Stochastic Analysis of Optimal Base Station Energy Saving in Cellular Networks with Sleep Mode

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***Abstract*—In this letter, we study the energy saving problem by switching off some macro Base Stations (BSs) under downlink coverage and uplink power constraints. Firstly, we derive the expressions of average coverage probability and users' power consumption with stochastic geometry tools. Then, based on the closed-form results, power adjustment and introducing micro BSs schemes are designed to keep the original coverage performance. In addition, we formulate a BS energy consumption minimization problem considering the users' power constraints, and jointly determine the optimal proportion of sleep macro BSs, transmission power of active macro BSs, and density of introduced micro BSs. Numerical results show that with our schemes, BSs' energy consumption can be significantly reduced while guaranteeing the downlink coverage and user power consumption performance.**

***Index Terms*—Cellular networks, energy saving, sleep mode, stochastic geometry, power adjustment, micro BS deployment.**

I. Introduction

With the increasing demand for mobile Internet services, it is becoming more and more difficult for a conventional cellular system to meet required capacity. To handle this problem, Micro BSs (MiBSs) are deployed to offload the traffic of Macro BSs (MaBSs) and improve the Spectral Efficiency (SE). MiBSs and MaBSs constitute the heterogeneous cellular networks. Besides, meeting traffic demands will cause a significant increase in operator energy consumption. The sharp rise in energy cost and carbon dioxide emission bring new focus on energy saving problem in all components, especially the BS, which consumes about 60-80% energy [1].

One important way to address the energy saving problem is to introduce active/sleep mode to BSs. When the system load is light enough, some BSs can be switched off and their traffic is accommodated by the active BSs. Obviously, with some BSs in sleep mode, Quality of Service (QoS) may be deteriorated. To save energy while guaranteeing acceptable QoS, researchers investigate the sleep operations considering delay [1], blocking probability [2], SE [3] and coverage performance [4]. Among them, [1] can be applied to arbitrary network topology and [2] is based on the hexagonal homogeneous cellular model where each MaBS is deployed at the center of a hexagonal lattice. The SE and coverage performance are dependent on the network topology. Unfortunately, the hexagonal model is idealized and intractable for interference characterization, especially which is hard to extend to heterogeneous cellular networks where MiBSs are randomly distributed. Using stochastic geometry, a more suitable and tractable model is proposed in [5], where the locations of BSs are modeled as a Poisson Point Process (PPP), and this model is used in [3][4].

In this paper, based on the PPP model, we take a step forward and study the energy saving problem through switching off some MaBSs in heterogeneous cellular networks. Different from previous work [4] which allows coverage performance loss, our proposed analysis provides a novel way to preserve the original coverage performance through adjusting transmission power or introducing additional MiBSs after switching off some MaBSs. Besides, we analyze the impact of sleep mode on uplink power consumption performance which determines the battery lifetime of devices and user experience. Based on the analysis, we formulate a BS energy consumption minimization problem while ensuring that uplink power consumption is below a given torrent value, and jointly determine the optimal proportion of sleep MaBSs, transmission power of active MaBSs and density of introduced MiBSs. Numerical results conform the effectiveness of our proposed schemes.

II. System Model and Assumptions

We consider a heterogeneous networks consisting of 2 tiers of BSs, where tier m and M represent MiBSs and MaBSs. MiBSs and MaBSs are modeled as independent homogeneous PPPs of intensities λ_m and λ_M. This model has been proved as a tractable yet accurate model recently [3-6]. We assume that BSs in tier i use the same link transmission power $\{P_i\}_{i=m,M}$. Besides, universal frequency reuse is applied and we assume that each BS allocates all the N channel resources to its served User Equipments (UEs). UEs are also located according to a homogeneous PPP. Rayleigh fading channel is assumed here. For a typical UE, its received power from a BS in tie i is $P_i h r^{-\alpha}$, where $h \sim \exp(1)$, $\alpha > 2$ is the path loss exponent and r is the communication distance. UE is associated to the strongest BS in terms of long-term average received power, i.e., it is connected to the tier $s = \arg\max_{i=m,M}\{P_i {r_i}^{-\alpha}\}$, where r_i denotes the distance between the UE and its nearest BS in the ith tier. We assume that all the UEs utilize distance-proportional fractional power control of the form $P_{ur}{r_s}^{\alpha}$ to maintain the average uplink received signal power P_{ur}. Thus, as UE connects to a farther BS, the transmission power increases, which is an important consideration for battery-powered mobile devices.

