量子信息导论
PHYS5251P

中国科学技术大学
物理学院/合肥微尺度物质科学国家研究中心

陈凯

2024.4

第三章 量子关联表现

1. 局域实在论
2. Bell不等式
3. 量子游戏（Quantum Games）
4. Bell不等式检验
5. 无不等式的Bell定理
6. 多体Bell不等式
7. 无漏洞Bell 不等式检验

Press release: The Nobel Prize in Physics 2022

English
English (pdf)
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KUNGL. VETENSKAPS AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

4 October 2022

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2022 to

Alain Aspect
Institut d'Optique Graduate School – Université Paris-Saclay and École Polytechnique, Palaiseau, France

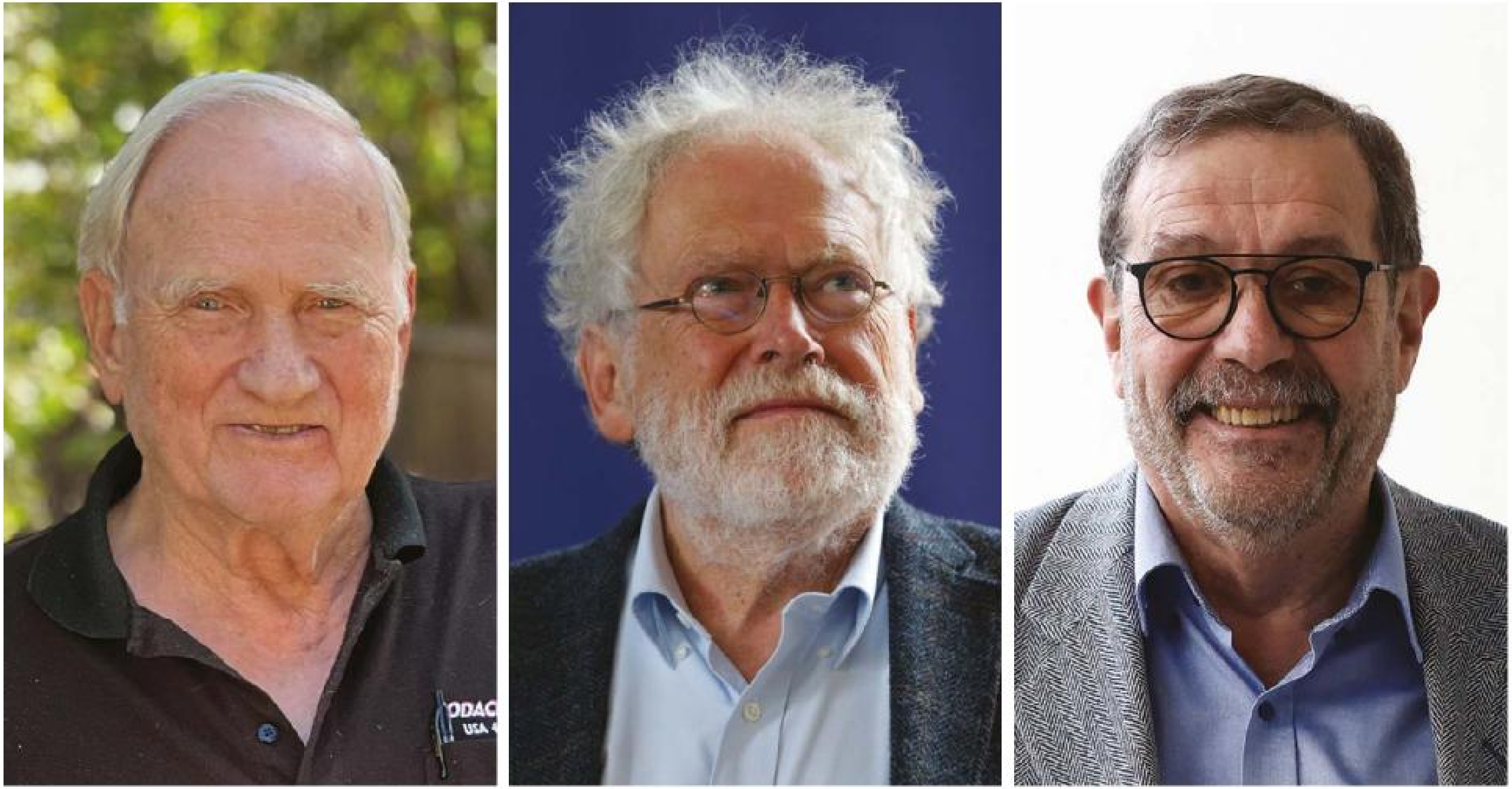
John F. Clauser
J.F. Clauser & Assoc., Walnut Creek, CA, USA

Anton Zeilinger
University of Vienna, Austria

"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

Entangled states – from theory to technology

Alain Aspect, John Clauser and Anton Zeilinger have each conducted groundbreaking experiments using entangled quantum states, where two particles behave like a single unit even when they are separated. Their results have cleared the way for new technology based upon quantum information.



From left: John Clauser, Anton Zeilinger and Alain Aspect won this year's physics Nobel prize.

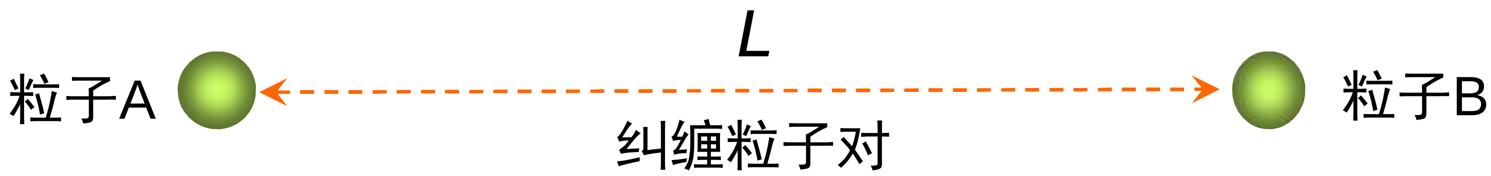
Nature | Vol 610 | 13 October 2022 | 241

PHYSICS NOBEL FOR 'SPOOKY' QUANTUM ENTANGLEMENT

Award goes to three physicists whose research laid the groundwork for quantum information science.

中国科学技术大学 陈凯

相对论定域性与量子非定域性



相对论定域性

测量时间：Δt

类空间隔：$L > c\Delta t$

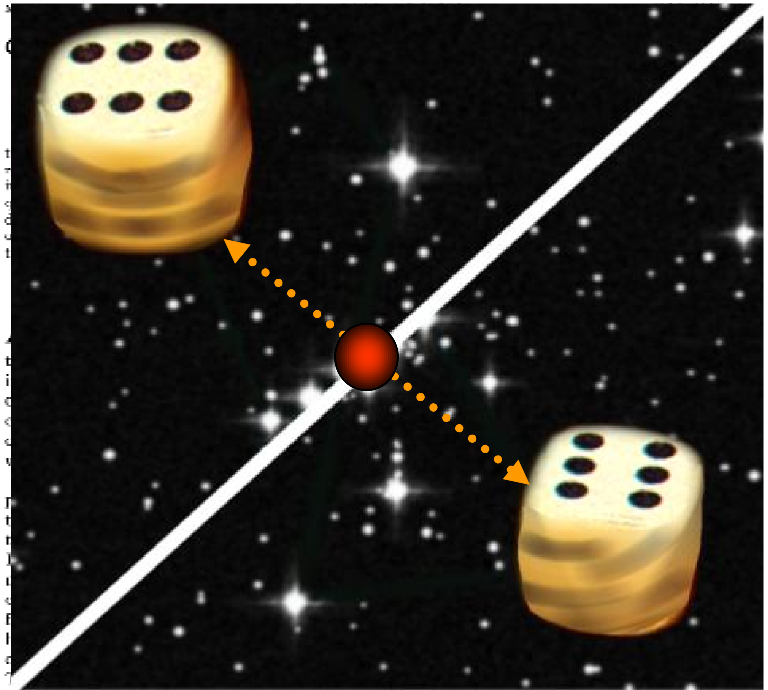
对一个粒子的测量
不会对另一个粒子产生影响

量子非定域性

量子纠缠

对一个粒子的测量
会瞬间改变另一个粒子的状态

EPR & Bohm



“遥远地点之间的诡异互动”——爱因斯坦

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0^A0^B\rangle + |1^A1^B\rangle)$$

Perfect correlation!

Plausible Propositions of EPR

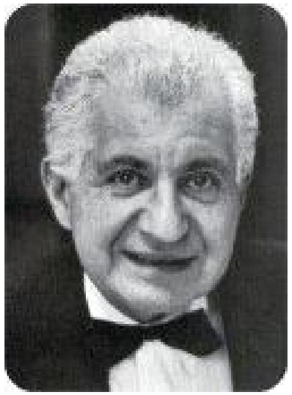
ã Perfect Correlation (Quantum Prediction)

ã Locality

ã Reality

ã Completeness

David Bohm



Boris Podolsky

Nathan Rosen

Einstein, Podolsky, and Rosen, Phys. Rev. 47, 777 (1935)

EPR

(i) Perfect correlation. If the spins of particle A and B are measured along the same direction, then with certainty the outcomes will be found to be opposite.

(ii) Locality. "Since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system."

(iii) Reality. "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

(iv) Completeness. "Every element of the physical reality must have a counterpart in the physical theory."

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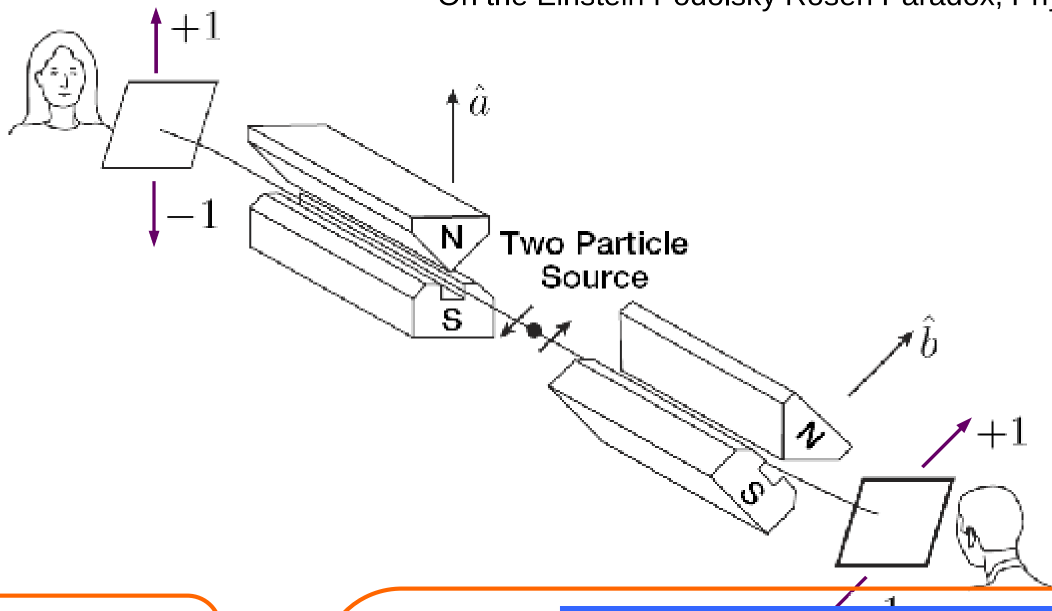
Bell不等式

On the Einstein Podolsky Rosen Paradox, Physics 1, 195 (1964)

John Bell



Bell's Inequality

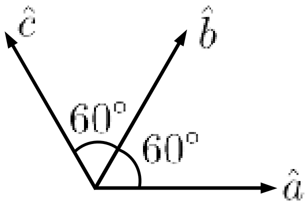
$$\left|C(\hat{a},\hat{b}) - C(\hat{a},\hat{c})\right| - C(\hat{b},\hat{c}) - 1 \leq 0$$

Correlation Function

QM $C(\hat{a},\hat{b}) = \langle\Psi^-|(\hat{a}\cdot\vec{\sigma})\otimes(\hat{b}\cdot\vec{\sigma})|\Psi^-\rangle = -\hat{a}\cdot\hat{b}$

LHV
$$\begin{aligned} C(\hat{a},\hat{b}) &= \int_\Lambda A(\lambda,\hat{a})B(\lambda,\hat{b})\rho(\lambda)d\lambda \\ &= -\int_\Lambda A(\lambda,\hat{a})A(\lambda,\hat{b})\rho(\lambda)d\lambda \end{aligned}$$

Quantum-Mechanically Violation



$$\left|C(\hat{a},\hat{b}) - C(\hat{a},\hat{c})\right| - C(\hat{b},\hat{c}) - 1 = 1/2$$

The most important breakthrough…

—诺贝尔物理学奖获得者Brian Josephson

The most profound discovery in science

—著名量子物理学家Henry Stapp

Bell不等式

$$|E(A_1,B_1)+E(A_1,B_2)+E(A_2,B_1)-E(A_2,B_2)|\leqslant 2$$

$E(A_i ,B_j)$ is the expectation value of the correlation experiment A_i,B_j.

