Forecasting isocurvature models with CMB lensing information:
Axion and curvaton scenarios

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(Received 6 April 2012; published 13 July 2012)

Some inflationary models predict the existence of isocurvature primordial fluctuations, in addition to
the well known adiabatic perturbation. Such mixed models are not yet ruled out by available data sets. In
this paper we explore the possibility of obtaining better constraints on the isocurvature contribution from
future astronomical data. We consider the axion and curvaton inflationary scenarios, and use Planck
satellite experimental specifications together with the Sloan Digital Sky Survey galaxy survey to forecast
for the best parameter error estimation by means of the Fisher information matrix formalism. In particular,
we consider how cosmic microwave background (CMB) lensing information can improve this forecast.
We found substantial improvements for all the considered cosmological parameters. In the case of
isocurvature amplitude this improvement is strongly model-dependent, varying between less than 1% and
above 20% around its fiducial value. Furthermore, CMB lensing enables the degeneracy break between
the isocurvature amplitude and correlation phase in one of the models. In this sense, CMB lensing
information will be crucial in the analysis of future data.

DOI: 10.1103/PhysRevD.86.023002
PACS numbers: 98.70.Vc

I. INTRODUCTION

Since the early measurement of the first acoustic peak in
the cosmic microwave background (CMB) angular power
spectrum [1,2], a pure isocurvature model of primordial
fluctuations was ruled out [3]. In addition, recent CMB
data from the WMAP satellite found no evidence for non-
adiabatic primordial fluctuations [4]. These results are
consistent with a single scalar field inflationary model
prediction of perfectly adiabatic density perturbations.
However, small contributions from isocurvature primordial
fluctuations (in mixed models) cannot be excluded by the
current data.

The standard inflationary scenario driven by a single
field cannot account for isocurvature fluctuations. If we
want to take into account isocurvature fluctuations, a
multiple-field inflation has to be considered (for a general
formalism, see Gordon et al. [5], for example). We con-
sider the alternative scenario where perturbations to a light
field different from the inflaton (the curvaton) are respon-
sible for curvature perturbations and may also generate
isocurvature fluctuations [6–8]. In this case, the isocurva-
ture component is completely correlated or anticorrelated
with the adiabatic component. A second scenario is
also taken into account in this work, where quantum
fluctuations in a light axion field generate isocurvature
fluctuations. Unlike the first scenario, this isocurvature
component is fully uncorrelated with the adiabatic one.

It is important to point out that axion particles can be
produced in this scenario, which can contribute to the
present dark matter in the Universe (see Beltran et al. [9],
Bozza et al. [10], Hertzberg et al. [11] and references
therein).

There are already many studies in the literature con-
straining the isocurvature contribution using different
data sets [CMB, large-scale structure (LSS), type Ia
supernovae (SN), Lyman-α forest and baryon acoustic
oscillations (BAO)] (see, for example, Bean et al. [12],
Beltran et al. [13], Carbone et al. [14], Crotty et al. [15],
Komatsu et al. [4], Larson et al. [16], Li et al. [17],
Mangilli et al. [18]). We intend to use the well-known
Fisher information matrix formalism (for a short guide
see Coe [19]) to estimate whether better constraints to the
isocurvature contribution can be obtained in the near
future using measurements of the CMB temperature and
polarization power spectrum from the Planck satellite, as
well as the large-scale matter distribution observed by the
Sloan Digital Sky Survey (SDSS), using CMB lensing
information. We will see how this new information could
improve the error prediction for some cosmological
parameters, especially those related to the isocurvature
mode.

The paper is organized as follows: in Sec. II we describe
briefly the isocurvature models and the notation that will be
used throughout the paper. We give a small introduction on
CMB lensing in Sec. III. In Sec. IV, we briefly review the
Fisher information matrix formalism for the CMB (with
and without lensing information) and for a galaxy survey.
Finally, we present our results in Sec. V, followed by our discussion and conclusions in Sec. VI.