We build the power consumption model of BS as follows:

$$P_{i,tot} = a_i P_i N + b_i, i = m, M \tag{1}$$

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where the coefficient $\{a_i\}_{i=m,M}$ accounts for power consumption that scales with the average radiated power. The term $\{b_i\}_{i=m,M}$ models the static power consumed by signal processing, battery backup and cooling.

An operator would save energy through turning some BSs into sleep mode so that they consume negligible energy when the system traffic is low enough to be served by less BSs. Because MaBSs' power consumption is generally much larger than that of MiBSs, we just apply sleep mode to MaBSs. A possible and simple implementation of sleep mode is random sleep strategy where each MaBS is turned off with probability β and continues to operate in active mode with probability $1-\beta$. According to the PPP thinning property, this procedure leads to the fact that the remaining active MaBSs follow a new PPP with $(1-\beta)\lambda_M$.

After switching off some MaBSs, the coverage and UE power consumption performance will be deteriorated. We first discuss the impact of switching off MaBSs on the coverage and uplink power consumption performance in Section III. Then based on the results, the optimal β and energy saving under QoS performance constraints are given in Section IV.

III. Analysis on Downlink Coverage and Uplink Power Consumption

A. Downlink coverage performance analysis

The coverage probability of a UE is defined as

$$P_C = \mathbb{P}[SINR > \gamma], \tag{2}$$

where γ is the outage threshold. Note that the outage probability is $1-P_C = \mathbb{P}[SINR < \gamma]$, which is also the cumulative distribution function (cdf) of the UE's Signal to Interference plus Noise Ratio (SINR). (2) can be calculated by:

$$P_C = \mathbb{E}_{x_i}(\mathbb{P}[SINR(x_i) \geq \gamma]), \tag{3}$$

where $\mathbb{P}[SINR(x_i) \geq \gamma]$ is the coverage probability given that the UE is associated with the BS located at point x_i (belonging to ith tier) and $\mathbb{E}_{x_i}(.)$ represents taking the expectation with respect to x_i.

Lemma 1. *The coverage probability for a typical randomly located UE in the heterogeneous networks consisting of MaBSs and MiBSs is:*

$$\begin{aligned} P_C =& \pi(\lambda_m P_m^{2/\alpha} + \lambda_M P_M^{2/\alpha}) \int_{t=0}^{\infty} \exp\big(-(\lambda_M P_M^{2/\alpha} + \lambda_m P_m^{2/\alpha}) \\ & \mathcal{Z}(\gamma,\alpha) t\big) \exp(-\gamma \sigma^2 t^{\frac{\alpha}{2}}) dt, \end{aligned} \tag{4}$$

where $\mathcal{Z}(\gamma,\alpha) = 1 - 2\gamma {}_2F_1[1; 1-2/\alpha; 2-2/\alpha; -\gamma]$ *and* ${}_2F_1[\boldsymbol{.}]$ *denotes the Gauss hypergeometric function.*

Proof: After achieving the location distributions of served BS x_i and $\mathbb{P}[SINR(x_i) \geq \gamma]$, [6] gives out the result as follows (Theorem 1 in [6]):

$$\begin{aligned} P_C =& \lambda_M \int_{\mathrm{r}=0}^{\infty} 2\pi r \exp(-(\lambda_M + \lambda_m P_m/{P_M}^{2/\alpha}) \\ & \mathcal{Z}(\gamma,\alpha) r^2) \exp(-(\gamma/P_M)\sigma^2 r^\alpha) dr \\ & + \lambda_m \int_{\mathrm{r}=0}^{\infty} 2\pi r \exp(-(\lambda_M P_M/{P_m}^{2/\alpha} + \lambda_m) \\ & \mathcal{Z}(\gamma,\alpha) r^2) \exp(-(\gamma/P_m)\sigma^2 r^\alpha) dr. \end{aligned} \tag{5}$$

The BS density and BS transmission power, which decide the system energy consumption, are separated in (5). For the first part, we use transformation $t = r^2/P_M^{2/\alpha}$. For the second part we use transformation $t = r^2/P_m^{2/\alpha}$. Then after some algebraic manipulations, we get the formula (4), which combines BS density and BS transmission power together as a term, i.e, $\lambda_m P_m^{2/\alpha} + \lambda_M P_M^{2/\alpha}$. □

We can get the coverage probability after MaBS sleep mode is applied through replacing λ_M with $(1-\beta)\lambda_M$ in formula (4). Obviously, the performance will be seriously deteriorated. Besides, from formula (4), we find that P_C is dependent on the term $\lambda_m P_m^{2/\alpha} + \lambda_M P_M^{2/\alpha}$. Only if we keep this combination unchanged, then the coverage probabilities can be maintained. Therefore, we have the following interesting result.