$$|\mathrm{Tr}(\mathcal{B}_{\mathrm{CHSH}}\rho)|\leqslant 2$$

$$\mathcal{B}_{\mathrm{CHSH}}=\mathbf{A}_1\otimes(\mathbf{B}_1+\mathbf{B}_2)+\mathbf{A}_2\otimes(\mathbf{B}_1-\mathbf{B}_2)$$

$$\mathbf{A}_1=\mathbf{a}_1\cdot\sigma,\ \mathbf{A}_2=\mathbf{a}_2\cdot\sigma\ \text{(similarly for } \mathbf{B}_1 \text{ and } \mathbf{B}_2\text{)}$$

Quantum formalism predicts the Cirel'son inequality (Cirel'son, 1980)

$$|\langle\mathcal{B}_{\mathrm{CHSH}}\rangle_{QM}|=|\mathrm{Tr}(\mathcal{B}_{\mathrm{CHSH}}\rho)|\leqslant 2\sqrt{2}$$

Bell不等式

Bell made two key assumptions:

1. *Each measurement reveals an objective physical property of the system. This means that the particle had some value of this property before the measurement was made, just as in classical physics. This value may be unknown to us (just as it is in statistical mechanics), but it is certainly there.*

2. *A measurement made by Alice has no effect on a measurement made by Bob and vice versa. This comes from the theory of relativity, which requires that any signal has to propagate at the (finite) speed of light.*

Bell不等式

$$
\begin{aligned}
& E(A_1B_1) + E(A_1B_2) + E(A_2B_1) - E(A_2B_2) \\
= \quad & E(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2) \\
= \quad & E(A_1(B_1 + B_2) + A_2(B_1 - B_2)) \; .
\end{aligned}
$$

The outcome of each experiment is ±1, which leads to two cases:

- ***$B_1 = B_2$. In this case $B_1 - B_2 = 0$ and $B_1 + B_2 = \pm 2$, so $A_1(B_1 + B_2) + A_2(B_1 - B_2) = \pm 2A_1 = \pm 2$.***
- ***$B_1 = -B_2$. In this case $B_1 + B_2 = 0$ and $B_1 - B_2 = \pm 2$, so $A_1(B_1 + B_2) + A_2(B_1 - B_2) = \pm 2A_2 = \pm 2$.***

Bell不等式

In either case, $A_1B_1+A_1B_2+A_2B_1-A_2B_2 = \pm 2$. We therefore obtain the following Bell's inequality:

$$
\begin{aligned}
& E(A_1B_1) + E(A_1B_2) + E(A_2B_1) - E(A_2B_2) \\
= \quad & E(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2) \\
= \quad & \sum_{a_1,a_2,b_1,b_2} p(a_1,a_2,b_1,b_2)(a_1b_1 + a_1b_2 + a_2b_1 - a_2b_2) \\
\leq \quad & 2\,.
\end{aligned}
$$

纯态和Bell不等式

$$|\psi^{AB}\rangle = a|00\rangle + b|11\rangle$$

$$|\psi^{AB}\rangle = a(|00\rangle + |11\rangle) + (b-a)|11\rangle$$

引入幺正变换和辅助量子态

$$|0^A\rangle|\psi^{AB}\rangle$$

$$\begin{aligned} U^A|0\rangle|0\rangle &= |0\rangle|0\rangle\,, \\ U^A|0\rangle|1\rangle &= \alpha|0\rangle|1\rangle + \beta|1\rangle|0\rangle \end{aligned}$$

纯态和Bell不等式

When Alice applies the unitary operation locally to her qubits, we obtain

$$\begin{aligned} U^A \otimes I^B(|0^A\rangle|\psi^{AB}\rangle) &= a|000\rangle + b(\alpha|011\rangle + \beta|101\rangle) \\ &= |0\rangle(a|00\rangle + b\alpha|11\rangle) + b\beta|101\rangle \end{aligned}$$

Therefore, if we tailor the unitary transformation so that a = bα, then if Alice measures her ancillary qubit in the state |0>, the state that she shares with Bob is maximally entangled.

So what we have shown is that by a local unitary transformation followed by a measurement, Alice can convert any nonmaximally entangled pure state into a maximally entangled pure state (with some nonzero probability).

混合态和Bell不等式

Mixed states may not violate Bell's inequalities

The Werner states are defined as mixtures of Bell states, where the degree of mixing is determined by a parameter F (which really stands for "fidelity"):

$$\varrho_{\mathrm{W}} = F|\Psi^-\rangle\langle\Psi^-| + \frac{1-F}{3}(|\Psi^+\rangle\langle\Psi^+| + |\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-|)$$

where $0 \leq F \leq 1$. When $F = 1/2$, we can write it as

$$\begin{aligned}\varrho_{\mathrm{W}} &= \frac{1}{6}(|\Psi^-\rangle\langle\Psi^-| + |\Psi^+\rangle\langle\Psi^+|) + \frac{1}{6}(|\Psi^-\rangle\langle\Psi^-| + |\Phi^+\rangle\langle\Phi^+|) \\ &+ \frac{1}{6}(|\Psi^-\rangle\langle\Psi^-| + |\Phi^-\rangle\langle\Phi^-|)\end{aligned}$$

混合态和Bell不等式

Mixed states may not violate Bell's inequalities

The Werner states for F=1/2 is separable.

An equal mixture of any two maximally entangled states is a separable state.

$$(1/2)(|\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-|)$$

is equivalent to

$$(1/2)(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

- The Werner states are entangled for F > ½;
- The Werner states violates Bell's inequalities when F > 0.78;
- The Werner states does not violate any Bell's inequalities when F   5/8=0.625 when the correlations result from projective measurements.

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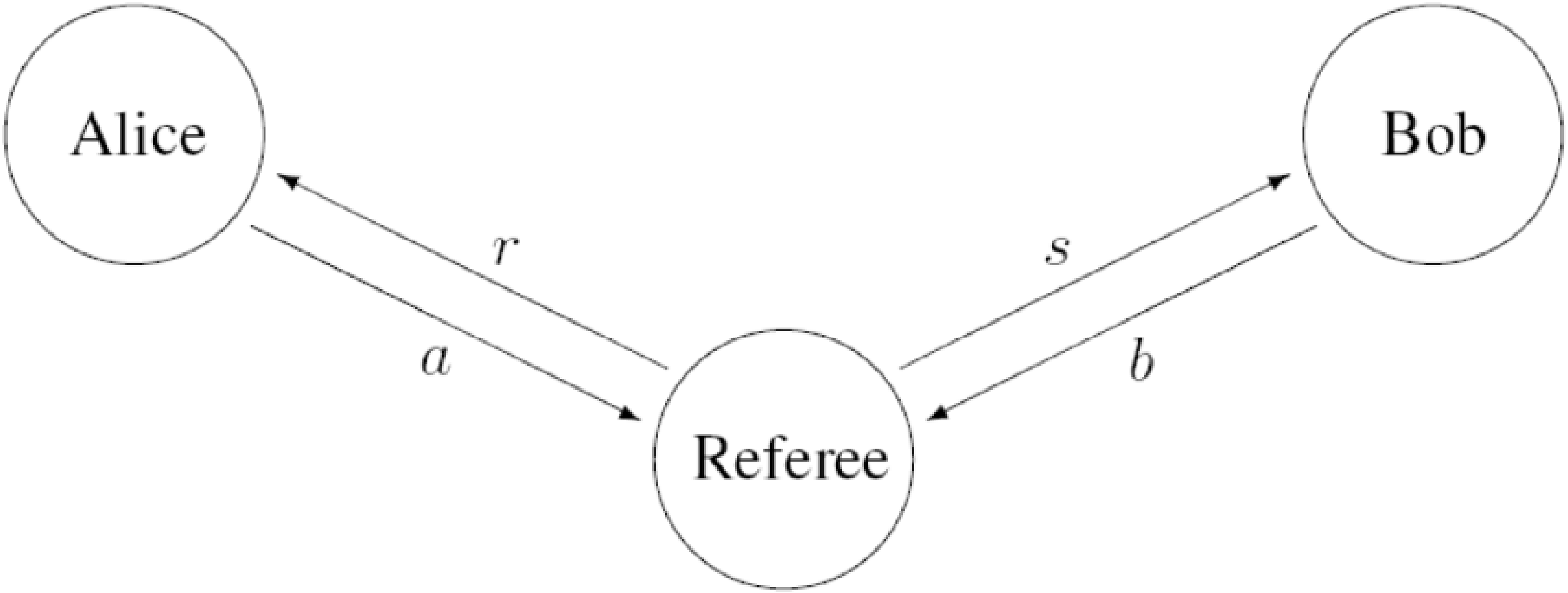
Nonlocal games



Here, the referee chooses a pair of questions (r; s) (according to some prespecied distribution), sends r to Alice and s to Bob, and Alice and Bob answer with a and b, respectively. The referee evaluates some predicate on (r; s; a; b) to determine if they win or lose.

The GHZ game

rst	$a \oplus b \oplus c$
000	0
011	1
101	1
110	1

They win if $a \oplus b \oplus c = r \vee s \vee t$ and lose otherwise.

GHZ game

The winning conditions can be expressed by the four equations

$$a_0 \oplus b_0 \oplus c_0 = 0$$
$$a_0 \oplus b_1 \oplus c_1 = 1$$
$$a_1 \oplus b_0 \oplus c_1 = 1$$
$$a_1 \oplus b_1 \oplus c_0 = 1$$

Adding the four equations modulo 2 gives 0 = 1, a contradiction. This means it is not possible for a deterministic strategy to win every time, so the probability of winning can be at most 3/4

GHZ game

Suppose that the three players share the entangled state

$$|\psi\rangle = \frac{1}{2}|000\rangle - \frac{1}{2}|011\rangle - \frac{1}{2}|101\rangle - \frac{1}{2}|110\rangle$$

Each player will use the same strategy:

1. *If the question is q = 1, then the player performs a Hadamard transform on their qubit of the above state. (If q = 0, the player does not perform a Hadamard transform.)*
2. *The player measures their qubit in the standard basis and returns the answer to the referee.*

GHZ game

There are two cases:

Case 1: r s t = 000. In this case the players all just measure their qubit, and it is obvious that the results satisfy a⊕b⊕c = 0 as required.

Case 2: r s t Î{011; 101; 110}. All three possibilities will work the same way by symmetry, so let us assume r s t = 011. Notice that

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle\left(\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle\right) - \frac{1}{\sqrt{2}}|1\rangle\left(\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle\right)$$
$$= \frac{1}{\sqrt{2}}|0\rangle|\phi^-\rangle - \frac{1}{\sqrt{2}}|1\rangle|\psi^+\rangle .$$

$$(H\otimes H)|\phi^-\rangle = |\psi^+\rangle \qquad \text{and} \qquad (H\otimes H)|\psi^+\rangle = |\phi^-\rangle$$

$$(I\otimes H\otimes H)|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle|\psi^+\rangle - \frac{1}{\sqrt{2}}|1\rangle|\phi^-\rangle = \frac{1}{2}(|001\rangle + |010\rangle - |100\rangle + |111\rangle)$$

When they measure, the results satisfy a⊕b⊕c=1 as required. We have therefore shown that there is a quantum strategy that wins every time.

The CHSH game

The referee chooses questions r s Î{00; 01; 10; 11} uniformly, and Alice and Bob must each answer a single bit: a for Alice, b for Bob.

rs	$a \oplus b$
00	0
01	0
10	0
11	1

They win if $a \oplus b = r \wedge s$ and lose otherwise.

By similar reasoning to the GHZ game, the maximum probability with which a classical strategy can win is ¾.

Andreas Winter, *Quantum mechanics: The usefulness of uselessness*, Nature 466, 1053–1054 (2010)

The CHSH game

The referee chooses questions r s Î {00; 01; 10; 11} uniformly, and Alice and Bob must each answer a single bit: a for Alice, b for Bob.

$$|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$$

Define

$$|\phi_0(\theta)\rangle = \cos(\theta)\,|0\rangle + \sin(\theta)\,|1\rangle\,,$$
$$|\phi_1(\theta)\rangle = -\sin(\theta)\,|0\rangle + \cos(\theta)\,|1\rangle \qquad \theta \in [0, 2\pi)$$

If Alice receives the question 0, she will measure her qubit with respect to the basis $\{|\phi_0(0)\rangle\,, |\phi_1(0)\rangle\}$

and if she receives the question 1, she will measure her qubit with respect to the basis $\{|\phi_0(\pi/4)\rangle\,, |\phi_1(\pi/4)\rangle\}$

The CHSH game

The referee chooses questions r s Î {00; 01; 10; 11} uniformly, and Alice and Bob must each answer a single bit: a for Alice, b for Bob.

$$|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$$

Define

$$|\phi_0(\theta)\rangle = \cos(\theta)\,|0\rangle + \sin(\theta)\,|1\rangle\,,$$
$$|\phi_1(\theta)\rangle = -\sin(\theta)\,|0\rangle + \cos(\theta)\,|1\rangle \qquad \theta \in [0, 2\pi)$$

Bob uses a similar strategy, except that he measures with respect to the basis: If Bob receives the question 0, he will measure her qubit with respect to the basis $\{|\phi_0(\pi/8)\rangle\,, |\phi_1(\pi/8)\rangle\}$

and if he receives the question 1, he will measure his qubit with respect to the basis $\{|\phi_0(-\pi/8)\rangle\,, |\phi_1(-\pi/8)\rangle\}$

The CHSH game

Then Alice's and Bob's observables are

$$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_z \qquad \text{and} \qquad A_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_x$$

$$B_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = H \qquad \text{and} \qquad B_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$$

How well does the strategy?
Consider:

$$\boxed{\frac{1}{4} \langle \psi | A_0 \otimes B_0 + A_0 \otimes B_1 + A_1 \otimes B_0 - A_1 \otimes B_1 | \psi \rangle}$$

This is the probability that Alice and Bob win minus the probability they lose.

