Volume-Hiding Range Searchable Symmetric Encryption for Large-Scale Datasets

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Abstract—Searchable Symmetric Encryption (SSE) is a valuable cryptographic tool that allows a client to retrieve its outsourced data from an untrusted server via keyword search. Initially, SSE research primarily focused on the efficiencysecurity trade-off. However, in recent years, attention has shifted towards range queries instead of exact keyword searches, resulting in significant developments in the SSE field. Despite the advancements in SSE schemes supporting range queries, many are susceptible to leakage-abuse attacks due to volumetric profile leakage. Although several schemes exist to prevent volume leakage, these solutions prove inefficient when dealing with largescale datasets. In this paper, we highlight the efficiency-security trade-off for range queries in SSE. Subsequently, we propose a volume-hiding range SSE scheme that ensures efficient operations on extensive datasets. Leveraging the order-weighted inverted index and bitmap structure, our scheme achieves high search efficiency while maintaining the confidentiality of the volumetric profile. To facilitate searching within large-scale datasets, we introduce a partitioning strategy that divides a broad range into disjoint partitions and stores the information in a local binary tree. Through an analysis of the leakage function, we demonstrate the security of our proposed scheme within the ideal/real model simulation paradigm. Our experimental results further validate the practicality of our scheme with real-life large-scale datasets.

Index Terms—Searchable Symmetric Encryption, Range Query, Volume-Pattern Leakage, Large-Scale Datasets

I. INTRODUCTION

In the past decade, driven by the rapid advancements in cloud computing and network technologies, an increasing number of individuals and organizations have shown a willingness to outsource their data to public cloud services. These cloud services offer a stable storage environment, enabling clients to access their outsourced data conveniently over the Internet, anytime and from anywhere. Given the data's relocation to the cloud, a primary concern for clients revolves around data privacy, especially when the outsourced data includes sensitive information. A straightforward approach to counter data leakage is to encrypt all data before outsourcing. However, traditional symmetric encryption methods like AES come at the cost of data searchability. To surmount this challenge, a cryptographic concept called Searchable

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Symmetric Encryption (SSE) was introduced [1][2]. SSE enables clients to privatize their data during outsourcing while maintaining the capability to search encrypted data. This dual functionality aims to address the trade-off between security and searchability.

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A general SSE scheme is designed for keyword-based search over encrypted documents. Since then, a considerable amount of research has emerged on this subject, raising issues related to rich queries, such as Boolean queries [1]–[3], fuzzy keyword queries [4]–[6], and range queries [7]–[12]. In this paper, we mainly focus on the range queries in SSE. Following the typical range SSE schemes [8], [10], a common approach is to transform a range query into a keyword-based search. More specifically, the scheme by Demertzis *et al.* [8] applies *range covering techniques* to reduce a range query to a multi-keyword search. Another scheme by Wang *et al.* [10] further reduces a range query to a two-keyword search by designing an *order-weighted inverted index*.

Regardless of keyword-based search or range query, to assure high search efficiency, an SSE scheme needs to construct a specific index, such as an inverted index, and normally leaks some information called *leakage profile* [13]-[15]. The leakage profile can be categorized into three types: search-pattern leakage (whether two search tokens are generated from the same keyword), access-pattern leakage (which identifiers are associated with the search token), and volume-pattern leakage (the size of identifiers for the search token). In the early stages of research, it was considered acceptable to leak these patterns. However, recent studies [15]-[20] have shown that the server can reconstruct the database by relying solely on volume-pattern leakage, especially in the case of range queries. In other words, compared to general SSE (which only supports keyword search), range SSE is more vulnerable to volume-leakage attacks. This is mainly due to the fact that volume information can reflect the size of the queried range to some extent, and the volume for a range can be regarded as the sum of volume for its subranges.

As a concrete example in Fig. 1, three range queries Q_1, Q_2 and Q_3 are issued by the client. After executing the search algorithm, the server can observe the number of records (i.e., volume pattern) corresponding to each range query. According to the volume patterns, the server can infer that the relations among these ranges' sizes may be $|Q_1| < |Q_3| < |Q_2|$. Moreover, because the volume corresponding to Q_2 is equal to the sum volume corresponding to Q_1 and Q_3 , the server can further infer that the relations among these ranges may be $Q_2 = Q_1 \cup Q_3$. Once the server observes enough volume

	Comm		(15	
Schemes	Size of	Storage Overhead	Volume-Pattern	Large-Scale
Schemes	Query Tokens	(Server's Side)	Preservation	Datasets Support
Scheme-A [9]	$O(\log \tilde{q})$	$O(d \cdot \lceil \log m \rceil)$	×	×
Scheme-B [9]	$O(\log \tilde{q})$	$O(d \cdot m)$	\checkmark	×
Range SSE-I [10]	O(q)	O(d+m)	×	\checkmark
Range SSE-II [10]	O(1)	$O(d+m^2)$	×	×
FBDSSE-RQ [11]	$O(\log q)$	$O(d \cdot m)$	\checkmark	×
Our Proposed Scheme	O(1)	O(m)	\checkmark	\checkmark

TABLE I Comparison with Prior Arts

Note that d and m represent the number of documents and the number of keywords in the dataset, respectively. \tilde{q} is the size of a range query, q is the number of distinct values inside a range query. Loosely speaking, $\tilde{q} \ge q$, cf., [10].

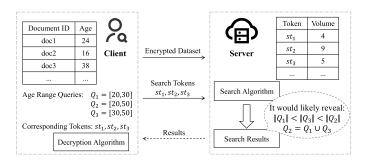


Fig. 1. An Example of Volume Leakage in Range SSE

patterns, it can attempt to reconstruct the whole dataset by leveraging existing attack algorithms [15]–[18].

Recently, the range SSE scheme by Zuo et al. [11] unintentionally prevent volume-pattern leakage by adopting the bitmap index structure. But Wang and Chow [12] argued that the bitmap index structure has inherent limitations when applying it to a large-scale dataset. Besides, according to the research [8], the Best Range Cover (BRC) technique employed in [11] would lead to extra leakage. In fact, how to support large-scale datasets is another important issue in range SSE. No matter the *order-weighted inverted index* or the range covering technique, there always exists at least one keyword which contains the whole records. This would result in low efficiency on index generating or introducing a high burden on storage. A trivial approach to support large-scale datasets is taking each value as a keyword instead of adopting particular transforming techniques. But this method would cause significant inefficiency in search due to a huge number of search tokens (cf., Range SSE-I [10]). All these factors thus lead to the following question: Can we design a range SSE to prevent volume-leakage attacks while keeping high security and search efficiency on large-scale datasets?

In this paper, we design a novel inverted index by utilizing the advantages of the order-weighted inverted index [10] and bitmap index [11]. Inspired by the idea of "searching locally" in [21], we construct a local search tree for retrieving document identifiers locally to prevent extra leakage. For supporting large-scale datasets, we adopt the partitioning strategy to split the whole range into disjoint subranges. In this way, we build a volume-hiding range SSE scheme with the optimal trade-off between search efficiency and storage overhead. Specifically, our proposed scheme achieves the optimal size of query tokens (i.e., O(1)) for any range queries, and O(m) storage overhead on the server's side, where m is the number of keywords in the datasets. The comparison of our proposed scheme with prior arts is given in TABLE I. Our contributions are listed as follows.

