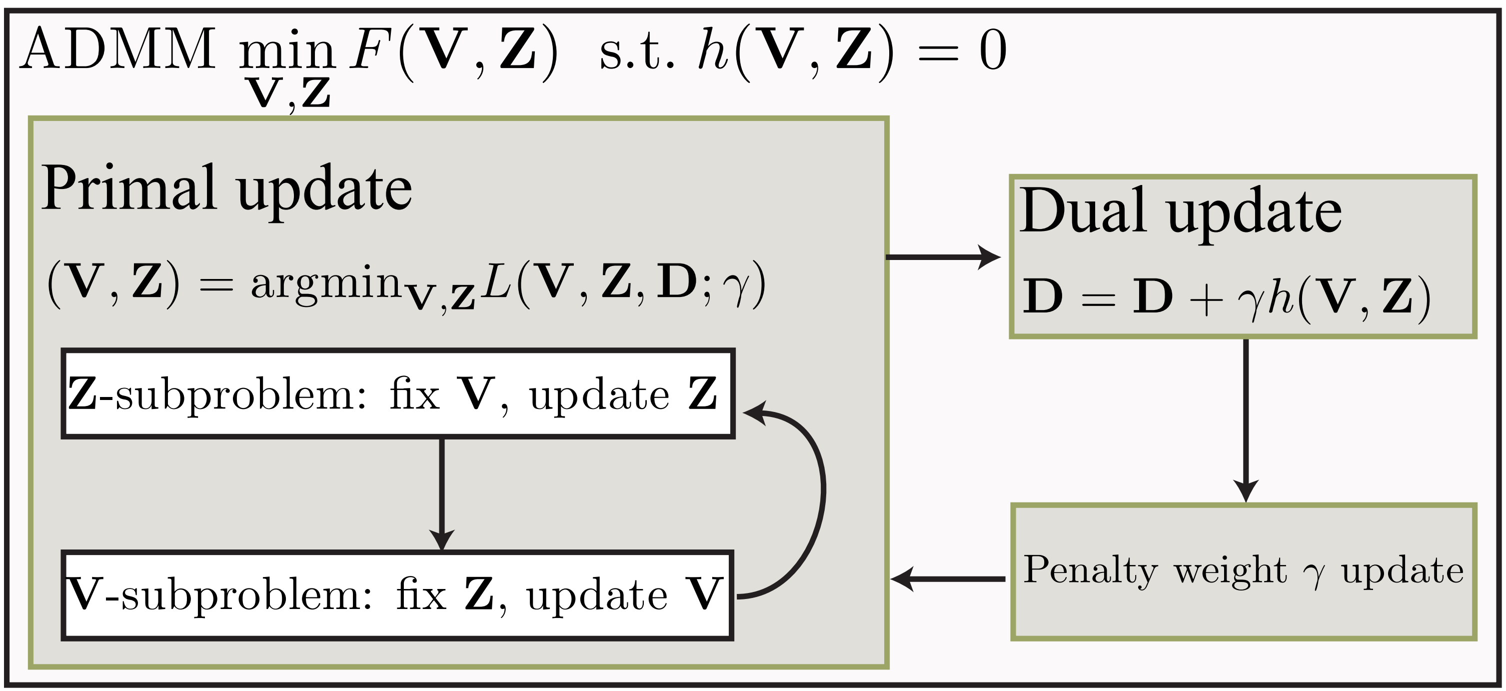
Dictionary Update

- Fix the connectivities, update the vertex positions

$$\min_{\mathbf{V}} \frac{1}{n}\|\mathbf{P}-\mathbf{V}\mathbf{B}\|_{2,q}+\frac{1}{l}\|\nabla\mathbf{V}\|_2^2$$

Dictionary Update

- A sparse optimization problem
  - ADMM (Alternating Direction Method of Multipliers)
  - Parallelable