The CHSH game

From

$$\langle\psi|A_0\otimes B_0|\psi\rangle = \langle\psi|A_0\otimes B_1|\psi\rangle = \langle\psi|A_1\otimes B_0|\psi\rangle = -\langle\psi|A_1\otimes B_1|\psi\rangle = \frac{1}{\sqrt{2}}$$

we know that the probability of winning minus the probability of losing is 1/$\sqrt{2}$.

This means the probability of winning is

$$\frac{1}{2}+\frac{1}{2\sqrt{2}} = \cos^2(\pi/8)$$

Thus Alice and Bob will answer correctly with probability $\cos^2$(p/8)»0.85, which is better than an optimal classical strategy that wins with probability ¾.

Is it possible to do better?

Tsirelson's bound

For any choice of observables A_0, A_1, B_0 and B_1 with eigenvalues in [-1,1] and any state,

$$\langle\psi|A_0 \otimes B_0 + A_0 \otimes B_1 + A_1 \otimes B_0 - A_1 \otimes B_1|\psi\rangle \leq 2\sqrt{2}$$

Using the fact that $\|A_0\|, \|A_1\|, \|B_0\|, \|B_1\| \leq 1$

$$\begin{aligned}
&\langle\psi|A_0 \otimes B_0 + A_0 \otimes B_1 + A_1 \otimes B_0 - A_1 \otimes B_1|\psi\rangle \\
&\qquad \leq \|(A_0 \otimes B_0 + A_0 \otimes B_1 + A_1 \otimes B_0 - A_1 \otimes B_1)|\psi\rangle\| \\
&\qquad \leq \|(A_0 \otimes (B_0 + B_1))|\psi\rangle\| + \|(A_1 \otimes (B_0 - B_1))|\psi\rangle\| \\
&\qquad \leq \|(I \otimes B_0)|\psi\rangle + (I \otimes B_1)|\psi\rangle\| + \|(I \otimes B_0)|\psi\rangle - (I \otimes B_1)|\psi\rangle\| \\
&\qquad = \||\phi_0\rangle + |\phi_1\rangle\| + \||\phi_0\rangle - |\phi_1\rangle\|
\end{aligned}$$

where $|\phi_b\rangle = (I \otimes B_b)|\psi\rangle$

Tsirelson's bound

By making use of

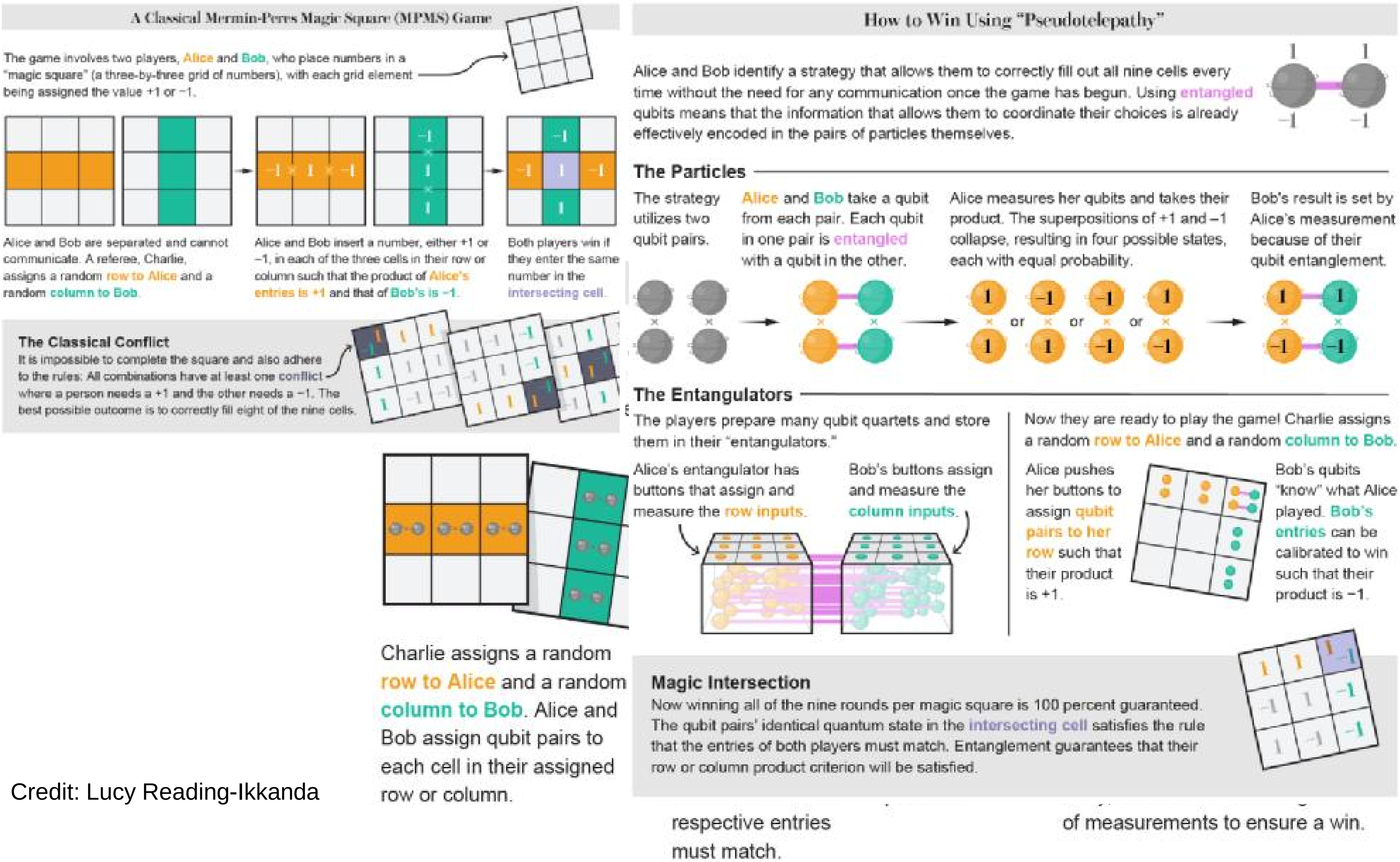
$$\| |\phi_b\rangle \| \leq 1$$

One has

$$\| |\phi_0\rangle + |\phi_1\rangle \| + \| |\phi_0\rangle - |\phi_1\rangle \| \leq \sqrt{2 + 2\Re \langle \phi_0|\phi_1\rangle} + \sqrt{2 - 2\Re \langle \phi_0|\phi_1\rangle} = \sqrt{2+2x} + \sqrt{2-2x}$$

for $x \in [-1,1]$. the maximum of this expression occurs at $x = 0$, giving $2\sqrt{2}$ as required. This lead to the best quantum strategy

Mermin-Peres magic square game



Credit: Lucy Reading-Ikkanda

https://www.scientificamerican.com/article/researchers-use-quantum-telepathy-to-win-an-impossible-game/

Experimental demonstration of Mermin-Peres magic square game

PHYSICAL REVIEW LETTERS 129, 050402 (2022)

Experimental Demonstration of Quantum Pseudotelepathy

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Quantum pseudotelepathy is a strong form of nonlocality. Different from the conventional nonlocal games where quantum strategies win statistically, e.g., the Clauser-Horne-Shimony-Holt game, quantum pseudotelepathy in principle allows quantum players to with probability 1. In this Letter, we report a faithful experimental demonstration of quantum pseudotelepathy via playing the nonlocal version of Mermin-Peres magic square game, where Alice and Bob cooperatively fill in a 3×3 magic square. We adopt the hyperentanglement scheme and prepare photon pairs entangled in both the polarization and the orbital angular momentum degrees of freedom, such that the experiment is carried out in a resource-efficient manner. Under the locality and fair-sampling assumption, our results show that quantum players can simultaneously win all the queries over any classical strategy.

DOI: 10.1103/PhysRevLett.129.050402

PHYSICAL REVIEW LETTERS 129, 050402 (2022)

TABLE I. Nine deterministic optimal classical strategies. Here, $\# = +1$ for Alice and $\# = -1$ for Bob. When receiving queries x and y, Alice and Bob select one table via preshared randomness and reply with the xth row and yth column, respectively. If they uniformly select tables, they would have on average a winning probability 8/9 for each query.

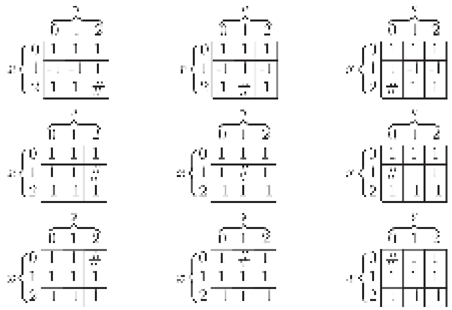
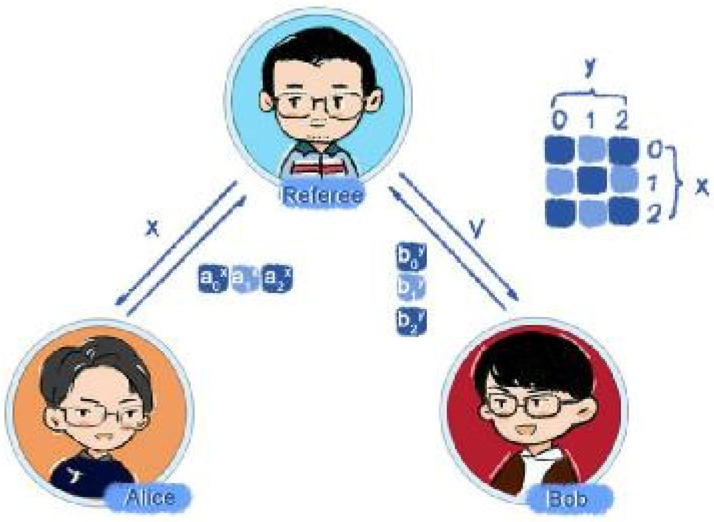


TABLE II. Optimal quantum strategy. The X, Y, and Z are three Pauli matrices. When receiving queries x and y, Alice and Bob select the xth row (yth column) of observables to measure their systems. They win all queries with probability 1.

$x \backslash y$	0	1	2
0	$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
1	$X \otimes I$	$I \otimes X$	$X \otimes X$
2	$-X \otimes Z$	$-Z \otimes X$	$Y \otimes Y$

Experimental demonstration of Mermin-Peres magic square game

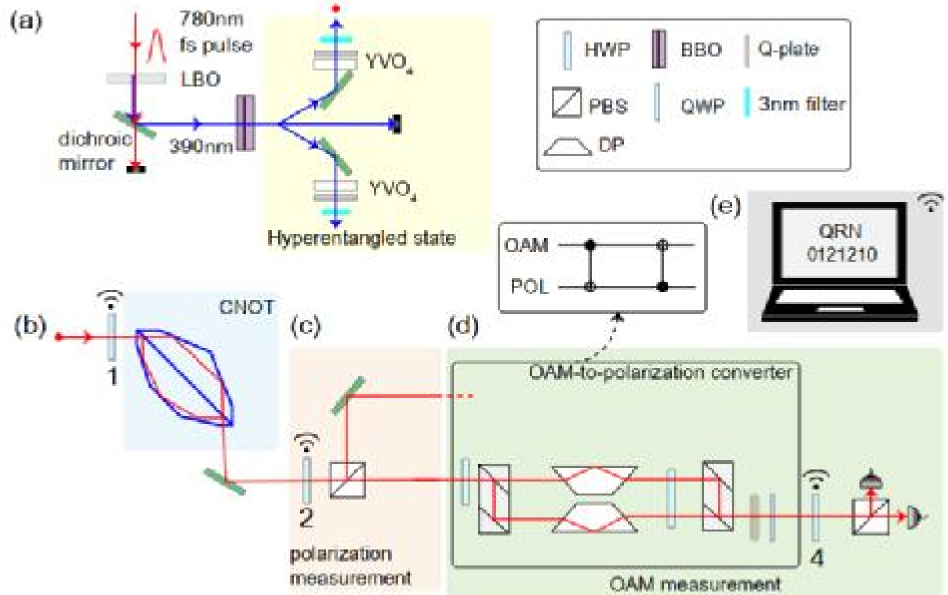
Preparation of hyperentangled photon pairs
(polarization vs. OAM)



Winning condition:

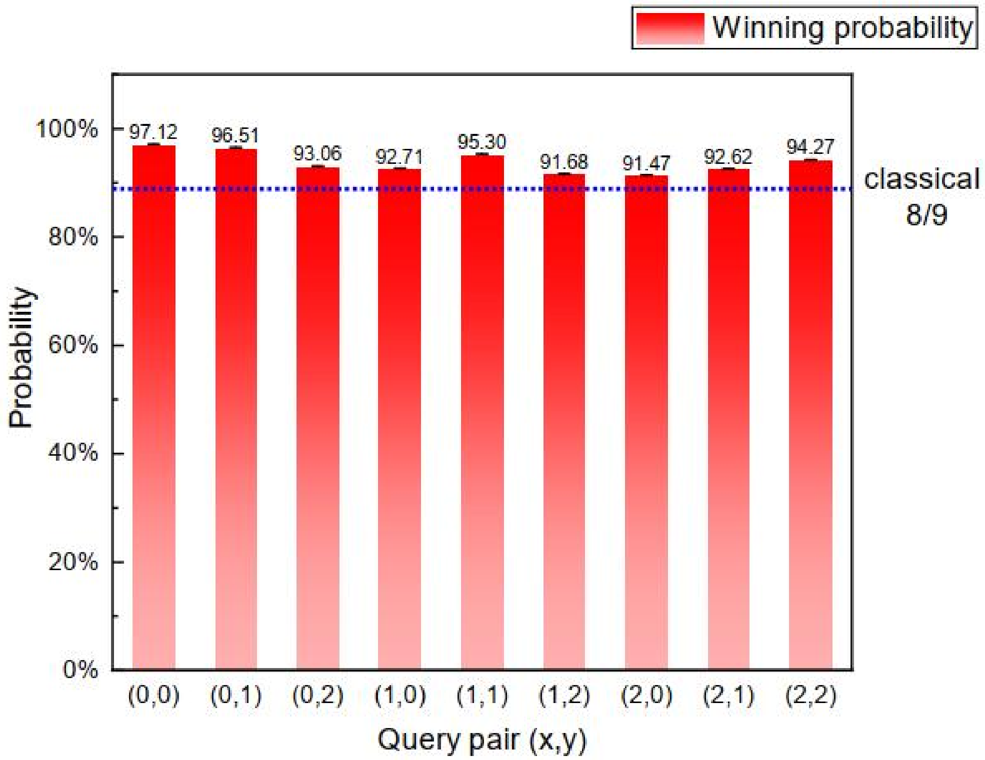
The overlapped entry the same

$$a_y^x = b_x^y$$



Jia-Min Xu*, Yi-Zheng Zhen*, Yu-Xiang Yang, Zi-Mo Cheng, Zhi-Cheng Ren, **Kai Chen**#, Xi-Lin Wang#, and Hui-Tian Wang#, **Phys. Rev. Lett** 129,050402 (2022).

Results: Classical VS Quantum



1075930 rounds

1009610 Win!

average winning probability:

0.9383(±0.0002)

All query pair is won with a probability higher than 8/9.

SCIENTIFIC AMERICAN.

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QUANTUM PHYSICS

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A new playful demonstration of quantum pseudotelepathy could lead to advances in communication and computation

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Science

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Reality doesn't exist until you measure it, quantum parlor trick confirms

Two players leverage quantum rules to achieve a seemingly telepathic connection

20 JUL 2022 • 5:50 PM • BY ADRIAN CHO

It only looks like telepathy, but a quantum game harpoons our usual sense of reality. KATERYNA KOVARZH/ISTOCK

SHARE:

Cabello says the work shows a new wrinkle in what quantum rules make possible by mobilizing two sources of quantum advantage at the same time: one linked to nonlocality and the other linked to contextuality. Investigating the two effects simultaneously

https://www.science.org/content/article/reality-doesn-t-exist-until-you-measure-it-quantum-parlor-trick-confirms

https://www.scientificamerican.com/article/researchers-use-quantum-telepathy-to-win-an-impossible-game/

Bell不等式检验

VOLUME 49, NUMBER 2 PHYSICAL REVIEW LETTERS 12 JULY 1982

Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

Alain Aspect, Philippe Grangier, and Gérard Roger
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(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

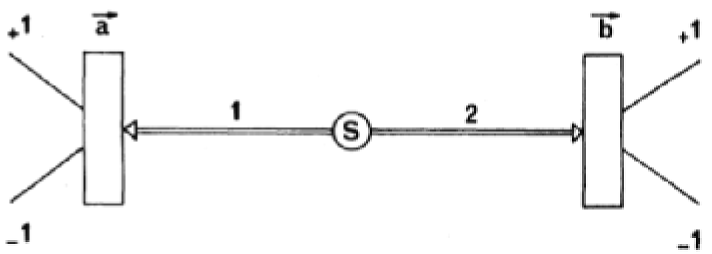

FIG. 1. Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. Two-spin-$\frac{1}{2}$ particles (or photons) in a singlet state (or similar) separate. The spin components (or linear polarizations) of 1 and 2 are measured along $\vec{a}$ and $\vec{b}$. Quantum mechanics predicts strong correlations between these measurements.

A. Aspect *et al.*, *Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities*, Phys. Rev. Lett. **49**, 91 (1982).

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Bell不等式检验

A typical CHSH experiment

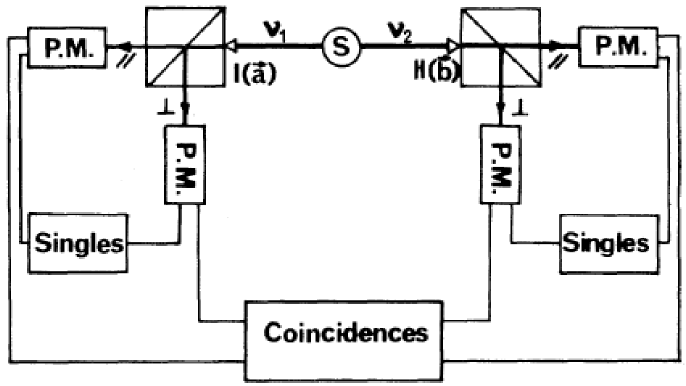
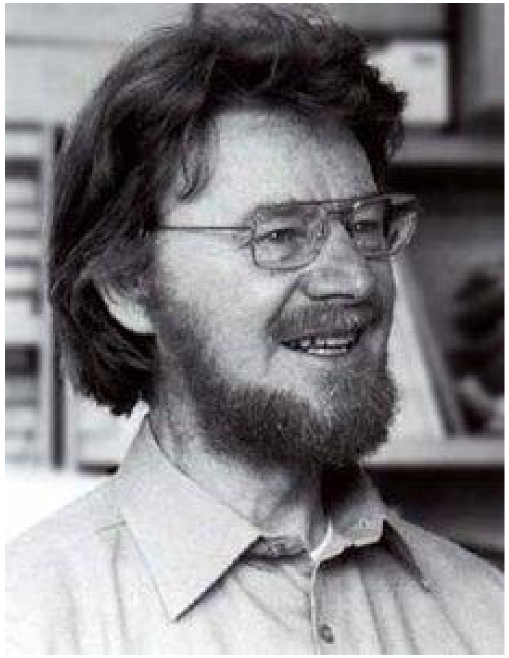


FIG. 2. Experimental setup. Two polarimeters I and II, in orientations $\vec{a}$ and $\vec{b}$, perform true dichotomic measurements of linear polarization on photons ν_1 and ν_2. Each polarimeter is rotatable around the axis of the incident beam. The counting electronics monitors the singles and the coincidences.



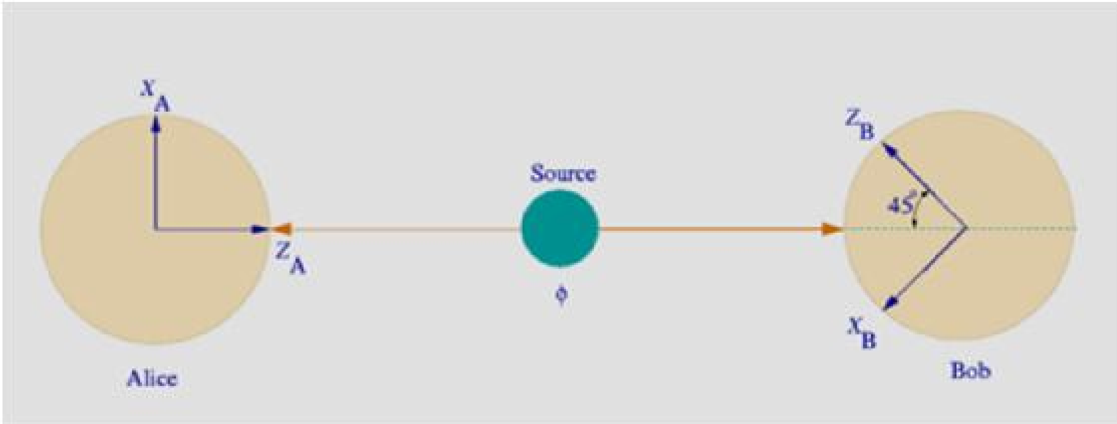
John Bell (1928-1990)

Alain Aspect

$$S_{\text{expt}} = 2.697 \pm 0.015$$

A. Aspect *et al.*, *Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities*, Phys. Rev. Lett. **49**, 91 (1982).

Bell不等式检验



Shimony：

Most of the dozens of experiments performed so far have favored Quantum Mechanics, but not decisively because of the 'detection loopholes' or the 'communication loophole.' The latter has been nearly decisively blocked by a recent experiment and there is a good prospect for blocking the former.

2004 Stanford Encyclopedia overview article

Bell不等式检验: two-qubit

An n-qubit state can be written as

$$\rho = \frac{1}{2^n} \sum_{i_1 \cdots i_n = 0}^{3} t_{i_1 \cdots i_n} \sigma_{i_1}^1 \otimes \cdots \otimes \sigma_{i_n}^n$$

The set of real coefficients forms a correlation tensor T_r

In particular, for the two-qubit system the 3x3-dimensional tensor is given by

$$t_{ij} := \mathrm{Tr}[\rho(\sigma_i \otimes \sigma_j)]$$

Bell不等式检验: two-qubit

An 2-qubit state can be written as

$$\varrho = \frac{1}{4}\left(I \otimes I + \mathbf{r}\cdot\boldsymbol{\sigma} \otimes I + I \otimes \mathbf{s}\cdot\boldsymbol{\sigma} + \sum_{n,m=1}^{3} t_{nm}\sigma_n \otimes \sigma_m\right)$$

$$\mathcal{B}_{\mathrm{CHSH}} = \hat{\mathbf{a}}\cdot\boldsymbol{\sigma} \otimes (\hat{\mathbf{b}} + \hat{\mathbf{b}}')\cdot\boldsymbol{\sigma} + \hat{\mathbf{a}}'\cdot\boldsymbol{\sigma} \otimes (\hat{\mathbf{b}} - \hat{\mathbf{b}}')\cdot\boldsymbol{\sigma}$$

$$|\langle \mathcal{B}_{\mathrm{CHSH}} \rangle_{\varrho}| \leqslant 2$$

One has

$$2\sqrt{M(\varrho)} = \langle \mathcal{B}_{\max} \rangle_{\varrho} = \max_{\mathcal{B}_{\mathrm{CHSH}}} |\langle \mathcal{B}_{\mathrm{CHSH}} \rangle_{\varrho}|$$

$$M(\varrho) := \max_{\hat{\mathbf{c}},\hat{\mathbf{c}}'} (\|T_{\varrho}\hat{\mathbf{c}}\|^2 + \|T_{\varrho}\hat{\mathbf{c}}'\|^2) = u + \tilde{u}$$

Here u and ũ are the two largest eigenvalues of $T^T_r\, T_r$

Horodecki, R.; Horodecki, P.; Horodecki, M.
Violating Bell inequality by mixed spin-1/2 states: necessary and sufficient condition,
Physics Letters A, Volume 200, Issue 5, May 1995, Pages 340-344

Clauser-Horne-Shimony-Holt 不等式

Mathematics

$$x, x', y, y' = \pm 1 \Longrightarrow xy + xy' + x'y - x'y' = \pm 2$$

Clauser *et al.*,
Phys. Rev. Lett. 23, 880 (1969)

The CHSH Inequality

$$\left| C(\hat{a}, \hat{b}) + C(\hat{a}, \hat{b}') + C(\hat{a}', \hat{b}) - C(\hat{a}', \hat{b}') \right| \leq 2$$

Without perfect correlation!

Quantum-Mechanically Violation

$$|\Psi\rangle = |\Psi^-\rangle, \ \hat{b} \perp \hat{b}', \ \hat{a} = \frac{1}{\sqrt{2}}(\hat{b} + \hat{b}'), \ \hat{a}' = \frac{1}{\sqrt{2}}(\hat{b} - \hat{b}')$$

All entangled pure states violate the CHSH inequality!

