Energy-Aware Cellular Deployment Strategy Under Coverage Performance Constraints

Jinlin Peng, Peilin Hong, and Kaiping Xue

Abstract—**The last ten years have witnessed explosive growth in mobile data traffic, which leads to rapid increases in energy consumption of cellular networks. One potential solution to this issue is to seek out a green deployment strategy. In this paper, we investigate the energy-efficient deployment strategy under coverage performance constraints for both homogeneous and heterogeneous cellular networks. Unlike just considering the base station (BS) density in previous work, we jointly optimize the BS density and the BS transmission power. First, we derive the relation between the average coverage probability and deployment strategy (i.e., BS density and BS transmission power) with stochastic geometry tools. Then, based on the expression results, we formulate a network energy consumption minimization framework considering coverage performance constraints and jointly determine the optimal macro BS (MaBS) density, MaBS transmission power, and micro BS (MiBS) density. With practical data sets, numerical simulation results show the following: 1) compared with homogeneous network deployment, heterogeneous network deployment has the advantage in energy efficiency performance, and 2) our joint BS density and BS transmission power optimization strategy exceeds the existing strategy, which just considers the BS density optimization in terms of energy efficiency.**

Index Terms—**Cellular networks, stochastic geometry, energy efficiency, BS density, transmission power.**

I. Introduction

In recent years, the increasing use of wireless connectivity via smart-phones and laptops has led to an exponential surge in network traffic, which presents cellular network operators with several challenges. One of the challenges is how to provide high quality of service (QoS) for the explosive data traffic. To handle this problem, both LTE [1] and WiMAX [2] standard groups have introduced micro Base Stations (MiBSs) concept, such as pico BSs, femto BSs, and relay nodes, in traditional homogeneous cellular networks. These MiBSs and macro BSs (MaBSs) constitute the heterogeneousness cellular networks. Deploying such MiBSs aims at offloading the traffic of MaBSs, improving indoor coverage and cell-edge user equipment performance, and boosting spectral efficiency per area unit via spatial reuse [3]–[5].

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The authors are with the Information Network Lab, Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei 230027, China (e-mail: pjinlin@mail.ustc.edu.cn; plhong@ustc.edu.cn; kpxue@ustc.edu.cn).

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Besides, meeting traffic demands will cause a significant increase in operator energy consumption. Potential harmful effects to the environment caused by carbon dioxide (CO_2) emissions and the sharp rising energy cost bring focus on developing more energy efficient underlying network infrastructures. It is estimated that 3% of the world's electrical energy consumption and 2% of CO_2 emissions are caused by the information and communication technology (ICT) industry [6]. And about a tenth of this can be attributed to cellular systems. Pushed by such needs of energy reduction, the operators have been seeking all kinds of ways to improve energy efficiency in all components of cellular networks, especially BS, which is reported to consume about 60–80% energy [7]. In this regard, European Commission has started projects within its seventh Framework Programme to address the energy efficiency of mobile communication systems, viz. "Energy Aware Radio and NeTwork TecHnologies (EARTH)" [8] and "Towards Real Energy-efficient Network Design (TREND)" [9].

In this paper, we pursue a unified study on QoS and energy efficiency performance from the perspective of network deployment. It should be noted that there are many studies on improving energy efficiency while considering QoS constraints with dynamic BS sleeping [10]–[15] and green deployment techniques [16]–[20]. However, most of them just investigate the sleep operations or BS density optimization with consideration of blocking probability, delay or flow-level performance constraints. Firstly, few of them consider the coverage performance. Although we can decrease the BS density to cut down the system energy consumption through sleep operations or deployment planning, it may bring coverage holes which are not covered by less BSs. It has a bad effect on UEs' experience when UEs move to coverage holes or when sessions are established in coverage holes. Thus, the coverage performance is an important factor and should be considered before some BSs are turned off or at the system planning stage. Considering the above tradeoff between system coverage and energy consumption performance, we investigate the network energy consumption minimization problem under coverage performance constraints in this paper. More importantly, to the best of our knowledge, all of the existing works focus on the BS density optimization, and there is no work jointly considering the BS transmission power and BS density, both of which couple with each other and have effects of different intensities on the system coverage and energy consumption performance. With transmission power adjustment, we can guarantee the same coverage requirement with different optimal BS densities, which finally leads to different energy consumption performance. Therefore, through jointly optimizing BS transmission power and BS density, we may achieve a higher energy efficiency while guaranteeing the coverage performance.

This paper firstly derives and analyzes the relations among the average coverage probability, network energy consumption and deployment strategy (i.e., BS density and BS transmission power) with stochastic geometry tools. Then based on the expression results, we formulate a theoretical framework which minimizes the network energy consumption under coverage performance constraints, and jointly solve the optimal solutions for both homogeneous and heterogeneous cellular networks. This work at least can give the answers to the following questions, and hence provides some new insights into the deployment problem in cellular networks: 1) Given the predefined QoS requirement, what are the optimal MaBS density, MaBS transmission power, and MiBS density? 2) In addition to providing better QoS, can heterogeneous network deployment achieve higher energy efficiency than homogeneous network deployment?

A. Related Work

The energy consumption problem in cellular networks has become more and more crucial. As one potential solution to improve energy efficiency, dynamic BS sleeping has been studied by a lot of researchers [10]–[15]. Several algorithms and schemes have been proposed to minimize the number of active BSs while satisfying the traffic load requirement in [10] and [11]. With some BSs in sleep mode, the QoS may be deteriorated. To save energy while maintaining acceptable QoS, [12]–[15] investigate the sleep operations considering blocking probability, delay, spectral efficiency and flow-level performance, respectively. However, almost all of the above works ignore the coverage performance and none of them focuses on joint BS density and BS transmission power optimization, which are the focus of this paper.

There are also many works [16]–[20] on cellular network deployment. Through simulations without any theoretical verification, [16] points out that heterogeneous network deployment can bring up to 50% BS energy reduction. References [17]–[19] focus on designing practical deployment algorithm in theory. Mostly, it is an NP-hard mixed integer programming problem with too many parameters and constraints. Hence, to achieve the optimal solution, many numerical methods, such as genetic algorithm [17], tabu search [18], and Lagrange relaxation [19], have been proposed. Unfortunately, each these solution is problem-specific and can be viewed as the subsequent step of BS density optimization which will be discussed in this paper. Reference [20] attempts to find out the maximal inter-BS distance for CDMA cellular networks based on the idealized regular (hexagonal) cellular network model. However, this work is difficult to extend to heterogeneous cellular networks.

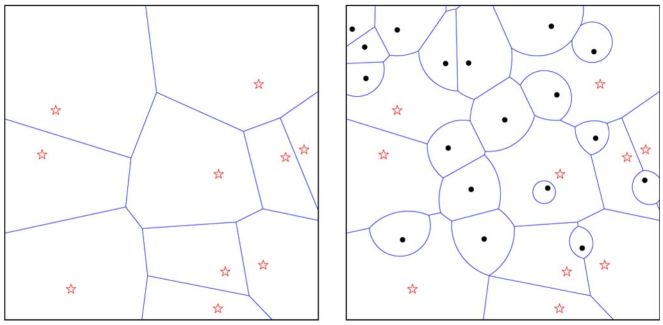
To derive and analyze the relations among the average coverage probability, network energy consumption and deployment strategy, we model locations of BS and user as homogeneous Poisson Point Processes (PPPs) [21], which has been proved as a tractable yet accurate model recently [22], [23]. Besides, some researchers have used this stochastic geometry model to analyze and solve some practical problems in multi-tier cellular networks: frequency reuse, spectrum allocation, biasing and load balancing and access policy [24]–[27]. Some more detail descriptions about PPP model can be found in [28]. In this paper, we use this model to deal with the energy saving problem in cellular networks.

There are some similar works building on stochastic geometry model to address the energy saving problem in cellular networks [29]–[32]. Considering BS sleep mode, [29] minimizes the energy consumption while ensuring that the outage probability is below a given threshold through switching off the fit proportion of MaBSs. Reference [30], which only considers the homogeneous cellular network scenario, further applies the dynamic power control for the remaining active MaBSs to remain the same coverage probability as before MaBSs are switched off. The studies most relevant to ours are [31], [32], where the optimal BS density for both homogeneous and heterogeneous cellular networks to minimize energy cost is analyzed from the perspective of network deployment. Unfortunately, they just use the fixed BS transmission power and do not discuss the impact of transmission power. With adapting the BS transmission power dynamically, the BS density should be re-optimized and the network energy consumption can be significantly reduced, as shown in this paper.