II. ISOCURVATURE NOTATION

In this paper, we consider the standard isocurvature cold dark matter (CDM) mode generated during inflationary time (For a more general case that also consider other generated modes, as for example the baryon mode, see Bucher et al. [20]), that reads

$$S_{\text{CDM}} = \frac{3 \delta \rho_{\text{CDM}}}{\rho_{\text{CDM}}} - \frac{3 \delta \rho_{\gamma}}{\rho_{\gamma}}. \quad (1)$$

The adiabatic density perturbation, believed to be the major factor responsible for the formation of the observable structure in the Universe, is defined as

$$\delta \rho_{\gamma} = \frac{1}{4} \frac{\delta \rho_{\gamma}}{\rho_{\gamma}} + \frac{1}{4} \frac{\delta \rho_{\nu}}{\rho_{\nu}} = \frac{1}{3} \frac{\delta \rho_{\text{CDM}}}{\rho_{\text{CDM}}}, \quad (2)$$

for a universe that consists of photons, massless neutrinos, baryons and CDM at early times. Following Bean et al. [12], we write the isocurvature contribution to the purely adiabatic CMB power spectra in the form

$$C_i = (1 - \alpha)C_i^{\text{ad}} + \alpha C_i^{\text{iso}} + 2\beta \sqrt{\alpha(1 - \alpha)}C_i^{\text{cross}}, \quad (3)$$

where the parameter $\alpha$ accounts for the isocurvature amplitude, while $\beta$ stands for the isocurvature correlation phase, given by $\beta = \cos \theta, -1 \leq \cos \theta \leq 1$. In the same way, the matter power spectrum, $P(k)$, can be decomposed into purely adiabatic, purely isocurvature and their cross correlation contribution:

$$P(k) = (1 - \alpha)P(k)^{\text{ad}} + \alpha P(k)^{\text{iso}} + 2\beta \sqrt{\alpha(1 - \alpha)}P(k)^{\text{cross}}, \quad (4)$$

where

$$P_i(k) = T_i(k)\left(\frac{k}{k_0}\right)^{n_i-1}, \quad i = \text{ad}, \text{iso} \text{ or cross}. \quad (5)$$

As stated by Bean et al. [12] it is reasonable to assume a cross spectrum independent of scale, $n_{\text{cross}} = \frac{1}{2}(n_{\text{ad}} + n_{\text{iso}})$. We take the pivot value $k_0 = 0.002 \text{Mpc}^{-1}$ as used by the WMAP team.

III. CMB LENSING

As new experiments are developed, the precision in CMB measurements makes small effects distinguishable by observations. CMB lensing is one of these effects and it has important quantitative contributions that should be taken into account. It is known that the CMB photons are deflected during their travel between the last scattering surface and the observer by gravitational potentials $\Psi(\chi, \eta)$ dependent on the comoving distance $\chi$ and the conformal time $\eta$. For CMB temperature anisotropy, this is quantitatively written as

$$\frac{\Delta T(\hat{n})}{T} = \frac{\Delta T(\hat{n'})}{T} = \frac{\Delta T(\hat{n} + d)}{T}, \quad (6)$$

where the temperature $T$ of the lensed CMB in a direction $\hat{n}$ is equal to the unlensed CMB in a different direction $\hat{n}'$. Both these directions, $\hat{n}$ and $\hat{n}'$, differ by the deflection angle $d$ as it can be seen in the third equality above. To first order, the deflection angle is simply the lensing potential gradient, $d = \nabla \psi$. In the same way, the effect of lensing in CMB polarization is written in terms of the Stokes parameters $Q(\hat{n})$ and $U(\hat{n})$ (for a review in CMB polarization theory, see Cabella and Kamionkowski [21]):

$$[Q + iU](\hat{n}) = [Q + iU](\hat{n} + d). \quad (7)$$

To use the CMB lensing information we have to measure the lensing potential that is defined as

$$\psi(\hat{n}) = -2 \int_0^{\chi^c} d\chi \frac{\chi^c - \chi}{\chi^c} \Psi(\chi; \eta_0 - \chi). \quad (8)$$

$\chi^c$ being the comoving distance and $\eta_0 - \chi$ is the conformal time at which the photon was at position $\chi \hat{n}$.

We can study CMB lensing properties through the lensing potential, and the temperature and polarization power spectra, as well as their cross correlation (for a review of CMB lensing see Lewis and Challinor [22]).

