SPRITE: A Novel Strategy-proof Multi-unit Double Auction Framework for Spectrum Allocation in Wireless Communications

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Abstract

Auction is widely used for spectrum resource allocation in wireless communications. Many existing works assume that the spectrum resource is single-unit and indivisible, which greatly limits users' capability to utilize the spectrum. Furthermore, most of them fail to take into account of buyer/seller's distinctive demands in auction and consider spectrum allocation as a single unit or single-sided auction.

In this paper, we consider the multi-unit double auction problem under the context that multiple buyers/sellers have different demands to buy/sell. Particularly, we present a novel strategy-proof multi-unit double auction framework (SPRITE). SPRITE establishes a series of bid-related buyer group construction and winner determination strategies. It improves the spectrum reusability and achieves sound spectrum utilization, fairness, and essential economic properties at the same time. Furthermore, we theoretically prove the correctness, effectiveness and economic properties of SPRITE and show that SPRITE is strategy-proof. In our evaluation study, we show that SPRITE can achieve multi-unit spectrum auction with better auction efficiency compared with existing double auction mechanisms. To the best of our knowledge, SPRITE is the first multi-unit double auction approach for wireless spectrum allocation.

Keywords: Wireless communication, spectrum auction, multi-unit, strategy-proof

Ⅰ Introduction

As the increasing popularity of wireless devices and applications, the ever-increasing demand of traffic poses a great challenge in spectrum allocation and usage. However, large companies and organizations occupy many spectrum resources by means of long-term and regional leases [1] without considering spectrum reuse, while new applicants, e.g., non-contract users, new applicant, etc., are in great shortage of spectrum resources. Therefore, it is imperative to provide an effective solution to redistribute the under-utilized spectrum resources to the ones in shortage of spectrum.

Auction, in which the spectrum owners could gain utilities to lease their idle spectrum in economic perspective [2,3] while new applicants could gain access to the spectrum, may serve as such a promising way that could better increase the efficiency, effectiveness and economic properties of the spectrum. However, in traditional one-to-many single-sided auction style (similar to FCC method) the rare resources are in centralized control of the seller/buyer, which is "resource dominant side" that has the rights to establish rule of transactions. This auction style may cause collusion or market manipulation problem.

Compared to single-sided auction [4,5], double auction mechanism is more suitable for spectrum redistribution owing to its fairness and allocation efficiency. Both of the buyer and seller group will lose their relative dominant position in double auction procedure, and their relationship becomes supply and demand coordination [6]. Consequently, double auction mechanism is more likely to achieve maximum spectrum reuse under the premise of protecting the profits of buyers and sellers. TRUST [7] is regarded as the first work to tightly integrate spectrum allocation and pricing components by using double auction mechanism. However, it only considers single-unit double auction issue, and thus lacks the ability to support auction in multi-radio wireless networks, which is taken as the enabling technology of next generation wireless network communications [8,9].

Besides, buyers in spectrum auction could share the same spectrum if they don't interference with each other in spectrum auction, e.g., heterogeneous geo-location could enable spectrum reuse. However, different buyer groups stand for different purchasing power and different payoff. It is hard to ensure that all the buyers bid truthfully by using the existing clearing price rule. This requires double auction mechanism consider economic effects not only in the process of transaction set construction (just like TRUST) but also in buyer group construction section.

To solve these issues, we propose a novel Strategy-Proof multi–unit double spectrum auction (SPRITE) which satisfies the economic properties, spectrum reuse and market clearing. The framework of the SPRITE mechanism can be described in Fig. 1. Compared with existing traditional single-unit double auction approaches, the major contributions of SPRITE can be identified as follows:

- SPRITE jointly considers economic properties with spectrum allocation problem. It could constitute a NASH Equilibrium through the whole auction process, better improves spectrum reuse, and further leads buyers and sellers participating the auction in an honest and fair manner.
- SPRITE provides a novel clearing price mechanism to assure the strategy-proof property and other essential

economic properties, which is significantly different from traditional spectrum auction methods.

- Compared with single-unit auction, SPRITE is the first work that achieves multi-unit double auction that satisfies the needs of users in multi-radio wireless networks and improves auction efficiency at the same time.

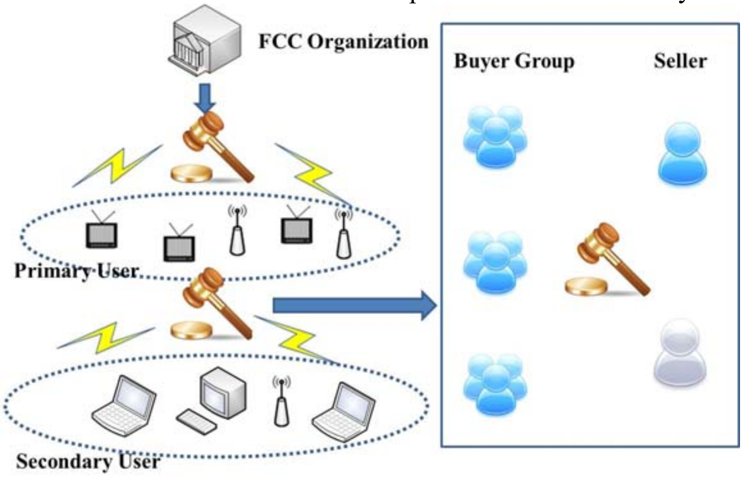


Fig. 1. Framework of SPRITE mechanism

The rest of paper is organized as follows: Section II introduces preliminaries and surveys the most related work. Section Ⅲ proposes the algorithm design of SPRITE. In section Ⅳ, we prove the correctness, effectiveness, and economic properties of our design. Section Ⅴ evaluates the performance of our approach. In the last section, we conclude the paper.

Ⅱ Preliminaries and Background

A. Assumptions and Terminologies

Suppose that each seller contributes multi-unit homogeneous channels to sell and each buyer have multi-unit channels to buy. We assume the double auction process is sealed-bid and private. Thus the bidders will not collude. We also assume that all multi-unit bids are "divisible": A buyer/seller willing to buy/sell q units at a specified price-per-unit would also be willing to trade q' at that price, where $q' < q$.