B. Contributions and Organization

In this paper, we focus on the energy efficient deployment strategy for both homogeneous and heterogeneous cellular networks under coverage performance constraints. In summary, we have made following contributions in this paper:

- With stochastic geometry tools, we derive and analyze the expressions of average coverage probability for two kinds of user association schemes. Although similar expressions of average coverage probability have been given in [23], [33], through some transformations and defining an energy-related deployment factor, we find some interesting results: the average coverage probability increases with the increase of energy-related deployment factor and eventually converges to a fixed value.
- Based on the above observations, we formulate a theoretical framework which jointly optimizes BS density and BS transmission power to minimize the area power consumption under coverage performance constraints. Besides, the optimal solutions are given out, which provides some new insights into the cellular deployment. Different from the existing works without considering transmission power, we can guarantee the same coverage requirement with different optimal BS densities through adjusting transmission power, which finally achieves a higher energy efficiency performance.
- Finally, through extensive simulations with practical data sets, we draw these conclusions: 1) Compared to homogeneous network deployment, heterogeneous network deployment has absolute advantage in energy efficiency performance. 2) Our joint BS density and BS transmission power optimization strategy exceeds the existing strategy which just considers BS density optimization in terms of energy efficiency.



(a) (b)

Fig. 1. Close-up view of coverage regions in homogeneous cellular networks and two-tier heterogeneous cellular networks. Red stars and black points represent MaBSs and MiBSs respectively. (a) Close-up view of coverage regions in homogeneous cellular networks. (b) Close-up view of coverage regions in two-tier heterogeneous cellular networks. ($P_M = 20P_m$ and $\lambda_m = 2\lambda_M$.)

The rest of the paper is organized as follows: In Section II, we present our system model and assumptions. We derive and analyze the relations between the coverage performance and deployment strategy in Section III. Then, we design the optimal deployment strategy under the coverage performance constraints for both homogeneous and heterogeneous cellular networks in Section IV. Numerical simulation results are presented and discussed in Section V. Finally, we conclude our work in Section VI.

II. System Model

A. Network Topology Model

As shown in Fig. 1(b), we consider a heterogeneous cellular networks consisting of 2 tiers of BSs, where tier m and M represent MiBSs and MaBSs, respectively. MiBSs and MaBSs are modeled as independent homogeneous PPPs Φ_m and Φ_M with intensities λ_m and λ_M respectively. We assume that BSs in tier i use the same link transmission power $\{P_i\}_{i=m,M}$. Users are also located according to a homogeneous PPP. In this paper, we focus on designing the network deployment parameters, i.e., λ_M, λ_m, P_M, and P_m. Note that the heterogeneous cellular networks degenerates to homogeneous cellular networks as shown in Fig. 1(a) when $\lambda_m = 0$ or $P_m = 0$.

There are several reasons for choosing this BS and user location model: 1) Though the PPP model is not an exact fit, it reasonably approximates the variability introduced by practical constraints of MaBS locations and the potential random location of MiBSs and users [32]. 2) This model is tractable and easy to handle, and provides tight bounds for the performance parameters in planned infrastructure-based networks and coordinated spectrum access networks [28]. 3) This model is suitable to analyze network planning and dynamic BS sleeping, as the independent thinning of a PPP are still a PPP. Some more detail descriptions about PPP model can be found in [28].

B. Channel Model

Without loss of generality, we conduct analysis on a typical user located at the origin. The fading loss between a BS located at x_i (belonging to ith tier) and the typical user is denoted as h_{x_i}, which is assumed to be i.i.d exponential (Rayleigh fading), i.e., $h_{x_i} \sim \exp(1)$. The standard path loss function is given by $l(x_i) = \|x_i\|^{-\alpha}$, where $\alpha > 2$ is the path loss exponent. Hence, the received power at a typical user from a BS located at point x_i is $P_i h_{x_i}\|x_i\|^{-\alpha}$. The resulting signal to interference plus noise ratio (SINR) expression assuming the user connects to this BS is:

$$SINR(x_i) = \frac{P_i h_{x_i}\|x_i\|^{-\alpha}}{I_{x_i} + \sigma^2}, \tag{1}$$

where $I_{x_i} = \sum_{j=m,M}\sum_{x_j\in\Phi_j\backslash x_i} P_j h_{x_j}\|x_j\|^{-\alpha}$ is the interference, and σ^2 is the constant additive noise power.

C. MiBS Operating Mode and User Association Scheme

Each user associates with which BS depends on the MiBS operating mode and association schemes. In this paper, we consider that all MiBSs operate in *open access* in heterogeneous cellular networks, i.e., any user is allowed to connect to MiBSs. Besides, we consider the following two user association schemes, namely *instantaneous SINR based scheme* and *average received power based scheme*:

- *Instantaneous SINR based scheme* [23]: User connects to tier i if the instantaneous SINR exceeds a given threshold γ. In heterogeneous cellular networks, to make sure at most one tier can provide a signal exceeding the threshold and thus serve user, γ should be larger than 1.
- *Average received power based scheme*: With this scheme, user is associated to the strongest BS in terms of long-term average received power, i.e., it is connected to tier $i = \arg\max_{i=m,M}\{P_i\|r_i\|^{-\alpha}\}$, where r_i denotes the distance between the user and its nearest BS in the ith tier. The resulting coverage regions are shown in Fig. 1.

We assume that universal frequency reuse is applied and each BS allocates all the channel resources (e.g., time and spectrum resources) equally to the users it serves.

D. BS Power Consumption Model

In order to quantify the energy demand of system, we refer to [34] to build the BS power consumption model, which considers the power consumption of power amplifier (PA), signal processing, A/D converter, antenna, cooling, power supply loss, and battery backup. The formula of MaBS's power consumption is specified as follow:

$$P_{M,tot} = N_{PApS}\left(\frac{P_{M,TX}}{\mu_{PA}} + P_{SP}\right)(1+C_C)(1+C_{PB}), \tag{2}$$

where N_{PApS} is the number of PAs, $P_{M,TX} = N \cdot P_M$ is the total transmission power of N links/channels, μ_{PA} is PA efficiency, P_{SP} is signal processing overhead, C_C is cooling loss, and C_{PB} is battery backup and power supply loss. With the fixed parameter settings, the power consumption of MaBS can be simply written as the similar linear form to [16]:

$$P_{M,tot} = a_M \cdot N \cdot P_M + b_M, \tag{3}$$

where the coefficient a_M accounts for power consumption that scales with the average radiated power, and the term b_M models the static power consumed by signal processing, battery backup and cooling.

The above power consumption model is also applicable for MiBSs. The difference is the detail parameter values:

$$P_{m,tot} = a_m \cdot N \cdot P_m + b_m. \tag{4}$$

E. Some Extensions

Although the above system model we considered is relatively simple, it can easily extended to (at least but not limited to) the following more general cases:

- Shadowing. It should be noted that the shadowing is not considered in this paper. Through scaling the BS densities with $\mathbb{E}\{\chi^{2/\alpha}\}$ where χ is shadowing, all the results can be easily extended to the case with shadowing as [14], [37].
- Multiple Antennas. Additional enhancements like multiple antenna communication could be extended to improve coverage performance. Although we do not explicitly consider antenna sectoring (that is multiple antenna communication situation), it can be easily incorporated in the current model if sectoring is done randomly. If the beam is partitioned into n equal sectors, the density of interfering BSs reduces by a factor of n because the probability that the beam of any BS would point towards a randomly chosen BS is $1/n$.
- Power Control. The topic discussed in this paper is deployment strategy problem. Therefore, the assumption that BSs in each tier use the same link transmission power is reasonable since BSs in each tier (e.g., MaBSs and MiBSs) have the same configurations. However, power control together with other schemes such as frequency reuse and resource allocation [38], [39] can further optimize the system performance. Therefore, we propose a simple way to approximately extend our work to consider the case where MaBSs or MiBSs use different link transmission power. That is, we assume MaBSs as well as MiBSs have K1 and K2 kinds of link transmission power configuration respectively and each BS is randomly configured with the link transmission power. Then the analysis model is also applicable just by extending the tier number from 2 to $\mathrm{K} = \mathrm{K1} + \mathrm{K2}$.

III. Impact of Deployment Strategy on Coverage Performance

In this section, we derive and analyze the relations between the average coverage probability and deployment strategy (i.e., BS density and BS transmission power) with stochastic geometry tools, which can guide the deployment strategy optimization in next section. Although similar expressions of average coverage probability have been given in [23], [33], they are not intuitive for designing deployment strategy because BS density and BS transmission power are separated. Through some transformations, we combine them together as one term, named energy-related deployment factor, and find some new interesting results as shown in Theorem 1, Theorem 2 and Theorem 3.

