# A Localized Backbone Renovating Algorithm for Wireless Ad Hoc and Sensor Networks

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***Abstract*—In this paper we propose and analyze a localized backbone renovating algorithm (LBR) to renovate a broken backbone in the network. This research is motivated by the problem of virtual backbone maintenance in wireless ad hoc and sensor networks, where the coverage area of nodes are disks with identical radii. According to our theoretical analysis, the proposed algorithm has the ability to renovate the backbone in a purely localized manner with a guaranteed connectivity of the network, while keeping the backbone size within a constant factor from that of the minimum CDS. Both the communication overhead and computation overhead of the LBR algorithm are $O(k)$, where $k$ is the number of nodes broken or added. We also conduct extensive simulation study on connectivity, backbone size, and the communicaiton/computation overhead. The simulation results show that the proposed algorithm can always keep the renovated backbone being connected at low communication/computation overhead with a relatively small backbone, compared with other existing schemes. Furthermore, the LBR algorithm has the ability to deal with arbitrary number of node failures and additions in the network.**

***Index Terms*—maximal independent set, backbone renovating**

## I. Introduction

Virtual backbone is an important issue in wireless ad hoc and sensor networks and has been widely applied in various research domains such as routing, coverage, interference management, energy saving, etc., e.g., CDS as a virtual backbone [1]–[5], or for coverage [6] and network topology control [7]–[9] for saving energy and reducing signal interference. In general, most of these approaches end up of forming a dominating set as a backbone, through which each node in the network either is on the backbone or has at least a backbone node as its neighbors.

However, in wireless ad hoc and sensor networks the network topology keeps changing all over the time due to node failures, additions, or periodically switch on/off. It is very likely that the constructed backbone quickly becomes defective. The dynamism of the network poses a great challenge for backbone management/maintenance. Therefore, it is imperative to provide an effective solution for backbone maintenance.

In maintaining a backbone in ad hoc and sensor networks, the localized approach is most favorable due to its efficiency and its support to scalable design and network dynamism. In this paper, we propose and analyze a localized backbone renovating (LBR) algorithm. This algorithm explores the geometric properties of unit-disk graphs and renovates a backbone at an ultra low $O(k)$ computation overhead and $O(k)$ communication overhead, where $k$ is the number of nodes broken or added.

The major contributions of this paper are identified below:

- In this paper, we proposes a purely localized backbone renovating algorithm (LBR) with ultra low communication and computation overhead.
- The proposed LBR algorithm has the capability of providing a renovated backbone with guaranteed connectivity of the network. It is proved that unless the network is no longer connected, the proposed algorithm can always keep the renovated backbone being connected.
- The proposed LBR algorithm has the ability to deal with arbitrary number of node failures and additions in the networks in a purely localized manner.
- We have conducted extensive simulation study under various scenarios. The results show that the LBR algorithm can effectively repair the backbone in an efficient manner compared with other existing centralized and localized approaches.

The rest of the paper is organized as follows: Section II presents the related works. The preliminaries, models and assumptions are introduced in Section III. Section IV further derives some geometric properties of unit-disk graphs that serve as the basis of the localized backbone renovating algorithm. Section V is devoted to the localized backbone renovating algorithm design. Section VI provides our theoretical analysis on LBR. Section VII reports our simulation study and comparison results, followed by the conclusions in Section VIII.

## II. Related Work

In the following we briefly overview the related works of backbone construction and maintenance in unit-disk graph and summarize the most related research.

Finding a CDS in the network is a popular approach for backbone construction. The study of NP-Completeness of finding an MCDS in general graphs is proposed in [10]. This problem remains NP-hard in unit-disk graphs [11]. For a detailed literature survey, we refer the readers to [12] and the references therein.

Wan proposes the first MIS based CDS construction algorithm [13]–[15]. A similar approach is proposed in [16] to construct and connect an MIS simultaneously. A PTAS for MCDS in unit-disk graphs is proposed in [17]. [18] proposed a distributed algorithm for producing a tree-like backbone with $O(n)$ computation complexity and $O(nlogn)$ communication overhead. [19] selects the nodes with wider communication range, more energy, etc., then uses a steiner tree to connect the dominating set. [20] constructs the backbone via algebraic connectivity and introduce a new metric, connectivity efficiency, as a benchmark when constructing the backbone. In [21], by setting a timer at each node, the nodes with higher node degree have higher probability to be included in the backbone, which finally produces a spanning tree. [22] aims to construct a backbone with the longest lifetime based on a weight matrix of energy efficiency.

In unit-disk graphs and general graphs, the size relationship between MCDS and MIS has been well studied, e.g., [9], [23]–[30]. [31] uses local neighbors information and takes node priority into consideration to construct a CDS, and used an iterative application of a selected local solution to maintain the CDS when the topology changed. In [32], a connected dominating set is built directly without calculating MIS. In [33], an MIS is constructed at first, and then the CDS constructed with gateway nodes. [34] proposed a protocol that is called Distributed Clustering Algorithm (DCA), which can produce a maximal independent that is also a minimal dominating set. [35] proposed a distributed algorithm for calculating a minimal dominating set by a sequential, centralized greedy way, whose execution time is polynomial, which is associated with the size of network. [29]presents a distributed algorithm which constructs a CDS $D$ of size at most $\alpha \cdot opt$ for some fixed constant $\alpha$ in a polynomial time. Compared with [29], [30] proposes a polynomial-time constant-approximation algorithm, GOC-MCDS-C, that produces a CDS $D$ whose size $|D|$ is within a constant factor from that of the minimum CDS. [36] proposed a local randomized greedy (LRG) algorithm, which calculates a minimal dominating set in poly-logarithmic time. However, it can‘t guarantee connectivity.

Another kind of approach focuses on cluster based topology and produces a independent set with the cluster heads. [37] selects the cluster head based on node degree, while in [38] the cluster head is selected based on the normalized link failure frequency and the mobility of the nodes. These algorithms usually start from a single-leader, whose election costs $O(n \log n)$ in message complexity [39]. To improve this, multiple-leader based algorithms are proposed in [9], [40], [41]. To connect all nodes in MIS, [40] requires that each node $u$ in the MIS computes a shortest path to all independent neighbors (the nodes in $I$ whose distance to $u$ is either two or three hops) with a higher id. This connection algorithm results in a CDS with size at most $192 \cdot opt + 48$. By further exploring the geometric properties of neighboring independent nodes, [9] proposes a connection algorithm to generate a CDS with size at most $147 \cdot opt + 33$.

Note that [9], [40], [41] are the most related work since both propose to connect an MIS in a localized fashion. There exist other distributed or centralized algorithms to connect an MIS. For example, a distributed spanning tree can be constructed to connect all nodes in an MIS [42]; or a Steiner tree with minimum number of Steiner points can be applied to connect an MIS [43].

## III. Preliminaries, Models and Assumptions

### A. Preliminaries

- *dominating set*: Given a graph $G(V,E)$, a dominating set $D$ of $G(V,E)$ is a subset of $V$ such that for $\forall u \in V - D$, there exists a $v \in D$ satisfying $uv \in E$.
- *connected dominating set*: If all nodes in $D$ induce a connected graph, $D$ is a *connected dominating set*.
- *minimum (connected) dominating set*: Among all (connected) dominating sets of $V$, the one with the smallest cardinality is called the *minimum (connected) dominating set*.
- *independent set*: An *independent set* $I$ of $V$ is a subset of $V$ such that $\forall u, v \in S$, $uv \notin E$.
- *maximal independent set (MIS)*: If adding any node $w \in V$ to $I$ breaks the independent property, $I$ is a *maximal independent set (MIS)*.