Notations: I: Group of all buyers;

J: Group of all sellers, where $I \cap J = \phi$;

f_i: The bid of buyer i, $i \in I$, f_i can be deemed as maximum price it is willing to pay for a channel;

K_i: Number of channels requirement for buyer i;

g_j: The bid of seller j, $j \in J$, g_j can be deemed as minimum price it is required to sell a channel;

K_j: Number of channels provided by seller j;

Vf_i: For a buyer, its true valuation of the channel;

P_{f_i}: If buyer i wins the auction, the price it needs to pay for each channel by bidding f_i;

U_{f_i}: The utility of a buyer i, $U_{f_i} = \sum_{K_i} Vf_i - \sum_{K_i} P_{f_i}$;

Vg_j: For a seller, its true valuation of the channel;

P_{g_j}: If seller j wins the auction, the actual payment it received for each channel;

U_{g_j}: The utility of a seller j, $U_{g_j} = \sum_{K_j} P_{g_j} - \sum_{K_j} Vg_j$;

ρ_{f_i}: The success rate for buyers i by bidding f_i;

We also assume that there are $\mathbf{n}$ participants, where $\mathbf{n} = |I| + |J|$, $|I|$, $|J|$ represents the number of buyers and sellers, respectively. To make a multi-unit double auction robust and practical, the mechanism should be strategy-proof, ex-post individual-rational and ex-post budget-balanced.

(1) Strategy-proof. In a double auction, if no buyer or seller can improve its own utility by bidding untruthfully, we say the auction is strategy-proof. In other words, truthfully bidding is the dominant strategy for each participant.

Proposition 1: For buyer: $U_{f_i} \geq U_{f_i'}$, f_i': Untruthful bid for buyers **(1)**

For seller: $U_{g_j} \geq U_{g_j'}$, g_j': Untruthful bid for sellers **(2)**

(2) Ex-post individually rational. No participant's expected utility is not less than its utility from non-participation.

Proposition 2: For buyer: $\sum_{K_i} f_i - \sum_{K_i} P_{f_i} \geq 0,\ i \in I$ **(3)**

For seller: $\sum_{K_j} P_{g_j} - \sum_{K_j} g_j \geq 0,\ j \in J$ **(4)**

(3) Ex-post budget-balanced. The expected payoff of the auctioneer is non-negative.

Proposition 3: Auctioneer's Expected Payoff: $\mathrm{EP} = \sum_{i=1}^{|I|} P_{f_i} - \sum_{j=1}^{|J|} P_{g_j} \geq 0$ **(5)**

(4) Auction efficiency. The valuation of the buyers is optimized.

Table 1. Comparing of different double auction mechanisms

Existing auction mechanism	Strategy-proof	Ex-post budget-balanced	Individual rationality	Spectrum reuse	Multi-unit goods trading
VCG	√	×	√	×	×
McAfee	√	×	√	×	×
BC-LP	√	√	√	×	×
Wurman	√	√	√	×	√
TRUST	√	√	√	√	×
SPRITE	√	√	√	√	√

B. Related Research

Double auction can be classified into two categories: continuous double auction (CDA) and periodic double auction (PDA). The PDA mechanism collects bids over a specified interval of a time, and then clears the market at the expiration of the bidding interval. In this paper, we consider the PDA model to study the dynamic spectrum auction.

The literature on strategy-proof mechanism design starts from the classical method by Vickrey-Clarke-Groves (VCG) mechanism. The improved VCG double auction [10] mechanism is strategy-proof, ex-post individual-rational, and efficient. However, the VCG auction method is not budget balanced with multiple buyers and sellers. McAfee [11] proposes a strategy-proof, budget-balanced double auction mechanism for a simple exchange environment, in which all the participants exchange only one unit good. In [12], the author designed an extended version, in which all participants could exchange multi-unit goods. Wurman [13] examines a general family of auction mechanisms that admit multiple buyers and sellers, and proposes a mechanism which transforms the buyers' multi-unit goods demand to single-unit transaction. Babaioff [14] proposes a budget-balanced and strategy-proof double auction mechanism for a bilateral exchange scenario with single output restriction, where each buyer desire for a bundle of goods. Chu and Shen [15] propose an asymptotically efficient truthful double auction mechanism called BC-LP, which achieves bundle of commodities transaction for buyers. TRUST [7] is the first strategy-proof and spectrum reuse double auction method and the most related work to the study in this paper, but it only refers to one-to-one channel trading. （To be added）

Ⅲ Algorithm Design

In this part, we propose the design and details of our proposed multi-unit double spectrum auction mechanism. Particularly, our mechanism can be separated into three steps. Firstly, we categorize the buyers into different groups based on each buyer's bid and its channel request. The buyer group formation algorithm should maximize each group's total bid and achieve the auction efficiency simultaneously. After that, we construct the bid set on the basis of each participant's bid and decide the transaction set. Finally, we determine the clearing price for the winning buyers and sellers according to the market clearing demand.

A. Buyer Group Construction

(1) Conflicting graph construction

The buyer group is formed by a set of non-conflicting buyers. Namely buyers within each other's communication range cannot use the same spectrum due to interference. Specially, we model the buyers' interference relationship in the network as an undirected conflict graph $G=(V,E)$, where each buyer in the network is represented by a node in the graph, V represents the node set in the graph, E represents the edge set in the graph. There is an edge $(i_1,i_2)\in E$ between nodes i_1 and i_2, if two nodes i_1 and i_2 interfere with each other. The node weight is defined as the buyer's per-channel bid price. Fig.2 shows an example of conflict graph.