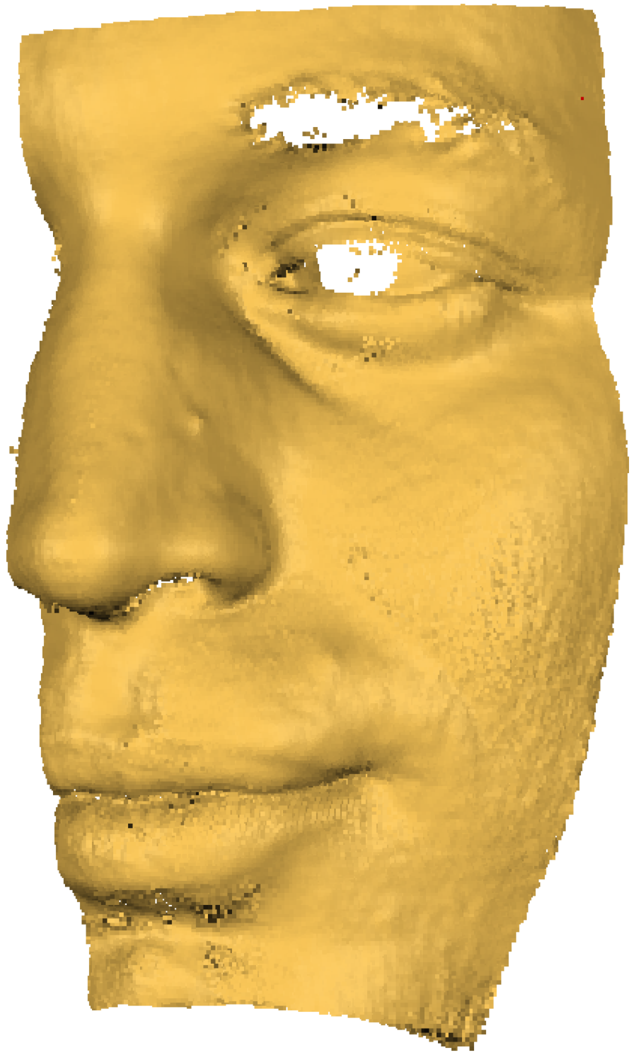


Recap

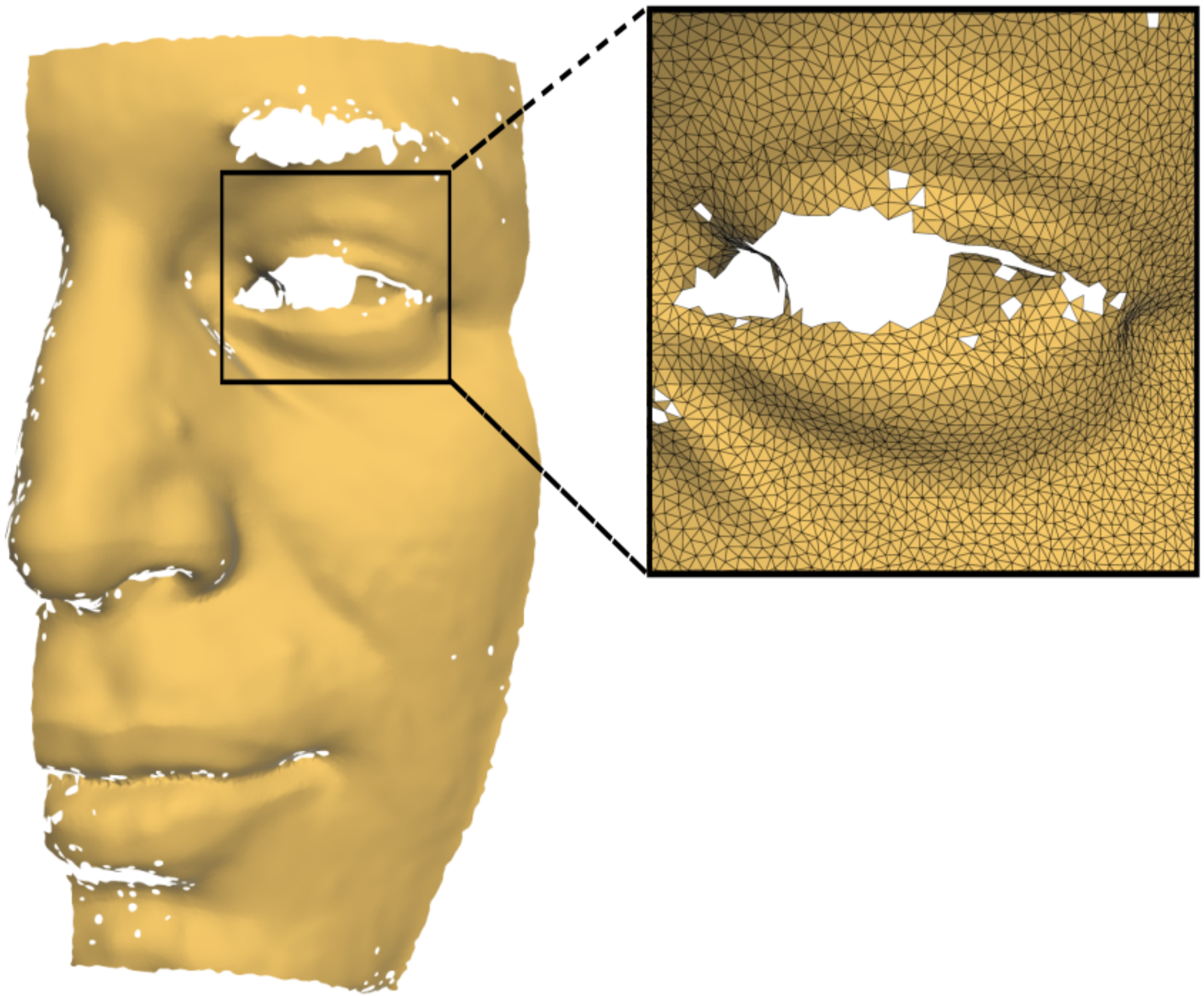
repeat
    Update matrix **B** (Sparse Coding);
    Update **V** (Dictionary Update);
until convergence