N. Gisin, Phys. Lett. A 154, 201 (1991)

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Greenberger-Horne-Zeilinger

GHZ 态

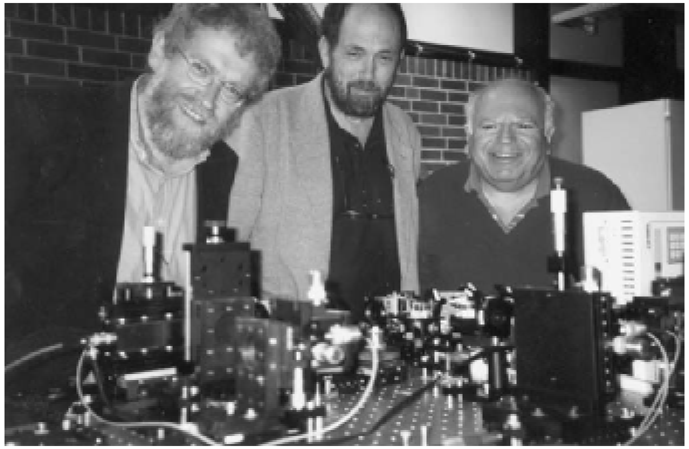
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle|0\rangle - |1\rangle|1\rangle|1\rangle)$$
$$= \frac{1}{2}(|+\rangle|-\rangle|+\rangle + |-\rangle|+\rangle|+\rangle + |+\rangle|+\rangle|-\rangle + |-\rangle|-\rangle|-\rangle)$$
$$= \frac{1}{2}(|+\rangle|+'\rangle|+'\rangle + |-\rangle|-'\rangle|+'\rangle + |+\rangle|-'\rangle|-'\rangle + |-\rangle|+'\rangle|-'\rangle)$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle),\ \sigma_x|\pm\rangle = \pm|\pm\rangle$$
$$|\pm'\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle),\ \sigma_y|\pm'\rangle = \pm|\pm'\rangle$$

Greenberger *et al.*,
Am. J. Phys. 58, 1131 (1990)

LHV

$$A(\lambda,\hat{x})B(\lambda,\hat{x})C(\lambda,\hat{x}) = -1$$
$$A(\lambda,\hat{x})B(\lambda,\hat{y})C(\lambda,\hat{y}) = +1$$
$$A(\lambda,\hat{y})B(\lambda,\hat{x})C(\lambda,\hat{y}) = +1$$
$$A(\lambda,\hat{y})B(\lambda,\hat{y})C(\lambda,\hat{x}) = +1$$



D. Greenberger, M. Horne, and A. Zeilinger in front of the GHZ experimental design at Anton Zeilinger’s lab in Vienna.

Bell's theorem without inequalities

$$|\Psi\rangle_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B|0\rangle_E - |1\rangle_A|1\rangle_B|1\rangle_E\right)$$

$$\begin{aligned}
X_A \otimes X_B \otimes X_E|\Psi\rangle_{\mathrm{GHZ}} &= -|\Psi\rangle_{\mathrm{GHZ}}, \\
X_A \otimes Y_B \otimes Y_E|\Psi\rangle_{\mathrm{GHZ}} &= |\Psi\rangle_{\mathrm{GHZ}}, \\
Y_A \otimes X_B \otimes Y_E|\Psi\rangle_{\mathrm{GHZ}} &= |\Psi\rangle_{\mathrm{GHZ}}, \\
Y_A \otimes Y_B \otimes X_E|\Psi\rangle_{\mathrm{GHZ}} &= |\Psi\rangle_{\mathrm{GHZ}}.
\end{aligned}$$

$$\begin{aligned}
x_A x_B x_E &= -1, \\
x_A y_B y_E &= +1, \\
y_A x_B y_E &= +1, \\
y_A y_B x_E &= +1.
\end{aligned}$$

But these relations are not mutually consistent!

Bell test

Correlation functions

$$E(a_i, b_j) = \langle \psi | \vec{\sigma} \cdot \hat{n}_{a_i} \otimes \vec{\sigma} \cdot \hat{n}_{b_j} | \psi \rangle$$

For a maximally entangled state $|\psi\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$

$$E(a_i, b_j) = -\cos\theta_{a_i b_j} = -\cos(\theta_i^a - \theta_j^b)$$

With appropriate angles $q_1^a = \frac{p}{2}, q_2^a = 0,\ q_1^b = \frac{p}{4}, q_2^b = \frac{3p}{4}$

Bell test

$$E_{11}(q_1^a, q_1^b) = -\cos(q_1^a - q_1^b) = -\cos\frac{p}{4} = -\frac{1}{\sqrt{2}}$$

$$E_{12}(q_1^a, q_2^b) = -\cos(q_1^a - q_2^b) = -\cos\left(-\frac{p}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$E_{21}(q_2^a, q_1^b) = -\cos(q_2^a - q_1^b) = -\cos\left(-\frac{p}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$E_{22}(q_2^a, q_2^b) = -\cos(q_2^a - q_2^b) = -\cos\left(-\frac{3p}{4}\right) = \frac{1}{\sqrt{2}}$$

$$E_{11} + E_{12} + E_{21} - E_{22} = -2\sqrt{2}$$

One verifies that the CHSH inequality is violated!

纯态的一般结果

Gisin′s theorem: every pure bipartite entangled state in two dimensions violates the CHSH inequality.

N. Gisin, Phys. Lett. A 154, 201 (1991);

N. Gisin and A. Peres, Phys. Lett. A 162, 15 (1992).

Volume 154, number 5,6 PHYSICS LETTERS A 8 April 1991

Bell's inequality holds for all non-product states

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Received 4 February 1991; accepted for publication 7 February 1991
Communicated by J.P. Vigier

We prove that any non-product state of two-particle systems violates a Bell inequality.

In 1964 Bell [1] surprised many physicists by proving that there are states of two-quantum-particle systems that do not satisfy a certain inequality which he derived from very plausible assumptions about locality and realism in the spirit of Einstein. A huge literature has covered lots of aspects, ranging from philosophy to experimental physics, of the new field opened by Bell's 1964 paper. See, for instance, the valuable mark review of Clauser and Shimony [2], and the more recent reviews by Greenberger and co-workers [3], and by Mermin [4]. The two latter reviews also contain the more recent results on a version of Bell's result without inequalities, but valid only for systems with more than two particles.

It is well known that not all states of two-particle systems violate the Bell inequality [#1], the product states, for instance, do satisfy the inequality. In this brief note I prove that the product states are the only states that do not violate any Bell inequality. When I had the chance to discuss this equivalence between "states that violate the inequality" and "entangled states" (i.e. "non-product states") with John Bell last September, just before his sudden tragic death, I was surprised that he did not know this result. This motivates me to present today this little note which I have had on my shelves for many years and which may be part of the "folklore", known to many people but (apparently) never published. I would like to dedicate this Letter to John Bell, not only as the person who discovered the inequality and thus opened the field of "experimental methaphysics", but also as the man who taught me so much during our discussions and who amazed me many times by his capability to immediately focus on the central point under investigation.

Theorem. Let $\psi \in \mathscr{H}_1 \otimes \mathscr{H}_2$. If ψ is entangled (i.e. ψ is not a product), then ψ violates the Bell inequality, that is there are projectors a, a', b, b', such that

$$|P(a,b) - P(a,b')| + P(a',b) + P(a',b') > 2\,,$$

where

$$P(a,b) = \langle (2a-1) \otimes (2b-1) \rangle_\psi\,.$$

Proof. Let $\{\varphi_i\}$ and $\{\theta_i\}$ be orthonormal bases of $\mathscr{H}_1$ and $\mathscr{H}_2$, respectively, such that

$$\psi = \sum_i c_i \varphi_i \otimes \theta_i\,,$$

for some real c_i, with $c_1 \neq 0 \neq c_2$. Notice that the above sum runs over only one index (polar or Schmidt decomposition); the existence of two non-zero c_i's comes from the entanglement of ψ. One has

$$\psi = \chi + \chi_\perp\,,$$

where

$$\chi = c_1 \varphi_1 \otimes \theta_1 + c_2 \varphi_2 \otimes \theta_2 \in \mathbb{C}^2 \otimes \mathbb{C}^2$$

and $\chi_\perp \perp \chi$.

[#1] There are many Bell inequalities, we shall use one due to Clauser, Horne, Shimony and Holt [5].

纯态的一般结果

Popescu and Rohrlich showed that any *n*-partite pure entangled state can always be projected onto a two-partite pure entangled state by projecting $n-2$ parties onto appropriate local pure states.

Popescu, S., and D. Rohrlich, 1992, Phys. Lett. A 166, 293

Open problem: Whether the Gisin theorem can be generalized without postselection for an arbitrary *n*-partite pure entangled state?

Kai Chen, Sergio Albeverio, and Shao-Ming Fei, Phys. Rev. A 74, 050101 (2006)

Sixia Yu *et al.*, Phys. Rev. Lett. 109, 120402 (2012)

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Multi-partite Bell inequalities

- *Mermin-Ardehali-Belinskii-Klyshko [MABK] type (1990 ~1993)*

$$F=\int d\lambda\, \rho(\lambda) \frac{1}{2i}\left(\prod_{j=1}^{n}(E_x^j+iE_y^j)-\prod_{j=1}^{n}(E_x^j-iE_y^j)\right)$$

$$F\leq 2^{n/2},\quad n \text{ even},$$

$$F\leq 2^{(n-1)/2},\quad n \text{ odd}$$

- *Werner, Wolf, Zukowski, Brukner [WWZB] (2001)*

$$B=\sum_s \beta(s)\prod_{k=1}^{n} A_k(s_k)$$

$$=\frac{1}{2}B_0[A_n(0)+A_n(1)]+\frac{1}{2}B_1[A_n(0)-A_n(1)]$$

$$\mathrm{tr}(\rho B):=\mathrm{tr}\left[\rho\sum_s \beta(s)\otimes_{k=1}^{n}A_k(s_k)\right]\leqslant 1$$

Multi-partite Bell inequalities

The WWZB inequalities are given by linear combinations of the correlation expectation values

$$\sum_k f(k)E(k) \leqslant 2^n \quad f(k) = \Sigma_s S(s)(-1)^{\langle k,s\rangle}$$

$$s = s_1 \cdots s_n \in \{-1,1\}^n$$

$$S(s_1 \cdots s_n) = \pm 1; \; \langle k,s\rangle = \Sigma_{j=1}^n k_j s_j$$

Correlation function

$$E(k) = \langle \Pi_{j=1}^n A_j(k_j)\rangle_{\mathrm{av}}$$

There are 2^{2^n} *different functions S(s), and correspondingly* 2^{2^n} *inequalities.*

R. F. Werner and M. M. Wolf, Phys. Rev. A 64, 032112 (2001).

M. Zukowski and C. Brukner, Phys. Rev. Lett. 88, 210401 (2002).

Multi-partite Bell inequalities

In particular putting

$$S(s_1 \cdots s_n) = \sqrt{2}\cos[-\pi/4 + (s_1 + \cdots + s_n - n)\pi/4]$$

one recovers the Mermin-type inequalities, and for n=2 the CHSH inequality follows.

Fortunately, the set of linear inequalities is equivalent to a single nonlinear inequality

$$\sum_s \left| \sum_k (-1)^{\langle k,s \rangle} E(k) \right| \leqslant 2^n$$

which characterizes the structure of the accessible classical region for the correlation function for n-partite systems, a hyperoctahedron in 2^n dimensions, as the unit sphere of the Banach space

特殊多粒子态

The generalized GHZ states

$$|\psi\rangle = \cos\alpha|0,\ldots,0\rangle + \sin\alpha|1,\ldots,1\rangle$$

with $0 \le \alpha \le \pi/4$

When $\sin 2\alpha \le 1/\sqrt{2^{N-1}}$ and N odd

these states are proved to satisfy all the standard inequalities. This is rather surprising as they are a generalization of the GHZ states which maximally violate the MABK inequalities.

Bell Inequalities for 3 qubits

3-qubit Zukowski-Brukner inequality

$$Q(A_1B_1C_2)+Q(A_1B_2C_1)+Q(A_2B_1C_1)-Q(A_2B_2C_2)\le 2$$

3-qubit Bell inequality developed by Chen-Wu-Kwek-Oh

$$\begin{aligned}&Q(A_1B_1C_1)-Q(A_1B_2C_2)-Q(A_2B_1C_2)-Q(A_2B_2C_1)+2Q(A_2B_2C_2)\\&-Q(A_1B_1)-Q(A_1B_2)-Q(A_2B_1)-Q(A_2B_2)+Q(A_1C_1)+Q(A_1C_2)\\&+Q(A_2C_1)+Q(A_2C_2)+Q(B_1C_1)+Q(B_1C_2)+Q(B_2C_1)+Q(B_2C_2)\le 4\end{aligned}$$

where $Q(A_iB_jC_k)$ *are three-particle correlation functions defined as* $Q(A_iB_jC_k)=\langle A_iB_jC_k\rangle_{avg}$ *after many runs of experiments.*
Similar definition for two-particle correlation functions

$$Q(A_iB_j)=\langle A_iB_j\rangle_{avg}\qquad Q(A_iC_k)=\langle A_iC_k\rangle_{avg}\qquad Q(B_jC_k)=\langle B_jC_k\rangle_{avg}$$

J. L. Chen, C. F. Wu, L. C. Kwek, and C. H. Oh, Phys. Rev. Lett. 93, 140407 (2004)

Bell Inequalities for 3 qubits

All pure 2-entangled states of a three-qubit system violate a Bell inequality for probabilities.

Numerical evidence!