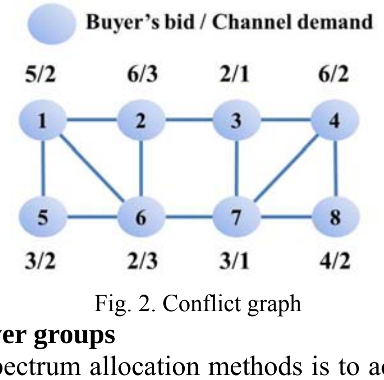


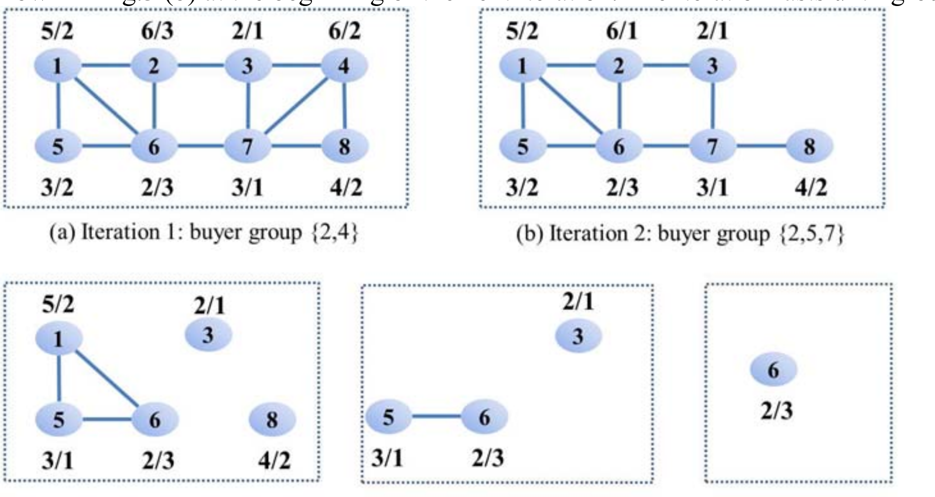
Fig. 2. Conflict graph

(2) Construction of non-conflicting buyer groups

The primary goal of the traditional spectrum allocation methods is to achieve maximum spectrum reuse, that is to say, let more user share the same channel together. The construction of buyers' *maximum independent set* is deemed as an effective method to achieve the goal. Nevertheless, the role of auctioneer is to devise a fair and reasonable mechanism for each agent in a robust double auction mechanism. Consider each buyer as a selfish and rational agent, who only wants to win the auction with lower payment, but none of them care about the spectrum reuse issue. However, the bidding success rate for each buyer will be increased along with the total bid of buyer group that it joined. We will prove in Theorem 6 that there is no relationship between charge and the total bid of group for the winning buyers in the auction. Thus, all buyers are willing to join into a group with higher bid so as to improve its competitiveness in the auction process. At the same time, the group with the highest bid could be regarded as the max-weight independent set.

As analyzed above, finding maximum non-conflicting buyer bid group problem can be considered as choosing the max-weight independent set problem. The max-weight independent set (MWIS) problem is the following: given a graph with positive weights on the nodes, find the heaviest set of mutually nonadjacent nodes. MWIS is a well-studied combinatorial optimization problem that naturally arises in many applications. It is known to be NP-hard, and hard to approximate []. Due to the real-time problem is not an essential issue in our spectrum auction framework, enumeration method is adopted to solve this problem. Let $C \subseteq V$ represents an independent set in $G = (V, E)$. We use $min(C)$ to denote the minimum weight of vertices in the independent set and the weight of the independent set is defined as $min(C)$ multiplied by the number of vertices in the independent set.

Algorithm 1 shows the detailed procedure of **n**on-**c**onflicting **b**uyer group **c**onstruction (NCBC). Let $Max\_weight\_IS(i)$ denotes the max-weight independent set including buyer i, and $Bid(MG_k)$ is the bid of MG_k. In the k[th] iteration of NCBC, we first traverse all the remaining buyers in the buyer group, and compute the bid of each independent set formed by the remaining buyers in buyer group. And then, the independent set with maximum $Bid(MG_k)$ will be chosen and nominated as the winner in this iteration. As shown in the conflicting graph Fig.3, there are 8 candidate buyer groups in the first iteration, {1,4}, {2,4}, {3,5,8}, {4,2}, {5,2,4}, {6,3,8}, {7,2,5}, {8,2,5}. The buyer group {2,4} possess the highest group bid and will be chosen as the winner in Iteration 1. Let $|i|$ and $\omega(MG_k)$ represent buyer *i*'s number of channel demand and minimum buyer's channel demand in MG_k respectively. During the kth iteration, we use $|i|$ to subtract $\omega(MG_k)$ for each buyer *i* in MG_k, if the updated value of $|i|$ equal to 0, then we will delete buyer i from group *I*. Therefore, we will delete buyer 4 in the end of iteration 1, and get the conflicting graph shown in Fig.3 (b) at the beginning of the next iteration. The iteration lasts until group *I* is empty.



(c) Iteration 3: buyer group {1,8} (d) Iteration 4: buyer group {3,5} (e) Iteration 5: buyer group {6}

Fig. 3. Buyer groups formation procedure

Algorithm1. Non-Conflicting Buyer Group Construction
$Max\_bid = 0;$
for $i = 1$ *to* remaining buyers in group I *do*
$Temp\_MG = Max\_weight\_IS(i);$
$Temp\_Bid = Bid(Max\_weight\_IS(i));$
if $(Temp\_Bid > Max\_bid)$
$Max\_bid = Temp\_Bid;$
$MG_i = Temp\_MG;$
end if
end for
foreach buyer i *in* MG_k
$\|i\| = \|i\| - \omega(MG_k)$
end for
delete the buyers (*Satisfied* $\|i\| - \omega(MG_k) = 0$) *in set* MG_k *from group I*
k increased by 1, *goto* 1 *while group I nonempty*

Algorithm 1

Theorem 1: Solution of buyer group formation game can be characterized by Nash Equilibrium (NE).