The lensing signal was detected for the first time by cross-correlating WMAP data to radio galaxy counts in the National Radio Astronomy Observatory Very Large Array sky survey (NVSS) [23]. In other words, $C_{\psi\psi} \neq 0$; however, it is still not possible to obtain the lensing potential power spectrum, $C_{\psi\psi}$, from current data. The detection of the gravitational lensing using CMB temperature maps alone and the measurement of the power spectrum of the projected gravitational potential were already done using the Atacama Cosmology Telescope and the South Pole Telescope [24] [25]. While waiting for more precise data sets, much work is being done to CMB lensing reconstruction techniques (e.g., Bucher et al. [26] Carvalho and Tereno [27], Hu [28], Okamoto and Hu [29], Smith et al. [31]). In this paper, we use the CAMB software package [32] to obtain the numerical lensed and unlensed power spectra ($C^{TT}$, $C^{EE}$, $C^{BB}$, $C^{TE}$ and $C^{dd}$, $C^{Td}$) for each cosmological model. We then used these predictions to forecast how CMB lensing information will help us constrain some isocurvature models when more precise future experiments will be available and the lensing potential can be extracted from the data.

IV. METHOD

In order to search how an isocurvature contribution would affect the measurements of cosmological parameters, we apply the Fisher information matrix formalism to a Planck-like experiment [33], considering both temperature and polarization for the lensed and unlensed CMB spectrum, and to the SDSS. We consider both the axion and the curvaton scenarios in a $\Lambda$CDM model.
A. Information from CMB

The Fisher information matrix for the CMB temperature anisotropy and polarization is given by the approximation in [34]:

\[ F_{ij} = \sum_i \sum_X \frac{\partial C_X^i}{\partial p_i} (\text{Cov}_i^{-1})_{XY} \frac{\partial C_Y^j}{\partial p_j}, \]  

(9)

where \( C_X^i \) is the power in the \( i \)th multipole, \( X \) stands for \( TT \) (temperature), \( EE \) (E-mode polarization), \( BB \) (B-mode polarization) and \( TE \) (temperature and E-mode polarization cross-correlation). We will not include primordial B-modes in the analysis since the measurement of the primordial \( C_{ij}^{BB} \) by Planck is expected to be noise-dominated. Therefore, our covariance matrix becomes

\[ \text{Cov}_i = \frac{2}{(2l + 1)f \, \text{sky}} \begin{bmatrix} \Xi_{TTTT} & \Xi_{TTEE} & \Xi_{ETTE} \\ \Xi_{ETET} & \Xi_{EEEE} & \Xi_{EEEE} \\ \Xi_{TTTE} & \Xi_{ETEE} & \Xi_{TTTT} \end{bmatrix}. \]

(10)

Explicit expressions for the matrix elements are given in the Appendix.

For the lensed case we have to perform a correction in the covariance matrix elements taking into consideration the power spectrum of the deflection angle and its cross correlation with temperature, \( C_{ij}^{dd} \). We also change in this case the unlensed CMB power spectra, \( C_X^i \), for the lensed ones, \( \tilde{C}_X^i \). When we include these corrections, the covariance matrix becomes

\[ \text{Cov}_i = \frac{2}{(2l + 1)f \, \text{sky}} \begin{bmatrix} \tilde{\xi}_{TTTT} & \tilde{\xi}_{TTEE} & \tilde{\xi}_{ETTE} & \tilde{\xi}_{TTTd} \\ \tilde{\xi}_{ETET} & \tilde{\xi}_{EEEE} & \tilde{\xi}_{EEEE} & \tilde{\xi}_{ETTE} \\ \tilde{\xi}_{TTTE} & \tilde{\xi}_{ETEE} & \tilde{\xi}_{TTTT} & \tilde{\xi}_{TTdd} \\ \tilde{\xi}_{TTdT} & 0 & 0 & \tilde{\xi}_{TTdd} \\ \tilde{\xi}_{ddTT} & 0 & 0 & \tilde{\xi}_{ddTd} \\ \tilde{\xi}_{dTTd} & 0 & 0 & \tilde{\xi}_{dddd} \\ 0 & 0 & 0 & 0 & \tilde{\xi}_{BBBB} \end{bmatrix}. \]

(11)

Full expressions for the corrections to the covariance matrix can be found in the Appendix. Note that in this case we are taking into consideration the B-mode polarization generated by the CMB gravitational lensing from the E-mode polarization. In both cases, we used \( f \, \text{sky} = 0.65 \).