For any vertex $u$ in a maximal independent set $I$, the length of the shortest path from $u$ to its closest vertex in $I$ is either two hops or three hops.

### B. Network Model

In this paper, we model the ad hoc and sensor network as a unit-disk graph $G(V,E)$, a widely adopted model for wireless ad hoc and sensor networks in which nodes can communicate with each other if their distance is at most 1 unit. Specifically, $V$ represents the set of sensors and $E$ represents the set of edges. An edge $uv \in E$ if and only if $u, v \in V$ and the Euclidean distance between $u$ and $v$ is no larger than 1 unit. This assumption is reasonable as in ad hoc and sensor networks the topology is determined by the transmission range, which is usually fixed.

We assume that in the network there already exists an MIS and a corresponding backbone that are generated by any approach available. For example, the algorithms proposed in [9], [40], [41] can be applied here. Let $u$ be any vertex in MIS, $N_u$ be the node set of one-hop neighbors of $u$, $I$ denote the node set of MIS, and $C$ denote the set of nodes that are on the backbone but not in the MIS $I$ (i.e., $C$ is the set of nodes that connect the MIS nodes on the backbone). Let $N_u^{(I)}$ $\subset$ MIS denote the set of nodes in MIS that are two hops or three hops away from $u$, and $I_u$ denote the set of MIS nodes within three hops of $u$. We assume $N_u^{(I)}$ and $I_u$ are available to $u$.

When a node $v$ fails or is added to the network, we assume there is a message broadcasted to $v$'s neighbors in three-hop distance.

## IV. Geometric Properties of Unit-Disk Graphs

Based on the definition, an edge in a unit-disk graph exists between two nodes if and only if their Euclidean distance is at most 1. We have identified the following properties:

*Lemma 4.1:* Let $uv$ and $st$ be two crossing edges in a unit-disk graph $G(V, E)$, as shown in Fig 1. Then at least one of $u$, $v$, $s$, $t$ has direct edges to the other three vertices in $G$.

*Proof:* Assume all the four edges in the quadrilateral $usvt$ have length greater than 1. That is, none of the four edges $us$, $sv$, $vt$, and $tu$ exists in $G$. Since $|sv| > 1$, $|vt| > 1$, and $|st| \leq 1$, we have either $\angle stv > \pi/3$ or $\angle tsv > \pi/3$ or both. Without loss of generality, assume $\angle tsv > \pi/3$. Then $\angle usv > \pi/3$, which means either $|uv| > |sv|$ or $|uv| > |us|$. Since $|us| > 1$ and $|sv| > 1$, we have $|uv| > 1$, a contradiction. Therefore at least one of the four edges of $usvt$ must have length at most 1.

Without loss of generality, assume $|sv| \leq 1$. If $|vt| \leq 1$, then $v$ can reach $s$, $t$, and $u$ directly in $G$. Now let's assume $|vt| > 1$. Let $o$ be the crossing point of edges $uv$ and $st$. Based on the triangle inequality, we have $|ov| + |ot| > |vt|$ and $|os| + |ou| > |us|$. Therefore $|uv| + |st| > |vt| + |us|$. Since $|uv| \leq 1$, $|st| \leq 1$, and $|vt| > 1$, we have $|us| < 1$, indicating $s$ can reach $u$, $v$, and $t$ directly in $G$.

From the above analysis, we conclude that at least one of $u$, $s$, $v$, $t$ can reach the other three vertices directly if $uv$ and $st$ intersect in a unit-disk graph $G$. ■

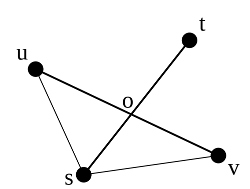



Fig. 1. $uv$ and $st$ are two crossing edges in a unit-disk graph $G$. Then at least one of $u$, $v$, $s$, and $t$ can reach the other three vertices directly in $G$.

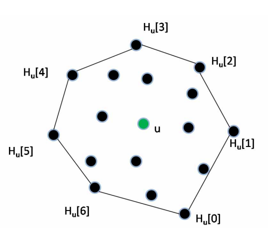



Fig. 2. The array $H_u$, in which elements are sorted in counter-clockwise.

*Lemma 4.2:* Let $u, v, s, t$ be four vertices in any MIS of a unit-disk graph $G$ such that there exists a path $P_{uv}$ with length at most three hops to connect $u$ and $v$ and a path $P_{st}$ with length at most three hops to connect $s$ and $t$. Let $P$ be the set of intersecting nodes in $P_{uv}$ and $P_{st}$. Then $u, v, s, t$ can reach each other by traversing only vertices in $P$.

*Proof:* Let $v_1, v_2, v_3, v_4$ be the four vertices in $P_{uv}$ and $P_{st}$ such that the two edges $v_1v_2$ and $v_3v_4$ cross. From Lemma 4.1, we know that one of these four vertices can reach the other three directly. Without loss of generality, assume $v_1$ can reach $v_2, v_3, v_4$ directly. Then by passing through $v_1$ and other vertices in $P$, $u, v, s, t$ can reach each other. Three example scenarios are illustrated in Fig 3. ■

Note that the path length constraint of this Lemma can be relaxed. Actually in a unit-disk graph $G$, every pair of nodes in two crossing paths can reach each other by traversing only vertices in these two paths.

*Lemma 4.3:* Let $u, v$ be two vertices in any MIS of a unit-disk graph $G$ such that there exists a path $P_{uv}$ with length at most three hops to connect $u$ and $v$. Considering the straight line segment $\overline{uv}$, every point on $\overline{uv}$ is covered in the transmission range of the nodes in $P_{uv}$.

*Proof:* As shown in Fig 4(a), we only need to consider the extreme case that $P_{uv}$ is a three hops path with maximum length. It is obvious that other paths with smaller length are always within the polygon $uvyx$, and cover all the points that $P_{uv}$ covers on $\overline{uv}$. Let $x, y$ be the intermediate nodes in $P_{uv}$. Take $x$ and $y$ as the center, draw a circle with radius 1 unit crossing $\overline{uv}$ at point $a$ and $b$ respectively. Take $u$ and $v$ as the center, draw a circle with radius 1 unit crossing $\overline{uv}$ at point $c$ and $d$ respectively. Let $\alpha$ and $\beta$ denote $\angle xuc$ and $\angle yvd$ respectively.