The coverage probability of a user is defined as:

$$P_C = \mathbb{P}[SINR \geq \gamma], \tag{5}$$

where γ is the outage threshold. Note that the outage probability is $1 - P_C = \mathbb{P}[SINR < \gamma]$, which is also the cumulative distribution function (cdf) of the user's SINR. Based on this, our following work can be extended to guarantee SINR or spectral efficiency distribution performance, rather than just coverage performance. (5) can be calculated by:

$$P_C = \mathbb{E}_{x_i}\left(\mathbb{P}\left[SINR(x_i) \geq \gamma\right]\right), \tag{6}$$

where $\mathbb{E}_{x_i}(.)$ represents taking the expectation with respect to x_i, and $\mathbb{P}[SINR(x_i) \geq \gamma]$ is the coverage probability given that the user is associated with the BS located at point x_i (belonging to ith tier), which can expressed as:

$$\begin{aligned}
\mathbb{P}\left[SINR(x_i) \geq \gamma\right] &= \mathbb{P}\left[\frac{P_i h_{x_i} l(x_i)}{I_{x_i} + \sigma^2} \geq \gamma\right] \\
&= \mathbb{P}\left[h_{x_i} \geq \frac{\gamma\left(I_{x_i} + \sigma^2\right)}{P_i l(x_i)}\right] \\
&\overset{(a)}{=} \exp\left(\frac{-\gamma\sigma^2}{P_i l(x_i)}\right)\mathbb{E}_{I_{x_i}}\left(\exp\left(\frac{\gamma I_{x_i}}{P_i l(x_i)}\right)\right) \\
&= \exp\left(\frac{-\gamma\sigma^2}{P_i l(x_i)}\right)\mathcal{L}_{I_{x_i}}\left(\frac{\gamma}{P_i l(x_i)}\right),
\end{aligned} \tag{7}$$

where (a) follows from $h_{x_i} \sim \exp(1)$, and $\mathcal{L}_{I_{x_i}}(s)$ is the Laplace transform of the cumulative interference from all the tiers I_{x_i}, which is given by:

$$\begin{aligned}
\mathcal{L}_{I_{x_i}}(s) &= \prod_{j=m,M} \mathbb{E}_{\Phi_j}\left[\prod_{x_j \in \Phi_j / x_i} \mathbb{E}_{h_{x_j}}\left[\exp\left(-sP_j h_{x_j} l(x_j)\right)\right]\right] \\
&= \prod_{j=m,M} \mathbb{E}_{\Phi_j}\left[\prod_{x_j \in \Phi_j / x_i} \frac{1}{1 + sP_j l(x_j)}\right].
\end{aligned} \tag{8}$$

Obviously, (6) and (8) are dependent on the location distributions of served BS x_i and interference BSs x_j. Though we apply the PPP model, with different user association schemes, there are different distribution results and hence cause different coverage results. The coverage probabilities of heterogeneous networks under *instantaneous SINR based scheme* and *average received power based scheme* are given out in Theorem 1 and Theorem 2, respectively.

Theorem 1: (Instantaneous SINR based scheme). For heterogeneous networks consisting of MaBSs and MiBSs, when $\gamma > 1$, the coverage probability for a typical randomly located user under the instantaneous SINR based scheme is:

$$P_{SINR}(A) = \pi A \int_{t=0}^{\infty} \exp\left(-AC(\alpha)\gamma^{2/\alpha} t\right) \exp\left(-\gamma\sigma^2 t^{\frac{\alpha}{2}}\right) dt, \tag{9}$$

where $A = \lambda_m P_m^{2/\alpha} + \lambda_M P_M^{2/\alpha}$, and $C(\alpha) = (2\pi^2/\alpha)\csc(2\pi/\alpha)$.

Proof: Combine (6)–(8) with lemma 1 in [23] (i.e., a typical user can connect to at most one BS when $\gamma > 1$) and Campbell Mecke Theorem, and through some algebraic manipulations similarly to [23], we can get the following result (the more details can be found in [23]):

$$\begin{aligned}P_{SINR} &= \lambda_M \int_{r=0}^{\infty} 2\pi r \exp\left(-\left(\lambda_M P_M^{2/\alpha} + \lambda_m P_m^{2/\alpha}\right) C(\alpha) (\gamma/P_M)^{2/\alpha} r^2\right) \\ &\quad \times \exp\left(-(\gamma/P_M)\sigma^2 r^\alpha\right) dr \\ &+ \lambda_m \int_{r=0}^{\infty} 2\pi r \exp\left(-\left(\lambda_M P_M^{2/\alpha} + \lambda_m P_m^{2/\alpha}\right) C(\alpha) (\gamma/P_m)^{2/\alpha} r^2\right) \\ &\quad \times \exp\left(-(\gamma/P_m)\sigma^2 r^\alpha\right) dr. \qquad (10)\end{aligned}$$

The result consists of two parts. For the first part, we use transformation $t = r^2/P_M^{2/\alpha}$. For the second part we use transformation $t = r^2/P_m^{2/\alpha}$. Then after some algebraic manipulations, we get the formula (9), which completes the proof. □

Theorem 2: (Average received power based scheme). The coverage probability for a typical randomly located user in the heterogeneous networks consisting of MaBSs and MiBSs under the average received power based scheme is:

$$P_{RP}(A) = \pi A \int_{t=0}^{\infty} \exp\left(-A\,(1+\mathcal{Z}(\gamma,\alpha,1))t\right) \exp\left(-\gamma\sigma^2 t^{\frac{\alpha}{2}}\right) dt, \qquad (11)$$

where $\mathcal{Z}(\gamma,\alpha,1) = -2\gamma_2 F_1[1; 1-2/\alpha; 2-2/\alpha; -\gamma]$, and ${}_2F_1[\cdot]$ denotes the Gauss hypergeometric function.

Proof: Combine (6)–(8) with lemma 1 (i.e., the probability that a typical user is associated with the ith tier is $\mathcal{B}_i = \lambda_i P_i^{2/\alpha}/(\lambda_m P_m^{2/\alpha} + \lambda_M P_M^{2/\alpha})$) and lemma 2 (i.e., the PDF of the distance between a typical user and its serving BS $\|x_i\|$ given that the user is associated with the ith tier is $f_{\|x_i\|}(x) = (2\pi\lambda_i/\mathcal{B}_i) x \exp\{-\pi x(\lambda_M (P_M/P_i)^{2/\alpha} + \lambda_m (P_m/P_i)^{2/\alpha})\}$.) in [33], and through some algebraic manipulations similarly to [33], we can get the following result (the more details can be found in [33]):

$$\begin{aligned}P_{RP} &= \lambda_M \int_{r=0}^{\infty} 2\pi r \exp\left(-\left(\lambda_M + \lambda_m P_m/P_M^{2/\alpha}\right)\left(1+\mathcal{Z}(\gamma,\alpha,1)\right) r^2\right) \\ &\quad \times \exp\left(-(\gamma/P_M)\sigma^2 r^\alpha\right) dr \\ &+ \lambda_m \int_{r=0}^{\infty} 2\pi r \exp\left(-\left(\lambda_M P_M/P_m^{2/\alpha} + \lambda_m\right)\left(1+\mathcal{Z}(\gamma,\alpha,1)\right) r^2\right) \\ &\quad \times \exp\left(-(\gamma/P_m)\sigma^2 r^\alpha\right) dr. \qquad (12)\end{aligned}$$

Similarly to the proof of Theorem 1, we use transformations $t = r^2/P_M^{2/\alpha}$ and $t = r^2/P_m^{2/\alpha}$ for the first part and the second part, respectively. Then after some algebraic manipulations, we get the formula (11), which completes the proof. □

Remark 1: The above results (9) and (11) can be simplified further for the homogeneous case by setting $\lambda_m = 0$ or $P_m = 0$, in which case they reduce to remarkably simple expressions. Besides, Theorem 1 and Theorem 2 show that no matter which kind of user association scheme is applied, the average coverage probability is dependent on the term $A = \lambda_m P_m^{2/\alpha} + \lambda_M P_M^{2/\alpha}$, which is defined as energy-related deployment factor in this paper.

Obviously, the network energy consumption is an increasing function of both the P_M (P_m) and λ_M (λ_m) because the larger P_M (P_m) means larger transmission power and the larger λ_M (λ_m) represents more BSs. What's about the monotonicity of coverage probability function on energy-related deployment factor A? With larger A (P_M, P_m, λ_M, and λ_m), although the received signal is stronger, the interference user suffers also becomes more serious. Interestingly, we can find that the average coverage probability also increases with increasing energy-related deployment factor A (P_M, P_m, λ_M, and λ_m) as shown in Theorem 3, which will guide us optimize P_M (P_m) and λ_M (λ_m) in next section.