- We design a novel index, referred to as the orderaccumulated inverted index, by combining the orderweighted inverted index and bitmap index. Our proposed order-accumulated index achieves the optimal size of query tokens (i.e., O(1)) and can also hide the volume pattern, thereby avoiding volume-leakage attacks.
- To further support large-scale datasets, we split the whole range into disjoint partitions, and design a local search tree to store the partition information. By leveraging the advantage of the local search tree, the client only needs to send at most two search tokens for any range query and retrieve document identifiers locally, which significantly decreases the storage overhead on the server's side.
- Based on the leakage function and the simulation paradigm, we provide formal security proof of our proposed scheme. Besides, we apply real-world datasets to our experiments and conduct comprehensive analyses in terms of computation and storage overhead. By comparing to the state-of-the-art works, we claim that our proposed scheme is more secure and efficient, especially on large-scale datasets.

The rest of this paper is organized as follows. Section II introduces the related work. The problem statement is given in Section III. In Section IV, we describe the SSE scheme and related index structures. Section V illustrates our proposed schemes in detail. Then, we give a formal security analysis and performance evaluation in Section VI and Section VII, respectively. Finally, we conclude this paper in Section VIII.

II. RELATED WORKS

Secure range query has attracted extensive attention in the database research domain, and it can be achieved straightforwardly with the aid of Order Preserving Encryption (OPE) [22], e.g., CryptDB [23]. Due to the property of preserving the order for encrypted values, the schemes based on OPE have high compatibility with existing database management systems. However the security of OPE is still debatable, an attacker can easily launch an inference attack through the leakage of order information [24], [25]. To provide a stronger privacy assurance for range queries, some studies [26], [27] adopt a two-server architecture and choose the Paillier cryptosystem instead of OPE. Although these schemes can achieve higher search efficiency and stronger privacy assurance compared with the scheme based on OPE, the assumption that two servers are required to be non-colluding seems too strong. Intuitively, no one can assure that two servers will never collude in practice [28].

Another line of secure range query is to utilize SSE for a better balance between privacy protection and search efficiency. The concept of SSE was first introduced by Song, Wagner, and Perrig [29] in 2000. To formalize the security definitions and fulfill the practical requirement, Curtmola et al. [30] presented the semantic security definitions of SSE and adopted an inverted-index structure to improve keyword search efficiency. After that, a series of studies have begun to concentrate on providing richer search functionality, such as Boolean queries [1]-[3], fuzzy queries [4]-[6], and range queries [7]–[12]. As one of the most well-known SSE schemes with support for Boolean queries, OXT protocol [1] first achieves multiple keyword conjunctive queries in sublinear time. Later, Faber et al. [7] further extended OXT protocol to support range queries. The core idea is to convert a range query into disjunctions of exact keywords, and then invoke OXT protocol to execute multiple keywords. A similar method is also conducted in research by Demertzis et al. [8], but Demertzis et al. argued that OXT protocol is not suitable for handling keyword disjunctive queries since the search time is linear with the number of documents. In addition, they formalize range queries in the context of SSE [8] and also propose numerous range SSE schemes with various trade-offs between efficiency and security. Different from the above two schemes, the research [10] achieves efficient range queries by constructing an order-weighted inverted index instead of the original inverted index.

Nevertheless, to balance efficiency and privacy, all the above SSE schemes need to reveal some information about the query and the corresponding response, which are usually called *leakage profiles* [13]–[15]. The leakage profiles can be categorized into three types: *search-pattern leakage*, *access-pattern leakage*, and *volume-pattern leakage*. Recent studies [15]–[20] emphasize that current range SSE schemes are vulnerable to attacks based on volume-pattern leakage. Although Zuo *et al.* [11] proposed a dynamic range SSE scheme that can inherently avoid leaking the volume pattern due to the natural property of bitmap structure. However, Wang and Chow [12] pointed out that the bitmap structure adopted in [11] restricts the size of files, which would lead to impracticality for large-scale datasets.

A naïve method to resist volume-leakage attacks is adopting padding countermeasures in general SSE schemes [31]–[33]. However, the padding strategies cannot be directly integrated in the range SSE schemes, especially when the dataset's size is very large. The reason is that, in the existing range SSE schemes [7]–[12], there always exists at least one keyword which is mapped to the total documents in the dataset. Therefore, directly applying padding strategies (i.e., each keyword contains the same size of corresponding documents) will result in too much storage overhead. One recent line of works [34]–[36] has focused on the design of volumehiding encrypted multi-maps that can provide better storage overhead than the naïve padding method. Unfortunately, Ando and George [36] argued that the length of each encrypted response produced by a minimally-leaking encrypted multimap scheme must be at least the maximum response length. That is to say, when applying volume-hiding encrypted multimaps into existing range SSE schemes, the response length must be at least as long as the number of the whole documents.

In a relatively recent work [37], Ren *et al.* designed a hybrid index framework to eliminate volume-pattern leakage for range queries by utilizing the Trusted Execution Environment (TEE) such as an SGX-enabled storage server. Like many other TEEassisted applications (e.g., [38]), these schemes are vulnerable to side-channel attacks [39]. So far, there still lacks a volumehiding range SSE scheme that supports large-scale datasets without relying on TEE.

III. PRELIMINARIES

A. SSE Scheme Syntax

In a typical SSE scheme, a client can outsource its encrypted documents to an untrustworthy cloud server preserving with keyword searchability. Following the definitions in [1], [8], each document has a unique identifier (denoted by ind) and contains a set of keywords (denoted by W). Assuming that the number of total documents is d, we use a database DB = $(ind_i, W_i)_{i=1}^d$ to represent the list of identifier-keywords pairs. We denote the document identifiers that contain the keyword w by DB(w). The client first encrypts DB and then uploads the encrypted database EDB to the cloud server, where the client first obtains the search token according to the search keyword, then the cloud server uses the search token to retrieve document identifiers from EDB. In general, an SSE scheme contains the following five algorithms:

- $K \leftarrow \text{Setup}(1^{\lambda})$: is a probabilistic algorithm run by the client. Given a security parameter λ , this algorithm outputs a secret key K.
- EDB ← BuildIndex(K, DB): is a probabilistic algorithm run by the client. Upon inputting a database DB and the secret key K, this algorithm outputs an encrypted database EDB.
- st ← TokenGen(K, w): is a deterministic algorithm run by the client. Upon inputting a search keyword w and the secret key K, this algorithm outputs a search token st.
- RS ← Search(st, EDB): is a deterministic algorithm run by the server. For a search token st and the encrypted database EDB, this algorithm outputs a set of search results RS.
- *I* ← Dec(*K*, *RS*): is a deterministic algorithm run by the client. For the secret key *K* and the search result *RS*, this algorithm outputs a set of document index *I*.

We say that an SSE scheme is correct if for any K output by Setup (1^{λ}) , given any DB, after running EDB \leftarrow IndexBuild(K, DB), $st \leftarrow$ TokenGen(K, w), $RS \leftarrow$ Search(st, EDB), and $I \leftarrow \text{Dec}(K, RS)$, the set I contains the same document identifiers as that in DB(w). To improve the search efficiency, a common approach is to use the inverted index as the EDB's structure, where each (encrypted) keyword is regarded as an index and mapped to a set of (encrypted) identifiers.