Iterative Results

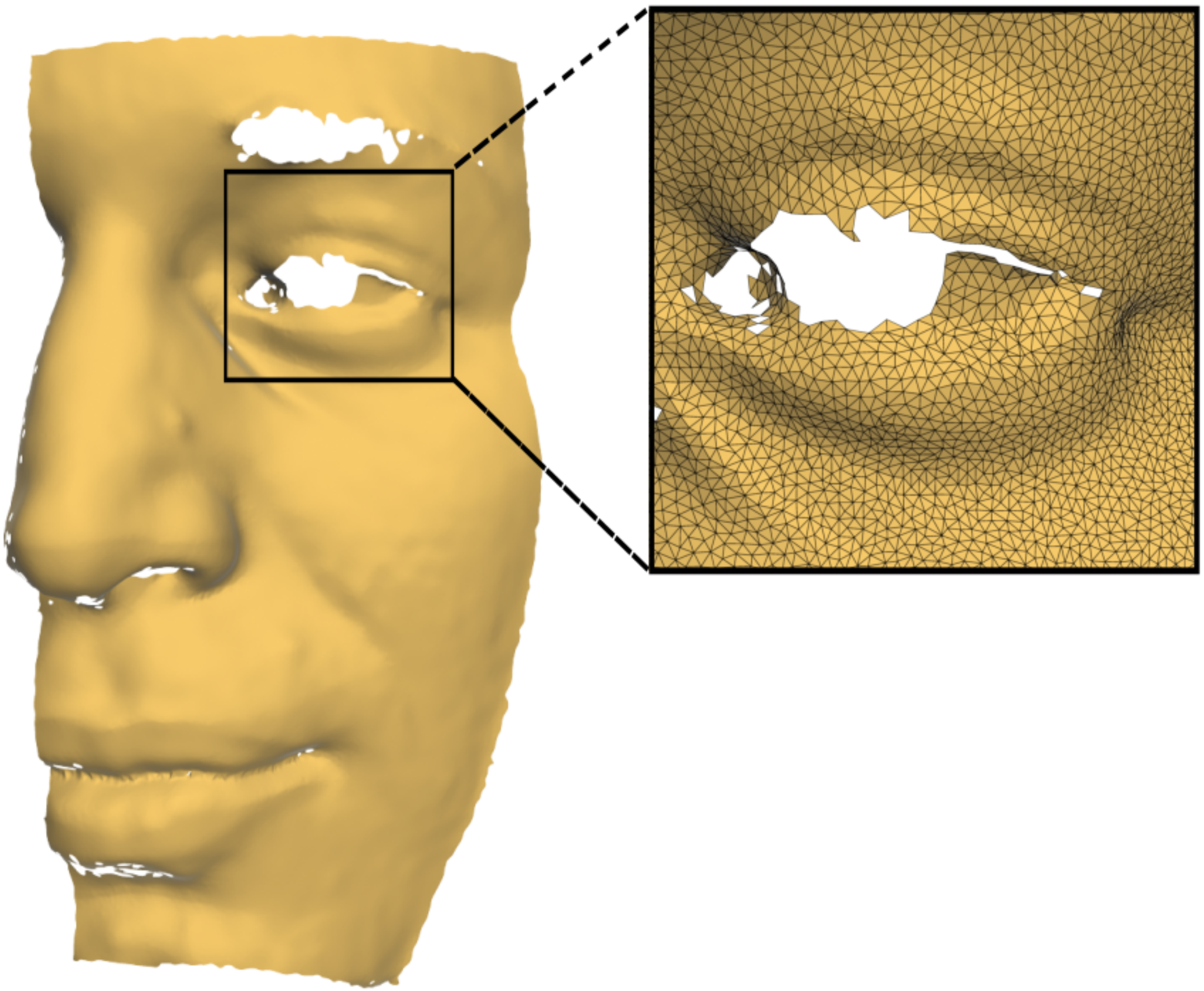
Input Point Cloud



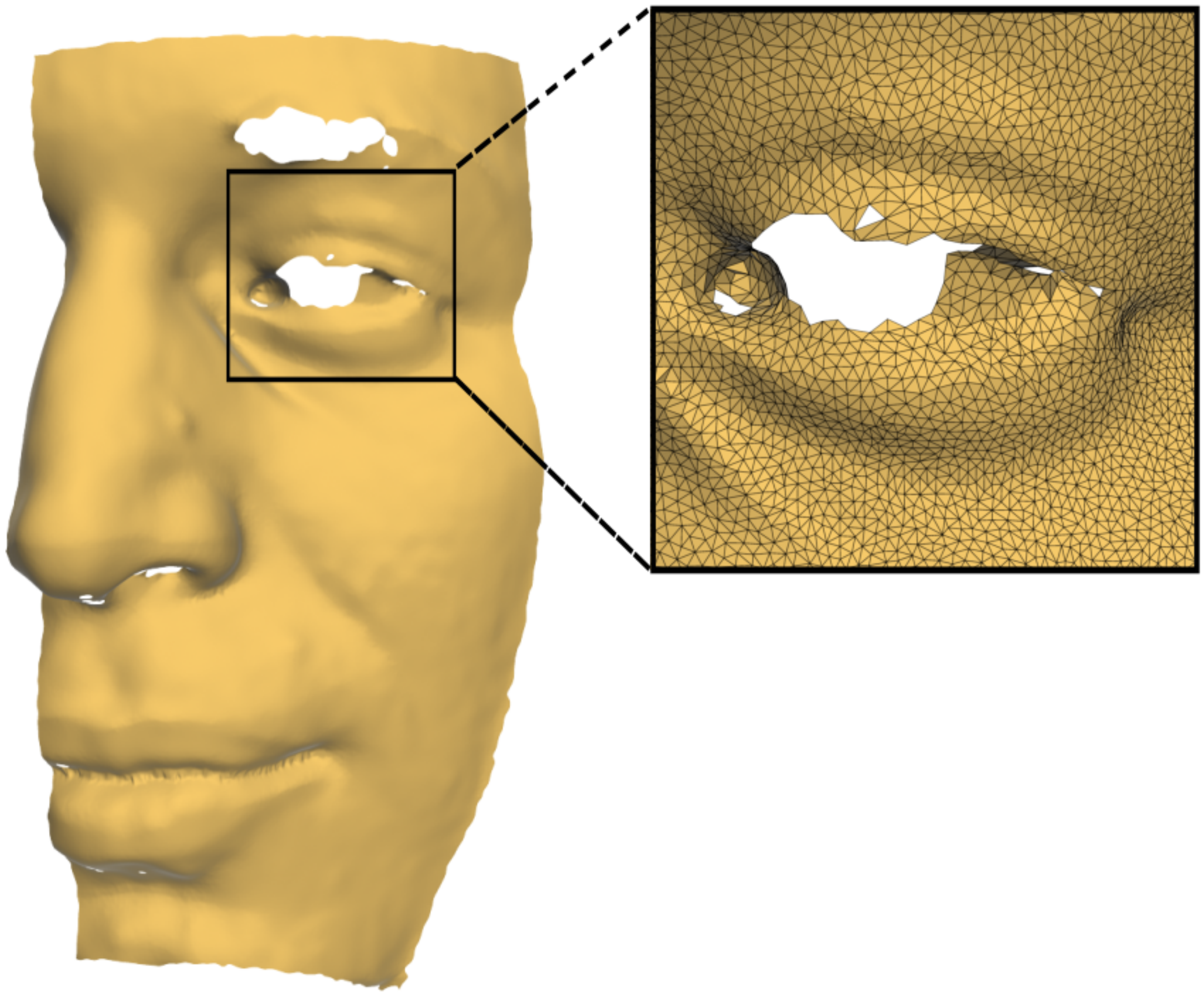
Initial Result



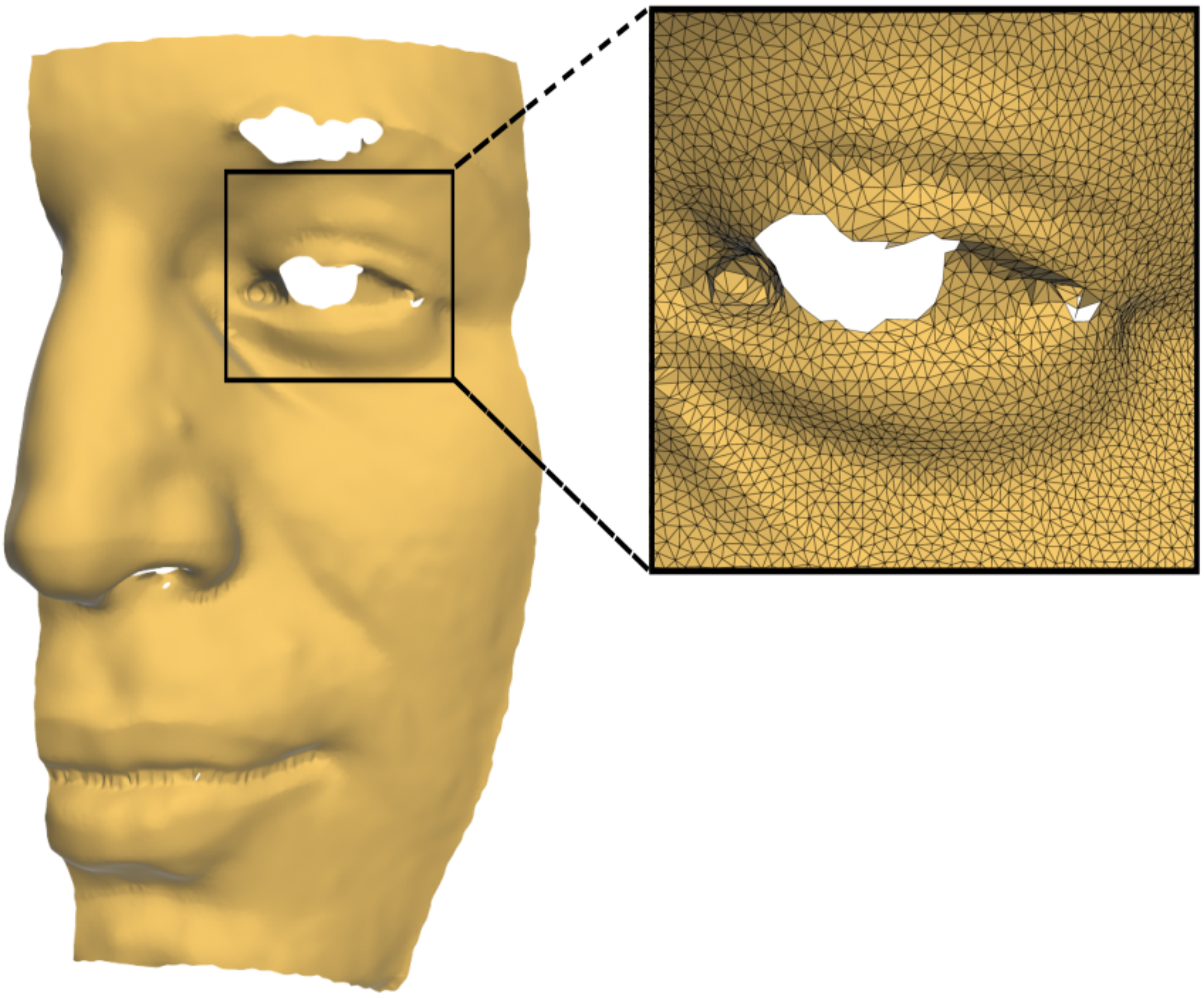
