Homework 1, Spring 2023:

Problem 1.1:

If $f \in L^\infty$ and $\|\tau_y f - f\|_\infty \to 0$ as $y \to 0$, then f agrees a.e. with a uniformly continuous function. ($\tau_y f(x) = f(x-y)$.)

Problem 1.2:

(a) Prove that for all $0 < \varepsilon < t < \infty$ we have

$$\left| \int_\varepsilon^t \frac{\sin(\xi)}{\xi} \, \mathrm{d}\xi \right| \leq 4.$$

(b) If f is an odd L^1 function on the line, conclude that for all $t > \varepsilon > 0$ we have

$$\left| \int_\varepsilon^t \frac{\widehat{f}(\xi)}{\xi} \, \mathrm{d}\xi \right| \leq 4\|f\|_1.$$

(c) Let $g(\xi)$ be a continuous odd function that is equal to $1/\log(\xi)$ for $\xi \geq 2$. Show that there does not exist an L^1 function whose Fourier transform is g.