B. Information from galaxy survey

The Fisher information matrix for the matter power spectrum obtained from galaxy surveys is given by [35]

\[ F_{ij} = \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{\partial \ln P(k)}{\partial p_i} \frac{\partial \ln P(k)}{\partial p_j} V_{\text{eff}} k^2 dk (2\pi)^2. \]

(12)

\[ V_{\text{eff}}(k) = \int \frac{\bar{n}(r) P_g(k)}{1 + \bar{n}(r) P_g(k)} d^3 r. \]

(13)

We know that \( P_g(k) = b^2 P(k) \) and using the specifications of the SDSS experiment for the bright red galaxy (BRG) sample, called luminous red galaxies (LRG) in more recent papers, we assume a linear and scalar independent bias \( b = 2 \) [35,36]. It is assumed that the expected number density of galaxies, \( \bar{n}(r) \), is independent of \( r \), \( n = 10^7 V_s / V_r \), in a volume-limited sample to a depth of \( 10^3 \) Mpc. The survey has an angle of \( \pi \) steradians; therefore the survey volume becomes, \( V_s = 10^3 \pi / 3 \) [35,37,38].

V. RESULTS

For the purpose of our analysis, we consider as free cosmological parameters the adimensional value of the Hubble constant, \( h \), the density of baryons and cold dark matter, \( \Omega_b h^2 \) and \( \Omega_c h^2 \), the spectral index of scalar adiabatic perturbations, \( n_\text{ad} \), the aforementioned isocurvature parameters, \( \alpha \) and \( \beta \), and the equation of state of dark energy \( w \), assumed as a constant. We first performed the forecast for Planck alone, with and without considering CMB lensing. Then, we introduced the forecast for SDSS, combining the results as (assuming that SDSS and CMB results can be well approximated as independent ones)

\[ F_{ij}^{\text{Total}} = F_{ij}^{\text{Planck}} + F_{ij}^{\text{SDSS}}. \]

(14)

We considered 3 different scenarios to constrain the cosmological parameters.

First, we use an axion scenario considering that a high \( n_{\text{iso}} = 1.9 \pm 1 \) is favored by Lyman-\( \alpha \) data [13]. Kasuya and Kawasaki [39] proposed an axion model capable of generating isocurvature fluctuations with an extremely blue spectrum with \( 1 < n_{\text{iso}} \leq 4 \). Taking into consideration that Planck could measure the existence of an isocurvature contribution with a high spectral index motivates our parameter forecast of such a model. We choose a fixed \( n_{\text{iso}} = 2.7 \) considering that Beltran et al. [13] tested the robustness of their result finding an extreme model with \( n_{\text{iso}} = 2.7 \). Bean et al. [12] found this same value for the isocurvature spectral index for their best fit model considering an adiabatic-plus-isocurvature CDM contribution, however, for a generally correlated isocurvature component with respect to the adiabatic one. It was shown, however, that the chosen pivot scale affects the \( n_{\text{iso}} \) likelihood [40]. The previous mentioned articles [12,13] used a pivot scale \( k_0 = 0.05 \) Mpc\(^{-1}\) that favors artificially large \( n_{\text{iso}} \) according to Kurki-Suonio et al. [40]. They chose instead \( k_0 = 0.01 \) Mpc\(^{-1}\) and found that the likelihood for \( n_{\text{iso}} \) peaks at approximately 3. It was also shown that for \( k_0 < 0.01 \) Mpc\(^{-1}\) the results do not change drastically, concluding that our choice for \( n_{\text{iso}} = 2.7 \) is valid.

In this case our fiducial model is given by \( h = 0.736, \Omega_b h^2 = 0.02315, \Omega_c h^2 = 0.1069, n_{\text{ad}} = 0.982 \) (\( n_{\text{iso}} = 2.7 \) fixed) \( \alpha = 0.06, \beta = 0 \) and \( w = -1 \). Fisher contours
and all the 1 sigma errors are shown in Fig. 1 and Table I. Our first approach was to let $/C12$ vary, obtaining in this way a lower and upper limit to the isocurvature correlation phase. Even if not predicted by the chosen inflationary scenario (axion type), we can still constrain a possibly nonzero measurement of $/C12$ where this scenario is still valid. A second approach was to keep $/C12$ fixed, as is showed in Table II. It can be noticed that there is not a significant change in the constraints on the other cosmological parameters between Tables I and II, especially for $/C11$.