In the context of range SSE, the syntax definition is slightly different. For the sake of simplicity, many studies (e.g. [8], [10], [11]) only consider single-dimensional range queries, because the multi-dimensional range queries can be reduced to several independent single-dimensional range queries. Following this assumption, each document is characterized by an attribute (e.g. age), whose value v is regarded as the document's keyword. That is, $DB = (ind_i, w_i)_{i=1}^d$ and $W = \bigcup_{i=1}^{d} \{w_i\} = \{v_1, v_2, \dots, v_m\}, \text{ where } v_1 < v_2 < \dots < v_n\}$ $\ldots < v_m$, m is the number of distinct values in DB. Since each document only corresponds to one keyword, it is clear to infer that $m \leq d$. The range query is denoted by $Q = [v_{\ell}, v_r]$, where $\ell, r \in \{1, 2, \dots, m\}$ and $\ell \leq r$. Therefore, the corresponding inverted index I can be parsed as $(v_i, I_i)_{i=1}^m$, where $I_i = \mathsf{DB}(v_i)$ is the collection of identifiers containing value v_i .

B. Order-Weighted Inverted Index

As a special construction of the inverted index, the orderweighted inverted index is originally designed for range queries in [10]. An example of order-weighted inverted index is shown in Fig. 2.

Inverted Index			Order-Weighted Inverted Index			
Keyword Document Identifiers		Keyword	Document Identifiers			
	6	{ind ₂ }	6	$\{ind_2 1\}$		
	18	$\{ind_2, ind_6, ind_7\}$	18	{ind ₂] 2, ind ₆] 1, ind ₇] 1}		
	21	$\{ind_4\}$	21	{ind ₂] 2, ind ₄] 1, ind ₆] 1, ind ₇] 1}		
	28	{ind ₆ , ind ₇ }	28	{ind ₂ 2, ind ₄ 1, ind ₆ 2, ind ₇ 2}		
	33	$\{ind_2\}$	33	{ind ₂ 3, ind ₄ 1, ind ₆ 2, ind ₇ 2}		
			1			

Fig. 2. An Example of Order-Weighted Inverted Index

At a high level, given an inverted index $\mathbf{I} = (v_i, I_i)_{i=1}^m$, the order-weighted inverted index can be represented as $(v_i, I_i^*)_{i=1}^m$, where $I_i^* = \bigcup_{j=1}^i \{I_j\}$ means the collection of identifiers containing values no greater than v_i . Each document identifier in order-weighted inverted index is attached with a weight information (e.g., $\operatorname{ind}_i || \omega_i$, where ω_i is ind_i 's weight). The weight information represents the number of occurrences of the corresponding document identifier. This structure has the benefit in reducing the size of search tokens (token size is O(1)). That is to say, for any range query $Q = [v_\ell, v_r]$, the client only needs to generate two search tokens: one is for $v_{\ell-1}$, and the other is for v_r . The search result can be calculated by $I_r^* \setminus I_{\ell-1}^*$. For more details of the order-weighted inverted index, we refer the readers to [10].

C. Bitmap Index

Bitmap index has been widely used to represent document identifiers in several SSE schemes [9], [11], [40]–[42]. As illustrated in Fig. 3, assume that there are 8 documents, then we can set a bit string bs with length 8. Consider an inverted index $(v_i, I_i)_{i=1}^m$, we can use a bit string bs to represent I_i . That is, if there exists document identifier ind_j in I_i , we set the j-th bit of bs to 1. Otherwise, it is set to 0.

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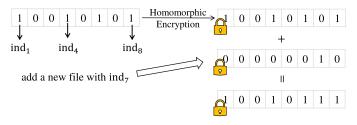


Fig. 3. An Example of the Bitmap Index

This index is usually encrypted by an additively homomorphic encryption scheme so that the server can update the bit string in the ciphertext domain. Since for each keyword v_i , the length of bs is identical, which implies that the bitmap index can inherently prevent the volume-pattern leakage.

IV. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Definition

In our model, there are two participants, i.e., a client and a cloud server (as shown in Fig. 1). The client first outsources its encrypted documents to the cloud server. Then the client can generate search tokens to retrieve the documents from the server. From the view of the client, the cloud server is considered to be honest-but-curious, which means the cloud server will execute the protocol honestly, but it will also try to infer what range the client is searching for or what identifiers have been retrieved by leveraging the leakage profile. Assume that d is the number of documents, we use $DB = (ind_i, w_i)_{i=1}^d$ to represent the list of identifier-keywords pairs. Following the range SSE syntax, each document contains a unique identifier ind and a distinct value $v \in \{v_1, v_2, \ldots, v_m\}$, where m is the number of distinct values in DB. The collection of all keywords is denoted by $W = \bigcup_{i=1}^{d} w_i = \{v_1, v_2, \dots, v_m\}.$ Notations used in this paper are given in TABLE II.

B. Design Goal

Our goal is to design a volume-hiding range SSE scheme which can support efficient operations on large-scale datasets. Despite the bitmap index can prevent volume-pattern leakage due to its inherent property, it also has limitations when the number of documents is large, especially in the case of range SSE. Our design aims to resolve the issue of the trade-off between privacy preservation and large-scale dataset support. In particular, the following objectives should be achieved in our work:

• **Privacy Preservation**: Our design aims to protect *query privacy* (i.e., what range the client is searching for) and *data privacy* (i.e., what data is being retrieved for each

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TABLE II	
NOTATIONS	

Symbol	Description
λ	The security parameter
BT	The local binary tree
$DB = (ind_i, w_i)_{i=1}^d$	A database held by the client, and $ DB = d$
$\mathbf{I} = (v_i, I_i)_{i=1}^m$	The inverted index of DB, and $ \mathbf{I} = m$
EDB	The encrypted database of I
bs	The bit string used to represent the
	document identifiers
L	The length of bs
e	The encrypted bit string
F	A secure Pseudorandom Function (PRF)
H_1, H_2	The keyed hash functions
K	The secret key held by the client
$C = \{C^{(1)}, C^{(2)}, \ldots\}$	A collection used to record each subrange's
	information
N	A node in BT with a left child $N.left$ and
	a right child N.right
$Q = [v_\ell, v_r]$	A range query with lower bound v_ℓ and
	upper bound v_r
ST	The search tokens list generated by the
	client
RS	The search result returned by the server
a b	String concatenation of a and b

query) from the cloud server. Considering that the cloud server would launch volume-leakage attacks to infer the query privacy and data privacy, we need to hide the volume pattern to avoid these leakage-abuse attacks.

• Large-Scale Dataset Support: Our design should be highly scalable in terms of supporting large documents. In addition, no matter how large the dataset is, or what range is queried, the protocol should maintain high efficiency on both the client's side and the server's side.

C. Security of Range SSE

Similar to the previous studies [8], [10], [11], we define a leakage function \mathcal{L} to depict the information which would be learned by the cloud server. The rigorous definition of leakage function \mathcal{L} is given in Section VI. Based on the leakage function, the security of our proposed scheme can be formalized by the simulation paradigm [30] (also referred to as the ideal/real model). That is to say, for a range scheme II, the behavior in the real world is essentially the same as the protocol in II. While in the ideal world, the behavior is simulated by a simulator \mathcal{S} which exploits the leakage information (i.e., the outputs of \mathcal{L}) as the input. Intuitively, to prove that II is secure, we only need to prove that for every probabilistic polynomial time (PPT) adversary \mathcal{A} , the probability to distinguish these two worlds is negligible.