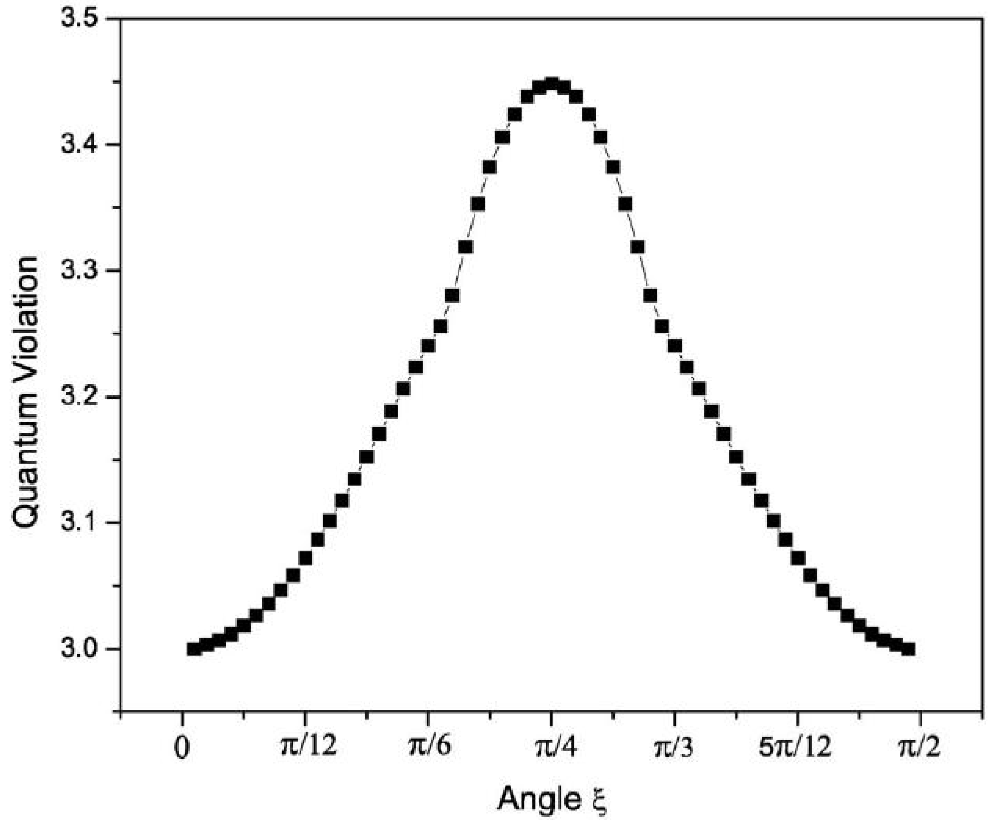


FIG. 1. Numerical results for the generalized GHZ states $|\psi\rangle_{\mathrm{GHZ}} = \cos\xi|000\rangle + \sin\xi|111\rangle$, which violate a Bell inequality for probabilities (6) except $\xi = 0$ and $\pi/2$. For the GHZ state with $\xi = \pi/4$, the Bell quantity reaches its maximum value $\frac{3}{8}(4 + 3\sqrt{3})$.

J. L. Chen, C. F. Wu, L. C. Kwek, and C. H. Oh, Phys. Rev. Lett. 93, 140407 (2004)

New Bell Inequalities for N qubits

$$\mathcal{B} = \mathcal{B}_{N-1} \otimes \frac{1}{2}(A_N + A'_N) + \mathbb{1}_{N-1} \otimes \frac{1}{2}(A_N - A'_N),$$

$$\mathcal{B}_{N-1} = \frac{1}{2^{N-1}} \sum_{s_1,\ldots,s_{N-1}=-1,1} S(s_1, \ldots, s_{N-1})$$

$$\times \sum_{k_1,\ldots,k_{N-1}=1,2} s_1^{k_1-1} \cdots s_{N-1}^{k_{N-1}-1} \otimes_{j=1}^{N-1} O_j(k_j),$$

$$O_j(1) = A_j$$
$$O_j(2) = A'_j \quad \text{with} \quad k_j = 1,2.$$

$$|\langle \mathcal{B} \rangle_{\mathrm{LHV}}| = \frac{1}{2} |\langle \mathcal{B}_{N-1}(A_N + A'_N) + (A_N - A'_N) \rangle_{\mathrm{LHV}}| \leq 1$$

- *They recover the standard Bell inequalities as a special case;*
- *They provide an exponentially increasing violation for GHZ states*
- *They essentially involve only two measurement settings per observer*
- *They yield violation for the generalized GHZ states in the whole region of for any number of qubits*

Kai Chen, Sergio Albeverio, and Shao-Ming Fei, Phys. Rev. A 74, 050101(R) (2006)

Multipartite Bound Entangled States that Violate Bell's Inequality

$$\rho_N = \frac{1}{N+1}\left(|\Psi\rangle\langle\Psi| + \frac{1}{2}\sum_{k=1}^{N}(P_k + \bar{P}_k)\right)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0^{\otimes N}\rangle + e^{i\alpha_N}|1^{\otimes N}\rangle)$$

Denoted by P_k a projector on the state

$$|\phi_k\rangle = |0\rangle_{A_1}|0\rangle_{A_2}\ldots|1\rangle_{A_k}\ldots|0\rangle_{A_{N-1}}|0\rangle_{A_N}$$

Fact: (i) the states are bound entangled, i.e., nonseparable and nondistillable if the number of parties $N \geq 4$; (ii) the states violate the Mermin-Klyshko inequality if the number of parties $N \geq 8$ and thus cannot be described by a LHV model.

This implies that (i) violation of Bell's inequality is not a sufficient condition for distillability and (ii) some bound entangled states cannot be described by a local hidden variable model.

W. Dür, Phys. Rev. Lett. 87, 230402 (2001)

Bipartite Bound Entangled States that Violate Bell's Inequality

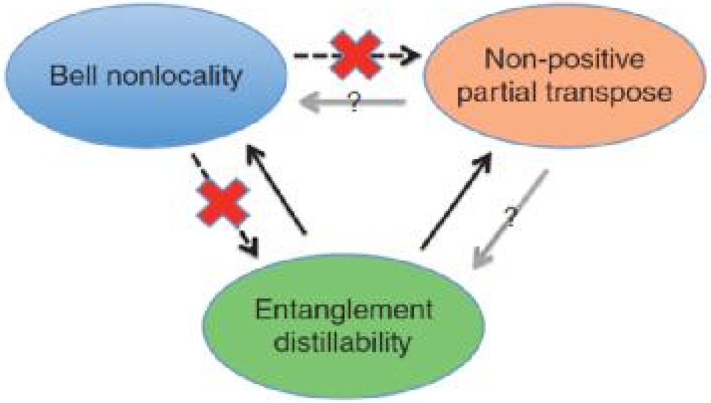


Figure 1 | Relation between different fundamental manifestations of quantum entanglement. Bell nonlocality, non-positivity under partial transposition, and entanglement distillability represent three facets of the phenomenon of entanglement. Understanding the connection between these concepts is a longstanding problem. It is well known that entanglement distillability implies both nonlocality[25] and non-positive partial transpose[17]. Peres[21] conjectured that nonlocality implies non-positivity under partial transposition and entanglement distillability; hence represented by the dashed arrows. The main result of the present work is to show that this conjecture is false, as indicated by the red crosses. To complete the diagram, it remains to be seen whether non-positive partial transpose implies distillability, one of the most important open questions in entanglement theory[45,46]. If this conjecture turns out to be false, it would remain to be seen whether non-positive partial transpose implies Bell nonlocality.

Vértesi, T. & Brunner, N. Disproving the Peres conjecture by showing Bell nonlocality from bound entanglement. Nat. Commun. 5:5297 doi: 10.1038/ncomms6297 (2014).

Quadratic Bell Inequalities as Tests for Multipartite Entanglement

Denote the spin observables on particle j, j=1, . . . , n, as A_j, A'_j . Further, S_n stands for the set of all n-particle states, and S_n^{n-1} for its subset of those states which are at most n-1-partite entangled.

For arbitrary quantum states

$$\forall\ \rho \in S_n: \qquad \langle S_n^+ \rangle_\rho^2 + \langle S_n^- \rangle_\rho^2 \leq 2^{2n-1}$$

Consider an *n*-particle state of the form $\rho_{1,\ldots,n-1} \otimes \rho_n$

$$\begin{aligned}
\langle S_n^+ \rangle^2 + \langle S_n^- \rangle^2 &= \langle S_{n-1}^+ A_n - S_{n-1}^- A_n' \rangle^2 + \langle S_{n-1}^- A_n + S_{n-1}^+ A_n' \rangle^2 \\
&= (\langle A_n \rangle \langle S_{n-1}^+ \rangle - \langle A_n' \rangle \langle S_{n-1}^- \rangle)^2 + (\langle A_n \rangle \langle S_{n-1}^- \rangle + (\langle A_n' \rangle \langle S_{n-1}^+ \rangle)^2 \\
&= (\langle A_n \rangle^2 + \langle A_n' \rangle^2)(\langle S_{n-1}^+ \rangle^2 + \langle S_{n-1}^- \rangle^2) \leq 2 \sup_{\rho \in S_{n-1}} \langle S_{n-1}^+ \rangle^2 + \langle S_{n-1}^- \rangle^2 \leq 2^{2n-2}
\end{aligned}$$

Jos Uffink, Phys. Rev. Lett. 88, 230406 (2002)

Tight Multipartite Bell's Inequalities Involving Many Measurement Settings

4 x 4 x 2 inequalities

$$\langle (C_1 + C_2)[A_1(B_1 + B_2) + A_2(B_1 - B_2)] +$$
$$(C_1 - C_2)[A_3(B_3 + B_4) + A_4(B_3 - B_4)]\rangle_{\mathrm{avg}} \leq 4$$

Let A_i with i Î {1; 2; 3; 4} stand for the predetermined local realistic values for the first observer under the local setting B_j with j Î {1; 2; 3; 4} for similar values for the second observer, and C_k with k Î {1; 2; 3; 4} for the values for the third observer (for the given run of the experiment). We assume that A_i, B_j, and C_k can take values 1 or -1.

其推广的Bell不等式的一般构造可以被Generalized GHZ states破坏！

W. Laskowski *et al.*, Phys. Rev. Lett. 93, 200401 (2004)

Bell-Klyshko Inequalities to Characterize Maximally Entangled States of n Qubits

$$\mathcal{B}_n = \mathcal{B}_{n-1} \otimes \frac{1}{2}(A_n + A'_n) + \mathcal{B}'_{n-1} \otimes \frac{1}{2}(A_n - A'_n),$$

$$\mathcal{B}'_n = \mathcal{B}'_{n-1} \otimes \frac{1}{2}(A_n + A'_n) - \mathcal{B}_{n-1} \otimes \frac{1}{2}(A_n - A'_n).$$

QM gives

$$\| \mathcal{B}_n \| \le 2^{(n-1)/2}$$

Bell-Klyshko Inequalities $\langle \mathcal{B}_n \rangle \le 1$

Theorem: *A state* $|\varphi\rangle$ *of* n *qubits maximally violates Eq.*(3), *that is,*

$$\langle \varphi | \mathcal{B}_n | \varphi \rangle = 2^{(n-1)/2},$$

if and only if it can be obtained by a local unitary transformation of the GHZ state $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0 \cdots 0\rangle + |1 \cdots 1\rangle)$, *i.e.*,

$$|\varphi\rangle = U_1 \otimes \cdots \otimes U_n |\mathrm{GHZ}\rangle$$

Ze-Qian Chen, Phys. Rev. Lett. 93, 110403 (2004)

Bell Inequalities for Hyperentangled States

Hyperentanglement has been demonstrated in recent experiments with two photons entangled in 2 degrees of freedom (polarization and path) and in 3 degrees of freedom (polarization, path, and time-energy)

Consider two particles 1 and 2 prepared in the state

$$|\psi\rangle^{(j)} = \tfrac{1}{2}(|00\rangle_1^{(j)}|00\rangle_2^{(j)} + |01\rangle_1^{(j)}|01\rangle_2^{(j)} + |10\rangle_1^{(j)}|10\rangle_2^{(j)} - |11\rangle_1^{(j)}|11\rangle_2^{(j)}).$$

$$|\Psi\rangle = \bigotimes_{j=1}^{N} |\psi\rangle^{(j)}$$

$$X_k^{(j)} = \sigma_x^{(j)} \otimes \mathbb{1}^{(j)}, \qquad Y_k^{(j)} = \sigma_y^{(j)} \otimes \mathbb{1}^{(j)},$$
$$Z_k^{(j)} = \sigma_z^{(j)} \otimes \mathbb{1}^{(j)},$$

$$x_1^{(j)} = \mathbb{1}^{(j)} \otimes \sigma_x^{(j)}, \qquad y_2^{(j)} = \mathbb{1}^{(j)} \otimes \sigma_y^{(j)},$$
$$z_2^{(j)} = \mathbb{1}^{(j)} \otimes \sigma_z^{(j)},$$

$$\begin{aligned}
\beta = {} & \langle X_1^{(1)} X_2^{(1)} z_2^{(1)} \ldots X_1^{(N-1)} X_2^{(N-1)} z_2^{(N-1)} X_1^{(N)} X_2^{(N)} z_2^{(N)} \rangle \\
& - \langle X_1^{(1)} X_2^{(1)} z_2^{(1)} \ldots X_1^{(N-1)} X_2^{(N-1)} z_2^{(N-1)} Y_1^{(N)} Y_2^{(N)} z_2^{(N)} \rangle \\
& + \langle X_1^{(1)} X_2^{(1)} z_2^{(1)} \ldots X_1^{(N-1)} X_2^{(N-1)} z_2^{(N-1)} X_1^{(N)} x_1^{(N)} Y_2^{(N)} y_2^{(N)} \rangle \\
& + \langle X_1^{(1)} X_2^{(1)} z_2^{(1)} \ldots X_1^{(N-1)} X_2^{(N-1)} z_2^{(N-1)} Y_1^{(N)} x_1^{(N)} X_2^{(N)} y_2^{(N)} \rangle \\
& - \langle X_1^{(1)} X_2^{(1)} z_2^{(1)} \ldots Y_1^{(N-1)} Y_2^{(N-1)} z_2^{(N-1)} X_1^{(N)} X_2^{(N)} z_2^{(N)} \rangle + \ldots \\
& + \langle Y_1^{(1)} x_1^{(1)} X_2^{(1)} y_2^{(1)} \ldots Y_1^{(N)} x_1^{(N)} X_2^{(N)} y_2^{(N)} \rangle,
\end{aligned}$$