Proof: Since a buyer group with higher bid has higher winning probability, buyers always want to join into a group with higher bid. Assume that there exists a buyer i in buyer group MG_k, but joining into MG_k is not the dominant strategy for buyer i. According to the buyer group construction process, if there exists a buyer group MG'_k including i which makes $Bid(MG'_k) > Bid(MG_k)$, then MG'_k must be generated before MG_k. There are two possible cases: 1) $Bid(MG'_k)$ will be decreased when buyer i joins. Although buyer i could obtain the maximum value when joining $MG_k^{'}$, but the profit of others in $MG_k^{'}$ will be decreased. Thus $MG_k^{'}$ will not admit buyer i's entry.

2) Some buyers in $MG_k^{'}$ have joined into other groups whose bid price is higher than $MG_k^{'}$. In this case, group $MG_k^{'}$ does not exist and buyer i also cannot join into $MG_k^{'}$.

Thus, joining into MG_k is the dominant strategy for buyer i, which contradicts the assumption. Therefore, NCBC for buyer group formation game can be characterized by Nash Equilibrium.

Corollary 1: NCBC is fair.

(3) Decision of buyer groups' properties

We use MG_1, MG_2, …, MG_n to denote the formed non-conflicting buyer groups. A buyer group can be viewed as a super buyer. There are two parameters will be used to describe the super buyer's characteristics. One is super buyer's bid for unit commodity, $Bid(MG_k) = MinBid(MG_k) * the\ number\ of\ buyers\ in\ MG_k$, while the $MinBid(MG_k) = min\{f_i | i \in MG_k\}$; The other is one super buyer's channel demand $\omega(MG_k)$, and the $\omega(MG_k) = min\{K_i | i \in MG_k\}$. After 5 rounds all the buyers in Fig. 2 are grouped into five different super buyers, Table 2 shows the buyer group formation results by executing Algorithm 1.

Table 2. Buyer groups formation result (Alg. 1)

Round	Buyer Group	Group Bid	Channel Demand
1	{2,4}	12	2
2	{2,5,7}	9	1
3	{1,8}	8	2
4	{3,5}	4	1
5	{6}	2	3

B. Decision of bid set and transaction set

Without loss of generality, we require that all participants' price-per-unit bids are arranged in descending order in the bid set. We use positive quantities correspond to demands of buyers and negative quantities correspond to offers to sell for each seller. Here transaction set means the remaining super buyers and sellers at the end of a time round. Considering a scenario with *M* selling offers and *N* buying offers after bid set has been established. The *(M+1)*st*-price* means the *(M+1)*st highest offer among all *(M+N)* bids. We use $rank(b_L)$ represents the position of a bid b_L in the bid set.

The *(M+1)*st*-pricing* rule is given under the following two conditions:

Condition 1: The *(M+1)*st*-price* and *M*th*-price* belong to different participants in the bid set.

If the *(M+1)*st*-price* comes from a seller, the transaction set construction rule can be depicted as following

$$\{K_L < 0 | rank(b_L) \geq M+1\} \cup \{K_L > 0 | rank(b_L) < M+1\} \quad \textbf{(6)}$$

If the *(M+1)*st*-price* comes from a buyer, the transaction set construction rule will be

$$\{K_L < 0 | rank(b_L) > M+1\} \cup \{K_L > 0 | rank(b_L) < M+1\} \quad \textbf{(7)}$$

Condition 2: The *(M+1)*st*-price* and *M*th*-price* come from the same participant in bid set.

If the *(M+1)*st*-price* comes from a seller, the transaction set is the same as that in Condition 1. However, if the *(M+1)*st*-price* comes from a buyer, the transaction set will be different from that in Condition 1. Particularly, after initial transaction set is constructed based on rule in Condition 1, we delete the residual buyer's bid that equals to the *(M+1)*st*-price* in the bid set, and also delete the same number of seller's bid in descending order.

For example, if there are four sellers are willing to sell 8 channels, of which seller1 sells only one channel at price 13, seller2 sells 3 channels at price 10, seller3 sells 2 channels at price 7 and seller4 sells 1 channels at price 6. After the *(M+1)*st*-rule* has been executed, bid set and transaction set can be depicted as following.

bid set:

$$\left\{\underbrace{13}_{-1}, \underbrace{(12,12)}_{2}, \underbrace{(10,10,10)}_{-3}, \underbrace{9}_{1}, \underbrace{(8,8)}_{2}, \underbrace{(7,7)}_{-2}, \underbrace{6}_{-1}, \underbrace{4}_{1}, \underbrace{(2,2,2)}_{3}\right\}$$

transaction set:

$$\left\{\underbrace{(12,12)}_{2}, \underbrace{9}_{1}, \underbrace{(7,7)}_{-2}, \underbrace{6}_{-1}\right\}$$

C. Choosing clearing price and market clearing

Most of the well-known double auction mechanisms are categorized into two classes: Discriminatory Auction (DA) and Uniform Price Auction (UPA). The main differences are the clearing price decision mechanism between these two classes. The payments and charges for all the winning bidders in DA will be its actual bidding price, and the highest rejected price will be nominated as clearing price for winning bidders in UPA. The McAfee auction mechanism adopted in TRUST and the Wurman's multi-unit double auction mechanism in our work all belong to uniform price auction. However, different from the traditional auction methods, buyer participates into the spectrum auction in a grouped form. Thus, the strategy-proof property will not hold in our SPRITE framework by applying clearing price mechanisms in DA or UPA directly.

As we have mentioned above, if bidding truthful is the dominant strategy for each agent, we can say the auction is strategy-proof. Let parameter $U_{f_i'}$ represents the profit of buyer i by bidding f_i'. If buyer i lose in the auction, then $U_{f_i'}$=0; If buyer wins the auction by bidding a value greater than Vf_i, the $U_{f_i'}$<0, and $U_{f_i'}$>0 when buyer i wins the auction by bidding a value less than Vf_i. We now prove the reason of untruthfulness by introducing existing clearing price mechanisms.

Theorem 2：SPRITE is not strategy-proof if it chooses the clearing price mechanism in UPA directly.