Theorem 1. *For heterogeneous cellular networks, after switching off $\beta\lambda_M$ MaBSs, we can keep the original coverage performance with any of the following schemes:*

- *(Power Adjustment scheme, PA): increasing the transmission power of remaining active MaBSs from P_M to $P_M(1-\beta)^{-\alpha/2}$;*
- *(MiBS Deployment scheme, MD): additionally introducing $\beta\lambda_M(P_M/P_m)^{2/\alpha}$ MiBSs;*
- *(Joint Power adjustment and MiBS deployment scheme, JPM): adjusting the transmission power of remaining active MaBSs from P_M to P_M' and additionally introducing λ_m' MiBSs, where P_M' and λ_m' satisfy*

$$\lambda_M {P_M}^{2/\alpha} = \lambda_m' {P_m}^{2/\alpha} + (1-\beta)\lambda_M {P_M'}^{2/\alpha}. \tag{6}$$

Remark 1: Although it is impossible to deploy MiBSs as soon as switching off $\beta\lambda_M$ MaBSs, we can pre-deploy some MiBSs. After switching off $\beta\lambda_M$ MaBSs, we can switch on λ_m' additional pre-deployed MiBSs which is equivalent to introducing λ_m' MiBSs. Noted that PA and MD are two special cases of JPM when $\lambda_m' = 0$ and $P_M' = P_M$, respectively. Thus, we only focus on JPM in the following parts. Besides, with these schemes, despite keeping the same coverage performance, the SINR and SE distribution will also not change after switching off $\beta\lambda_M$ MaBSs. However, it should be noted that the UE capacity will become lower due to less available resources after switching off some MaBSs. Therefore, the sleep mode can only be applied when the system traffic is low enough to be served by less BSs.

B. Uplink power consumption performance analysis

After some BSs are switched off, their traffic load should re-associate to the active BSs, which are usually much farther away. Hence, another important performance that will be seriously impacted is the uplink power consumption. To the best of our knowledge, there is no discussion on this topic, which relates to the battery lifetime of devices and UE experience, hence is meaningful. UEs' power consumption is dependent on the distance from them to their serving BSs. Therefore, we first analyze the distance distribution, then derive the average uplink power consumption performance.

Lemma 2. *For heterogeneous cellular networks, the probability density function (pdf) $f_{r_s}(x)$ of the distance r_s between* *a typical UE and its serving BS is*

$$f_{r_s}(x) = 2\pi\lambda_M x \exp\{-\pi(\lambda_M + \lambda_m(\frac{P_m}{P_M})^{\frac{2}{\alpha}})x^2\} + 2\pi\lambda_m x \exp\{-\pi(\lambda_m + \lambda_M(\frac{P_M}{P_m})^{\frac{2}{\alpha}})x^2\}. \tag{7}$$

Proof: The pdf of r_m and r_M can be written as [5]:

$$f_{r_m}(x) = 2\pi\lambda_m x e^{-\pi\lambda_m x^2}; \quad f_{r_M}(x) = 2\pi\lambda_M x e^{-\pi\lambda_M x^2}. \tag{8}$$

Thus, the cdf $\mathbb{P}(r_s \le R)$ of r_s can be expressed as

$$\begin{aligned}&\mathbb{P}(r_M \le R, r_M < r_m(\frac{P_M}{P_m})^{\frac{1}{\alpha}}) + \mathbb{P}(r_m \le R, r_M > r_m(\frac{P_M}{P_m})^{\frac{1}{\alpha}}) \\ &= \frac{\lambda_M}{\lambda_M + \lambda_m(\frac{P_M}{P_m})^{\frac{-2}{\alpha}}}(1-\exp\{-\pi(\lambda_M + \lambda_m(\frac{P_M}{P_m})^{\frac{-2}{\alpha}})R^2\}) \\ &+ \frac{\lambda_m}{\lambda_m + \lambda_M(\frac{P_M}{P_m})^{\frac{2}{\alpha}}}(1-\exp\{-\pi(\lambda_m + \lambda_M(\frac{P_M}{P_m})^{\frac{2}{\alpha}})R^2\}).\end{aligned} \tag{9}$$

(7) can be obtained from the derivative of (9) with R. □

Combining the pdf $f_{r_s}(x)$ and distance-proportional fractional power control, we can get the closed-form expression of average UE power consumption as Theorem 2.