There are three different cases:

**Case 1: $\overline{ac}$ and $\overline{bd}$ are within $u$'s transmission range and $v$'s transmission range, respectively, as shown in Fig 4(b).**

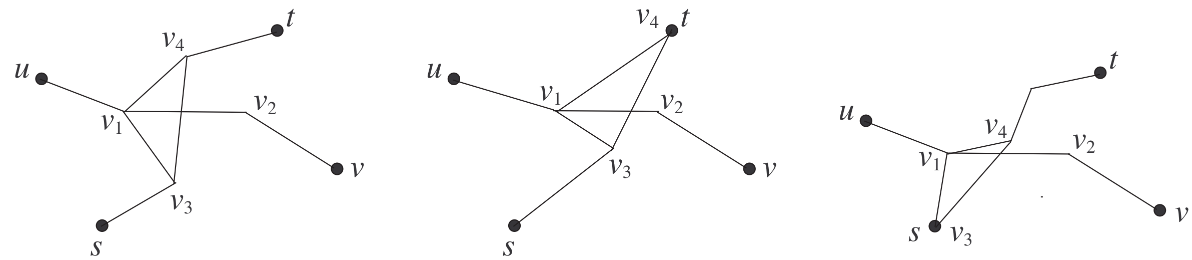
Suppose $u$'s transmission range and $v$'s transmission range do not overlap. Since $\alpha > \pi/3$ and $\beta > \pi/3$. Denote the projection of $\overline{xc}$ and $\overline{yd}$ on $\overline{uv}$ by $Proj_{\overline{xc}}$ and $Proj_{\overline{yd}}$ respectively. Obviously the length of $Proj_{\overline{xc}}$ and $Proj_{\overline{yd}}$ is larger than $1/2$. We have $|Proj_{\overline{xc}}| + |Proj_{\overline{yd}}| > 1$. Therefore, $|Proj_{\overline{xc}} + |Proj_{\overline{yd}} + \overline{cd}| > 1$. Since $Proj_{\overline{xc}} + Proj_{\overline{yd}} + \overline{cd}$ is the projection of $xy$ on $\overline{uv}$, $|Proj_{\overline{xc}} + |Proj_{\overline{yd}} + \overline{cd}| < |xy| = 1$. Contradict to previous derivation. Therefore, $u$'s transmission range and $v$'s transmission range must overlap. So every point on $\overline{uv}$ is covered in the transmission range of the nodes in $P_{uv}$.

**Case 2: Either $\overline{ac}$ or $\overline{bd}$ is within $u$'s transmission range or $v$'s transmission range.**

Suppose $u$'s transmission range and $v$'s transmission range do not overlap, namely $|\overline{uv}| > 1$. Without loss of generality, we assume $\overline{bd}$ is within $v$'s transmission range, as shown in Fig 4(c). Obviously $\beta > \pi/3$ and $|\overline{yu}| \leq |\overline{xy}| + |\overline{xu}| = 2$. We have $|\overline{yu}|^2 = |\overline{uv}|^2 + 1 - 2|\overline{uv}| \cos\beta \leq 2$. Therefore, $|\overline{uv}| \leq 1$, which contradicts the assumption $|\overline{uv}| > 1$. Therefore, $u$'s transmission range and $v$'s transmission range must overlap. So every point on $\overline{uv}$ is covered in the transmission range of the nodes in $P_{uv}$.

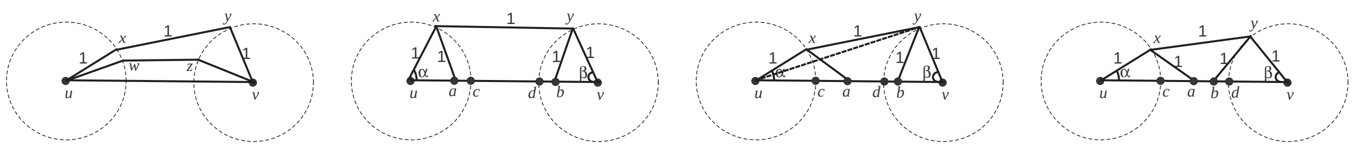
**Case 3: $\overline{ac}$ and $\overline{bd}$ are outside of $u$'s transmission range and $v$'s transmission range.**

We assume $\overline{ac}$ and $\overline{bd}$ do not overlap, as shown in Fig 4(d). Otherwise all the points in $\overline{cd}$ are covered by either $x$ or $y$, and thus the proof is trivial.




(a) (b) (c)

Fig. 3. Case study for Lemma 4.2. $P_{uv}$ and $P_{st}$ are two crossing paths in a unit-disk graph $G$. $v_1, v_2, v_3$ and $v_4$ are the four vertices of the two crossing edges. Then $u, v, s, t$ connect to each other by traversing only nodes in $P_{uv}$ and $P_{st}$.




(a) An extreme case.

(b) Case 1: $\overline{ac}$ and $\overline{bd}$ are within $u$ and $v$'s transmission range, respectively.

(c) Case 2: $\overline{ac}$ is outside $u$'s transmission range and $\overline{bd}$ is within $v$'s transmission range.

(d) Case 3: $\overline{ac}$ and $\overline{bd}$ are outside of $u$'s transmission range and $v$'s transmission range.

Fig. 4. Case study for Lemma 4.3

Obviously $\alpha < \pi/3$ and $\beta < \pi/3$. We also have $\angle uxa = \pi - 2\alpha$ and $\angle vyb = \pi - 2\beta$. Therefore

$$\begin{aligned} & \angle xyb + \angle yxa \\ = \; & 2\pi - \angle uxa - \angle vyb - \alpha - \beta \\ = \; & \alpha + \beta \\ < \; & 2\pi/3 \end{aligned}$$

Since $\overline{ac}$ and $\overline{bd}$ do not overlap, we have $|xb| > |xa| = 1$ and $|xy| = |by| = 1$. In $\angle xby$, we have $|xb| > |xy| = |by| = 1$. Therefore $\angle xyb > \pi/3$. Similarly we have $\angle yxa > \pi/3$. Thus $\angle xyb + \angle yxa > 2\pi/3$. Contradict.

Therefore, $\overline{ac}$ and $\overline{bd}$ must overlap and are covered in the transmission range of the nodes in $P_{uv}$. ■

*Corollary 4.1:* All the points within the convex polygon $uvst$ are covered by the transmission range of the nodes in $P_{uv}$.

*Lemma 4.4:* Let $u, v, s, t$ be four vertices in any MIS of a unit-disk graph $G$ such that $u$ and $v$ are within at most three hops, and $s$ and $t$ are within at most three hops. If the line segment $\overline{uv}$ crosses the line segment $\overline{st}$, given any arbitrary path $P_{st}$ with length at most three hops to connect $u$ and $v$ and path $P_{uv}$ with length at most three hops to connect $s$ and $t$, they must be connected.

*Proof:* Given an arbitrary path $P_{uv}$. Consider the convex polygon $uvyx$ formed by $P_{uv}$ and $\overline{uv}$, as shown in Fig 5. There are three different cases:

**Case 1: $t$ is within the convex polygon $uvyx$, as shown in Fig 5(a).**

$t$ is covered by the transmission range of the nodes in $P_{uv}$ according to Corollary 4.1. Therefore $P_{uv}$ and $P_{st}$ are connected.

**Case 2: $P_{st}$ crosses $P_{uv}$, as shown in Fig 5(b).**
The proof is trivial according to Lemma 4.2.

**Case 3: $P_{st}$ does not cross $P_{uv}$, as shown in Fig 5(c).**
Since $v$ is within the convex polygon formed by $\overline{st}$ and $P_{st}$, same result applies here according to Case 1. ■

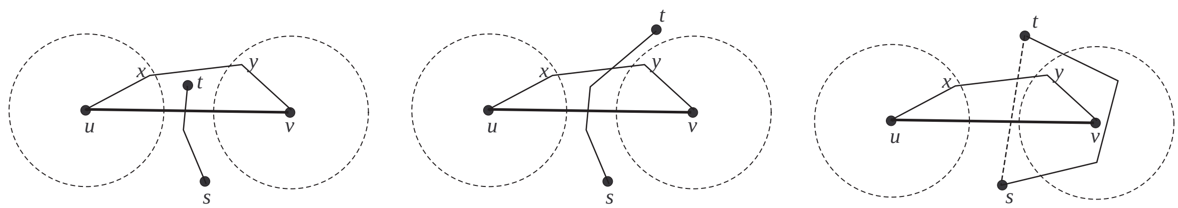
## V. LOCALIZED BACKBONE RENOVATING ALGORITHM

In this section, a backbone expansion procedure is proposed first. Then we introduce our localized backbone renovating (LBR) algorithm. Specifically, during backbone renovating there are two scenarios to consider: 1. node failure; 2. node addition. In both scenarios we choose to update either $I$ or $C$, or both. The detailed design of LBR algorithm is elaborated in the following sections.