#iter = 1



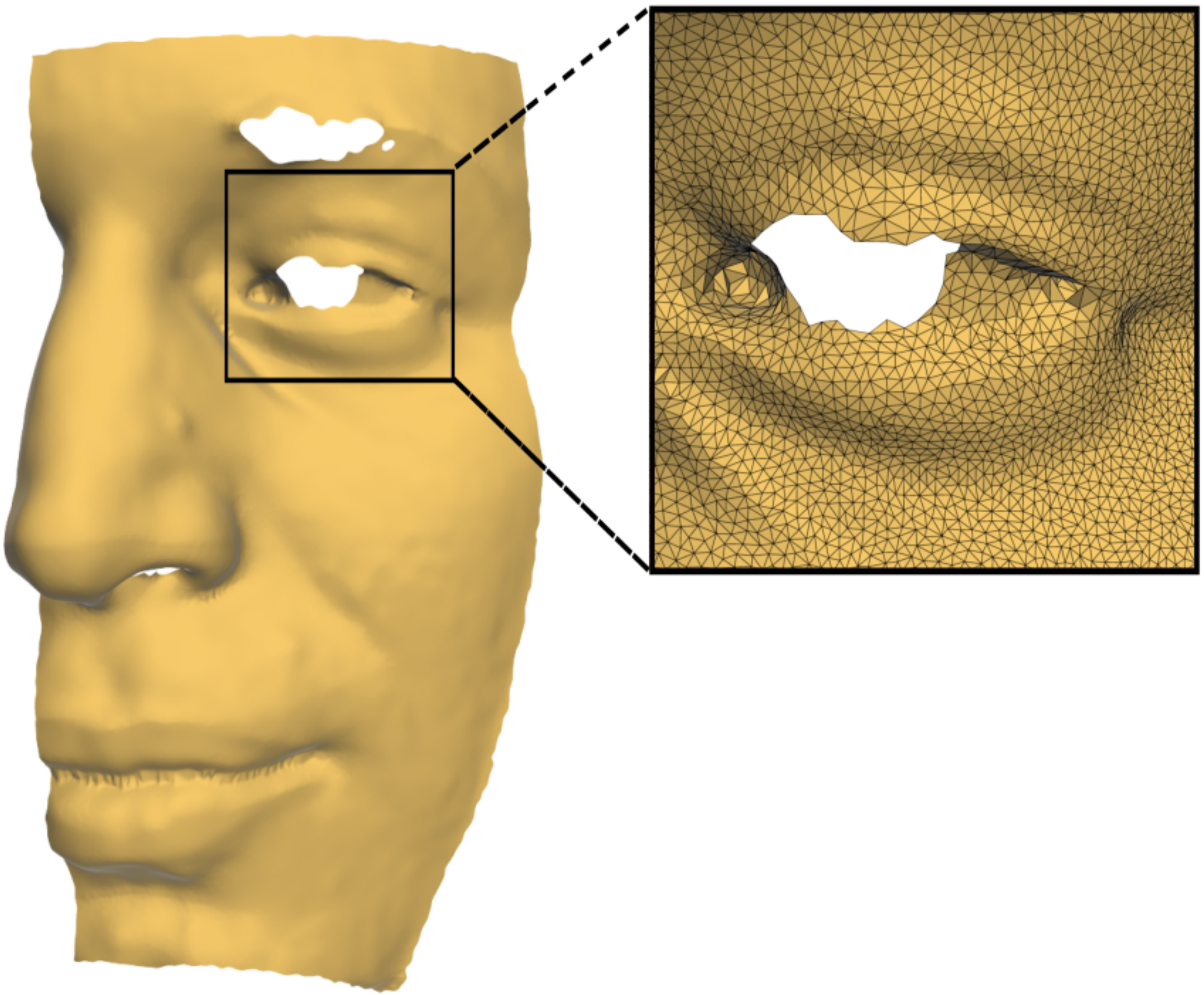
#iter = 2



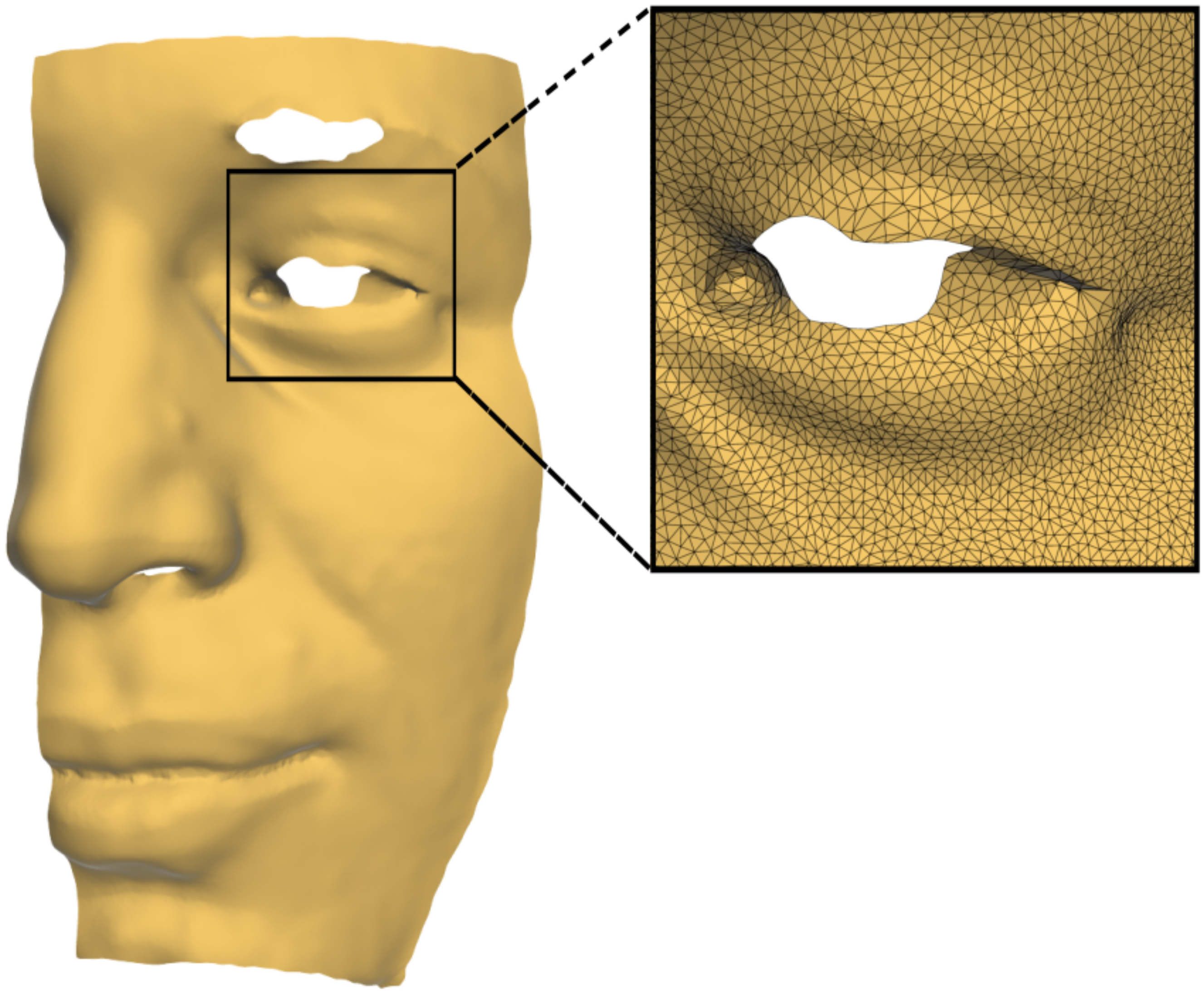
#iter = 3



#iter = 4



Final Result



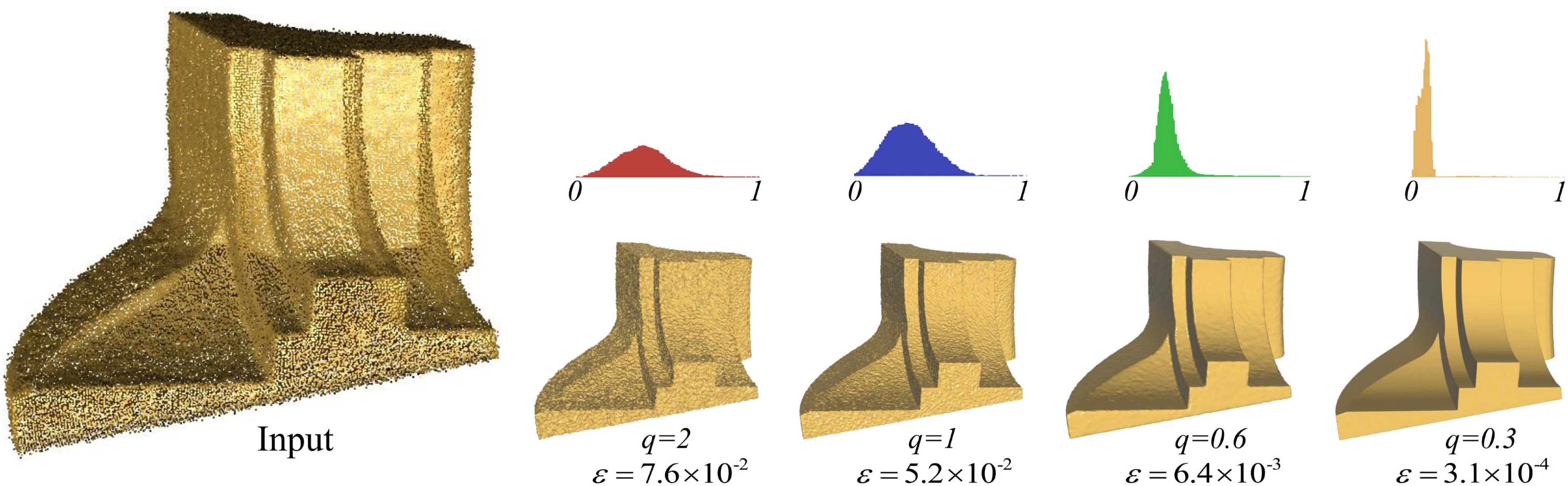