We use $\operatorname{Real}^{\Pi}_{\mathcal{A}}$ and $\operatorname{Ideal}^{\Pi}_{\mathcal{A},S}$ to denote two security games in real world and ideal world, respectively. The definitions of $\operatorname{Real}^{\Pi}_{\mathcal{A}}$ and $\operatorname{Ideal}^{\Pi}_{\mathcal{A},S}$ are described as follows.

Real^Π_A: Upon inputting a security parameter λ, the challenger invokes Setup(λ) to generate a secret key

K. For a database DB provided by the adversary \mathcal{A} , the challenger returns EDB \leftarrow IndexBuild(K, DB) to \mathcal{A} . Then adversary \mathcal{A} sends a polynomial number of range queries to the challenger. For each range query Q, the challenger returns a corresponding search tokens list ST. Eventually, \mathcal{A} outputs a bit $b \in \{0, 1\}$ as the result of the game.

Ideal^Π_{A,S}: For a database DB provided by the adversary A, the simulator S returns EDB by exploiting the leakage function L(DB). Then adversary A sends a polynomial number of range queries to S. For each range query Q, the simulator S returns a corresponding search tokens list ST by exploiting the leakage function L(Q,DB). Eventually, A outputs a bit b ∈ {0,1} as the result of the game.

Definition 1 (Security of Range SSE). For a range SSE scheme Π and the security games described above, the scheme is \mathcal{L} -adaptively-secure if for any probabilistic polynomial time (PPT) adversary \mathcal{A} , there is a PPT simulator \mathcal{S} such that

$$|\Pr[\mathbf{Real}^{\Pi}_{\mathcal{A}}(\lambda) = 1] - \Pr[\mathbf{Ideal}^{\Pi}_{\mathcal{A},\mathcal{S}}(\lambda) = 1]| \le \mathsf{nelg}(\lambda), \ (1)$$

where $nelg(\lambda)$ is a negligible function.

V. THE PROPOSED SCHEME

A. Overview

In our system, the client first partition the range $[v_1, v_m]$ into disjoint subranges. For each subrange, we utilize a bitmap index to represent the document identifiers, and ensure that the number of documents in each subrange could not exceed the bit string's capacity. We also use a collection C to record the subranges' information. For example, assuming $C = \{C^{(1)}, C^{(2)}, \ldots\}, C^{(i)}$ records the *i*-th subrange's information. The information includes the subrange's boundary values (denoted by $[C_{min}^{(i)}, C_{max}^{(i)}]$) and the volume information within the subrange (denoted by $C_{info}^{(i)}$).

Inverted Index		Order-Accumulated Inverted Index (bitmap length = 8)								
Keyword	Document Identifiers	fiers Keyword		Document Identifiers						
6	{ind ₂ , ind ₈ }	6	1	1	0	0	0	0	0	0
18	{ind ₇ }	18	1	1	1	0	0	0	0	0
21	$\{ind_9, ind_{13}\}$	21	1	1	1	1	1	0	0	0
28	{ind ₁₀ }	28	1	1	1	1	1	1	0	0
33	$\{ind_6, ind_{15}\}$	33	1	1	1	1	1	1	1	1
or the subra	nge [6, 33], $C_{min}^{(i)} = 6, C_{max}^{(i)}$. = 33	↓ [ind:	, ir	ıd _s ,	ind	7, ir	ıd۹,	,	↓ ind

Fig. 4. An Example of Our Order-Accumulated Inverted Index Structure

According to collection C, the client can build a binary tree BT for local search. As shown in Algorithm 1, for each node N, we use $[N_{min}, N_{max}]$ to represent the boundary of the range associated with N, where $N_{min} = N.left_{min}$ and $N_{max} = N.right_{max}$. After building the local search tree BT, the client begins to construct an order-accumulated inverted index for each subrange. A concrete example of the orderaccumulated inverted index is given in Fig. 4. The idea behind

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Algorithm 1: Local Binary Tree

BinaryTree.BuildTree(C): **Input:** A Collection C Output: A Binary Tree BT **Client:** if |C| = 0 then 1 Return \perp 2 else if |C| = 1 then 3 Generate a root node and set this node as BT; 4 Associate $C^{(1)}$ to this node; 5 Return BT 6 else 7 Generate 2^{ℓ} leaf nodes ; $//2^{\ell-1} < |C| \le 2^{\ell}$ 8 Assign each element in C to each leaf node; 9 for $i = \ell - 1$ to 0 do 10 Generate 2^i nodes: 11 for each node N do 12 13 Set its left and right child to two consecutive unassigned nodes of previous level; $N_{min} := N.left_{min};$ 14 $N_{max} := N.right_{max};$ 15 end 16 17 end Set the root node as BT; 18 19 end Return BT 20 21 end BinaryTree.LocalSearch(Q, BT): **Input:** Range Query $Q = [v_{\ell}, v_r]$ and binary tree BT Output: C_s Client: Temp \leftarrow Empty Set; 22 for $v \in \{v_\ell, v_r\}$ do 23 Let N be the root node; 24 while N is not a leaf node do 25 if $v \leq N.left_{max}$ then 26 N := N.left;27 28 else N := N.right;29 end 30 end 31 32 Temp := Temp $\cup N$; end 33 Parse Temp as $\{C^{(left)}, C^{(right)}\}$; // $left \leq right$ 34 $C_s := \{C^{(left)}, C^{(left+1)}, \dots, C^{(right)}\};$ 35 Return C_s 36 37 end

our proposed order-accumulated inverted index is from the fact that each document is characterized by only a keyword value in the range query scenario. Compared to the original order-weight inverted index, we remove the redundant weight information and use the bitmaps to represent the document identifiers, so that the volume pattern of each keyword can be hidden. After that, the client encrypts the order-accumulated inverted indexes as EDB and sends it to the server.

To search the documents in a range $Q = [v_{\ell}, v_r]$, the client first performs the local search based on BT (cf., Algorithm 1). If a node of BT precisely matches the queried range, then the client can directly obtain the bitmap index locally without interacting with the server. Otherwise, the client needs to generate a search token(s) list ST and send ST to the server.

Upon receiving ST, the server starts to execute the search

algorithm with each token in ST and returns the final results to the client. Finally, the client decrypts the final results and obtains the bitmap index from BT. The whole procedure is depicted in Fig. 5. More specifically, we divide the whole procedure into three phases: *Setup*, *Building*, and *Search*.

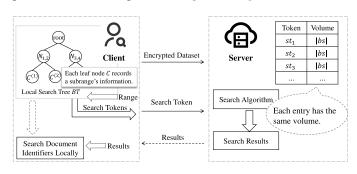


Fig. 5. Overview of Our Proposed Scheme

B. Setup Phase

In this phase, the client first initializes the secret key Kand the bitmap's length L according to the security parameter λ (e.g., 80bits) and the max volume \mathcal{V} of database DB. In our system, we require that L is not less than λ or \mathcal{V} , i.e., $L \geq \max{\{\lambda, \mathcal{V}\}}$. Afterward, the client generates two hash functions H_1, H_2 , a secure Pseudorandom Function (PRF) F, an empty map EDB and a binary tree BT. Note that EDB is an inverted index where we use a bitmap to represent the document identifiers, and the binary tree BT stores the range information. The procedure is described in Algorithm 2.