### *A. Backbone Expansion with Convex-hull*

Given a node $u$ in MIS $I$, let $H_u$ denote the convex hull of the nodes in $u \cup N_u^{(I)}$, where $H_u$ is an array that records the nodes on the boundary of the convex hull of $u$, as shown in Fig.2. The convex hull can be easily calculated by Graham's Scan algorithm. Note that the number of nodes on the boundary of convex hull $H_u$ is limited by a constant number 18 [27], the execution of Graham's Scan algorithm costs a constant time for the computation of convex hull $H_u$.

Let node $u \in V$ compute the shortest path to connect $u$ and the nodes of $N_u^{(I)}$ on the boundary of convex-hull $H_u$. All the




(a) Case 1: $t$ is within the convex polygon $uvyx$. (b) Case 2: $P_{st}$ crosses the line segment $\overline{uv}$. (c) Case 3: $P_{st}$ does not cross the line segment $\overline{uv}$.

Fig. 5. Case study for Lemma 4.4

intermediate nodes that connect $u$ and the nodes of $N_u^{(I)}$ on the convex-hull $H_u$ form a set $C_u$, as shown in Fig.2. $\forall u \in V$ $\cap C^u$ form the set $C$. It is worth pointing out that $C \cup I$ is proved to be a backbone in [27]. In the following the word *backbone* refers to the expanded backbone.

### *B. Localized Backbone Renovation with Node Failure*

Given an arbitrary node $v$ fails in the network, there are three cases: (a). $v \in I$, namely $v$ belongs to the MIS; (b). $v \in C$, namely $v$ belongs to the backbone but $v$ is not in the MIS; (c). $v$ belongs to neither $I$ nor $C$, namely $v$ does not belong to the backbone. In the following we sketch the basic idea of our algorithm to deal with the three cases.

*1) Case 1: $v \in I$, namely $v$ belongs to the MIS:* In this case, there are four steps to renovate the backbone:

- Step 1. The MIS is renovated by $I' = I \cup MDS(S)$, where $S$ denotes the set of $v$'s one-hop neighbors that are not adjacent to any node in $I$, and $MDS(S)$ denotes the minimum dominating set of $S$.

  *Remark 5.1:* Note that the local topology information (e.g., $N_v^I$) is available to $\forall u \in S$, where $|S|$ is no greater than the node degree of $v$ (usually a small constant number), node $u \in S$ could easily compute a uniquely determined $MDS(S)$. Then the nodes $u \in N_v^I$ update their $I_u$. We have $I' = I \cup MDS(S)$.

  *Remark 5.2:* Note that though we use $I' = I \cup MDS(S)$, this update is not necessarily taken all over the network. Instead, it is only taken by nodes within three hops of $v$, i.e., only the nodes within $v$'s three hops update their $I_u$ with $I'_u = I_u \cup MDS(S) \cap N_u^I$.
- Step 2. The node $u \in I$ where $v \in H_u$ (i.e., $v$ is on the boundary of the convex hull of $u$) renovates its convex hull based on $N_u^{I'} \setminus v$. Specifically, node $u$ launches Graham Scan algorithm with $N_u^{I'} \setminus v$ to update its convex hull and the shortest paths to the nodes on convex hull, denoted as $H'_u$ and $C'_u$.

  *Remark 5.3:* It is worth pointing out that all the nodes in $H_u$ except $v$ will remain in $H'_u$. The detailed proof is given in Lemma 6.1.
- Step 3. Every newly added node $u \in MDS(S)$ computes its convex hull $H_u$ and the corresponding $C_u$ to connect to the nodes in $H_u$. According to Step 1 and Step 2, the set $C$ is renovated by $C' = (\bigcup_{u \in I' \setminus I} C_u) \cup C \setminus C_v$.
- Step 4. $I' \cup C'$ contributes the renovated backbone. It is worth pointing out that $I' \cup C'$ is a CDS. The detailed proof will be given in Lemma 6.4.

*2) Case 2: $v \in C$, namely $v$ is on the backbone but $v$ does not belong to MIS:* Let $u, w \in I$ denote two nodes in the MIS that are connected through node $v$ on the backbone, where $u \in N_w^I$ and $w \in N_u^I$. If the alternate shortest path between $u$ and $w$ is not greater than 3, $u$ and $w$ will be connected with this alternate shortest path. The corresponding intermediate nodes on the path between $u$ and $w$ will be updated in $C'$.

If the alternate shortest path between $u$ and $w$ is greater than 3, i.e., $u \notin N_w^I$ and $w \notin N_u^I$, $u$ and $w$ begin to update their convex hulls based on Section V-A. All the nodes in $H_u$ except $w$ will remain in $H'_u$, and vice versa. The detailed proof is given in Lemma 6.1.

*3) Case 3: $v \notin I \cup C$, namely $v$ does not belong to the backbone:* Since the backbone is a connected dominating set, for the nodes that do not belong to the backbone, they must be dominated by the backbone. In other words, they are all one-hop neighbors adjacent to the backbone nodes. Therefore, it doesn't need to take any action when node $v$ fails in this case.

According to Lemma 6.9, $I' \cup C'$ is the renovated backbone and connects all the nodes in the network.

### *C. Localized Backbone Renovation with Node Addition*

Given an arbitrary node $v$ added into the network, there are two cases: (a). $v \in N_u$, where $u \in I$, namely $v$ has a neighbor $u$ in MIS; (b). $\nexists u \in I$ such that $v \in N_u$, namely $v$ is not adjacent to any node in MIS. In the following we sketch the basic idea of our algorithm to deal with the two cases.

*1) Case 1: $\nexists u \in I$ such that $v \in N_u$, namely $v$ is not adjacent to any node in MIS:*

- Step 1. The MIS $I$ is renovated first by adding $v$ to $I$. Let $I'$ denote the renovated MIS, we have $I' = I \cup v$. Specifically, $v$ selects itself as a new MIS node by broadcasting this notification to its neighbors within three hops and then collecting the local topology information from these nodes.

  *Remark 5.4:* Note that though we have $I' = I \cup v$, this information is not necessary to be broadcast over the whole network. Instead, every node only needs to know the topology changes within its three hops. Therefore, only the nodes $u$ within $v$'s three hops update their local MIS information $I'_u$.

- Step 2. The set $C$ is renovated. Let $C'$ denote the renovated set $C$, we have $C' = C \cup C_v$. Specifically, $v$ computes $H_v$ with $N_v^I$ based on Graham Scan algorithm and connects to the nodes on the boundary of convex-hull $H_v$ via the shortest path. Then all the intermediate nodes that connect $v$ and the nodes on the boundary of convex-hull $H_v$ form the set $C_v$. $I' \cup C'$ contributes the renovated backbone.
- Step 3. $I' \cup C'$ contributes the renovated backbone.

*2) Case 2: $v \in N_u$, where $u \in I$, namely $v$ has a neighbor $u$ in MIS:* Since $v$ has a neighbor $u$ in MIS, it must be dominated by $u$ on the backbone. Therefore, it doesn't need to take any action when node $v$ is added into the network in this case.