For any EPR-type local realistic theory

$$\beta_{\mathrm{EPR}} = 2^N$$

$$\beta_{\mathrm{QM}} = 4^N$$

A. Cabello, Phys. Rev. Lett. 97, 140406 (2006)

Bell Inequalities for Multipartite Arbitrary Dimensional Systems

$$\mathcal{B} = \frac{1}{2^3} \sum_{n=1}^{d-1} \left\langle \prod_{j=1}^{3} (A_j^n + \omega^{n/2} B_j^n) \right\rangle + \text{c.c.}$$

$$\mathcal{B} \leq \frac{3d}{4} - 1, \quad \text{if } d \text{ is even.}$$

Consider three observers and allow each to independently choose one of two variables. The variables are denoted by A_j and B_j for the jth observer. Each variable takes, as its value, an element in the set S ={1,w, w^2, ..., w^{d-1}} where the elements of S are the dth roots of unity over the complex field.

W. Son, Jinhyoung Lee, and M. S. Kim, Phys. Rev. Lett. 96, 060406 (2006)

Asymptotic Violation of Bell Inequalities and Distillability

A bipartite state ρ is distillable if, and only if, there exists a positive integer m and a SLO map Ω such that $\Omega[\rho^{\otimes m}]$ violates CHSH.

Result 5.—Consider an N-partite state ρ, an integer m, and a SLO map Ω such that the WWZB inequality β is asymptotically violated by the amount $\beta[\Omega(\rho^{\otimes m})]$ in the range

$$1 < 2^{(N-G-1)/2} < \beta[\Omega(\rho^{\otimes m})] \leq 2^{(N-G)/2}. \qquad (8)$$

Then, pure-state entanglement can be extracted from ρ when the parties join into groups of at most G people.

Stochastic local operations without communication (SLO)

L. Masanes, Phys. Rev. Lett. 97, 050503 (2006)

entangled $\Longleftrightarrow$ nonsimulable in general,

distillable $\Longleftrightarrow$ nonsimulable in the asymptotic scenario.

The second equivalence is only proved for the case $K = M = 2$.

Consider N separated parties, denoted by n 1; . . . ; N, each having a physical system which can be measured with one among M observables with K outcomes each.

MAY 9, 2018

The BIG Bell Test—Global physics experiment challenges Einstein with the help of 100,000 volunteers

by ICFO

The BIG Bell Test Initiative, November 30th, 2016. Credit: ICFO

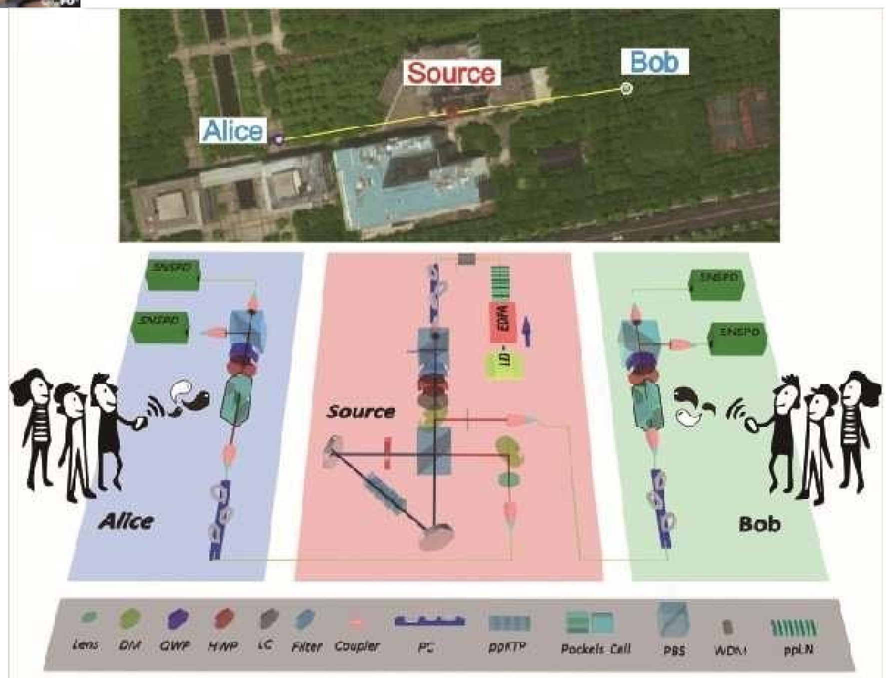
On November 30th, 2016, more than 100,000 people around the world contributed to a suite of first-of-a-kind quantum physics experiments known as The BIG Bell Test. Using smartphones and other internet-connected devices, participants contributed unpredictable bits, which determined how entangled atoms, photons, and superconducting devices were measured in 12 laboratories around the world. Scientists used the human input to close a stubborn loophole in tests of Einstein's principle of local realism. The results have now been analysed, and are reported in this week's *Nature*.

https://phys.org/news/2018-05-big-bell-testglobal-physics-einstein.html

The BIG Bell Test Initiative,
November 30th 2016. Credit: ICFO

https://phys.org/news/2018-05-big-bell-testglobal-physics-einstein.html

The setup of the experiment.
Credit: Jian-Wei Pan's Group



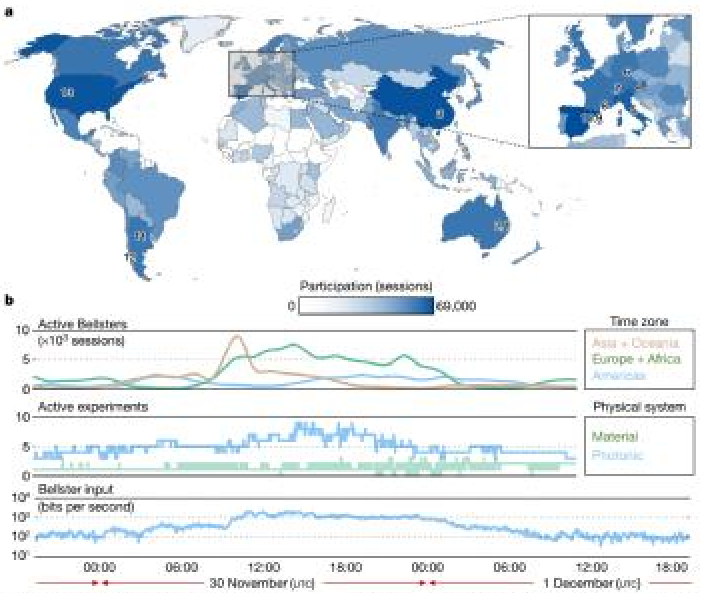


Fig. 2 | Geography and timing of the BBT. a, Locations of the 13 BBT experiments, ordered from east to west. The index numbers label the experiments, which are summarized in Table 1. Shading shows total sessions by country. Eight sessions from Antarctica are not shown. Map created by G. Colangelo using data from OpenStreetMaps, rendered in Wolfram Mathematica. **b,** Temporal evolution of the project. The top graph shows the number of live sessions versus time for different-continent groups, which exhibits a large drop in the local early morning in each region. The spike in the participation of the Asian group around 11:00 UTC coincides with a live-streamed event in Barcelona, hosted by D. Jiménez and the CosmoCaixa science museum, re-broadcast live in Chinese by L.-F. Yuan and the University of Science and Technology of China (USTC). The middle graph shows the number of connected laboratories versus time, divided into experiments using only photons and experiments with at least one material component (such as atoms or superconductors). The bottom graph shows the input bitrate versus time. The data flow remains nearly constant despite regional variations, with Asian Bellsters handing off to Bellsters from the Americas in the critical period 12:00–00:00 UTC. Session data from Google Analytics.

Challenging local realism with human choices

•The BIG Bell Test Collaboration

Nature volume 557, pages 212–216 (2018)

Table 1 | Experiments carried out as part of the BBT, ordered by longitude, from east to west

Experiment	Lead Institution	Location	Entangled system	Rate (bps)	Inequality	Result	Stat. sig.
(1)	Griffith University	Brisbane, Australia	Photon polarization	4	$S_{16} \leq 0.511$	$S_{16}=0.965\pm0.008$	57σ
(2)	University of Queensland & EQUS	Brisbane, Australia	Photon polarization	3	$\lvert S\rvert \leq 2$	$S_{AD}=2.75\pm0.05$	15σ
						$S_{BC}=2.79\pm0.05$	16σ
(3)	USTC	Shanghai, China	Photon polarization	10^3	PRBLG[30]	$I_0=0.10\pm0.05$	N/A
(4)	IQOQI	Vienna, Austria	Photon polarization	1.61×10^3	$\lvert S\rvert \leq 2$	$S_{HRN}=2.639\pm0.008$	81σ
						$S_{QRN}=2.643\pm0.006$	116σ
(5)	Sapienza	Rome, Italy	Photon polarization	0.62	$B \leq 1$	$B=1.225\pm0.007$	32σ
(6)	LMU	Munich, Germany	Photon–atom	1.7	$\lvert S\rvert \leq 2$	$S_{HRN}=2.427\pm0.0223$	19σ
						$S_{QRN}=2.413\pm0.0223$	18.5σ
(7)	ETHZ	Zurich, Switzerland	Transmon qubit	3×10^3	$\lvert S\rvert \leq 2$	$S=2.3066\pm0.0012$	$P<10^{-99}$
(8)	INPHYNI	Nice, France	Photon time bin	2×10^3	$\lvert S\rvert \leq 2$	$S=2.431\pm0.003$	140σ
(9)	ICFO	Barcelona, Spain	Photon–atom ensemble	125	$\lvert S\rvert \leq 2$	$S=2.29\pm0.10$	2.9σ
(10)	ICFO	Barcelona, Spain	Photon multi-frequency bin	20	$\lvert S\rvert \leq 2$	$S=2.25\pm0.08$	3.1σ
(11)	CITEDEF	Buenos Aires, Argentina	Photon polarization	1.02	$\lvert S\rvert \leq 2$	$S=2.55\pm0.07$	7.8σ
(12)	UdeC	Concepción, Chile	Photon time bin	5.2×10^4	$\lvert S\rvert \leq 2$	$S=2.43\pm0.02$	20σ
(13)	NIST	Boulder, USA	Photon polarization	10^5	$K \leq 0$	$K=(1.65\pm0.20)\times10^{-4}$	8.7σ

Descriptions of the experiments are given in Supplementary Information. Stat. sig., statistical significance; indicates the number of standard deviations assuming independent and identically distributed trials, unless otherwise indicated. Rate indicates the peak rate (in bits per second, bps) at which bits were used by the experiments. Owing to the limited rate of Bellster input, some experiments had dead times. B, K, S, S_{AD}, S_{BC}, S_{HRN} and S_{QRN} indicate Bell parameters for the respective experiments and S_{16} is the steering parameter (see Supplementary Information). I_0 indicates the minimum Pütz–Rosset–Barnea–Liang–Gisin measure of setting-choice independence, consistent with the observed BIV.

USTC, University of Science and Technology of China; EQUS, Centre for Engineered Quantum Systems; IQOQI, Institute for Quantum Optics and Quantum Information; INPHYNI, Institut de Physique de Nice; ICFO, Institut de Ciencies Fotoniques; LMU, Ludwig Maximilians Universität; ETHZ, ETH Zurich; CITEDEF, Institute of Scientific and Technical Research for Defence; UdeC, University of Concepción; NIST, National Institute of Standards and Technology.

中国科学技术大学 陈凯

第三章 量子关联表现

1. 局域实在论
2. Bell不等式
3. 量子游戏（Quantum Games）
4. Bell不等式检验
5. 无不等式的Bell定理
6. 多体Bell不等式
7. 无漏洞Bell 不等式检验

无漏洞Bell 不等式检验

Loophole-free Bell test – 2015

In 1935, Einstein asked a profound question about our understanding of Nature: are objects only influenced by their nearby environment? Or could, as predicted by quantum theory, looking at one object sometimes instantaneously affect another far-away object? We tried to answer that question, by performing a loophole-free Bell test.