Proof: For spectrum auction, we could only charge each buyer group the same payment by using the clearing price mechanism in UPA. Suppose buyer *i* formed into $Group_1$ when it bids Vf_i, and grouped into $Group_2$ when it bids f_i' untruthfully. If buyer i bids $f_i' > Vf_i$, we can easy get that $Bid(Group_1) \geq Bid(Group_2)$，thus $\rho_{f_i'} \geq \rho_{Vf_i}$ 。However, the charge for each buyer only related to the number of buyers in the group, thus the actual charge for each buyer may not be affected when it bids $f_i' > Vf_i$. The expected payoff can be regarded as $E(P_{Vf_i}) = E(P_{f_i'})$ and we can get:

$$\begin{aligned} U_{f_i'} - U_{Vf_i} &= \rho_{f_i'}(Vf_i - P_{f_i'}) - \rho_{Vf_i}(Vf_i - P_{Vf_i}) \\ &= \rho_{f_i'}(Vf_i - E(P_{f_i'})) - \rho_{Vf_i}(Vf_i - E(P_{Vf_i})) \\ &= \rho_{f_i'}(Vf_i - E(P_{f_i'})) - \rho_{Vf_i}(Vf_i - E(P_{f_i'})) \\ &= (\rho_{f_i'} - \rho_{Vf_i}) * (Vf_i - E(P_{f_i'})) \end{aligned}$$

iff $Vf_i < E(P_{f_i'})$, $U_{f_i'} - U_{Vf_i} < 0$. All the buyers are satisfied with $P_{f_i} \leq f_i$, thus we cannot assure $Vf_i < E(P_{f_i'})$ when $f_i' > Vf_i$. Therefore, bidding truthful is not the dominate strategy for buyers, and the proposed SPRITE is not strategy-proof.

Similarly, if we charge each buyer the bidding price of its own ($P_{f_i} = f_i$), then the payoff for the truthful buyer always goes 0 ($U_{Vf_i} \equiv 0$). When the buyer bids $f_i' < Vf_i$, the payoff $U_{f_i'}$ can be depicted as

$$U_{f_i'} = \rho_{f_i'}(Vf_i - P_{f_i'}) = \rho_{f_i'}(Vf_i - f_i') \geq 0$$

Therefore, we can get $U_{f_i'} \geq U_{Vf_i}$ and clearing price mechanism in DA could not be used in the proposed SPRITE spectrum auction directly.

Our SPRITE framework proposed a novel clearing price mechanism which combines the characteristics in UPA and DA. For buyers, all the buyer groups are deemed as super buyers, and adopt clearing price mechanism in DA. All the winning buyer groups will be charged its bidding price:

$$P_{MG_k} = Bid(MG_k) \tag{8}$$

For the single winning buyer, its actual charge can be descripted as following:

$$P_{f_i} = MinBid(MG_k),\ i \in MG_k \tag{9}$$

We choose the clearing price mechanism in UPA for the winning sellers in SPRITE, and the actual payment is given as:

$$P_{g_j} = \min(MG_k),\ MG_k \in Transaction\ set \tag{10}$$

The clearing price strategy in SPRITE could achieve the strategy-proof, ex-post budget-balanced and market clearing. We will give the proof details in the next paragraph.

Ⅳ Proofs and correctness of our algorithm

We now analyze the properties of our proposed strategy-proof multi-unit double spectrum auction mechanism in terms of strategy-proof, ex-post individually rational, ex-post budget-balanced and market clearing.

Observation 1: For each buyer, if buyer i wins the auction by bidding f_i, then it also wins by bidding $f_i' > f_i$; For each seller, if seller j wins the auction by bidding g_j, then it also wins by bidding $g_j' < g_j$.

Observation 2: The observation 1 shows the monotonicity of winning rules. It implies there existing critical value for winning sellers. For each seller j, if seller j wins the auction by bidding g_j or g_j', and these two biddings are all less than seller group's critical value, the payment to seller j is the same for both.

A reasonable auction framework should follow the strategy-proof guideline for each bidder, which requires each bidder could get the maximum payoff when they bid truthfully in the auction. Based on the combination of agent's actions, there are four possible outcomes lists in Table 3.

Table 3 Auction results based on different configurations

Case	1	2	3	4
Agent lies	lose	lose	win	win
Agent is truthful	lose	win	lose	win

Lemma 1: Buyers can't benefit from bidding $f_i' > Vf_i$ in case 3.

Proof:

There are two outcomes may happen when the buyer i raised its bid.

1)The buyer group formation process is not affected when the buyer i bid Vf_i and f_i' respectively. In other words, buyer i still be formed into the same super buyer group when buyer i raised its bid to f_i'. Because buyer i changes the auction results by bidding a fake price f_i', it demonstrates that the bid f_i' changes this group's bidding price. Namely, the strategy-proof bidding price Vf_i must be the lowest bid in its group. When buyer i wins the auction, the actual price charges buyer i should satisfied interval(Vf_i, f_i']. Thus, the $U_{f_i'}$<0 when buyer i bid f_i'. Because the utility for buyer i is U_{f_i} when it bids truthfully, $U_{f_i'}$<U_{f_i}=0. So buyer *i* can't benefit from bidding $f_i' > Vf_i$ in this case.

2) Buyer i formed into $Group_1$ when it bids Vf_i, and grouped into $Group_2$ when it bids untruthfully f_i'. On the basis of above analysis, $Bid(Group_2) > Bid(Group_1)$ holds. That is to say, buyer i cannot be formed into $Group_2$ when it bids Vf_i. It shows that if buyer i formed into $Group_2$, $Bid(Group_2)$ will be decreased. Namely, $Vf_i \leq MinBid(Group_2)$. Therefore, if buyer i wins the auction and be formed into $Group_2$ by bidding f_i', it should pay more than Vf_i for the designated channel. Then $U_{f_i'} < 0$, and bidding truthful is also the dominant strategy for buyer *i*.