According to Lemma 6.9, $I' \cup C'$ is the renovated backbone and connects all the nodes in the network.

### D. LBR Algorithm

This section provides the pseudo code of LBR algorithm.

## VI. Performance Analysis

*Lemma 6.1:* Given an arbitrary node $v$ on the boundary of node $u$'s convex hull $H_u$ that fails, all the other boundary nodes in $H_u$ will remain in the renovated $H'_u$.

*Proof:* It's obvious that the coverage area of convex-hull $H_u$ will shrink. According to the property of convex hull and the execution procedure of Graham Scan algorithm, all the nodes in $H_u$ except $v$ will remain in $H'_u$. ■

*Corollary 6.1:* Given an arbitrary node $v \in I$ fails, during the convex hull renovation executed at the MIS node $u \in I$ that $v \in H_u$, all the nodes in $H_u$ will remain in the renovated $H'_u$.

*Lemma 6.2:* Given an arbitrary node $v$ in the network fails or is added in the network, LBR terminates locally in a constant time.

*Proof:* According to the renovating procedure proposed in Section V-B and Section V-C, it is easy to find that LBR is a localized algorithm and thus terminates locally. For the execution time, we consider two cases here.

- Case 1: Node $v$ fails.
  According to Section V-B,
  1) if $v \in I$, there are four steps to renovate the backbone:
     - In Step 1, the MIS is renovated as $I' = I \cup MDS(S)$. Note that $|S|$ is no greater than the degree of $v$ (usually a small constant) and $|MDS(S)| \leq |S|$. Step 1 is expected to terminate in a constant time.
     - In Step 2, each MIS node $u \in I$ where $v \in H_u$ renovates its convex hull based on $N_u^{I'} \setminus v$. Let $k_1$ denote the number of nodes that need to update their convex hull and $k_2$ denote the size of a convex hull. It is obvious that $k_1$ and $k_2$ are bounded by the constants 30 and 18 respectively according to [27]. Since the input of Step 2 is a constant, Step 2

**Algorithm 1** Localized Backbone Renovating Algorithm

**Input:** $v$, $I$, $C$, $G(V, E)$
**Output:** The renovated backbone $I' \cup C'$.

**function** LBR($v$, $I$, $C$, $G(V, E)$)
  Case 1: $v$ fails
  **if** $v \in I$ **then** ▷ $v$ is an MIS node
    Step 1: $I' = I \cup MDS(S)$ ▷ The nodes within $v$'s three-hop distance update their $I_u$s with $I'_u = I_u \cup MDS(S) \cap N_u^I$, where $S$ denotes the set of $v$'s one-hop neighbors that are not adjacent to any node in $I$, and $MDS(S)$ denotes the minimum dominating set of $S$.
    Step 2: Renovate $H'_u \leftarrow H_u$ and $C'_u \leftarrow C_u$, $\forall u \in I$ where $v \in H_u$ ▷ Each MIS node $u \in I$ where $v \in H_u$ renovates its convex hull and corresponding $C_u$ based on $N_u^{I'} \setminus v$.
    Step 3: $C' = (\bigcup_{u \in I' \setminus I} C_u) \cup C \setminus v$ ▷ Every newly added node $u \in MDS(S)$ recomputes its convex hull $H_u$ and the corresponding $C_u$. The renovated $C'$ is updated by the newly generated $C_u$ according to Step 1 and Step 2.
  **end if**
  **if** $v \in C$ **then** ▷ $v$ is on the backbone but is not an MIS node.
    Step 1. Compute the shortest path $SP(u, w)$ between $u$ and $w$ ▷ $u, w \in I$ denote two nodes in the MIS that are connected through node $v$ on the backbone, where $u \in N_w^I$ and $w \in N_u^I$
    Step 2-1. If $|SP(u, w)| \leq 3$, update $C'$ with $SP(u, w)$ ▷ If the alternate shortest path between $u$ and $w$ is no greater than 3, update the set $C'$ with this shortest path $SP(u, w)$.
    Step 2-2. If $|SP(u, w)| > 3$, recompute $H'_u, H'_w$ and $C'_u, C'_w$ and update $C$ with $C'_u$ and $C'_w$ ▷ If the alternate shortest path between $u$ and $w$ is greater than 3, i.e., $u \notin N_w^I$ and $w \notin N_u^I$, $u$ and $w$ begin to update their convex hulls and then $C_u$ and $C_w$.
  **end if**
  **if** $v \notin I \cup C$ **then**
    No action is needed.
  **end if**

  Case 2: $v$ is a newly added node
  **if** $\nexists u \in I$ such that $v \in N_u$ **then** ▷ $v$ is not adjacent to any node in MIS
    Step 1: $I' = I \cup v$ ▷ The MIS $I$ is renovated by adding $v$ to $I$
    Step 2: Compute $H_v, C_v$ ▷ The set $C$ is renovated
  **else**
    No action is needed.
  **end if**
  Nodes in $I' \cup C'$ contribute the renovated backbone. Return.
**end function**

terminates in a constant time according to Graham Scan algorithm.

  - In Step 3, the set $C$ is renovated as $C' = (\bigcup_{u \in I' \setminus I} C_u) \cup C \setminus C_v$; Since the number of newly added nodes $u \in I' \setminus I$ is a small constant number and the size of a convex hull is limited by 18, the computation of the convex hulls and the corresponding $C_u$s costs a constant time.
  - In Step 4, $I' \cup C'$ contributes the new backbone, which costs a constant time for the local nodes within three-hop distance of $v$ to update their corresponding information of the renovated backbone.

2) if $v \in C$, either an alternate shortest path $SP(u, w)$ within three hops between $u$ and $w$ is computed, where $u, w \in I$ denotes two nodes in the MIS connected through node $v$ on the backbone and $u \in N_w^I$ and $w \in N_u^I$; or $u$ and $w$ need to update their convex hull. Both cost a constant time.
3) if $v \notin I \cup C$, no action is taken.

From the above we can see that in each step the renovation procedure can be finished in a constant time. Therefore, LBR terminates in a constant time when a node fails.

- Case 2: Node $v$ is a newly added node.
According to Section V-C,

1) if $v \in N_u$, where $u \in I$, no action is taken.
2) if $\nexists u \in I$ such that $v \in N_u$
  - In Step 1, the MIS $I$ is renovated by adding $v$ to $I$, which costs a constant time.
  - In Step 2, the set $C$ is renovated by $C' = C \cup C_v$. Specifically, the corresponding convex hull $H_v$ is generated with Graham Scan algorithm and $v$ is connected to the nodes in $H_v$ via the shortest path, both of which cost a constant time since the size of convex hull is limited by 18 [27];
  - In Step 3, $I' \cup C'$ contributes the new backbone, which costs a constant time.

From the above we can see that in each step the update procedure can be finished in a constant time. Therefore, LBR terminates in a constant time when a node is added.

Therefore, LBR terminates locally in a constant time. ■

*Corollary 6.2:* Given that an arbitrary node in the network fails or is added, the communication overhead of LBR is O(k), where $k$ is the number of nodes broken or added.

*Corollary 6.3:* Given that an arbitrary node in the network fails or is added, the computation overhead of LBR is O(k), where $k$ is the number of nodes broken or added.

*Corollary 6.4:* The computation complexity of LBR is $O(n)$, where $n$ is the number of nodes broken or added.