Theorem 3: For a typical randomly located user in the heterogeneous networks consisting of MaBSs and MiBSs, when $\alpha > 2$ and $\sigma^2 > 0$, the coverage probabilities $P_{SINR}(A)$ and $P_{RP}(A)$ are monotonically increasing functions of energy-related deployment factor A.

Proof: We take the analysis of $P_{SINR}(A)$ as an example here, $P_{RP}(A)$ can be easily obtained through similar process. Assume any two positive real numbers A_1 and A_2 satisfying $A_2 > A_1$, then we have

$$\begin{aligned}&P_{SINR}(A_2) \\ &= \pi A_2 \int_{t_2=0}^{\infty} \exp\left(-A_2 C(\alpha)\gamma^{2/\alpha} t_2\right) \exp\left(-\gamma\sigma^2 t_2^{\frac{\alpha}{2}}\right) dt_2 \\ &\overset{(a)}{=} \pi A_2 \frac{A_1}{A_2} \int_{t_1=0}^{\infty} \exp\left(-A_2 C(\alpha)\gamma^{2/\alpha} \frac{t_1 A_1}{A_2}\right) \\ &\quad \times \exp\left(-\gamma\sigma^2 \left(\frac{t_1 A_1}{A_2}\right)^{\frac{\alpha}{2}}\right) dt_1 \\ &= \pi A_1 \int_{t_1=0}^{\infty} \exp\left(-A_1 C(\alpha)\gamma^{2/\alpha} t_1\right) \\ &\quad \times \exp\left(-\gamma\sigma^2 t_1^{\frac{\alpha}{2}} \left(\frac{A_1}{A_2}\right)^{\frac{\alpha}{2}}\right) dt_1 \\ &\overset{(b)}{>} \pi A_1 \int_{t_1=0}^{\infty} \exp\left(-A_1 C(\alpha)\gamma^{2/\alpha} t_1\right) \exp\left(-\gamma\sigma^2 t_1^{\frac{\alpha}{2}}\right) dt_1 \\ &= P_{SINR}(A_1), \qquad (13)\end{aligned}$$

where (a) follows from the transformation $t_2 = t_1(A_1/A_2)$, and (b) makes use of the fact $\alpha > 2$, $\gamma\sigma^2 > 0$, and $(A_1/A_2) < 1$. □

With Theorem 3 and the fact that the coverage probability is always not larger than 1, we know that the coverage probability ultimately converges to a fixed value with the increase of A. The value of the converged probability is bounded as Lemma 1.

Lemma 1: When $A \to \infty$, the coverage probabilities are bounded as

$$\frac{\pi \exp\left(-\gamma\sigma^2 m^{\frac{\alpha}{2}}\right)}{C(\alpha)\gamma^{2/\alpha}} \le \lim_{A\to\infty} P_{SINR}(A) \le \frac{\pi}{C(\alpha)\gamma^{2/\alpha}}, \qquad (14)$$

$$\frac{\pi \exp\left(-\gamma\sigma^2 m^{\frac{\alpha}{2}}\right)}{1+\mathcal{Z}(\gamma,\alpha,1)} \le \lim_{A\to\infty} P_{RP}(A) \le \frac{\pi}{1+\mathcal{Z}(\gamma,\alpha,1)}. \qquad (15)$$

where m is an arbitrary positive constant.

Proof: We take the analysis of $P_{SINR}(A)$ as an example here, $P_{RP}(A)$ can be easily obtained through similar process.

$$\begin{aligned} &P_{SINR}(A)\\ &=\pi\int_{t=0}^{\infty}\exp\left(-C(\alpha)\gamma^{2/\alpha}t\right)\exp\left(-\gamma\sigma^2\left(\frac{t}{A}\right)^{\frac{\alpha}{2}}\right)dt\\ &=\pi\int_{t=0}^{mA}\exp\left(-C(\alpha)\gamma^{2/\alpha}t\right)\exp\left(-\gamma\sigma^2\left(\frac{t}{A}\right)^{\frac{\alpha}{2}}\right)dt\\ &\quad+\pi\int_{t=mA}^{\infty}\exp\left(-C(\alpha)\gamma^{2/\alpha}t\right)\exp\left(-\gamma\sigma^2\left(\frac{t}{A}\right)^{\frac{\alpha}{2}}\right)dt. \end{aligned} \tag{16}$$

When $A\to\infty$, the second term ultimately converges to 0 because $0\le\exp(-\gamma\sigma^2(t/A)^{\alpha/2})\le\exp(-\gamma\sigma^2m^{\alpha/2})\le1$. For the first term, due to $\exp(-\gamma\sigma^2m^{\alpha/2})\le\exp(-\gamma\sigma^2(t/A)^{\alpha/2})\le 1$, we have

$$\begin{aligned} &\lim_{A\to\infty}\pi\int_{t=0}^{mA}\exp\left(-C(\alpha)\gamma^{2/\alpha}t\right)\exp\left(-\gamma\sigma^2\left(\frac{t}{A}\right)^{\frac{\alpha}{2}}\right)dt,\\ &=\lim_{A\to\infty}\pi\int_{t=0}^{\infty}\exp\left(-C(\alpha)\gamma^{2/\alpha}t\right)\exp\left(-\gamma\sigma^2\left(\frac{t}{A}\right)^{\frac{\alpha}{2}}\right)dt,\\ &\in\left[\frac{\pi\exp\left(-\gamma\sigma^2m^{\frac{\alpha}{2}}\right)}{C(\alpha)\gamma^{2/\alpha}},\frac{\pi}{C(\alpha)\gamma^{2/\alpha}}\right]. \end{aligned} \tag{17}$$

Thus, the lemma 1 is proved. □

Remark 2: From lemma 1, let $m\to 0$, we have that the saturated coverage probabilities are near $\pi/C(\alpha)\gamma^{2/\alpha}$ and $\pi/(1+\mathcal{Z}(\gamma,\alpha,1))$, respectively. The physical meaning of convergence is that initially, when the transmission power or density of BS (i.e., energy-related factor) is small (i.e., noise-limited networks), that is noise-limited networks, increasing BS transmission power or density can enhance the received signal strength thus increase the coverage probabilities. However, with the increase of BS transmission power or density, the networks are becoming interference-limited. In dense networks, the improvement in received signal power by adding more BSs or increasing transmission power will also introduce more interference. This is the main reason why the coverage probability ultimately converges to a fixed value (which may be smaller than 1.) Besides, from formulas (9) and (11), we find that the parameters which affect the saturated coverage probability include the outage threshold γ, additive noise power σ^2 and path loss exponent α. Obviously, the saturated coverage probabilities are the monotonically decreasing functions of γ and σ^2. It is intuitive that the channel condition is worse with larger σ^2 and it is harder to satisfy the coverage requirement with larger outage threshold γ. However, they are not the monotonic functions of α since it affects both the received signal power and interference power.

The fact that coverage probabilities ultimately converge to a fixed value with the increase of A indicates that when A is large enough, increasing A is not meaningful to enhance coverage probabilities while consuming additional energy, which provides us with energy saving possibility. Based on this point, we formulate an area power consumption minimization framework under coverage performance constraints, and jointly determine the optimal MaBS density, transmission power of MaBS, and density of MiBS in next section.

IV. Optimal Deployment Strategy

After deriving and analyzing the coverage performance in previous section, we will formulate a theoretical framework and design the energy efficient deployment strategy for both homogeneous and heterogeneous cellular networks in this section.

A. Problem Formulation

There are a lot of metrics for energy efficiency, such as Area Power Consumption (APC, power per area unit measured in W/m^2) [40] and Energy Consumption Rating (ECR, energy per bit measured in J/bit) [41]. In this paper, we use the APC as energy efficiency metric. APC can be also equivalent to the network energy consumption which is the product of APC, constant area and time.

In order to pursue a unified study on coverage performance and energy efficiency performance, we formulate a theoretical framework which jointly determines the optimal BS density and BS transmission power to minimize the APC while guaranteeing that the coverage probability is higher than a given expected value $P_{\exp}$ as follows (noted as $\mathbf{P0}$):

$$\begin{aligned} \underset{\lambda_M,P_M,\lambda_m,P_m}{\text{minimize}}\quad & APC\\ \text{subject to}\quad & P_C(\lambda_M,P_M,\lambda_m,P_m)\ge P_{\exp},\\ & P_{M,\max}\ge P_M\ge 0,\\ & P_{m,\max}\ge P_m\ge 0,\\ & \lambda_{M,\max}\ge\lambda_M\ge 0,\\ & \lambda_{m,\max}\ge\lambda_m\ge 0, \end{aligned} \tag{18}$$

where

$$APC=\lambda_M(a_MNP_M+b_M)+\lambda_m(a_mNP_m+b_m). \tag{19}$$

The first constraint represents that the coverage probability should be higher than a given expected value $P_{\exp}$. Note that through setting γ and $P_{\exp}$, it can be extended to SINR or spectral efficiency distribution constraint. The last four constraints are the maximum value constraints of λ_M, P_M, λ_m, and P_m in practical systems.