Lemma 2: When buyer i wins in the auction, the $E(P_{f_i})$ will not be decreased with the $Bid(MG_k)$ where $i \in MG_k$.

Proof:

We can see from the formula (9), if $i \in MG_k$, the actual charge for buyer i is determined by the $MinBid(MG_k)$. At the same time, the total bid of buyer group MG_k is determined by $MinBid(MG_k)$ and number of buyers in MG_k. The lowest bid of a higher bid buyer group with large amount of buyers may smaller than the lowest bid of a lower bid buyer group with fewer buyers. Thus, the relationship between $MinBid(MG_k)$ and $Bid(MG_k)$ shows non-monotonicity. That is to say, joining into a buyer group with higher bidding price may not decrease its own payoff when the buyer wins in the auction.

Lemma 3: Buyers can't benefit from bidding $f_i' > Vf_i$ in case 4.

Proof:

No matter how buyer i bid, it always wins the auction in Case 4. So the calculation formula can be depicted as:

$$Uf_i' = \begin{cases} Uf_i + \rho_d \Delta V_d - \rho_a \Delta V_a, & \text{if } Pf_i \leq Vf_i \\ \leq 0, & \text{if } Pf_i > Vf_i \end{cases} \quad \textbf{(11)}$$

where U_{f_i} stands for the utility for buyer who bids truthfully. Compared to bidding truthfully, ρ_d and ΔV_d denote the decreasing probability and reduced cost of P_{f_i} when buyer bids untruthfully. And ρ_a and ΔV_a denote the increasing probability and additional cost respectively.

Based on Lemma 2, we can consider that $\rho_d \Delta V_d \approx \rho_a \Delta V_a$, formula (11) can be rewritten as:

$$Uf_i' = \begin{cases} Uf_i, & \text{if } Pf_i \leq Vf_i \\ \leq 0, & \text{if } Pf_i > Vf_i \end{cases} \quad \textbf{(12)}$$

It is clearly shows that $U_{f_i'} < U_{f_i}$, thus bidding truthful is the dominant strategy.

Lemma 4: Buyers can't benefit from bidding $f_i' < Vf_i$ in case 2 and case 4.

Proof:

When $f_i' < Vf_i$, $U_{f_i'} = U_{f_i} + \rho_d \Delta V_d - \rho_a \Delta V_a$. The abbreviation description of $U_{f_i'}$ can be written as $U_{f_i'} = U_{f_i}$. However, the increase of the buyer's bid may result in the appearance of Case2. Sum up Case 2 and Case 4, when $f_i' < Vf_i$, the profit for buyer i can be rewritten as:

$$Uf_i' = \begin{cases} Uf_i, & \text{if } Case4 \\ 0, & \text{if } Case2 \end{cases} \quad \textbf{(13)}$$

Owing to $U_{f_i'} \leq U_{f_i}$, bidding truthful is still the dominant strategy for buyers.

Theorem 3: SPRITE mechanism is truthful for buyers.

Proof:

The demonstrate process is categorized into buyers respectively on the basis of case 1~4.

Case 1: No matter how buyer bid, it always lose in the auction in Case 1. We can conclude that the utility for buyer always goes zero.

Case 2: On the basis of observation 1, this case happens only if the buyer decreases its bid in auction process, namely $f_i' < Vf_i$. Buyer's utility is zero if it lies in this case, and the truthful action makes its utility no less than zero.

Case 3: This condition happens only if $f_i' > Vf_i$. According to lemma 1, buyers can't benefit from bidding $f_i' > Vf_i$ in case 3.

Case 4: No matter how buyer bid, it always wins the auction in Case 4. If the buyer bid $f_i' > Vf_i$, bidding truthful is the dominant strategy according to Lemma 3. Similarly, bidding truthfully is still the dominate strategy for buyers when $f_i' < Vf_i$.

Theorem 4: SPRITE mechanism is truthful for sellers.

Proof:

Case 1 and Case 2: The proof procedure is the same with buyer case.

Case 3: This condition happens only if $g_j' < Vg_j$ on the basis of observation 1. We use the P_{g_j} stands for the payment to the auction winners. Owing to seller j loses in this case when it bids truthfully, we can get the conclusion that $P_{g_j} < Vg_j$. If the seller j wins the auction by bidding a lower price g_j', the payment to seller j must smaller than P_{g_j}. Thus, the payment to seller j also smaller than Vg_j, and the utility for seller j is negative when it bids untruthfully.

Case 4: SPRITE mechanism pays each seller the minimum buyer group's bid in the transaction set. It is the critical value of seller group we mentioned in observation 2. According to observation 2, no matter the bidding price is f_j or f_j', the payment for seller j is all the same if it is win in the auction. That is to say, utility will not change in both conditions.

Theorem 5: SPRITE mechanism is strategy-proof.

Proof:

As we have mentioned above, if no buyer or seller can improve its own utility by bidding untruthfully no matter how other agents bid, we can say the auction is strategy-proof. Thus, SPRITE mechanism is strategy-proof according to Theorem 3 and Theorem 4.

Theorem 6: SPRITE mechanism is ex-post individually rational.

Proof: For each seller, the proposed SPRITE mechanism pays each seller the minimum buyer group's bid in the transaction set. Thus, $P_{g_j} = MinBid(MG_k)$ ($MG_k \in Transaction\ set$) no less than anyone else winning seller's actual bid. We have SPRITE mechanism is ex-post individually rational for seller.

For each buyer, the actual price charges each winning buyer i is $MinBid(MG_i)$, where $i \in MG_i$. $MinBid(MG_i)$ represents the lowest bid in each winning buyer group. Therefore, $MinBid(MG_i)$ is no more than each winning buyer i's actual bid. SPRITE mechanism is also ex-post individually rational for buyer.

Theorem 7: SPRITE mechanism is ex-post budget-balanced.

Proof: In the designed transaction set, all participants' bids are sorted in descending order. Based on the definition of clearing price decision in SPRITE, all the winning buyers bid are no less than the bids of winning sellers. From Proposition 3, it's straightforward to show that $EP \geq 0$. Therefore, SPRITE mechanism is ex-post budget-balanced.