*Lemma 6.3:* Given an arbitrary node $v \in I$ fails while the network is still connected, all the other nodes in $I$ will remain on the renovated backbone.

*Proof:* According to Section V-B, during backbone renovating procedure, the only node that is removed from the backbone is $v$ itself. All the other nodes in $I$ remain in the newly renovated MIS $I'$. Since the backbone is $I' \cup C'$, all the other nodes in $I$ remain on the renovated backbone. ■

*Lemma 6.4:* Given an arbitrary node $v \in I$ fails, while the network is still connected, the renovated backbone is connected.

*Proof:* Obviously $I'$ is a dominating set. In the following we prove that $I' \cup C'$ is a connected dominating set, namely a backbone. According to Section V-B1, $S$ denotes the set of $v$'s one-hop neighbors that are not adjacent to any node in $I$, and $MDS(S)$ denotes the minimum dominating set of $S$, we have

- Case 1: If $MDS(S) = \emptyset$, namely $S = \emptyset$, we have $I' = I \setminus v$. Let $C''$ denote the set that is obtained by launching backbone expansion with $I'$. According to [27], since the network is still connected, $I' \cup C''$ can be proved to be a backbone. We then prove $C' = C''$.
According to the convex hull computation procedure, $\forall u \in I$ that is not involved in the convex hull renovation procedure, its $C_u$ is the same as that in $C''$.
$\forall u \in I$ that needs to update its convex hull $H_u$ and $C_u$ in Step 2 in Section V-B1, it is obvious that $H'_u$ is computed based on $I \setminus v$, (or more specifically $N_u^I \setminus v$), which is the same as the convex hull computed during backbone expansion with $N_u^{I'}$. Thus $C'_u$ is the same as that in $C''$. Therefore, we have $C' = C''$, and thus the renovated nodes in $I' \cup C'$ are connected and contribute a backbone.
- Case 2: $\forall u \in MDS(S)$, suppose $w \in I'$ is the farthest node to $u$ where $w \in N_u^{I'} \cap (I' \setminus MDS(S))$. If $w$ exists, $u$ must connect with $w$ according to the convex hull generation procedure given in Graham Scan algorithm. If $w$ does not exist, $u$ must connect with at least one node in $MDS(S)$. Let $M$ denote such set of nodes in $MDS$ that these nodes only connect with the nodes in $MDS(S)$. Since the network is connected, the nodes in $M$ must connect with at least one node in $I' \setminus MDS(S)$. Therefore, every node in $MDS(S)$ either directly connects to the nodes in $I' \setminus MDS(S)$, or indirectly connects to them through some intermediate nodes in $MDS(S)$.

(a) If $(I' \cup C') \setminus MDS(S)$ is connected, note that every node in $MDS(s)$ directly or indirectly connects to the nodes in $I' \setminus MDS(S)$, $I' \cup C'$ must be connected.

(b) If $(I' \cup C') \setminus MDS(S)$ is not connected, without loss of generality, we assume $(I' \cup C') \setminus MDS(S)$ consists of two disjointed components $A$ and $B$. Note that the network is connected, there must exist a path connecting $A$ and $B$ through $MDS(S)$. Let $c, d \in MDS(S)$ denote the closest node to $A$ and $B$ on the path, respectively, $a \in N_c^{I'}$ denotes the farthest node to $c$ where $a \in N_c^{I'} \cap A$, and $b \in N_d^{I'}$ denotes the farthest node to $d$ where $b \in N_c^{I'} \cap B$. According to the convex hull generation procedure given in Step 3 in Section V-B1, $c, d \in MDS(S)$ must connect with $a \in A$ and $b \in B$ respectively via the nodes in $I' \cup C'$. For contradiction we assume $c$ and $a$ are not connected by the nodes in $I' \cup C'$. We have $a \notin H'_c$ and $c \notin H'_a$ (otherwise $c$ and $a$ are connected via $H'_c$ ($H'_a$) and $C'_c$ ($C'_a$), which contradicts the assumption). Since $a$ and $c$ are within three hops of each other, $a$ must be enclosed by $H_c$ and $c$ must be enclosed by $H_v$. Let $c_1$ and $c_2$ be the two closest vertices in $H_c$ such that the polygon $P(c, c_1, c_2)$ encloses $a$. Similarly let $a_1$ and $a_2$ be the two closest vertices in $H_a$ such that the polygon $P(a, a_1, a_2)$ encloses $c$. It is obvious that the shortest paths $SP(c, c_1)$ and $SP(a, a_1)$ cross, and $SP(c, c_2)$ and $SP(a, a_2)$ cross. From Lemma 4.2, $c$ and $a$ can reach each other by traversing only vertices in $SP(c, c_1)$ and $SP(a, a_1)$, or in $SP(c, c_2)$ and $SP(a, a_2)$. This contradicts to our assumption that $c$ and $a$ cannot be connected via the nodes in $I' \cup C'$. Therefore $c$ and $a$ must be connected with each other via the nodes in $I' \cup C'$. So does the nodes $b$ and $d$. Similarly, Let $e, f \in MDS(S)$ denote the second closest node to $A$ and $B$ on the path, respectively, and $e$ and $f$ will connect with $A \cup c$ and $B \cup d$ respectively. And so on. Until $e$ and $f$ are within three hops of each other. Based on the proof above, we can easily find that $e$ and $f$ connect with each other according to Lemma 4.2

Based on the proof above, the renovated backbone $I' \cup C'$ is connected. ■

*Lemma 6.5:* Given an arbitrary node $v \in I$ is added into the network, while the network is still connected, the renovated backbone is connected.

*Proof:* According to Section V-C1, node $v$ indirectly connects to the backbone via its convex hull $H_v$. Therefore the renovated backbone is still connected. ■

*Corollary 6.5:* Given an arbitrary node $v \in I$ fails or is added into the network, while the network is still connected, the renovated backbone is connected.

*Lemma 6.6:* Given an arbitrary node $v$ in $C$ fails, while the network is still connected, the renovated backbone is connected.

*Proof:* If $v \in C$ fails, either an alternate shortest path between $u$ and $w$ should be computed, where $u, w \in I$ denotes two nodes in the MIS that are connected through node $v$ on the backbone, $u \in N^I_w$ and $w \in N^I_u$; or $u$ and $w$ need to update their convex hull. In the former case, the alternate shortest path connects the backbone. Therefore the renovated backbone is connected. In the latter case, let $C''$ denote the set that is obtained by launching backbone expansion with $I' = I$. Note that $\forall u \in I$ that needs to update its convex hull $H_u$ and $C_u$ according to Section V-B2, it is obvious that $H'_u$ is computed based on $I$ and the local topology is the same as that of backbone expansion. There $C'_u$ is the same as that in $C''$, $C' = C''$. The renovated nodes in $I' \cup C'$ are connected and contribute a backbone. ■

*Lemma 6.7:* Given an arbitrary node $v \in N_u$ where $u \in I$ is added, while the network is still connected, the renovated backbone is connected.

*Proof:* According to Section V-C2, node $v$ directly connects itself to the backbone (or more specifically node $u$). Therefore the renovated backbone is still connected. ■

*Lemma 6.8:* Given an arbitrary node on the backbone that fails, while the network is still connected, the renovated backbone is connected.

*Proof:* According to Corollary 6.5, Lemma 6.6 and Lemma 6.7, given an arbitrary node on the backbone fails, the renovated backbone provided by LBR is connected. ■

*Lemma 6.9:* Given an arbitrary node in the network fails or is added, while the network is still connected, the renovated backbone is connected.