Demo

Noisy Input

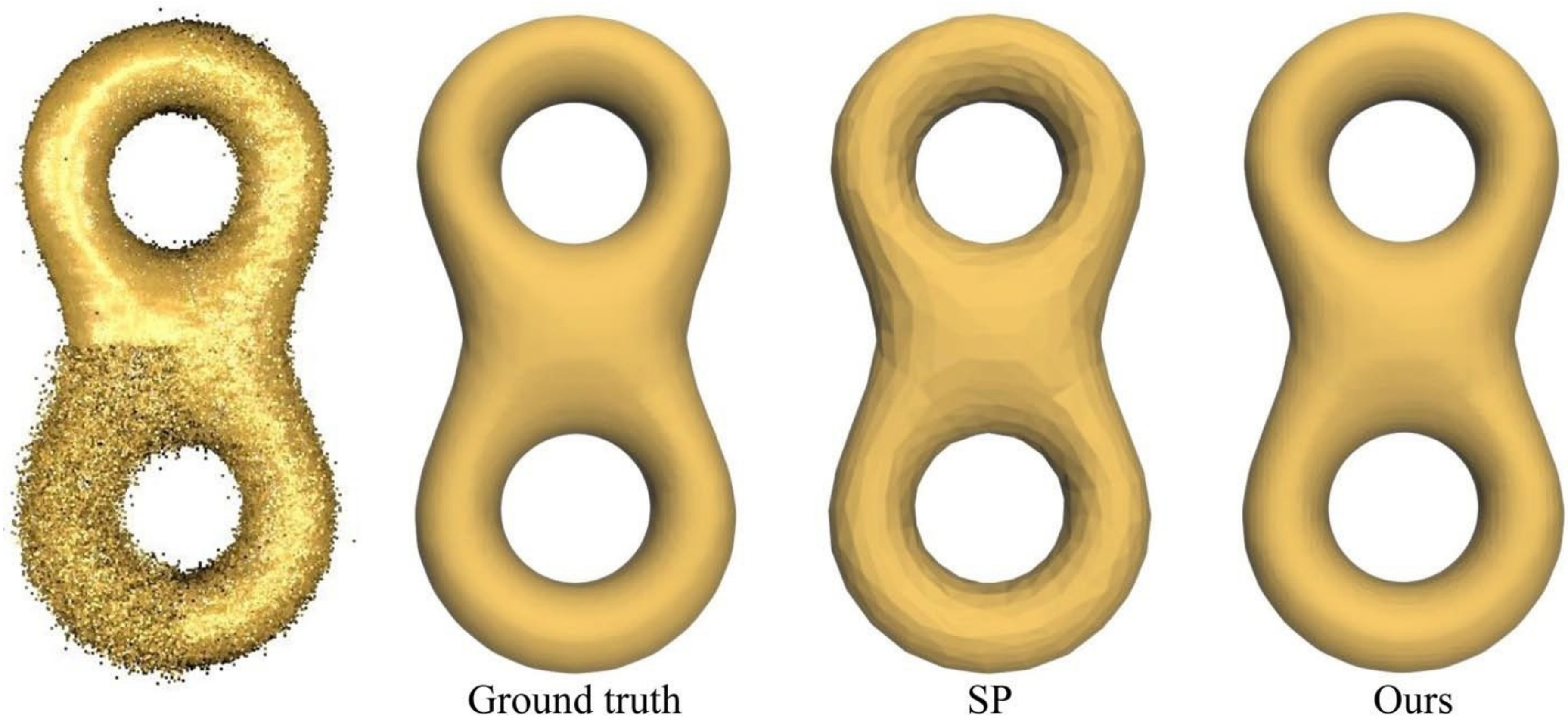
More Results

Different *q* Values

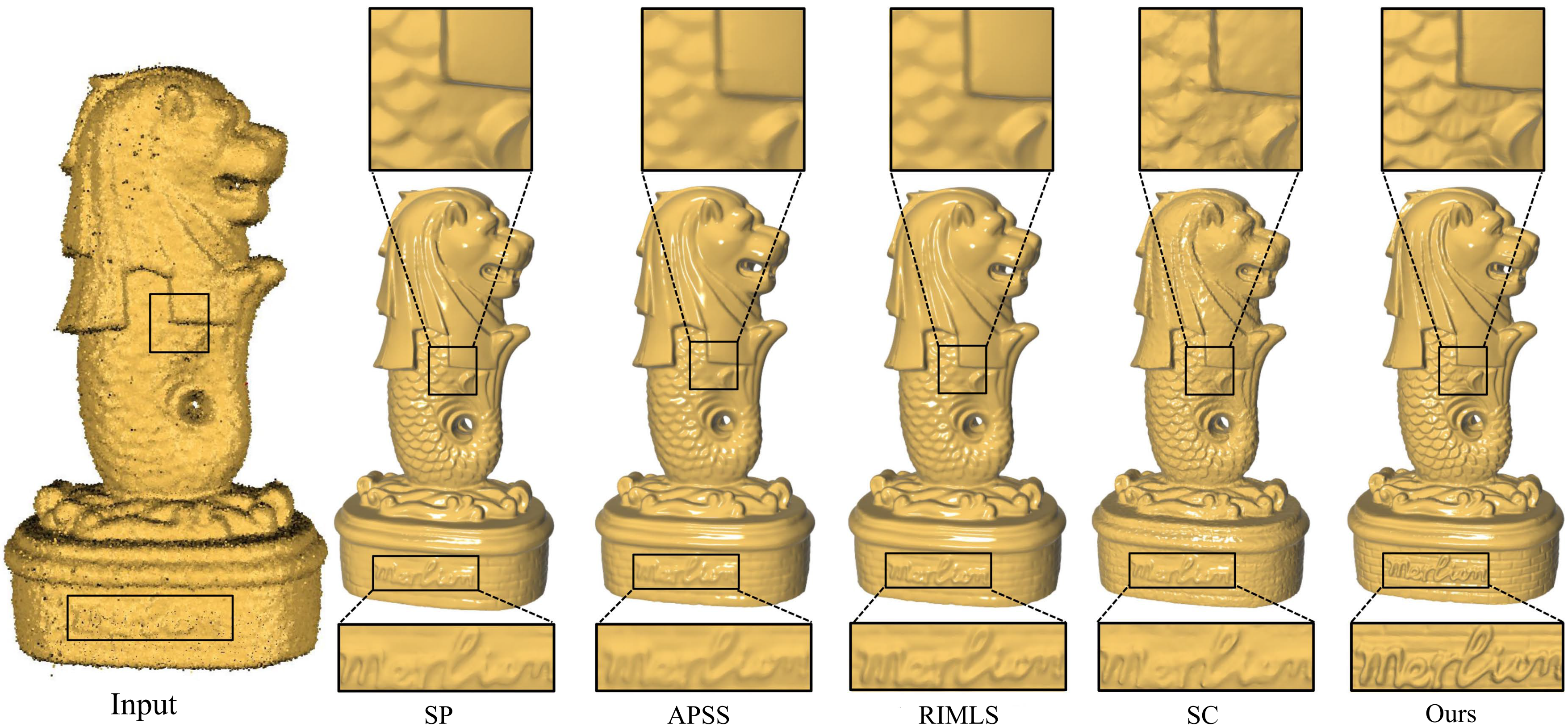
$$E_{\text{appr}} = \frac{1}{n}