We found the best upper limits for the isocurvature contribution from this axion type of nonadiabatic fluctuation, considering a $/C3CDM$ cosmological model, for the combined Planck (considering CMB lensing) + SDSS forecast with $/C11<0.061$ (95% confidence level [CL]) and $-0.0007 < /C12 < 0.0007$ (95% CL). If $/C12$ is not allowed to vary, the result for $/C11$'s upper limit $Q1$ is not significantly

![FIG. 1 (color online). Fisher contours for $/C3CDM$ model plus a contribution of initial isocurvature fluctuation with fiducials amplitude of $/C11 = 0.06$, correlation phase of $/C12 = 0$ and scalar spectral index of $n_{iso} = 2.7$ for the axion scenario. The red and blue contours represent 95.4% and 68% CL, respectively, for the unlesed CMB + SDSS (dashed lines) and for the lensed CMB + SDSS (solid lines) (see Table I).](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CMB alone Planck T + P</th>
<th>CMB alone Planck T + P + lens</th>
<th>$P(k)$ alone SDSS</th>
<th>CMB + $P(k)$ Planck + SDSS T + P</th>
<th>CMB + $P(k)$ Planck + SDSS T + P + lens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.031</td>
<td>0.021</td>
<td>0.43</td>
<td>0.0073</td>
<td>0.0068</td>
</tr>
<tr>
<td>$h^2\Omega_b$</td>
<td>0.00013</td>
<td>0.00012</td>
<td>0.038</td>
<td>0.00012</td>
<td>0.00012</td>
</tr>
<tr>
<td>$h^2\Omega_c$</td>
<td>0.0011</td>
<td>0.00089</td>
<td>0.14</td>
<td>0.00093</td>
<td>0.00079</td>
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<tr>
<td>$n_s$</td>
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<td>0.0045</td>
<td>0.70</td>
<td>0.0021</td>
<td>0.0016</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0011</td>
<td>0.00089</td>
<td>0.28</td>
<td>0.00060</td>
<td>0.00056</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.00077</td>
<td>0.00040</td>
<td>0.17</td>
<td>0.00048</td>
<td>0.00035</td>
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<tr>
<td>$w$</td>
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<td>0.048</td>
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<td>0.012</td>
<td>0.012</td>
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Percentage of the parameters’ fiducial values for each error above

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CMB alone Planck T + P</th>
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<th>$P(k)$ alone SDSS</th>
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<th>CMB + $P(k)$ Planck + SDSS T + P + lens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>4.21%</td>
<td>2.71%</td>
<td>58.42%</td>
<td>0.99%</td>
<td>0.92%</td>
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<tr>
<td>$h^2\Omega_b$</td>
<td>0.56%</td>
<td>0.52%</td>
<td>164.1%</td>
<td>0.52%</td>
<td>0.52%</td>
</tr>
<tr>
<td>$h^2\Omega_c$</td>
<td>1.03%</td>
<td>0.83%</td>
<td>130.96%</td>
<td>0.87%</td>
<td>0.74%</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.81%</td>
<td>0.45%</td>
<td>70%</td>
<td>0.21%</td>
<td>0.16%</td>
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<tr>
<td>$\alpha$</td>
<td>1.83%</td>
<td>1.48%</td>
<td>Not constrained</td>
<td>1.0%</td>
<td>0.93%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<td>$w$</td>
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<td>4.8%</td>
<td>Not constrained</td>
<td>1.2%</td>
<td>1.2%</td>
</tr>
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</table>
TABLE II. The same as Table I, but in this case $\beta = 0$ will be kept fixed.

<table>
<thead>
<tr>
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<th>CMB alone Planck T + P</th>
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<th>CMB + $P(k)$ Planck + SDSS T + P + lens</th>
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<tbody>
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<td>$h$</td>
<td>0.021</td>
<td>0.019</td>
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<td>0.0072</td>
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<td>$h^2 \Omega_b$</td>
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<td>0.00011</td>
<td>0.038</td>
<td>0.00011</td>
</tr>
<tr>
<td>$h^2 \Omega_c$</td>
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<td>$n_s$</td>
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<td>0.0013</td>
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<td>0.00094</td>
<td>0.00084</td>
<td>0.12</td>
<td>0.000562</td>
</tr>
<tr>
<td>$w$</td>
<td>0.045</td>
<td>0.042</td>
<td>0.91</td>
<td>0.012</td>
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Percentage of the parameters' fiducial values for each error above