*Proof:* According to Lemma 6.8, given an arbitrary node on the backbone fails or is added, the renovated backbone provided by LBR is connected. ■

*Corollary 6.6:* The renovated backbone is always connected if the network is connected.

*Lemma 6.10:* Let $h$ denote the cardinality of the convex hull $H_u$, the cardinality of the renovated backbone is at most $2h \cdot |I|$.

*Proof:* Since all nodes in $H_u$ are independent, their communication radius is at least one unit. In the extreme case, at most the number of $h$ vertices reside on $H_u$ whose distance is three hops from $u$ ($h$ is a constant number which is smaller than 18 since the maximum number of nodes in $H_u$ is $2\pi \times 3 = 6\pi < 19$). In other words, $u$ may be connected to at most $h$ nodes in $H_u$ through shortest paths. Since each shortest path is at most three hops, at most two intermediate nodes are introduced between $u$ and any node in $H_u$. Therefore each node $u$ will be charged for at most $2h$ intermediate vertices. This completes the proof. ■

Note that a maximal independent set of $V$ is also a dominating set of $V$. Multiple works (e.g., [15]) have proved the following result that relates the size of any MIS of a unit-disk graph $G$ to that of its MCDS.

*Lemma 6.11:* Let $I$ be any maximal independent set and $opt$ be any MCDS of a unit-disk graph $G$. Then $|I| \leq k \cdot |opt| + 1$ for $|opt| > 1$, $k \leq 4$.

*Lemma 6.12:* Let $h$ denote the cardinality of the convex hull $H_u$, which is usually a small constant. The size of the connected dominating set renovated by LBR is less than $8h \cdot opt + h + 1$, where $opt$ is the size of a MCDS.

*Proof:* This lemma follows from Lemma 6.11 and Lemma 6.10. ■

## VII. Simulation

In this section, We compare the performance of three different backbone maintenance algorithms with LBR, OST, a centralized algorithm that keeps a minimum spanning tree in entire network; AST, another centralized algorithm that keeps the minimum spanning trees computed with every MIS node as a root in the network; BF, a localized best-effort algorithm that tries to reconnect every broken part on the backbone with shortest path within three hops. It is worth pointing out that

BF may fail to renovate the backbone sometimes and cannot guarantee network connectivity after maintenance.

The metrics we used to evaluate the performance of LBR and other algorithms are the size of maintained backbone and the success rate that the renovated backbone is connected while the network is connected.

### A. Settings

In the simulation, nodes are randomly distributed in an area of $500m \times 500m$ and the results are averaged over 100 runs. The communication radius of each node is chosen from $[30m, 40m]$. According to our simulation settings, the radius of $30m$ indicates that the initial network is sparse (i.e., average node degree is about 6) and the radius of $40m$ indicates that the initial network is relatively dense (i.e., average node degree is about 10). Let $cn$ denote the number of changed nodes in the network, where $cn = [50, 100, 150, 200, 250, 300, 350, 400]$ and 0 represents the initial topology, $fr$ denotes the percentage of the number of failed nodes in $cn$, $ar$ denotes the percentage of the number of added nodes in $cn$. Obviously, $fr + ar = 100\%$.

The network topology changes over time in three ways. In the first topology changing situation, the initial number of nodes in the network is 300, $fr = 10\%$, and $ar = 90\%$. This setting can show the performance of these algorithms when the number of nodes in the network increases (i.e., the network becomes denser). In the second topology situation, the initial number of nodes is 500, $fr = 50\%$, and $ar = 50\%$. This setting can show the performance when the size of the network slightly changes. In the third situation, the initial number of nodes is 500, $fr = 90\%$, and $ar = 10\%$. This setting can show the performance when the number of nodes in the network decreases (i.e., the network becomes sparser). The failed nodes (newly added nodes) are randomly selected (deployed) in the network.

### B. Simulation Results

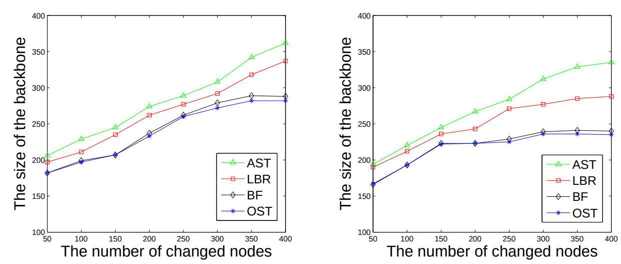
*1) Simulation Study on Backbone Size:* Fig.6, Fig.7, and Fig.8 illustrate the relationship between the size of the backbone and the number of changed nodes given $[ar = 90\%, fr = 10\%]$, $[ar = 50\%, fr = 50\%]$, and $[ar = 10\%, fr = 90\%]$, respectively, under different communication radii $30m$ and $40m$.

Both Fig.6 and Fig.8 show that as the number of nodes in the network increases (decreases), the size of the backbone increases (decreases) linearly in all the algorithms OST, AST, BF, and LBR. OST leads to the slowest backbone size increase (decrease) as it uses only one minimum spanning tree in the network. BF leads to the second slowest backbone size increase (decrease) as it repairs a spanning tree locally and thus leads to limited size increase (decrease) in the network. AST leads to the fastest backbone size increase (decrease) as it uses all possible minimum spanning trees rooted at the nodes in MIS. LBR leads to the medium increase (decrease) of backbone size among the three algorithms, as it repairs the backbone locally and terminates locally at a constant time, as shown in Lemma 6.2. LBR does not necessarily provide a minimum spanning tree or contribute the combination of all possible minimum spanning trees. Thus LBR leads to the medium increase (decrease) in both Fig.6 and Fig.8.

Fig.7 illustrates the relationship between the size of the backbone and the number of changed nodes given $ar = 50\%$ and $fr = 50\%$ under the communication radii $30m$ and $40m$, respectively. Both Fig.7(a) and Fig.7(b) show that as the number of nodes in the network remains stable, the size of the backbone remains stable in all algorithms OST, AST, BF, and LBR.

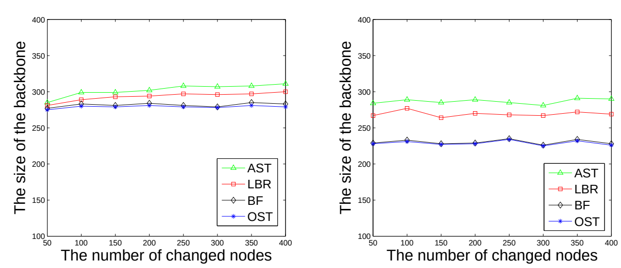
According to Fig.6, Fig.7, and Fig.8, OST and BF lead to the smallest backbone all the time as they use only one minimum spanning tree in the network. AST leads to the largest backbone as it uses all possible minimum spanning trees rooted at the nodes in MIS. LBR leads to a medium-size backbone among the three algorithms, as it repairs the backbone locally and terminates locally at a constant time, as shown in Lemma 6.2.

From Fig.6, Fig.7, and Fig.8, we can also find that the larger the communication radius, the denser the network, the smaller the renovated backbone, and vice versa. It is also interesting to observe that when the network becomes denser, the backbone size of LBR generally follows the trend of that of OST; when the network becomes sparser, the backbone size of LBR generally follows the trend of AST.