\|\mathbf{P} - \mathbf{V}\mathbf{B}\|_{2,q} = \frac{1}{n}\sum_{i=1}^{n}\|\mathbf{p}_i - \mathbf{V}\mathbf{b}_i\|_2^q$$



Robust to Noise

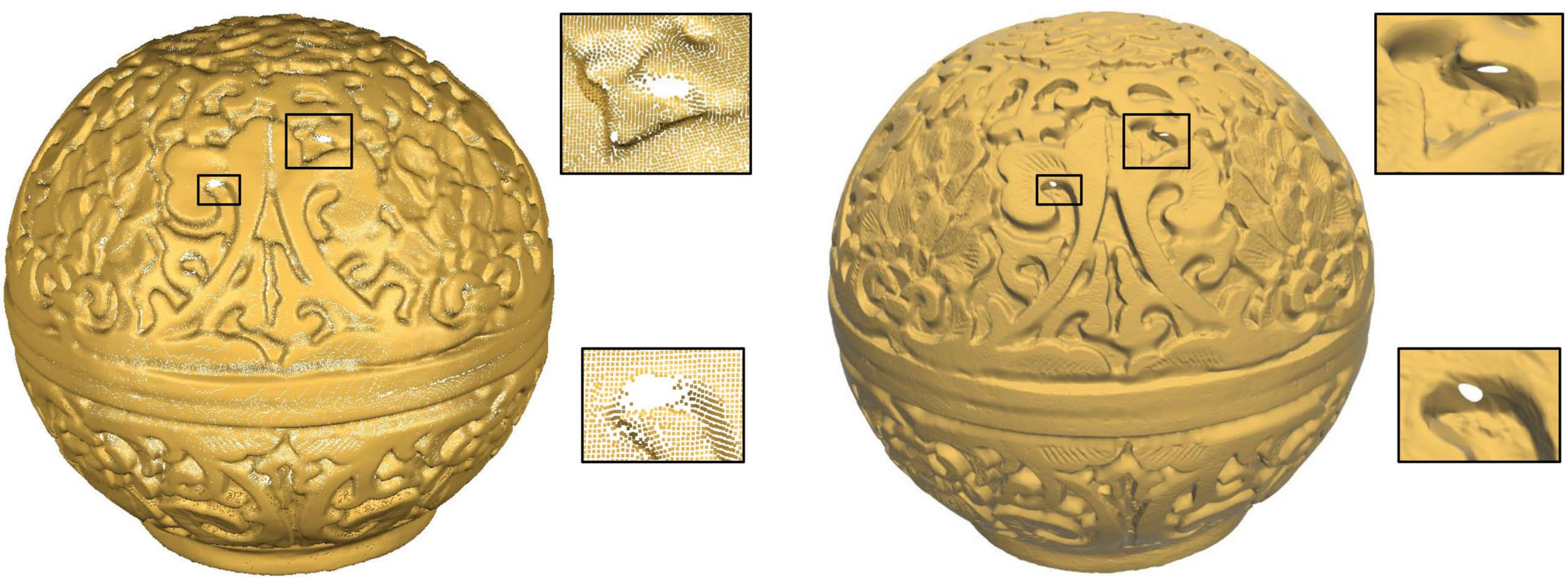


Feature Preserving



Limitations

- Nonconvex optimization
  - Cannot guarantee convergence theoretically
- Cannot fill large holes caused by missing data



Conclusion

- Model the surface reconstruction problem via dictionary learning
  - ✦ Insensitive to noises, outliers
  - ✦ Preserve geometric features
  - ✦ No need of normal information

Thank you!