Obviously, APC is the monotonically increasing function of λ_M, P_M, λ_m, and P_m. Besides, as shown in Theorem 3, P_C is also the monotonically increasing function of λ_M, P_M, λ_m, and P_m. Therefore, we have the following conclusion:

Lemma 2: If there are feasible solutions for $\mathbf{P0}$, then the optimal solutions must satisfy

$$\lambda_mP_m^{2/\alpha}+\lambda_MP_M^{2/\alpha}=A^*, \tag{20}$$

where A^* depends on the user association scheme and is calculated by (21) and (22) when instantaneous SINR and average received power based scheme are applied respectively.

$$P_{SINR}(A^*)=P_{\exp}, \tag{21}$$
$$P_{RP}(A^*)=P_{\exp}. \tag{22}$$

Lemma 2 can be easily proved by contradiction, which is omitted for brevity. It should be noted that with the monotonicity of $P_{SINR}(A)$ and $P_{RP}(A)$, A^* can be easily solved by bisection method [36]. With lemma 2, the first constraint of $\mathbf{P0}$ can be replaced by (20).

In the following parts, we will optimize BS density and BS transmission power for homogeneous and heterogeneous cellular networks, respectively.

B. Optimal Homogeneous Cellular Networks Deployment

In homogeneous cellular networks where $\lambda_m = 0$ and $P_m = 0$, $\mathbf{P0}$ is equivalent to (noted as $\mathbf{P1}$):

$$\begin{aligned} \underset{\lambda_M, P_M}{\text{minimize}} \quad & \lambda_M(a_M N P_M + b_M) \\ \text{subject to} \quad & \lambda_M P_M^{2/\alpha} = A^*, \\ & P_{M,\max} \geq P_M \geq 0, \\ & \lambda_{M,\max} \geq \lambda_M \geq 0. \end{aligned} \tag{23}$$

Using the equation constraint, the above two-variable problem can be simplified as single-variable problem. And the solution to this problem is achieved for:

$$P_M^* = \min\left(\max\left(\frac{2b_M}{a_M N(\alpha-2)}, \left(\frac{A^*}{\lambda_{M,\max}}\right)^{\alpha/2}\right), P_{M,\max}\right), \tag{24}$$

$$\lambda_M^* = A^* {P_M^*}^{-2/\alpha}. \tag{25}$$

C. Optimal Heterogeneous Cellular Networks Deployment

In $\mathbf{P0}$, there are four variables coupling with each other. To simplify the analysis, we treat P_m as a constant and try to achieve the optimal λ_M^*, P_M^*, and λ_m^* in the following contents. Although the optimization of P_m is ignored, we can set a lot of typical P_m values and compare the optimal APCs to achieve the optimal P_m^*. It may bring a certain amount of calculations which depends on the set of typical P_m values. However, it is acceptable for deployment planning which is offline.

With lemma 2, we have $\lambda_m = P_m^{-2/\alpha}(A^* - \lambda_M P_M^{2/\alpha})$ and the APC can be re-written as:

$$\begin{aligned} APC &= \lambda_M(a_M N P_M + b_M) + \lambda_m(a_m N P_m + b_m) \\ &= \lambda_M(a_M N P_M + b_M) - P_m^{-2/\alpha}\lambda_M(a_m N P_m + b_m)P_M^{2/\alpha} \\ &\quad + P_m^{-2/\alpha}A^*(a_m N P_m + b_m). \end{aligned} \tag{26}$$

Therefore, the heterogeneous cellular networks deployment problem can be formulated as (noted as $\mathbf{P2}$):

$$\begin{aligned} \underset{\lambda_M, P_M}{\text{minimize}} \quad & \lambda_M G(P_M) \\ \text{subject to} \quad & P_{M,\max} \geq P_M \geq 0, \\ & \lambda_{M,\max} \geq \lambda_M \geq 0, \\ & A^* \geq \lambda_M P_M^{2/\alpha} \geq A^* - \lambda_{m,\max} P_m^{2/\alpha}, \end{aligned} \tag{27}$$

where

$$\begin{aligned} G(x) &= a_1 x - a_2 x^{2/\alpha} + b_M, \\ a_1 &= a_M N, \\ a_2 &= P_m^{-2/\alpha}(a_m N P_m + b_m). \end{aligned} \tag{28}$$

Obviously, one challenge to solve the optimal deployment strategy problem is that λ_M and P_M couple with each other. Thus, we try to decouple them to simplify the problem. Fortunately, we have the following lemma.

Lemma 3: For $\mathbf{P2}$, given P_M, we have the optimal λ_M^*:

$$\lambda_M^* = \begin{cases} \max\left(\frac{A^* - \lambda_{m,\max} P_m^{2/\alpha}}{P_M^{2/\alpha}}, 0\right) & G(P_M) \geq 0; \\ \min\left(\frac{A^*}{P_M^{2/\alpha}}, \lambda_{M,\max}\right), & G(P_M) < 0. \end{cases} \tag{29}$$

With lemma 3, the optimal deployment strategy problem can be divided into the following four cases, each of which is single-variable problem and can be solved easily.

1) Case 1: $G(P_M) \geq 0$ and $A^* - \lambda_{m,\max} P_m^{2/\alpha} \leq 0$. In this case, from (29), we have $\lambda_{M,1}^* = 0$ and hence $\lambda_{m,1}^* = P_m^{-2/\alpha}(A^* - \lambda_{M,1}^* P_M^{2/\alpha}) = P_m^{-2/\alpha} A^*$. Then the original problem $\mathbf{P2}$ can be converted to the following problem, called $\mathbf{P2\text{-}1}$:

$$\begin{aligned} \underset{P_M}{\text{minimize}} \quad & 0 \\ \text{subject to} \quad & P_{M,\max} \geq P_M \geq 0, \\ & G(P_M) \geq 0. \end{aligned} \tag{30}$$

The physical meaning behind this case is that when MiBS is more energy efficient than MaBS and all the available MiBSs can support the predefined coverage performance requirement, we only need to deploy $P_m^{-2/\alpha} A^*$ MiBSs.

One difficult to handle $\mathbf{P2\text{-}1}$ (including $\mathbf{P2\text{-}2}\sim\mathbf{P2\text{-}4}$ in following contexts) lies in function $G(x)$. Fortunately, it has the following feature:

Lemma 4: If $G((\alpha a_1/2a_2)^{\alpha/(2-\alpha)}) \geq 0$, then $G(x) \geq 0$ when $x \geq 0$; otherwise, there are x_1 and x_2 satisfying

$$G(x_1) = G(x_2) = 0;$$

$$G(x) < 0, x \in (x_1, x_2);$$

$$G(x) > 0, x \in (0, x_1) \cup (x_2, \infty).$$

Proof: It can be proved by analyzing the derivation of $G(x)$ as follows:

$$\frac{dG(x)}{dx} = a_1 - \frac{2a_2}{\alpha} x^{2/\alpha - 1}. \tag{31}$$

Due to $\alpha > 2$, we have $\lim_{x\to 0}(dG(x)/dx) = -\infty$ and $\lim_{x\to\infty}(dG(x)/dx) = a_1 > 0$. What's more, $(dG(x)/dx)|_{x=(\alpha a_1/2a_2)^{\alpha/(2-\alpha)}} = 0$. Obviously, $dG(x)/dx$ is an increasing function of x. Hence, combining these facts together, we have $G(x)$ is a decreasing function when $x \in (0, (\alpha a_1/2a_2)^{\alpha/(2-\alpha)}]$ and an increasing function when $x \in (\alpha a_1/2a_2)^{\alpha/(2-\alpha)}, \infty)$. Combining this conclusion with $G(0) = b_M > 0$ and $\lim_{x\to\infty} G(x) = \infty$, we have lemma 4. □

If x_1 and x_2 exist, they can be solved by bisection method [36]. With this property, the constraint $G(P_M) \geq 0$ can be further simplified as a closed interval of P_M and **P2-1** can be solved easily. The detail is omitted for brevity here.

There may be no feasible solution for **P2-1**. If the solution exists and suppose it is $P^*_{M,1}$, then we have the optimal value $Y_1 = \lambda^*_{M,1} G(P^*_{M,1}) = 0$; otherwise, let $Y_1 = \infty$.