Theorem 2. *For heterogeneous cellular networks, the average uplink power consumption of a typical UE is*

$$\begin{aligned}P_u &= \int_0^\infty P_{ur} x^\alpha f_{r_s}(x)dx \\ &= \frac{P_{ur}\Gamma(\frac{\alpha}{2}+1)[\lambda_M {P_M}^{\frac{2+\alpha}{\alpha}} + \lambda_m {P_m}^{\frac{2+\alpha}{\alpha}}]}{\pi^{\frac{\alpha}{2}}(\lambda_M {P_M}^{\frac{2}{\alpha}} + \lambda_m {P_m}^{\frac{2}{\alpha}})^{\frac{\alpha}{2}+1}}.\end{aligned} \tag{10}$$

Remark 2: We can get the result after switching off $\beta\lambda_M$ MaBSs through replacing λ_M with $(1-\beta)\lambda_M$. Obviously, the performance will be seriously impacted. With (6), we find that given UEs' torrent value P_{exp}, JPM can guarantee the uplink power consumption is in UEs' tolerance range only if

$$(1-\beta){P'_M}^{\frac{\alpha+2}{\alpha}} + [{P_M}^{\frac{2}{\alpha}} - (1-\beta){P'_M}^{\frac{2}{\alpha}}]P_m \le A, \tag{11}$$

where

$$A \triangleq [\frac{P_{\exp}\pi^{\frac{\alpha}{2}}(\lambda_M {P_M}^{\frac{2}{\alpha}} + \lambda_m {P_m}^{\frac{2}{\alpha}})^{\frac{\alpha}{2}+1}}{\Gamma(\frac{\alpha}{2}+1)P_{ur}} - \lambda_m {P_m}^{\frac{2+\alpha}{\alpha}}]/\lambda_M.$$

IV. Optimal Energy Saving

In this section, we design the optimal proportion of sleep MaBSs β, active MaBSs' transmission power P'_M and introduced MiBSs' density λ'_m to minimize BS energy consumption under UE power consumption constraint for JPM. The problem is formulated as:

$$\begin{aligned}\textbf{P0:}\quad &\underset{\beta, P'_M, \lambda'_m}{\text{minimize}} \quad E_{net} \\ &\text{subject to} \quad (6), (11) \text{ and} \\ &\qquad 0 \le \beta \le 1, \\ &\qquad 0 \le P'_M \le P_{M,max}, \\ &\qquad 0 \le \lambda'_m \le \lambda_{m,max} - \lambda_m,\end{aligned}$$

where the last two constraints is the maximum value constraints of P'_M and $\lambda_m + \lambda'_m$ in practical systems and E_{net} is BS energy consumption, which can expressed as:

$$\begin{aligned}E_{net} = &(1-\beta)\lambda_M(a_M N P'_M + b_M) \\ &+ (\lambda_m + \lambda'_m)P_m^{2/\alpha}(a_m N P_m + b_m).\end{aligned} \tag{12}$$

Obviously, one challenge to solve **P0** is that β, P'_M and λ'_m couple with each other. Thus, we try to decouple them to simplify the problem. Use (6) to eliminate variable λ'_m and first ignore the uplink power consumption constraint (11), then the problem can be equivalent to

$$\begin{aligned}\textbf{P1:}\quad &\underset{\beta, P'_M}{\text{minimize}} \quad (1-\beta)G(P'_M) \\ &\text{subject to} \quad 0 \le \beta \le 1, \\ &\qquad 0 \le P'_M \le P_{M,max}, \\ &\qquad 0 \le \lambda_M \frac{{P_M}^{2/\alpha} - (1-\beta){P'_M}^{2/\alpha}}{{P_m}^{2/\alpha}} \le \lambda_{m,\max} - \lambda_m,\end{aligned}$$

where $G(x) = a_M N x - P_m^{-2/\alpha}(a_m N P_m + b_m)x^{2/\alpha} + b_M$.