Theorem 8: SPRITE mechanism achieves the market clearing.

Proof: Let S denotes the winning sellers' bid set and B stands for the winning buyers' bid set.

1) **The *(M+1)st-price* comes from a seller:**

On the basis of SPRITE transaction rules, Set S can be depicted as:

$S = \{b_i > 0 | rank(b_i) \geq M+1\}$ and $\overline{S} = \{b_i > 0 | rank(b_i) < M+1\}$

Then, we have the equation $|S| + |\overline{S}| = M$, where M represents the total quantity provided by sellers. From the rules, we can also get the description of Set B: $B = \{b_i < 0 | rank(b_i) < M+1\}$.

Because $\overline{S} \cup B = \{b_i | rank(b_i) < M+1\}$, and $\overline{S} \cap B = \emptyset$. Thus, we can get the conclusion that $|B| + |\overline{S}| = M$. It is easy to show that $|S| + |\overline{S}| = M$, combine two equations, we have $|S| = |B|$. In other words, quantity supplied by winning sellers equals the quantity demanded by winning buyers at the end of auction.

2) **The *(M+1)st-price* comes from a buyer:**

Based on transaction rules, the (M+1)st-price buyer cannot get into transaction set. Thus, the winning buyer bid set B is the same as condition 1. At the same time, the winning seller bid set is $S = \{b_i | rank(b_i) > M+1\}$. Because the (M+1)st-price comes from a buyer, the S can also depicted as $S \equiv \{b_i > 0 | rank(b_i) \geq M+1\}$. S is the same as condition 1. Therefore, SPRITE could achieve market clearing.

Ⅴ Simulation Study

In this section, we conduct simulation study to evaluate the performance of SPRITE under the metrics of spectrum utilization, number of transacted channels, per-channel utilization, average success rat, degradation, group rank score, and further compare SPRITE with the existing mechanism. All the simulation results are averaged over 1000 runs.

In our simulations, all the buyers are deployed in a square 100*100 area under either random topology or clustered topology (hot spot), and any two buyers within 20 unit distance will conflict with each other, thus the conflicting buyers cannot bid same channels. In the clustered topology, we set 50% of the whole buyers are distributed in a small area. In our simulation, all the bidders will bid at their true valuation, and the bids are uniformly distributed in an interval. To compare with TRUST [7], we implement two versions of TRUST: TRUST-1 (single round auction) and TRUST-2 (multi-round auction).

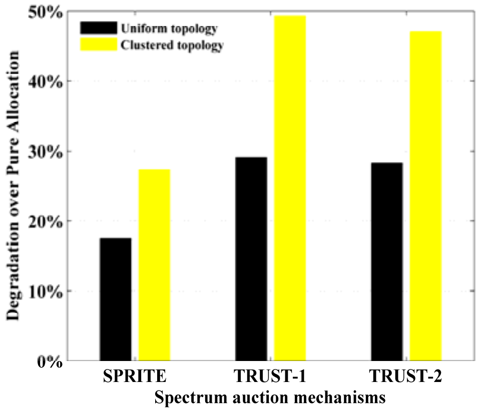
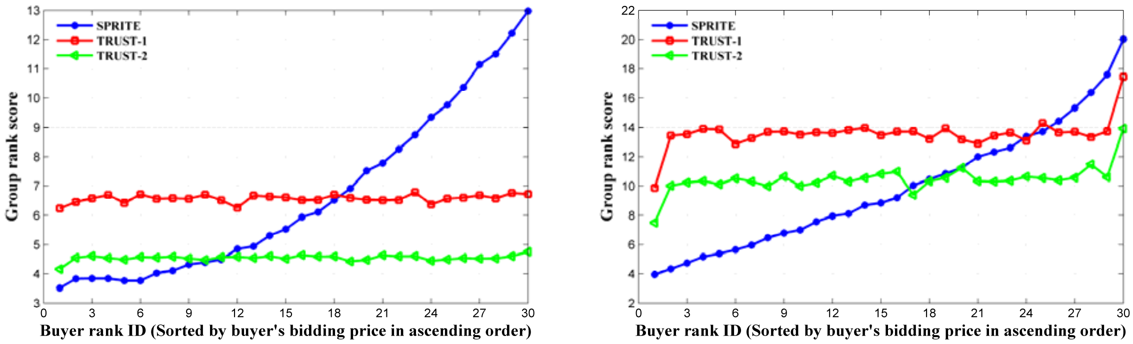


Fig. 4. Degradation



(a) Group rankings in random topology (b)Group rankings in clustered topology

Fig. 5. Group rankings in different topologies

We first consider the impact of economics factors on spectrum utilization. Traditional channel allocation algorithms are pursuing the maximization of the spectrum utilization, while the spectrum auction mechanisms still have to take bidder's purchasing power into consideration. Therefore, compared with the PA (Pure Allocation), various auction mechanisms will experience different degradation. We will use this parameter to reflect the impact of economics factors. In our simulation, we choose the greedy allocation way to represent pure allocation method. Assuming that there are 30 buyers and 5 sellers deployed in both uniform and clustered topologies. We set each buyer requires 1~3 channels in the auction process, and each seller provides 1~3 channels at the same time. The bidding prices for buyers are uniformly distributed in interval [10,35], and prices for selling channels are uniformly distributed in interval [35,60].

Fig. 4 plots the degradation performance of SPRITE, TRUST-1 and TRUST-2 under random and clustered topologies respectively. In Fig. 2, SPRITE suffers less degradation than TRUST-1 and TRUST-2. This is because SPRITE constructs the buyer groups with each buyer's bid, thus the groups with higher bidding price are more likely to be generated. Therefore, SPRITE could increase the buyer groups' opportunity to successfully bid the channels.