2) Case 2: $G(P_M) \geq 0$ and $A^* - \lambda_{m,\max} P_m^{2/\alpha} > 0$. In this case, from (29), we have $\lambda^*_{M,2} = (A^* - \lambda_{m,\max} P_m^{2/\alpha})/P_M^{2/\alpha}$ and hence $\lambda^*_{m,2} = P_m^{-2/\alpha}(A^* - \lambda^*_{M,2} P_M^{2/\alpha}) = \lambda_{m,\max}$. Then the original problem **P2** can be converted to the following problem, called **P2-2**:

$$\begin{aligned} \underset{P_M}{\text{minimize}} \quad & \frac{A^* - \lambda_{m,\max} P_m^{2/\alpha}}{P_M^{2/\alpha}} G(P_M) \\ \text{subject to} \quad & P_{M,\max} \geq P_M \geq 0, \\ & \lambda_{M,\max} \geq \frac{A^* - \lambda_{m,\max} P_m^{2/\alpha}}{P_M^{2/\alpha}} \geq 0, \\ & G(P_M) \geq 0. \end{aligned} \tag{32}$$

This case means that although MiBS is more energy efficient, after deploying all the available MiBSs which are not enough to support the predefined coverage performance, we need to additionally deploy some MaBSs. The density of MaBS depends on the optimal transmission power which is the solution of **P2-2**.

There may be no feasible solution for **P2-2**. If the solution exists and suppose it is $P^*_{M,2}$, then we have the optimal value $\lambda^*_{M,2} = (A^* - \lambda_{m,\max} P_m^{2/\alpha})/{P^*_{M,2}}^{2/\alpha}$ and $Y_2 = \lambda^*_{M,2} G(P^*_{M,2})$; otherwise, let $Y_2 = \infty$.

3) Case 3: $G(P_M) < 0$ and $A^*/P_M^{2/\alpha} \leq \lambda_{M,\max}$. In this case, from (29), we have $\lambda^*_{M,3} = A^*/P_M^{2/\alpha}$ and hence $\lambda^*_{m,3} = P_m^{-2/\alpha}(A^* - \lambda^*_{M,3} P_M^{2/\alpha}) = 0$. Then the original problem **P2** can be converted to the following problem, called **P2-3**:

$$\begin{aligned} \underset{P_M}{\text{minimize}} \quad & \frac{A^*}{P_M^{2/\alpha}} G(P_M) \\ \text{subject to} \quad & P_{M,\max} \geq P_M \geq 0, \\ & G(P_M) < 0, \\ & \frac{A^*}{P_M^{2/\alpha}} \leq \lambda_{M,\max}. \end{aligned} \tag{33}$$

This case means that when MaBS is more energy efficient than MiBS and the predefined coverage performance can be supported through optimizing the transmission power and density of MaBS, we only need to deploy MaBSs.

There may be no feasible solution for **P2-3**. If the solution exists and suppose it is $P^*_{M,3}$, then we have the optimal value $\lambda^*_{M,3} = A^*/{P^*_{M,3}}^{2/\alpha}$ and $Y_3 = A^*/{P^*_{M,3}}^{2/\alpha} G(P^*_{M,3})$; otherwise, let $Y_3 = \infty$.

4) Case 4: $G(P_M) < 0$ and $A^*/P_M^{2/\alpha} > \lambda_{M,\max}$. In this case, from (29), we have $\lambda^*_{M,4} = \lambda_{M,\max}$ and hence $\lambda^*_{m,4} = P_m^{-2/\alpha}(A^* - \lambda^*_{M,4} P_M^{2/\alpha}) = P_m^{-2/\alpha}(A^* - \lambda_{M,\max} P_M^{2/\alpha})$. Then the original problem **P2** can be converted to the following problem, called **P2-4**:

$$\begin{aligned} \underset{P_M}{\text{minimize}} \quad & \lambda_{M,\max} G(P_M) \\ \text{subject to} \quad & P_{M,\max} \geq P_M \geq 0, \\ & G(P_M) < 0, \\ & \frac{A^*}{P_M^{2/\alpha}} > \lambda_{M,\max}, \\ & \lambda_{M,\max} P_M^{2/\alpha} \geq A^* - \lambda_{m,\max} P_m^{2/\alpha}. \end{aligned} \tag{34}$$

This case means that when MaBS is more energy efficient than MiBS, after deploying all the available MaBSs which are not enough to support the predefined coverage performance, we need to additionally deploy some MiBSs. The density of MiBS depends on the optimal MaBS transmission power which is the solution of **P2-4**.

There may be no feasible solution for **P2-4**. If the solution exists and suppose it is $P^*_{M,4}$, then we have the optimal value $\lambda^*_{m,4} = P_m^{-2/\alpha}(A^* - \lambda_{M,\max} {P^*_{M,4}}^{2/\alpha})$ and $Y_4 = \lambda_{M,\max} G(P^*_{M,4})$; otherwise, let $Y_4 = \infty$.

Remark 3: From the above analysis, we find an interesting result that which is more energy efficient between MiBS and MaBS depends on the properties of positive and negative of $G(P_M)$. If case 1 and case 2 $(G(P_M) > 0)$ exist and have feasible solutions, it means that MiBS can be more energy efficient. Hence we attach a higher priority to deploy MiBSs in these two cases. Only if the available MiBSs can not support the predefined QoS requirement, we need to additionally deploy some MaBSs. On the contrary, if case 3 and case 4 $(G(P_M) < 0)$ exist and have feasible solutions, it means that MaBS can be more energy efficient. Hence we attach a higher priority to deploy MaBSs in these scenarios. Only if the available MaBSs can not support the predefined QoS requirement, we need to additionally deploy some MiBSs. Besides, all the cases show that the optimal BS density is dependent on BS transmission power, and hence optimizing MaBS transmission power is meaningful to reduce APC, which is ignored by the existing works [29], [31], [32].

After discussing the above four cases and solving **P2-1**∼**P2-4**, we can get the optimal deployment strategy for heterogeneous cellular networks (the solution for **P2**) as follows:

$$[\lambda^*_M, P^*_M, \lambda^*_m] = \left[\lambda^*_{M,i}, P^*_{M,i}, \lambda^*_{m,i}\right]. \tag{35}$$

where $i = \arg\min_{i=1,2,3,4}\{Y_i\}$.

V. Numerical Results and Discussions

A. Simulation Setup

In this section, through numerical simulation, we first investigate the impact of energy-related deployment factor A on the system coverage performance, then verify the APC performance by comparing the homogeneous scenario and heterogeneous scenario, our designed deployment strategy and the existing work.

TABLE I
SIMULATION PARAMETERS

Parameters	Assumption
Total BS bandwidth	20 MHz (N = 100) [35]
Additive noise power	σ^2 =-100 dBm
Pathloss exponent	$\alpha \in$ [3.5,4.5]
Max. available MaBS density	$\lambda_{M,max} = 10^{-6} m^{-2}$
Max. MaBS transmission power	$N \cdot P_{M,max}$ = 40 W (46 dBm) [16]
Power parameters of MaBS	$a_M = 22.6$, $b_M = 412.4$ W [16]
Max. available MiBS density	$\lambda_{m,max} \in [10^{-6}, 2 \times 10^{-6}]$ m^{-2}
MiBS transmission power	$N \cdot P_m = 2$ W (33 dBm) [16]
Power parameters of MiBS	$a_m = 5.5$, $b_m = 32$ W [15]
Outage Threshold	γ = -6.5 dB [42]

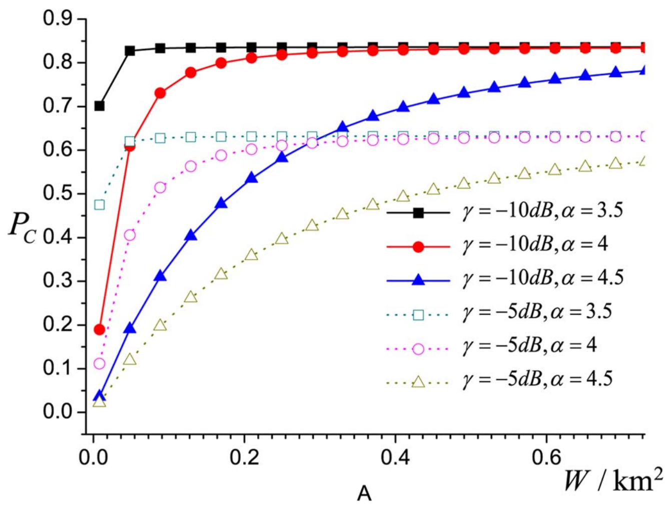


Fig. 2. Coverage probability as a function of energy-related deployment factor A (Average received power based scheme).