Algorithm 2: Setup

Input: Security Parameter λ , $\mathcal{V} = \max\{|\mathsf{DB}(w)|, \forall w \in \mathsf{W}\}$ Output: System ParametersClient:1Initialize L ($L \ge \max\{\lambda, \mathcal{V}\}$), $K \leftarrow \{0, 1\}^{\lambda}$;2Initialize two hash functions H_1, H_2 and a PRF F;3Initialize an empty map EDB, a binary tree BT;4Return ($L, K, \mathsf{EDB}, \mathsf{BT}, H_2, H_2, F$)5end

C. Building Phase

For DB = $(\text{ind}_i, w_i)_{i=1}^d$, the client first parses it as an inverted index $\mathbf{I} = (v_i, I_i)_{i=1}^m$. As we explained in Section IV, in the case of a range query, each document identifier corresponds to only a single keyword, and each keyword w has a distinct value v. Afterward, the client begins by partitioning the range $[v_1, v_m]$ into disjoint subranges and stores them in a collection C. Then, based on C, the client can build BT and EDB. Finally, the client retains BT locally and sends EDB to the cloud server. To alleviate the storage burden, we assert that the client only needs to store an offset value and the size of document identifiers for each collection C. This is due to the fact that the bitmap's non-zero entries are consecutive in our proposed order-accumulated inverted index. Therefore, the client can reconstruct the bitmap using the offset value and the size of document identifiers. The details of the procedure are shown in Algorithm 3.

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Algorithm 3: BuildIndex

Input: Database $\mathsf{DB} = (\mathsf{ind}_i, \mathsf{w}_i)_{i=1}^d$ Output: Encrypted Database EDB, Local Search Tree BT **Client:** Parse DB into an inverted index $\mathbf{I} = (v_i, I_i)_{i=1}^m$; 1 Initialize an empty set C, let ct = 1; 2 Initialize an offset value $\phi = 0$; 3 Initialize an empty inverted index \mathbf{I}^* and set $\mathbf{I}^*[v_0] = \emptyset$; 4 for $i \leftarrow 1$ to m do 5 if $|C^{(ct)}| + |I_i| \le L$ then 6 $\mathbf{I}^*[v_i] = I_i \cup \mathbf{I}^*[v_{i-1}];$ 7 else 8 9 $\mathbf{I}^*[v_i] = I_i;$ Append $C^{(ct)}$ into C: 10 ct = ct + 1;11 12 end Let $C_{info}^{(ct)} := (|\mathbf{I}^*[v_i]|, \phi);$ 13 $\phi := \phi + |\mathbf{I}^*[v_i]|;$ 14 end 15 $BT \leftarrow BinaryTree.BuildTree(C);$ 16 for $ct \leftarrow 1$ to |C| do 17 for v_i in range $[C_{min}^{(ct)}, C_{max}^{(ct)}]$ do 18 $K_1 || K_2 \leftarrow F(K, v_i);$ 19 Generate bs based on $C_{ind}^{(ct)}$ and $\mathbf{I}^*[v_i]$; 20 $e \leftarrow bs \oplus H_1(K_1, v_i);$ 21 $st \leftarrow H_2(K_2, v_i);$ 22 $\mathsf{EDB}[st] := e;$ 23 24 end 25 end Return EDB 26 27 end

D. Search Phase

For a range query $Q = [v_{\ell}, v_r]$, the client first obtains a subrange set C_s via local search tree BT. If v_{ℓ} and v_r are exactly the low and high bounds of C_s , then the client can generate the bitmaps within the range Q locally. Otherwise, the client needs to generate the search tokens and send them to the cloud server. Upon receiving the search tokens list ST, the server retrieves all encrypted bitmap index e corresponding to ST and sends search results to the client. Finally, the client decrypts the search result and restores all plain bitmap index via C_s . The details of the procedure is shown in Algorithm 4.

E. Discussions

Like many other SSE schemes [1]–[3] that focus on the search document index, our design only retrieves the bitmap index corresponding to the documents that match the queried range. This model is also deemed as structure-only SSE [43], which means the process of document retrieval is left out. This is because the client can retrieve the corresponding documents by PIR or ORAM techniques, after obtaining the document index. In our proposed scheme, in order to retrieve the documents, we need to establish a connection between the bitmap and the positions of documents stored on the server. For a bitmap bs with an offset value ϕ , assume that the *i*-th bit of bs corresponds to ind_j , we only need to set the position of document ind_j as $P'(K, i + \phi)$ in the setup phase, where P' is a pseudorandom permutation. In this way, the client can

Algorithm 4: Search

```
Input: Search Range Q = [v_{\ell}, v_r].
    Output: Bitmap Index Collection.
    Client:
          Initialize two empty set ST, I_{RS};
 1
          C_s \leftarrow \mathsf{BinaryTree.LocalSearch}(Q,\mathsf{BT});
 2
          // Parse C_s as \{C^{(s_1)}, C^{(s_2)}, \dots, C^{(s_t)}\};
          if v_{\ell} = C_{min}^{(s_1)} and v_r = C_{max}^{(s_t)} then

| for i \leftarrow 1 to t do
 3
 4
                      (\nu, \phi) \leftarrow C_{info}^{(s_i)};
bs_i = 1^{\nu} || 0^{L-\nu};
 5
 6
                       Append (bs_i, \phi) into I_{RS};
 7
                 end
 8
                 Return I<sub>RS</sub>
 9
          else
10
                 Initialize an empty set V;
11
12
                 Let C_{s'} := C_s;
                if v_{\ell} \neq C_{min}^{(s_1)} then

|K_1||K_2 \leftarrow F(K, v_{\ell});
13
14
                       st_{\ell} \leftarrow H_2(K_2, v_{\ell-1});
15
                       Append st_{\ell} into ST;
16
17
                       Append v_{\ell-1} into V;
                       C_{s'} \leftarrow C_{s'} \setminus C^{(s_1)};
18
                 end
19
                 if v_r \neq C_{max}^{(s_t)} then
20
                       K_1 || K_2 \leftarrow F(K, v_r);
21
                       st_r \leftarrow H_2(K_2, v_r);
22
                       Append st_r into ST;
23
                       Append v_r into V;
24
                       C_{s'} \leftarrow C_{s'} \setminus C^{(s_t)};
25
                 end
26
                 Send search tokens ST to Server;
27
          end
28
29
    end
    Server:
          Initialize an empty set RS;
30
          for st in ST do
31
                 e \leftarrow \mathsf{EDB}[st];
32
33
                 Append e into RS;
          end
34
          Send search result RS to Client.
35
36 end
     Client:
          for e in RS, and corresponding v in V do
37
                 K_1||K_2 \leftarrow F(K, v);
38
39
                 bs \leftarrow e \oplus H_1(K_1, v);
                 if v equals to v_{\ell-1} then
40
                       (\nu, \phi) \leftarrow C_{info}^{(s_1)};
41
                       Append (bs \oplus 1^{\nu} || 0^{L-\nu}, \phi) into I_{RS};
42
43
                 else
                       (\nu, \phi) \leftarrow C_{info}^{(s_2)};
44
                       Append (bs, \phi) into I_{RS};
45
                 end
46
47
          end
          Return I_{\mathsf{RS}} \cup \{(1^{\nu}||0^{L-\nu},\phi)\}_{\forall C^{(s_i)} \in C_{s'},(\nu,\phi) \leftarrow C_{info}^{s_i}}
48
49 end
```

calculate the positions of documents according to the bitmap index, and further retrieve the documents from the server.