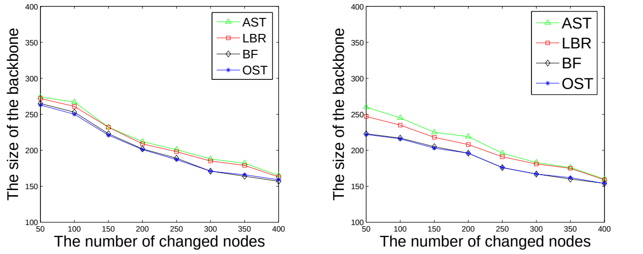

(a) Communication radius = 30m (b) Communication radius = 40m

Fig. 6. The size of backbone $vs.$ the number of changed nodes under the topology changing setting $ar = 90\%$ and $fr = 10\%$ under different communication radii [30m, 40m].




(a) Communication radius = 30m (b) Communication radius = 40m

Fig. 7. The size of backbone $vs.$ the number of changed nodes under the topology changing setting $ar = 50\%$ and $fr = 50\%$ under different communication radii [30m, 40m].




(a) Communication radius = 30m (b) Communication radius = 40m

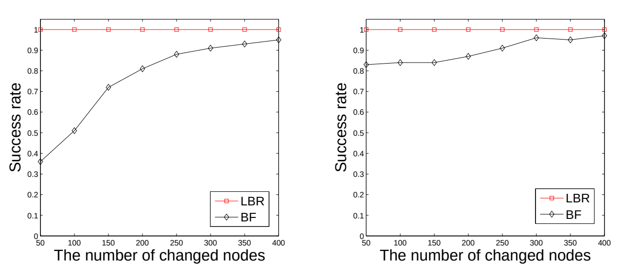
Fig. 8. The size of backbone $vs.$ the number of changed nodes under the topology changing setting $ar = 10\%$ and $fr = 90\%$ under different communication radii [30m, 40m].

*2) Simulation Study on Success Rate:* It is worth pointing out that the success rates of AST, OST are always 1 when the network is connected. This result is reasonable since both AST and OST are centralized algorithms and thus can always guarantee connectivity when the network is connected. Therefore, we simple compare the success rate of LBR and BF.

Fig.9, Fig.10, and Fig.11 illustrate the relationship between the success rate and the number of changed nodes given $[ar = 90\%, fr = 10\%]$, $[ar = 50\%, fr = 50\%]$, and $[ar = 10\%, fr = 90\%]$, respectively, under different communication radii $30m$ and $40m$.

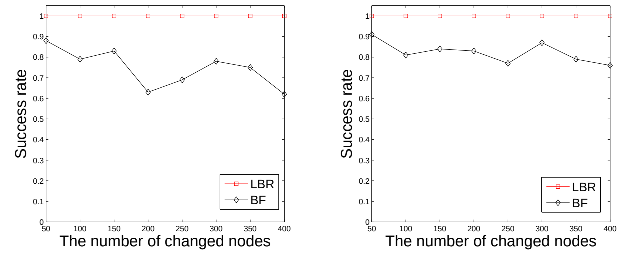
Both Fig.9 and Fig.11 show that as the number of nodes in the network increases (decreases), the success rate that the backbone renovated by $BF$ is connected increases (decreases) in BF. In Fig.10, it is also interesting to observe that when the number of nodes in the network remains stable, the success rate that the backbone renovated by $BF$ is connected is not stable and slowly decreases as the number of changed nodes increases. This indicates that $BF$ is not robust for backbone maintenance.

From Fig.9, Fig.10, and Fig.11, we can easily find that the backbone renovated by LBR is always connected if the network is connected. This is also proved in Lemma 6.9. In these figures, we can also find that the larger the communication radius, the denser the network, the higher the success rate that the backbone renovated by $BF$ is connected, and vice versa.



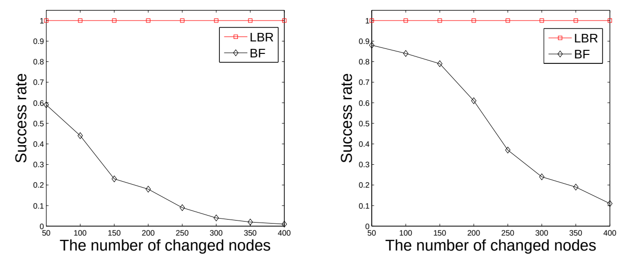

(a) Communication radius = 30m (b) Communication radius = 40m

Fig. 9. The success rate of maintaining the network connectivity via the renovated backbone $vs.$ the number of changed nodes under the topology changing setting $ar = 90\%$ and $fr = 10\%$ under different communication radii [30m, 40m].




(a) Communication radius = 30m (b) Communication radius = 40m

Fig. 10. The success rate of maintaining the network connectivity via the renovated backbone $vs.$ the number of changed nodes under the topology changing setting $ar = 50\%$ and $fr = 50\%$ under different communication radii [30m, 40m].



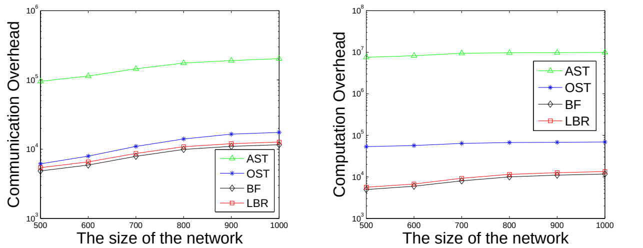

(a) Communication radius = 30m (b) Communication radius = 40m

Fig. 11. The success rate of maintaining the network connectivity via the renovated backbone $vs.$ the number of changed nodes under the topology changing setting $ar = 10\%$ and $fr = 90\%$ under different communication radii [30m, 40m].

*3) Communication and Computation Overhead:* This section studies the relationship between the size of the network and the communication/computation overhead of the four algorithms. Specifically, we set the initial number of nodes in the network to $[500, 1000]$. The number of changed nodes is set to 500, given the topology changing setting $ar = 50\%$ and $fr = 50\%$ with a communication radius 40m.

Fig.12(a) shows that as the size of the network increases, the communication overhead of the four algorithms increases. As shown in the graph, since AST and OST are centralized algorithm, they need to collect the global topology information, and thus have the highest communication overhead. OST has smaller communication compared with AST, since OST computes much less number of minimum spanning trees than AST. LBR and BF are localized algorithms and thus have much less communication overhead compared with AST and OST. However, LBR has slightly larger communication overhead than that of BF, because it needs to repair the topology within three hops instead of repairing only one path.

Fig.12(b) shows that as the size of the network increases, the computation overhead of centralized algorithms increases to some extents, because the backbone size of OST and AST, which determines their computation overhead, becomes stable when the size of the network (or more specifically, the network density) increases to some extent. We can also find that the computation overhead of LBR and BF slowly increases, because as the size of the network increases the number of nodes needed to repair increases.




(a) Communication overhead (b) Computation overhead

Fig. 12. The communication/computation overhead $vs.$ the number of nodes in the network under the topology changing setting $ar = 50\%$ and $fr = 50\%$ with communication radius 40m.

## VIII. Conclusion

In this paper we propose a localized backbone renovating algorithm (LBR) for backbone maintenance in wireless ad hoc and sensor networks. Our theoretical analysis shows that the LBR algorithm could renovate the backbone in a purely localized manner with guaranteed connectivity while keeping the backbone size within a constant factor from that of the minimum CDS. Unless the network is no longer connected, LBR can always keep the renovated backbone connected. Both theoretical analysis and simulation study also show that LBR has ultra low communication and computation overhead. Besides, LBR can deal with arbitrary number of node failures and additions, which provides good scalability to network management.

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