Since this evaluation depends on many variables, different cases are studied where some parameters vary, while the remaining ones, unless otherwise stated, are equal to their default representative values, summarized in Table I. It should be noted that we use a typical simulation scenario of a LTE system [35], where the total bandwidth is divided into many Resource Blocks (RBs). One RB pair consists of 12 subcarriers in the frequency domain (i.e., 180 kHz). It assumes that the total transmission power is evenly allocated to each RB. The power model parameters can be found in [16].

B. The Impact of Energy-Related Deployment Factor on the Coverage Performance

Figs. 2 and 3 depict how the energy-related deployment factor A affects the coverage probability under different channel environments and different user association schemes. We have the following four observations:

1) The coverage probability is an increasing function of energy-related deployment factor A for both channel environments and user association schemes, which corroborates the accuracy of our theoretical analysis as Theorem 3.
2) Given the coverage outage threshold, all the coverage probability curves eventually converge to a fixed value. For example, the coverage probability curves eventually converge to 0.836 and 0.633 when $\gamma = -10$ dB and $\gamma = -5$ dB under average received power based user association scheme in Fig. 2. This means that when A is large enough, increasing A is not meaningful for enhancing coverage probability while consuming too much energy. This is the point why we can save energy through designing deployment strategy. Besides, it should be noted that the outage probability is $1 - P_C = \mathbb{P}[SINR < \gamma]$, which is also the cumulative distribution function of user's SINR. Therefore, as shown in Figs. 2 and 3, the higher γ is, the smaller the eventual value is. Through setting fit threshold and expected probability values, our optimization framework can save system energy while guaranteeing the users' SINR or spectral efficiency distribution requirement.

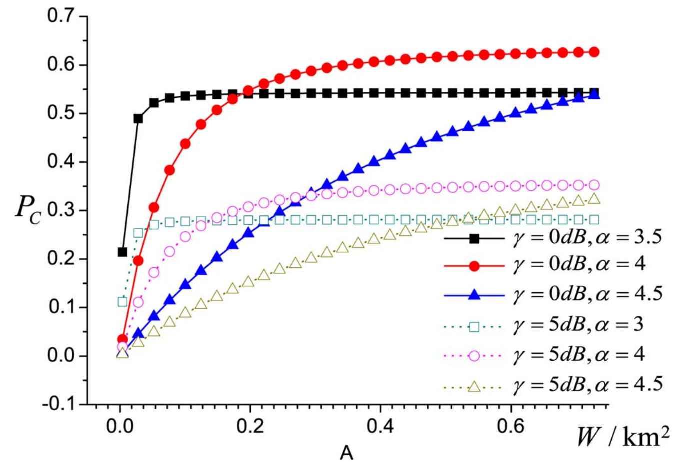


Fig. 3. Coverage probability as a function of energy-related deployment factor A (Instantaneous SINR based scheme).

3) The smaller α is, the higher the convergence rate is. The reason is that the smaller α means better channel environment. With better environment, the QoS requirement is more easy to be satisfied.
4) Compared Fig. 2 with Fig. 3, it is obviously that the instantaneous SINR based scheme is superior to the average received power based scheme. For example, the coverage probability curves eventually converge to close values, i.e, near 0.6 when $\gamma = 0$ dB for the former and $\gamma = 5$ dB for the latter. The reason is that instantaneous SINR based scheme ensures that user connects to the BS which provides highest SINR when $\gamma > 1$.

C. Performance Comparison Between Homogeneous Scenario and Heterogeneous Scenario

In this subsection, we compare the energy efficiency performance between homogeneous scenario and heterogeneous scenario with different configurations. The optimal deployment strategy and the corresponding energy consumption results are shown in Fig. 4. The homogeneous scenario and heterogeneous scenario are denoted as Ho and He, respectively. As shown in Fig. 4(a), the performance of heterogeneous scenario is superior to the homogeneous scenario performance, which accords with the conclusion in [16]. The main reason is that with the practical power model parameters in [35], MiBS is more energy-efficient than MaBS, i.e, $G(P_M) > 0$. Thanks to deploying some MiBSs, we can deploy less, even no MaBSs to guarantee the user's QoS requirement. The less energy each MiBS consumes, the larger the performance gap between Ho and He is.

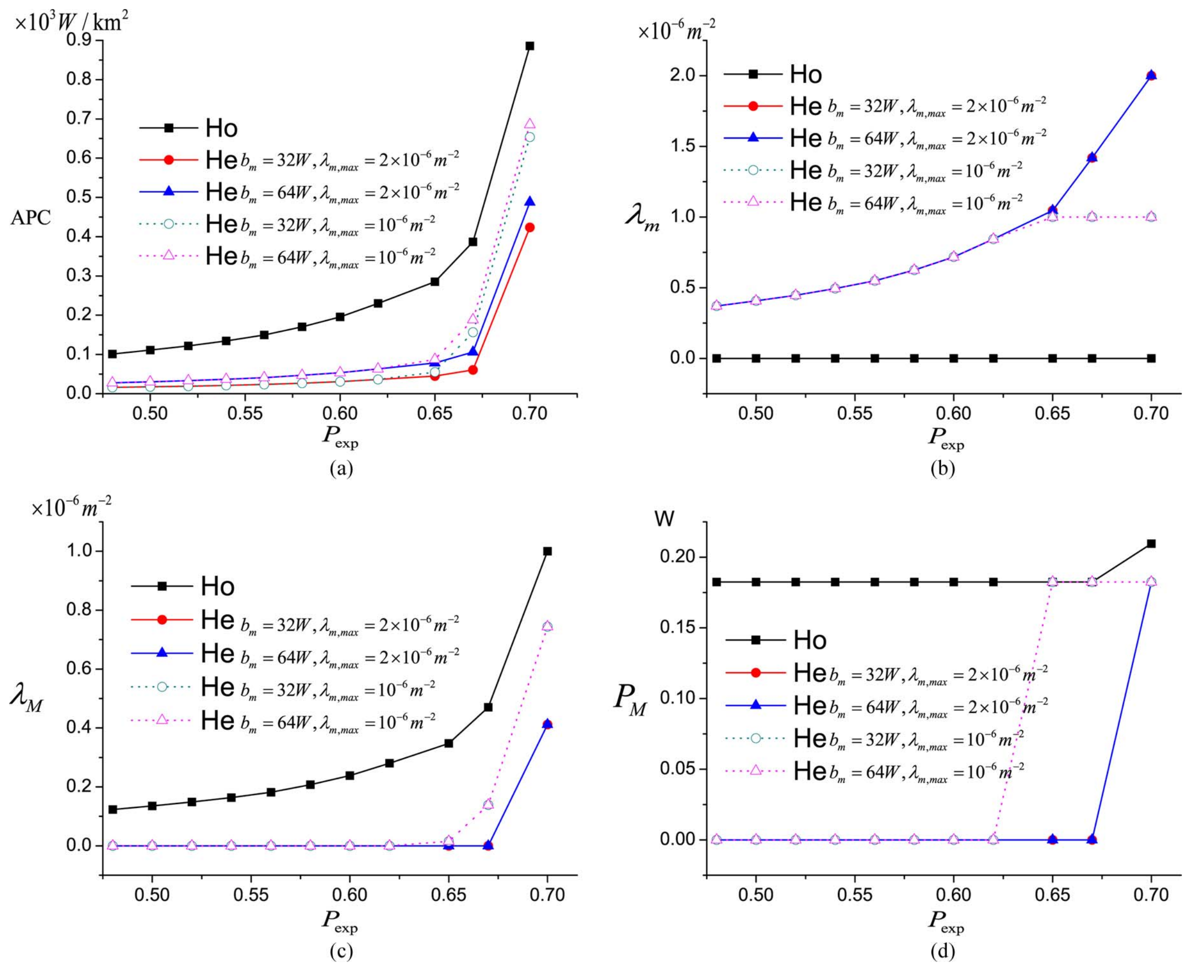


Fig. 4. Performance comparison between homogeneous scenario and heterogeneous scenario. ($\alpha = 4$). (a) Optimal energy consumption; (b) Optimal λ_m; (c) Optimal λ_M; (d) Optimal P_M.

Besides, the optimal energy consumption is an increasing function of $P_{\exp}$. The fact behind it is that we need to deploy more BSs to make sure coverage probability is at least $P_{\exp}$. When $P_{\exp}$ is small enough, such as less than 0.65, the available MiBSs are enough to provide the expected service quality. Thus, the optimal λ_M and P_M are 0 as shown in Fig. 4(c) and (d). As depicted in Fig. 4(b), the needed MiBS density increases as the increase of $P_{\exp}$. This trend terminates at the maximum available MiBS density. With less available MiBSs, the smaller $P_{\exp}$ which can be supported by MiBSs is. For example, the maximum $P_{\exp}$ which can be supported by MiBSs are 0.64 and 0.67 when $\lambda_{m,\max} = 10^{-6}$ m^{-2} and $\lambda_{m,\max} = 2 \times 10^{-6}$ m^{-2}, respectively. On the other hand, with less available MiBSs, we need to deploy more MaBSs when the available MiBSs can not support the QoS requirement as depicted in Fig. 4(c). The reason is that when MiBS is more energy efficient and MaBS is necessary, the optimal P_M for different configurations, which are the solutions of **P2-2**, are the same as shown in Fig. 4(d). Thus, more MaBSs are needed in less available MiBSs scenario.