Then we need to discuss the strength of our proposed scheme when applied to large-scale datasets. As we mentioned earlier, no matter the BRC technique applied in [8], [11] or the order-weighted inverted index utilized in [10], there always exists a keyword containing the whole document identifiers. To address this issue, in our proposed scheme, we design a hybrid index structure called order-accumulated inverted index by combining the bitmap index and order-weighted inverted index. Through integrating the partitioning strategy, each keyword in EDB corresponds to a bit string with a constant size, where the size depends on the partitioning strategy and is far smaller than the size in Zuo *et al.*'s scheme [11]. Concretely, our proposed scheme requires that each bit string's length $L \geq \max\{|\text{DB}(w)|, \forall w \in W\}$, while Zuo *et al.*'s scheme requires that each bit string's scheme requires that each bit string's length $L' \geq \sum |\text{DB}(w)|, \forall w \in W$. Although Zuo *et al.* also mentioned that they can divide a large bit string into several shorter ones by taking the method in [40], the size of total bit strings for each keyword has no changes.

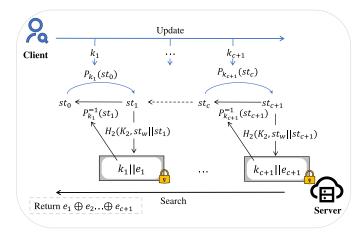


Fig. 6. An Extension of Our Scheme to Support Dynamic Updates

From the aspect of index storage overhead, it is easy to find that the size of EDB only depends on the length of the bitmap and the number of keywords, which implies that the number of document identifiers will have little impact on it. The token generation time is $O(\log |C|)$, i.e., the height of the local search tree BT. For any range query, the number of search tokens is at most 2. Since the search complexity is directly correlated with the number of search tokens, we claim that our proposed scheme achieves high search efficiency.

Our proposed scheme can also be extended to support dynamic datasets by employing the idea from existing dynamic SSE schemes (such as [44]–[46]). Taking the idea of [46] as an example, we only need to make slight changes to our proposed scheme for supporting dynamic updates, where the procedure for updating and searching is shown in Fig. 6. In the setup phase, the client needs to additionally initialize an empty map Σ , a hash function H_3 and a pseudorandom permutation P. The empty map Σ is used to store the states for each keyword, and P is used to establish a connection between every two states. For each keyword v, $\Sigma[v]$ records a tuple (st_c, c) , where c is a counter to record the number of updates and st_c is the state. In each state st_c , the client needs to generate a random k_{c+1} to calculate the next state, namely, $st_{c+1} \leftarrow P(k_{c+1}, st_c)$. The bit string bs corresponding to st_{c+1} is encrypted as $e_c \leftarrow bs \oplus H_1(K_1, v || c+1)$. Then the client just set $EDB[H_2(K_2, st || st_{c+1})] := (k_{c+1} || e_{c+1}) \oplus$

 $H_3(st||st_{c+1})$. As for the search phase, the server only needs to take $st||st_{c+1}$ as the input, and obtain the previous state by computing $st_c \leftarrow P^{-1}(k_{c+1}, st_{c+1})$. After retrieving $[e_1, e_2, \ldots, e_{c+1}]$, the server just needs to return the final result $RS = e_1 \oplus e_2 \oplus \ldots \oplus e_{c+1}$. One of the main challenges in dynamic SSE is preserving forward privacy, which means the newly added documents cannot be matched by the previous search tokens. In our extended version, the newly added document in state st_{c+1} cannot be matched by the previous search tokens because the server cannot calculate $P_{k_{c+1}}(st_c)$ without the knowledge of k_{c+1} in probabilistic polynomial time. Since our purpose is to design a volume-hiding range SSE for large-scale datasets, here we omit a full analysis of dynamic updates for brevity.

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VI. SECURITY ANALYSIS

A. Leakage Function

As we mentioned in Section IV, the security of our proposed scheme is defined on the basis of the leakage function. Therefore, before analyzing the security of our proposed scheme, we need to discuss the leakage function first. In particular, the leakage function of our proposed scheme is defined by $\mathcal{L} = \{\mathcal{L}^{\text{Index}}, \mathcal{L}^{\text{Query}}\}\)$, where $\mathcal{L}^{\text{Index}}$ represents the leakage information from the procedure of building index and $\mathcal{L}^{\text{Query}}\)$ represents the leakage information from the output of building index, the server only knows the number of keywords, i.e., m. For a range query Q, the server only knows the size of ST rather than the size of the matched identifiers. Thus, we use $\mu(Q) = |\text{ST}|\)$ to denote the token size pattern of Q. The definitions of $\mathcal{L}^{\text{Index}}$ and $\mathcal{L}^{\text{Query}}$ are described as follows:

- $\mathcal{L}^{Index}(DB) = (m),$
- $\mathcal{L}^{\mathsf{Query}}(\mathsf{EDB}, Q) = (\mu(Q)).$

Note that the token size pattern would reveal the information that whether a range query contains the minimal or maximal value of a subrange when the token size is 1. We argue that this leakage is acceptable and can be eliminated by randomly generating a "fake" token for the minimal (or maximal) value in each subrange.

B. Security Proof

Based on the leakage function, we now analyze the security of our proposed range SSE scheme.

Theorem 1. Let $\mathcal{L} = {\mathcal{L}^{Index}, \mathcal{L}^{Query}}$ be the leakage function defined in Section VI-A, our proposed scheme RSSE = (Setup, BuildIndex, Search) is \mathcal{L} -adaptively-secure, assuming that H_1 and H_2 are random oracles [47], and F is a secure PRF.

Proof. Similar to the proof in the previous works [10], [11], we define several games G_0, G_1, G_2, G_3, G_4 from $\operatorname{Real}_{\mathcal{A}}^{\mathsf{RSSE}}$ to $\operatorname{Ideal}_{\mathcal{A},\mathcal{S}}^{\mathsf{RSSE}}$. According to the definition in Section IV, we need to prove $\operatorname{Real}_{\mathcal{A}}^{\mathsf{RSSE}}$ and $\operatorname{Ideal}_{\mathcal{A},\mathcal{S}}^{\mathsf{RSSE}}$ are indistinguishable. A common approach is to prove that every two consecutive games from $\operatorname{Real}_{\mathcal{A}}^{\mathsf{RSSE}}$ to $\operatorname{Ideal}_{\mathcal{A},\mathcal{S}}^{\mathsf{RSSE}}$ are indistinguishable.