Lemma 3. *For* ***P1****, given* P'_M*, the optimal* β *is:*

$$\beta^* = \begin{cases}\min(1, H(P'_M)), & G(P'_M) \ge 0; \\ \max(0, 1-(\frac{P_M}{P'_M})^{2/\alpha}), & G(P'_M) < 0,\end{cases} \tag{13}$$

where $H(x) = 1 - \frac{{P_M}^{2/\alpha} - {P_m}^{2/\alpha}(\lambda_{m,max} - \lambda_m)/\lambda_M}{x^{2/\alpha}}$.

With (13), **P1** can be simplified as the following 4 cases, each of which can be solved easily due to single-variable.

1) *Case 1*: $1 < H(P'_M)$ and $G(P'_M) \ge 0$. In this case, from (13) we have $\beta_1 = 1$ and hence $\lambda'_{m,1} = \lambda_M(\frac{P_M}{P_m})^{2/\alpha}$ correspondingly. Consider the constraints (11), $1 < H(P'_M)$ and $G(P'_M) \ge 0$, solve **P1** with $\beta_1 = 1$. If the solution exists and suppose it is $P'_{M,1}$, then we have the optimal value $Y_1 = 0$; otherwise, let $Y_1 = \infty$.

2) *Case 2*: $1 \ge H(P'_M)$ and $G(P'_M) \ge 0$. In this case, $\beta_2 = H(P'_M)$ and $\lambda'_{m,2} = \lambda_{m,max} - \lambda_m$ correspondingly. Consider the constraints (11), $1 \ge H(P'_M)$ and $G(P'_M) \ge 0$, solve **P1** with $\beta_2 = H(P'_M)$. If the solution exists and suppose it is $P'_{M,2}$, then we have the optimal value $\beta_2 = H(P'_{M,2})$ and $Y_2 = (1-\beta_2)G(P'_{M,2})$; otherwise, let $Y_2 = \infty$.

3) *Case 3*: $P_M \le P'_M$ and $G(P'_M) < 0$. In this case, $\beta_3 = 1 - (\frac{P_M}{P'_M})^{2/\alpha}$ and $\lambda'_{m,3} = 0$ correspondingly. Consider the constraints (11), $P_M \le P'_M$ and $G(P'_M) < 0$, solve **P1** with $\beta_3 = 1 - (\frac{P_M}{P'_M})^{2/\alpha}$. If the solution exists and suppose it is $P'_{M,3}$, then we have $\beta_3 = 1 - (\frac{P_M}{P'_{M,3}})^{2/\alpha}$ and the optimal value $Y_3 = (1-\beta_3)G(P'_{M,3})$; otherwise, let $Y_3 = \infty$.

4) *Case 4*: $P_M > P'_M$ and $G(P'_M) < 0$. In this case, $\beta_4 = 0$. Consider the constraints (11), $P_M > P'_M$ and $G(P'_M) < 0$, solve **P1** with $\beta_4 = 0$. If the solution exists and suppose it is $P'_{M,4}$, then we have $\lambda'_{m,4} = \lambda_M \frac{P_M^{2/\alpha} - {P'_{M,4}}^{2/\alpha}}{{P_m}^{2/\alpha}}$ and the optimal value $Y_4 = G(P'_{M,4})$; otherwise, let $Y_4 = \infty$.

After discussing the above four cases, we can get the optimal solution to **P0** as follows:

$$[\beta^*, {P'_M}^*, {\lambda'_m}^*] = [\beta_i, P'_{M,i}, \lambda'_{m,i}], \tag{14}$$

where $i = \arg\min_{i=1,2,3,4}\{Y_i\}$.

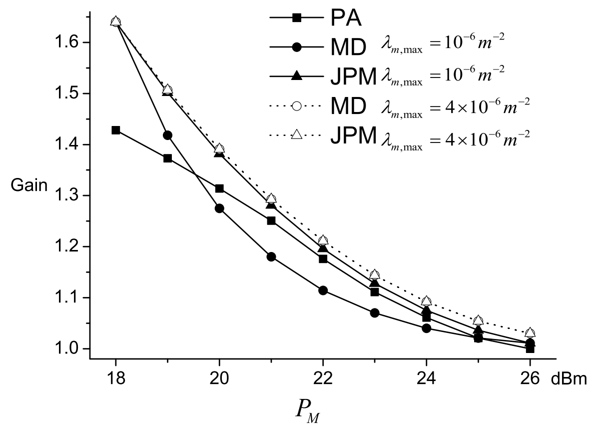

Fig. 1. The energy saving gain of three schemes as a function of P_M.