D. Performance Comparison Between Our Proposed Scheme and Existing Scheme

In this part, we investigate the energy efficiency gain of our proposed scheme which jointly optimizes the BS density and transmission power through comparing with the existing scheme which just considers the BS density optimization [29], [31], [32]. We denote our proposed novel scheme and the existing traditional scheme as Nov and Tra respectively. The results can be found in Fig. 5.

For Tra, we configure MaBSs with high transmission power $P_M = 0.4$ W and low transmission power $P_M = 0.05$ W, respectively. Besides, we consider two scenarios where the available MiBS densities are $\lambda_{m,\max} = 10^{-6}$ m^{-2} and $\lambda_{m,\max} = 2 \times 10^{-6}$ m^{-2}, respectively. From all curves in Fig. 5(a), we can observe that our proposed scheme consumes less system energy than the existing scheme. Due to the higher energy efficiency of MiBS, i.e, $G(P_M) > 0$, all the schemes deploy as much as MiBSs as shown in Fig. 5(b). When MiBSs are not much enough to satisfy user's requirement, then some MaBSs are deployed. Therefore, the optimal MiBS densities of both two schemes are equaled to each other. Although with the same MiBS density, our proposed scheme solves the optimal MaBS transmission power instead of using the fixed power as Tra and hence has higher energy efficiency. The optimal MaBS transmission power which is adjusted with the change of $P_{\exp}$ can be found in Fig. 5(d). When $P_{\exp}$ is small enough (smaller than 0.52), the optimal MaBS transmission power is 0 W. After $P_{\exp}$ increased, the optimal MaBS transmission power is 0.1459 W which is the solution of **P2-2** or **P2-3**. When $P_{\exp}$

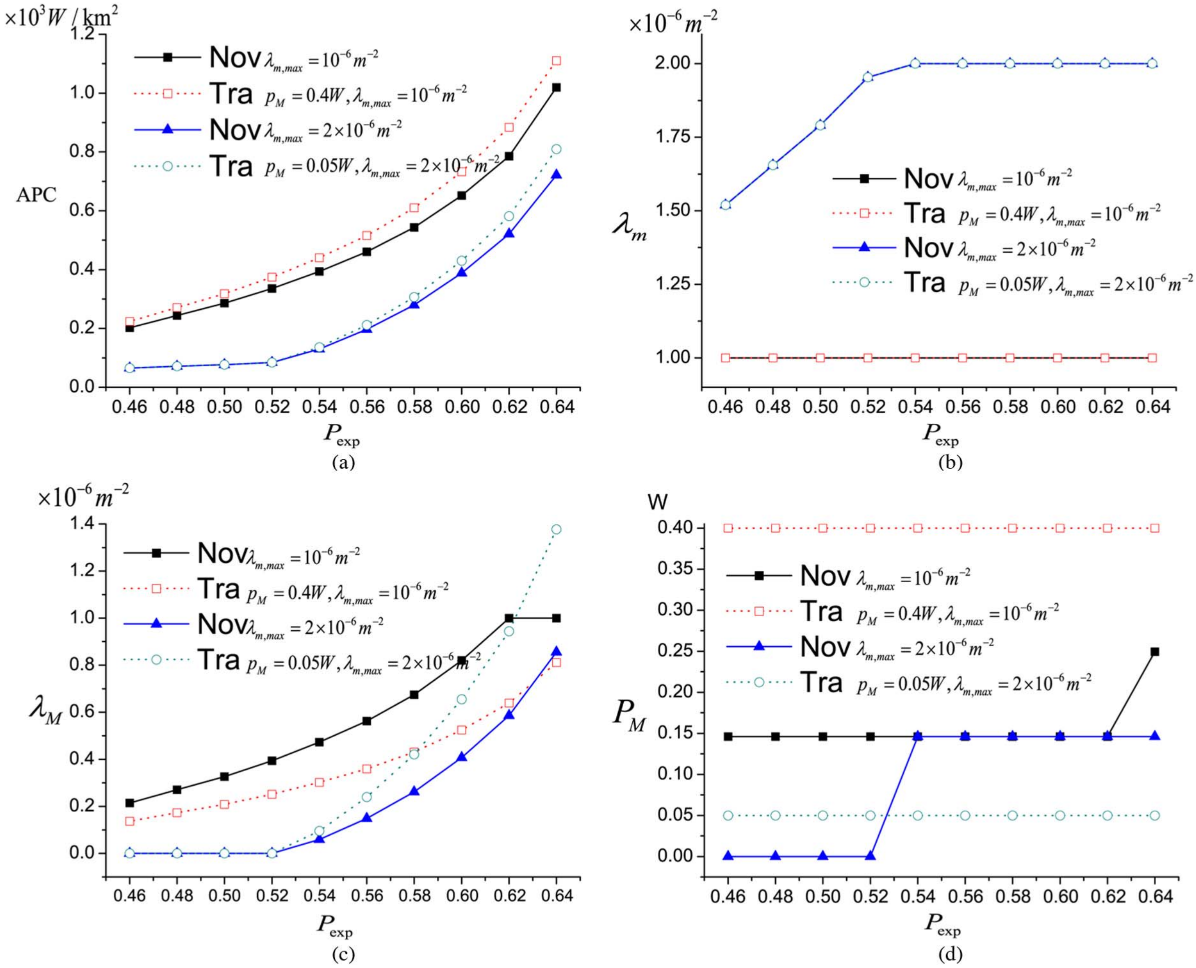


Fig. 5. Performance comparison between our proposed scheme and existing scheme. $(\alpha = 4.5)$. (a) Optimal energy consumption; (b) Optimal λ_m; (c) Optimal λ_M; (d) Optimal P_M.

is large enough (e.g., 0.62) and available MiBS is little (e.g., 10^{-6} m^{-2}), the optimal MaBS transmission power further increases (e.g., 0.25). With optimal MiBS density and optimal MaBS transmission power, the optimal MaBS can be easily achieved from (16) to make sure the coverage probability is higher than $P_{\exp}$ as depicted in Fig. 5(c).

VI. Conclusion

In this paper, we study the impact of deployment strategy on the system coverage and energy consumption performance in cellular networks. The expressions of average coverage probability are derived. From the results, we find that the system coverage performance first increases then converges to a fixed value with the increase of energy-related deployment factor $A = \lambda_m P_m^{2/\alpha} + \lambda_M P_M^{2/\alpha}$. Based on this point, we formulate the area power consumption minimization framework under coverage performance constraints, and jointly determine the optimal MaBS density, MaBS transmission power, and MiBS density. Interestingly, the optimal solutions show that given P_M, we should attach a higher priority to deploy MaBSs when $G(P_M) < 0$ and deploy MiBSs when $G(P_M) > 0$. Through simulation with practical data sets, numerical results confirm that: 1) Compared to homogeneous network deployment, heterogeneous network deployment has absolute advantage in energy efficiency performance; 2) Our joint BS density and BS transmission power optimization strategy consumes less energy than the existing strategy which just considers the BS density optimization.

Since our analysis is based on Rayleigh fading assumption, the future work will focus on considering more general fading, such as the Rician channel fading.

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Jinlin Peng was born in 1987. He received the B.S. degree from the University of Science and Technology of China (USTC), Hefei, China, in 2010. He is currently working toward the Ph.D. degree in communication and information systems with the Department of Electronic Engineering and Information Science, USTC. His research interests include green communication and cooperative communication.

Peilin Hong was born in 1961. She received the B.S. and M.S. degrees from the University of Science and Technology of China (USTC), Hefei, China, in 1983 and 1986, respectively. She is currently a Professor and an Advisor for Ph.D. candidates with the Department of Electronic Engineering and Information Science, USTC. Her research interests include next-generation Internet, policy control, IP QoS, and information security. She has authored/coauthored 2 books and over 100 academic papers in several journals and conference proceedings.

Kaiping Xue was born in 1980. He received the B.S. and Ph.D. degrees from the University of Science and Technology of China (USTC), Hefei, China, in 2003 and 2007, respectively. He is currently an Associate Professor with the Department of Information Security and the Department of Electronic Engineering and Information Science, USTC. His research interests include next-generation Internet, distributed networks, and network security.