Game G_0 : G_0 straightforwardly performs our proposed scheme. Namely, G_0 is the same as the experiment in **Real**^{RSSE}_A. Thus, we have

$$\Pr[G_0 = 1] = \Pr[\operatorname{\mathbf{Real}}_{\mathcal{A}}^{\mathsf{RSSE}}(\lambda) = 1].$$
 (2)

Game G_1 : In G_1 , we replace $K_1||K_2 \leftarrow F(K,v)$ by choosing K_1 and K_2 at random and using a map \mathbb{K} to store the pair $(v, K_1||K_2)$. More specifically, given an input v, if there exists an entry v in map \mathbb{K} , the corresponding $K_1||K_2$ is returned; otherwise, a random $K_1||K_2$ is returned and stored with the value v in \mathbb{K} . We denote this process by $K_1||K_2 \leftarrow \mathbb{K}(v)$ for simplicity.

Assuming that there is a PPT adversary \mathcal{A}_1 that can distinguish between G_0 and G_1 , then we can construct another PPT adversary \mathcal{B}_1 to distinguish $K_1 || K_2 \leftarrow F(K, v)$ from the random selection $K_1 || K_2 \leftarrow \mathbb{K}(v)$. Since F is a secure PRF, we have

$$\Pr[G_1 = 1] - \Pr[G_0 = 1] \le \mathsf{Adv}_{F,\mathcal{B}_1}^{\mathsf{prf}}(\lambda), \tag{3}$$

where $\operatorname{Adv}_{F,\mathcal{B}_1}^{\operatorname{prf}}(\lambda)$ is a negligible function that represents the advantage for the adversary \mathcal{B}_1 to distinguish F from a truly random function.

Game G_2 : We replace the hash function H_1 by a random oracle \mathbb{H}_1 such that $bs' \leftarrow \mathbb{H}_1(K_1, v_i)$. Since the length of bs' is L, the probability of a correct guess for bs' by the adversary is $1/2^L$. According to the assumption of random oracle, if the adversary \mathcal{A} issues polynomial number $p(\lambda)$ of queries, then we have

$$\Pr[G_2 = 1] - \Pr[G_1 = 1] \le p(\lambda)/2^L.$$
(4)

Game G_3 : Similarly, we replace the hash function H_2 by a random oracle \mathbb{H}_2 such that $st \leftarrow \mathbb{H}_2(K_2, v_i)$. As the length of search token st is λ , the probability of a correct guess for search token st by the adversary is also $1/2^{\lambda}$. According to the assumption of random oracle, if the adversary \mathcal{A} issues polynomial number $p(\lambda)$ of queries, then we have

$$\Pr[G_3 = 1] - \Pr[G_2 = 1] \le p(\lambda)/2^{\lambda}.$$
 (5)

Game G_4 : We replace $e \leftarrow bs \oplus \mathbb{H}_1(K_1, v_i)$ by just setting $e \leftarrow \mathbb{H}_1(K_1, v_i)$ in the building phase. According to the assumption of random oracle and the definition of perfectly secret [47], if the adversary \mathcal{A} issues polynomial number $p(\lambda)$ of queries, then we have

$$\Pr[G_4 = 1] - \Pr[G_3 = 1] \le p(\lambda)/2^L.$$
 (6)

Simulator S: Now we construct the simulator $S = \{S.Setup, S.BuildIndex, S.Search\}$. The details of the algorithm is described in Algorithm 5.

Note that the simulator S takes λ and the leakage function $\mathcal{L} = \{\mathcal{L}^{\text{Index}}, \mathcal{L}^{\text{Query}}\}$ as input, and returns a simulated EDB and a simulated token list ST to the adversary \mathcal{A} . Since the procedure of constructing EDB is the same as that in game G_4 , then we have

$$\Pr[\mathbf{Ideal}_{\mathcal{A},\mathcal{S}}^{\mathsf{RSSE}}(\lambda) = 1] = \Pr[G_4 = 1].$$
(7)

By using the standard hybrid argument technique [47], it is

Algorithm 5: Simulator S \mathcal{S} .Setup (1^{λ}) : **Input:** Security Parameter λ . Output: System Parameters. Client: Initialize $L \ (L \ge \lambda)$; Initialize two random oracles \mathbb{H}_1 and \mathbb{H}_2 ; 2 Initialize an empty map EDB, a binary tree BT; 3 **Return** (L, EDB, BT). 4 5 end $\mathcal{S}.\mathsf{BuildIndex}(\mathcal{L}^{\mathsf{Index}})$: **Input:** $\mathcal{L}^{\text{Index}}(\mathsf{DB}) = (m).$ Output: Encrypted Database EDB. **Client:** for $i \leftarrow 1$ to m do 6 generate a random value v_i ; 7 $K_1||K_2 \leftarrow \mathbb{K}(v_i);$ // Definied in G_1 8 $e \leftarrow \mathbb{H}_1(K_1, v_i);$ // Definied in G_4 9 $st \leftarrow \mathbb{H}_2(K_2, v_i);$ 10 // Definied in G_3 $\mathsf{EDB}[st] := e.$ 11 end 12 **Return** EDB to the adversary A. 13 14 end S.Search(\mathcal{L}^{Query}): **Input:** $\mathcal{L}^{\mathsf{Query}}(Q) = (\mu(Q)).$ Output: Document Identifiers. **Client:** $\mu(Q) = 0$ then if 15 16 Return \perp . else 17 Initialize an empty search token set ST; 18 for $i \leftarrow 1$ to $\mu(Q)$ do 19 Obtain a random v from \mathbb{K} ; 20 $K_1 || K_2 \leftarrow \mathbb{K}(v) ;$ // Definied in G_1 21 $st \leftarrow \mathbb{H}_2(K_2, v)$; // Definied in G_3 22 Append st to ST. 23 24 end **Return** ST to the adversary A. 25 26 end

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clear to see that

27 end

$$\Pr[\mathbf{Ideal}_{\mathcal{A},\mathcal{S}}^{\mathsf{RSSE}}(\lambda) = 1] - \Pr[\mathbf{Real}_{\mathcal{S}}^{\mathsf{RSSE}}(\lambda) = 1]$$

$$\leq \mathsf{Adv}_{F,\mathcal{B}_{1}}^{\mathsf{prf}}(\lambda) + p(\lambda)/2^{\lambda} + 2p(\lambda)/2^{L}$$

$$\leq \mathsf{Adv}_{F,\mathcal{B}_{1}}^{\mathsf{prf}}(\lambda) + 3p(\lambda)/2^{\lambda}, \quad (8)$$

which indicates that our proposed scheme is secure.

VII. PERFORMANCE EVALUATION

In this section, we evaluate the performance of our proposed scheme from various aspects by comparing it with the recently proposed schemes [10], [11]. All experiments are conducted on a computer with an Intel Core i5-10400 CPU (4.30 GHz) and 16 GB RAM running the Linux OS. We adopt the Python Crypto module to implement the cryptographic operations, such as the hash function and the PRFs. For the sake of simplicity, we simulate the instances of the client and server on the same machine.

We choose Gowalla location checkin dataset¹ as the test dataset. Specifically, the Gowalla location checkin dataset

¹http://snap.stanford.edu/data/loc-gowalla.html.