Fig. 5(a)-(b) shows the trend of group rank score corresponding to buyer's bid. We can see that in SPRITE the winning probability for a single buyer is corresponding to its bid, while in TRUST there is no such relationship because chooses rand division method in the buyer group construction process and thus the bidding price for a buyer does not have positive connection with the rank of group it belongs to. This indicates that SPRITE provides a more reasonable solution in the realistic environment. In the clustered topology, there are more buyer groups will be formed compared to uniform topology, thus the rank score curves are higher than uniform topology.

Fig. 6 shows that SPRITE better meets the requirements of fairness principle than TRUST, which demonstrates our theoretical analysis. Fig. 7 shows that the average success rate for buyers increases from 28% up to 43% along the increase of maximum bidding price. In the clustered topology, the average success rate for sellers reaches 86% when the buyer's maximum bid equal to sellers' highest offer. In random topology, the average success rate for seller reaches 93%. Fig. 8 shows that the higher the maximum bidding price of buyers, the better the buyer's purchasing power, the larger the number of transacted channels, and the higher the average success rate and spectrum utilization.

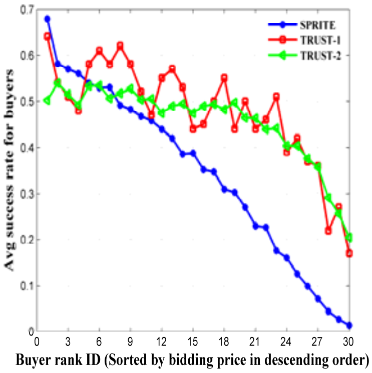


Fig. 6. Success rate of buyers

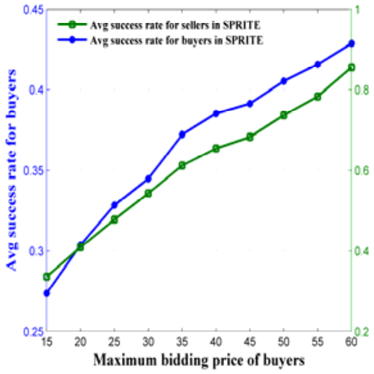


Fig. 7. Average success rate

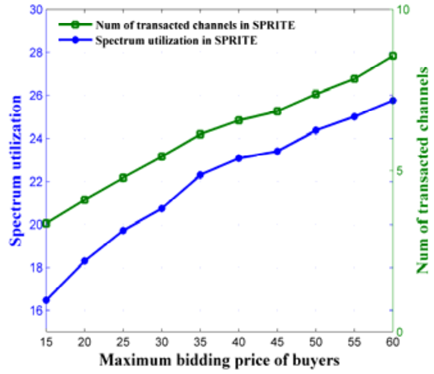


Fig. 8. Spectrum utilization and # of transacted channels

We will concentrate on the auction efficiency between SPRITE, TRUST and McAfee in Fig. 9. The auction efficiency is defined as number of transacted channels divided by the total channels provided by sellers. McAfee is regarded as the most classic double auction mechanism which does not consider spectrum reuse. In order to encourage channel trading, we change the sellers' bidding price interval at [20, 45] for facilitate comparison. We can learn from the comparing results that the auction efficiency of SPRITE and TRUST significantly outperform than and McAfee. As we analyze before, buyer group formation prompts buyer's purchasing power, thus the number of transacted channels for McAfee obviously less than SPRITE and TRUST. Fig. 9 demonstrates that the buyer group formation process could effectively improve the purchasing power for each buyer so as to let each buyer willing to join into group. At the same time, SPRITE performs better than TRUST because of the SPRITE maximizes buyer group's purchasing power. The auction efficiency converges to 98% when the number of buyers reaches to 30.

We can get the conclusion from Fig. 10 and Fig. 11 that the seller's utility in SPRITE obviously better than TRUST and McAfee. The maximization of buyer's purchasing power not only improves the auction efficiency, but also promotes the improvement of the actual payment to the seller. In addition, Fig. 11 shows the buyers' total purchasing power and total utility of sellers will be increased with increasing of number of buyers.

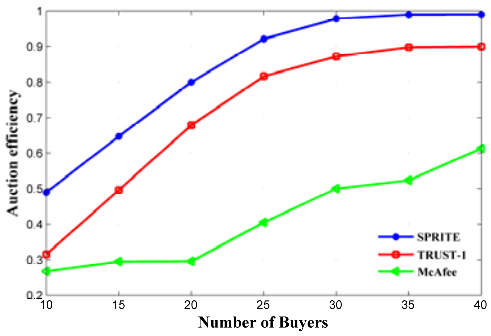


Fig. 9. Auction efficiency

Fig. 10. Utility of each seller

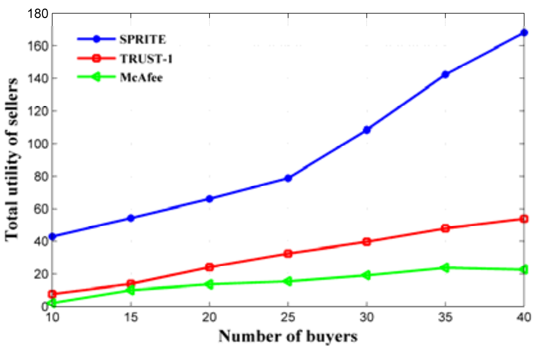


Fig. 11. Total utility of sellers

Ⅵ Conclusion

In this paper, we propose SPRITE mechanism, a strategy-proof multi-unit double auction framework for spectrum allocation in wireless networks. To our best knowledge, SPRITE is the first work that enables multi-unit commodities trading in spectrum allocation in wireless networks. It not only assures strategy-proof but also market clearing property. More importantly, buyers' winning probability is in a proper economic way, which is significantly different from existing double auction approaches, e.g., TRUST. Besides, the relationship among buyers could constitute a Nash Equilibrium. The correctness, effectiveness and economic properties of SPRITE are well studied in our theoretical analysis. The simulation study also show that SPRITE could achieve better performance under various metrics. The future work includes the extension of the concrete effectiveness analysis of spectrum double auction and the study of tradeoff between economic impacts and efficiency degradation.

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