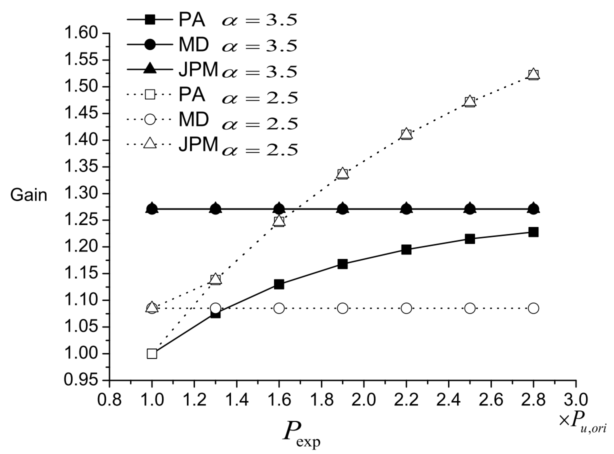

Fig. 2. The energy saving gain of three schemes as a function of P_{exp}. ($\lambda_{m,max} = 4 \times 10^{-6}\ m^{-2}$)

V. Numerical Results and Discussions

In this section, the performance is evaluated through simulation. We consider MaBSs whose initial density is $\lambda_M = 10^{-6}m^{-2}$, initial transmission power is P_M, and channel number is $N = 100$. For power model, we set $NP_{M,max} = 40$ W, $NP_m = 6.3$ W, $a_M = 2.66$, $b_M = 118.7$ W, $a_m = 3.1$, $b_M = 53$ W according to [7]. For the channel, we fix $\alpha = 3.5$ and additive noise power $\sigma^2 = -90$ dbm. To evaluate energy saving performance of our proposed schemes, we define the achieved energy saving gain as

$$Gain = \frac{E_{net,init}}{E_{net,i}}, i = PA, MD, JPM, \qquad (15)$$

where $\{E_{net,i}\}_{i=PA,MD,JPM}$ is the energy consumption of PA, MD and JPM, while $E_{net,init}$ is the energy consumption of the initial network with all the MaBSs in active mode.

The performance gain results of PA, MD and JPM are shown in Fig. 1 and Fig. 2. We can find that it is unclear that which technique is more energy efficient between PA and MD. Under different configuration parameters, there are different results. For example, PA is superior to MD when $\lambda_{m,max} = 10^{-6}\ m^{-2}$ and $P_M > 19$ dBm, but the result is opposite when $\lambda_{m,max} = 4 \times 10^{-6}\ m^{-2}$. However, *no matter which is more energy efficient, JPM always surpasses the PA and MD*. On the one hand, PA and MD are two special cases of JPM. Thus, the gain of JPM at least equals to the bigger value of PA and MD. On the other hand, JPM can use PA to further cut down the system energy consumption when no more MiBSs are available. This is the reason why there is a gap between JPM and PA/MD when $\lambda_{m,max} = 10^{-6}\ m^{-2}$ in Fig. 1. Besides, we also investigate the impact of P_{exp} on the performance in Fig. 2, where $P_{u,ori}$ is the original uplink average power consumption when all MaBSs are on. Introducing MiBSs can shorten the distance from UE to its serving BS. Thus, the gain of MD is independent on P_{exp}. However, for PA, P_{exp} limits the maximum proportion of sleep MaBSs, hence the higher P_{exp} is, the more gain PA can achieve as shown in Fig. 2. Therefore, *from the perspective of uplink power consumption, MD and JPM are better choices*. Finally, all curves in Fig. 1 and Fig. 2 show that *with any our proposed scheme, sleep mode can reduce system energy consumption significantly under UE power constraint while maintaining the original coverage performance.*

VI. Conclusions

In this letter, we have investigated the impact of MaBS sleep mode on the coverage and UE power consumption performance. We first derive the closed-form expressions. Then based on the results, three schemes are proposed to maintain the original coverage performance: PA, MD, and JPM. Besides, we formulate the BS energy consumption minimization problem under UE power consumption constraint, and achieve the optimal solutions. Numerical simulation results show that: 1) JPM is a general case of PA and MD, hence has the best performance; 2) From the view of UE power consumption performance, MD and JPM are better choices; 3) With any our proposed scheme, sleep mode can reduce energy consumption significantly under UE power consumption constraint while maintaining the original coverage performance.

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