1. B. Hensen *et al.*, "Loophole-free Bell Inequality Violation Using Electron Spins Separated by 1.3 Kilometres," **Nature 526, 682 (2015)**.
2. M. Giustina *et al.*, "Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons," **Phys. Rev. Lett. 115, 250401 (2015)**.
3. L. K. Shalm *et al.*, "Strong Loophole-Free Test of Local Realism," **Phys. Rev. Lett. 115, 250402 (2015)**.

无漏洞Bell 不等式检验

PRL 115, 250401 (2015) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons

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Local realism is the worldview in which physical properties of objects exist independently of measurement and where physical influences cannot travel faster than the speed of light. Bell's theorem states that this worldview is incompatible with the predictions of quantum mechanics, as is expressed in Bell's inequalities. Previous experiments convincingly supported the quantum predictions. Yet, every experiment requires assumptions that provide loopholes for a local realist explanation. Here, we report a Bell test that closes the most significant of these loopholes simultaneously. Using a well-optimized source of entangled photons, rapid setting generation, and highly efficient superconducting detectors, we observe a violation of a Bell inequality with high statistical significance. The purely statistical probability of our results to occur under local realism does not exceed 3.74×10^{-31}, corresponding to an 11.5 standard deviation effect.

DOI: 10.1103/PhysRevLett.115.250401 PACS numbers: 03.65.Ud, 42.50.Xa

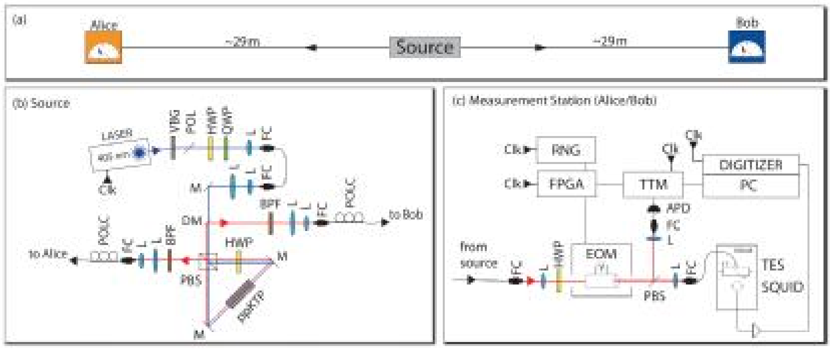

FIG. 1 (color). (a) Schematic of the setup. (b) Source: The source distributed two polarization-entangled photons between the two identically constructed and spatially separated measurement stations *Alice* and *Bob* (distance ≈58 m), where the polarization was analyzed. It employed type-II spontaneous parametric down-conversion in a periodically poled crystal (ppKTP), pumped with a 405 nm pulsed diode laser (pulse length: 12 ns FWHM) at 1 MHz repetition rate. The laser light was filtered spectrally by a volume Bragg grating (VBG) (FWHM: 0.3 nm) and spatially by a single-mode fiber. The ppKTP crystal was pumped from both sides in a Sagnac configuration to create polarization entanglement. Each pair was split at the polarizing beam splitter (PBS) and collected into two different single-mode fibers leading to the measurement stations. (c) Measurement stations: In each measurement station, one of two linear polarization directions was selected for measurement, as controlled by an electro-optical modulator (EOM), which acted as a switchable polarization rotator in front of a plate PBS. Customized electronics (FPGA) sampled the output of a random number generator (RNG) to trigger the switching of the EOM. The transmitted output of the plate PBS was coupled into a fiber and delivered to the TES. The signal of the TES was amplified by a SQUID and additional electronics, digitized, and recorded together with the setting choices on a local hard drive. The laser and all electronics related to switching or recording were synchronized with clock inputs (Clk). Abbreviations: APD, avalanche photodiode (see Fig. 2); BPF, bandpass filter; DM, dichroic mirror; FC, fiber connector; HWP, half-wave plate; L, lens; POL, polarizer; M, mirror; POLC, manual polarization controller; QWP, quarter-wave plate; SQUID, superconducting quantum interference device; TES, transition-edge sensor; TTM, time-tagging module.

PRL 115, 250402 (2015) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

Strong Loophole-Free Test of Local Realism*

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(Received 10 November 2015; published 16 December 2015)

We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using fast random number generators and high-speed polarization measurements. A high-quality polarization-entangled source of photons, combined with high-efficiency, low-noise, single-photon detectors, allows us to make measurements without requiring any fair-sampling assumptions. Using a hypothesis test, we compute p values as small as 5.9×10^{-9} for our Bell violation while maintaining the spacelike separation of our events. We estimate the degree to which a local realistic system could predict our measurement choices. Accounting for this predictability, our smallest adjusted p value is 2.3×10^{-7}. We therefore reject the hypothesis that local realism governs our experiment.

DOI: 10.1103/PhysRevLett.115.250402 PACS numbers: 03.65.Ud, 42.50.Xa, 42.65.Lm

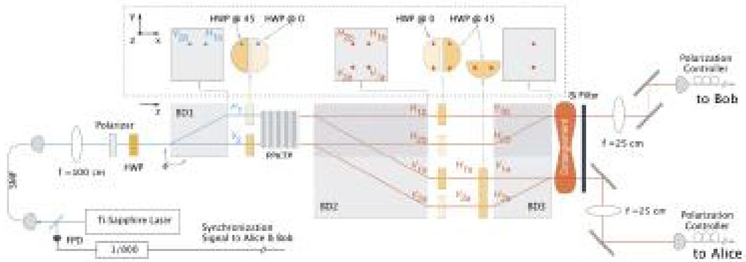


FIG. 1 (color online). Schematic of the entangled photon source. A pulsed 775-nm-wavelength Ti:sapphire picosecond mode-locked laser running at a 79.3-MHz repetition rate is used as both a clock and a pump in our setup. A fast photodiode (FPD) and divider circuit are used to generate the synchronization signal that is distributed to Alice and Bob. A polarization-maintaining single-mode fiber (SMF) then acts as a spatial filter for the pump. After exiting the SMF, a polarizer and half-wave plate (HWP) set the pump polarization. To generate entanglement, a periodically poled potassium titanyl phosphate (PPKTP) crystal designed for type-II phase matching is placed in a polarization-based Mach-Zehnder interferometer formed using a series of HWPs and three beam displacers (BD). At BD1 the pump beam is split into two paths (1 and 2): The horizontal (H) component of polarization of the pump translates laterally in the x direction, while the vertical (V) component of polarization passes straight through. Tilting BD1 sets the phase, ϕ, of the interferometer to 0. After BD1 the pump state is $(\cos(16°)|H_1\rangle + \sin(16°)|V_2\rangle)$. To address the polarization of the paths individually, semicircular wave plates are used. A HWP in path 2 rotates the polarization of the pump from vertical to horizontal. A second HWP at 0° is inserted into path 1 to keep the path lengths of the interferometer balanced. The pump is focused at two spots in the crystal, and photon pairs at a wavelength of 1550 nm are generated in either path 1 or 2 through the process of spontaneous parametric down-conversion. After the crystal, BD2 walks the V-polarized signal photons down in the y direction (V_{1s} and V_{2s}), while the H-polarized idler photons pass straight through (H_{1i} and H_{2i}). The x–y view shows the resulting locations of the four beam paths. HWPs at 45° correct the polarization, while HWPs at 0° provide temporal compensation. BD3 then completes the interferometer by recombining paths 1 and 2 for the signal and idler photons. The two down-conversion processes interfere with one another, creating the entangled state in Eq. (2). A high-purity silicon wafer with an antireflection coating is used to filter out the remaining pump light. The idler (signal) photons are coupled into a SMF and sent to Alice (Bob).

Bell 不等式的更多内涵

- *Quantum Communication Complexity*
- *Classifying N-Qubit Entanglement*
- *Maximal Violation of Bell Inequalities for Mixed States*
- *Error Correcting Bell Inequalities*
- *Stronger Quantum Correlations with Loophole-Free Postselection*
- *Violation of Bell's Inequality beyond Tsirelson's Bound*
- *Bell's Inequalities in quantum network scenarios*

Bell Inequalities的推广

- *Bell inequalities for M qubits (M>3)*

- *Bell inequalities for M qudits (M>3)*

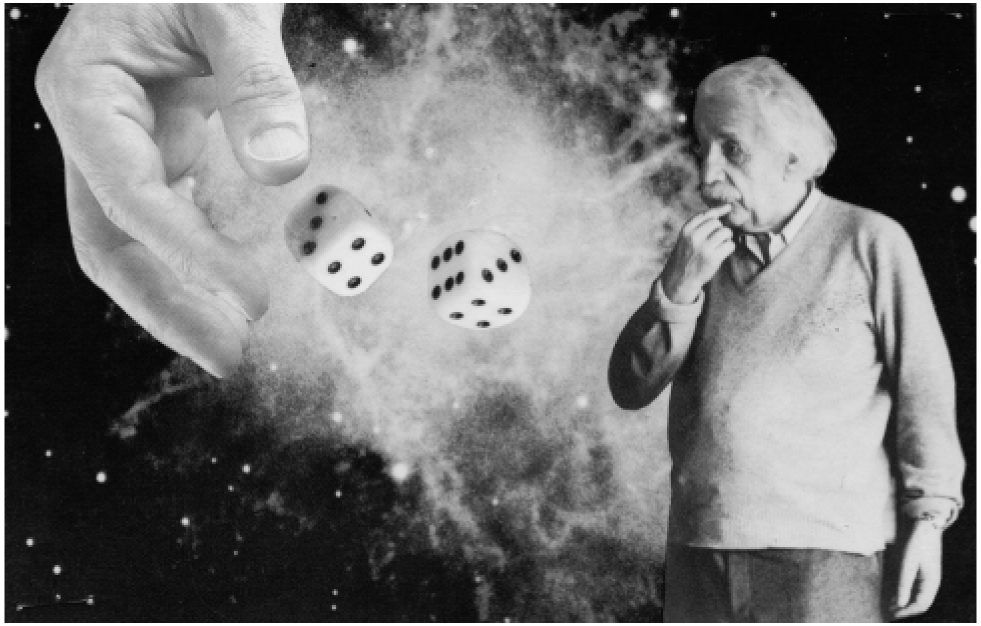
- M-qudit: M particles in *d*-dimensional Hilbert space

- M particles, arbitrary dimension, multiple settings, multiple outcomes

Bohr-Einstein debates

Einstein:

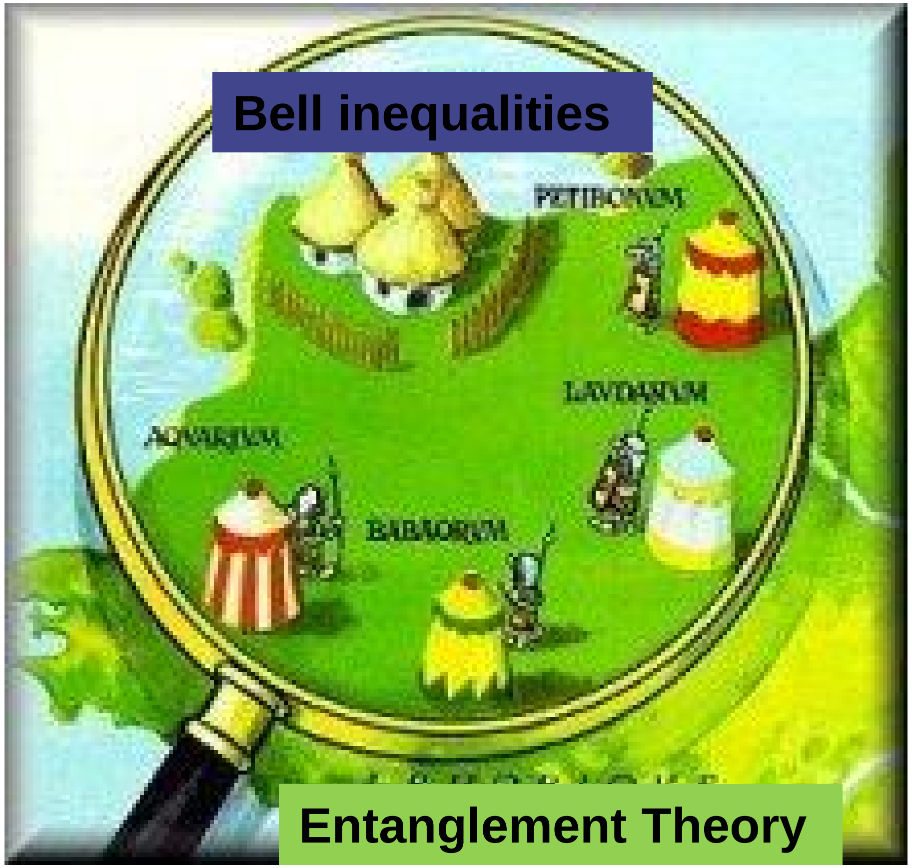
I can't believe God plays dice with the universe.



Bohr:

Albert, stop telling God what to do.

A bit of history



From Scarani

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谢谢