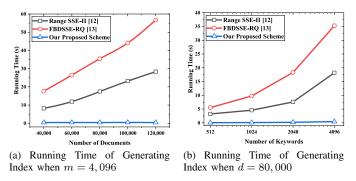
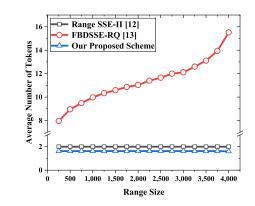


Fig. 7. Comparison of Running Time of Generating Index

contains 6, 442, 892 entries, and each entry is recorded in the form of (user, check-in time, latitude, longitude, location id). For our experiments, we treat each entry as a document and the value² of latitude in each entry as the keyword, which is similar to the experiment setting in [10]. In this way, we assign each entry a unique document identifier and parse the whole dataset as $DB = (ind_i, w_i)_{i=1}^{|DB|}$. The total number of documents is 6, 442, 892 and the number of keywords is 335, 362. Unless otherwise stated, the default bitmap's lenght *L* is set to 6, 264.

As shown in TABLE I, FBDSSE-RQ [11] outperforms Scheme-A and Scheme-B [9], and Range SSE-II [10] is a modified version of SSE-I [10]. Consequently, for our comparative experiments, we only need to choose Range SSE-II [10] and FBDSSE-RQ [11] to evaluate their performance in comparison to our proposed scheme. To compare with these two schemes, we generate several datasets with various sizes of documents and keywords from the whole dataset. Note that the number of keywords is denoted by m and the number of document identifiers is denoted by d. We first compare the time of generating encrypted index and local binary tree. The result is shown in Fig. 7. It is clear to see that our proposed scheme and Wang et al.'s scheme [10] are more efficient than Zuo et al.'s scheme [11] in generating index. This is because, in Zuo et al.'s scheme, the identifiers for each keyword are disorganized, and the bit string's length is extremely large, which results in time-consuming computation in generating the bitmap index for each keyword. In contrast, both Wang et al.'s scheme and our scheme use a specially constructed inverted index, which is more regular than the normal inverted index, thus leading to efficiency. When the number of keywords is fixed to 4,096, the running time of generating index grows linearly with the number of documents except for our proposed scheme (see Fig. 7a). This is due to the fact that the inverted index structure in our scheme only depends on the number of keywords and the bitmap's length. We also observe that when the number of documents is fixed to 80,000, the running time of our proposed scheme grows much slower than that of others (see Fig. 7b). The reason behind that is the local tree structure in our proposed scheme depends on the number of documents and the bitmap's length.

As illustrated in Fig. 8, we compare the average number of search tokens with respect to different range sizes. Here we set the number of documents to d = 80,000, and the number of



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Fig. 8. Number of Tokens for Different Range Sizes

keywords to m = 4,096. The range size represents the number of distinct values within the range. Assume that the range size is q, a straightforward solution is to transform each value into a search token, where the number of search tokens is also q. Zuo *et al.*'s scheme [11] adopts the BRC technique to reduce the number of search tokens from q to $O(\log q)$. Benefiting from the order-weighted inverted index, Wang *et al.*'s scheme [10] and our proposed scheme have a smaller number of search tokens. To be specific, the number of search tokens in Wang *et al.*'s scheme is always 2, and in our proposed scheme, the number of search tokens is even smaller, which is at most 2.

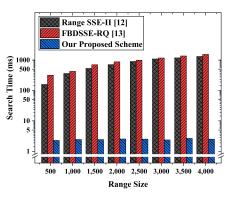


Fig. 9. Comparison of Search Time for Different Range Sizes

As for the storage overhead, TABLE III lists the size of the encrypted index (stored on the server's side) and local binary tree (stored on the client's side) for each scheme. We can see that when the number of keywords is fixed, the encrypted index size increases linearly with the number of documents except for our scheme. In other words, we eliminate the impact of large document identifiers by splitting the whole range into disjoint subranges. In this way, each keyword in our inverted index only needs to carry a small set of document identifiers within the corresponding subrange. That is to say, the encrypted index size in our scheme is only related to the number of keywords. In addition, we find that the encrypted index size in Wang et al.'s scheme [10] is extremely huge, which will cause inefficiency when uploading it to the server. In the comparison of the local binary tree, as for our scheme, since we only need to assign each subrange collection to each leaf node, the the storage of local binary tree in our scheme

²All keyword values are rounded to 4 decimal places.

Dataset Settings		Range SSE-II [10]		FBDSSE-RQ [11]		Our Proposed Scheme		
		Client	Server	Client	Client Server		Server	
	d = 60,000	193 KB	527 MB	193 KB	61 MB	37 KB	3,297 KB	
m = 4,096	d = 100,000	193 KB	928 MB	193 KB	100 MB	38 KB	3,297 KB	
	d = 120,000	193 KB	$1,164~\mathrm{MB}$	193 KB	120 MB	38 KB	$3,297~\mathrm{KB}$	
	m = 2,048	93 KB	309 MB	93 KB	41 MB	19 KB	1,649 KB	
d = 80,000	m = 4,096	193 KB	732 MB	193 KB	80 MB	37 KB	$3,297~\mathrm{KB}$	
	m = 8,192	401 KB	$1,538 \ \mathrm{MB}$	401 KB	159 MB	73 KB	6,593 KB	

TABLE III Comparison of Storage Overhead

 TABLE IV

 Performance of Our Proposed Scheme on Large-Scale Dataset

Dataset Setting	Running Time	Storage Overhead	Storage Overhead		
Dataset Setting	for BuildIndex	(Client's Side)	(Server's side)		
$m = 2^{16}, d = 10^6$	$=2^{16}, d=10^6$ 7.09 s		52 MB		

is lower than that in the other schemes.

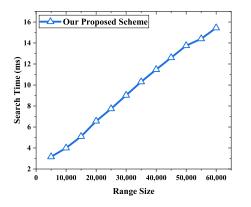


Fig. 10. Search Time of Our Proposed Scheme on Large-Scale Dataset

We also evaluate the search time for the three schemes. Considering that except for our proposed scheme, the other two schemes are not suitable for large-scale datasets. Thus, we choose a smaller dataset in this experiment, i.e., the number of keywords is 4,096, and the number of documents is 80,000. According to the results in Fig. 9, we found that all three range SSE schemes have an efficient search time. Nevertheless, our proposed scheme achieves the best performance among them.

To examine the performance of our proposed scheme on a large-scale dataset, we also conduct additional experiments under a large-scale dataset where the number of documents is set to 10^6 and the number of keywords is set to 2^{16} . The results are shown in TABLE IV and Fig. 10. We find that even in a dataset with a million records, the computation time of generating index is still efficient (around 7.09s), and the storage overhead on the server's side (i.e., the size of EDB) is around 52 MB, which is still small compared to the results of prior arts in TABLE III. As for the search time, the results in Fig. 10 also demonstrate that our proposed scheme remains highly efficient in search even the range size is extremely large.

VIII. CONCLUSION

In this paper, we proposed a novel range SSE scheme that avoids volume-pattern leakage and supports efficient operations on large-scale datasets. By leveraging the orderweighted inverted index and bitmap structure, we designed an order-accumulated inverted index that provides optimal token size and simultaneously conceals volume patterns. Through our partitioning strategy, the proposed scheme achieves high efficiency for range queries on large-scale datasets. Based on the leakage function, we demonstrated the security of our scheme under the ideal/real model simulation paradigm. Finally, we conducted a series of experiments that highlight the practicality of our proposed scheme compared to prior methods, especially for large-scale datasets.

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