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>2.85%</td>
<td>2.58%</td>
<td>58.42%</td>
<td>0.98%</td>
</tr>
<tr>
<td>$h^2 \Omega_b$</td>
<td>0.52%</td>
<td>0.47%</td>
<td>164.1%</td>
<td>0.47%</td>
</tr>
<tr>
<td>$h^2 \Omega_c$</td>
<td>1.03%</td>
<td>0.77%</td>
<td>121.61%</td>
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</tr>
<tr>
<td>$n_s$</td>
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<td>0.135%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.57%</td>
<td>1.40%</td>
<td>Not constrained</td>
<td>0.94%</td>
</tr>
<tr>
<td>$w$</td>
<td>4.5%</td>
<td>4.2%</td>
<td>91%</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

TABLE III. Marginalized errors for $\Lambda$CDM model plus a contribution of initial isocurvature fluctuation with fiducials amplitude of $\alpha = 0.06$, correlation phase of $\beta = 0$ and scalar spectral index of $n_{\text{iso}} = n_{\text{ad}} = 0.982$.

<table>
<thead>
<tr>
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<th>CMB + $P(k)$ Planck + SDSS T + P + lens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.062</td>
<td>0.036</td>
<td>0.31</td>
<td>0.0090</td>
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</tr>
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<td>0.0030</td>
<td>0.26</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>0.031</td>
<td>0.031</td>
<td>22.55</td>
<td>0.031</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.091</td>
<td>0.079</td>
<td>41.78</td>
<td>0.055</td>
</tr>
<tr>
<td>$w$</td>
<td>0.43%</td>
<td>0.93%</td>
<td>5.40</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Percentage of the parameters' fiducial values for each error above

<table>
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<tr>
<th>Parameter</th>
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<th>CMB alone Planck T + P + lens</th>
<th>CMB + $P(k)$ Planck + SDSS T + P</th>
<th>CMB + $P(k)$ Planck + SDSS T + P + lens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>6.93%</td>
<td>4.89%</td>
<td>42.12%</td>
<td>1.22%</td>
</tr>
<tr>
<td>$h^2 \Omega_b$</td>
<td>0.52%</td>
<td>0.47%</td>
<td>120.95%</td>
<td>0.47%</td>
</tr>
<tr>
<td>$h^2 \Omega_c$</td>
<td>1.03%</td>
<td>0.83%</td>
<td>88.87%</td>
<td>0.86%</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.33%</td>
<td>0.30%</td>
<td>26%</td>
<td>0.31%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>52.48%</td>
<td>52.48%</td>
<td>Not constrained</td>
<td>52.48%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$w$</td>
<td>13%</td>
<td>9.3%</td>
<td>Not constrained</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

023002-5
In this first scenario, we can also see in Fig. 1 that lensing information can break the degeneracy between $\Omega_{C11}$ and $\Omega_{C12}$.

On the other hand, for the second axion scenario, where the cosmological parameters’ values are the same as the above ones, except for $n_{iso} = 0.982$ fixed (this assumption was made by Bean et al. [12] and by the WMAP team following Dunkley et al. [41]), the isocurvature amplitude is better constrained when $\Omega_{C12}$ is kept fixed (see Tables III and IV, and Fig. 2 for the Fisher constraints). The value for the limit of $\Omega_{C11}$ for the Planck forecast only (without including CMB lensing and keeping $\Omega_{C12}$ fixed) is comparable to the value found by the WMAP team: $\Omega_{C11} < 0.11$ (95% CL) for Planck, against $\Omega_{C11} < 0.13$ (95% CL) for WMAP 7-year data only [16]. However, if we consider the CMB lensing in the analysis we can improve this limit, obtaining $\Omega_{C11} < 0.10$ (95% CL) for Planck. Finally, combining Planck (including CMB lensing) + SDSS, we have $\Omega_{C11} < 0.08$ (95% CL) against $\Omega_{C11} < 0.064$ (95% CL) found earlier with WMAP + BAO + SN [4]. If, on the other hand, $\beta$ is allowed to vary, we have that $\alpha < 0.12$ (95% CL) as our best constraint (Planck + lensing information + SDSS) and $-0.11 < \beta < 0.11$ (95% CL). Even though it is not better constrained, we found an upper limit to $\Omega_{C11}$ when $\Omega_{C12}$ is allowed to vary, giving us an extra bonus to also constrain the correlation phase.

Since the values chosen for $n_{iso}$ in these first two scenarios are in the limit of the error bars found using Lyman-$\alpha$ data, $n_{iso} = 1.9 \pm 1$, for the sake of completeness we tested another scenario for $n_{iso} = 1.9$ finding a significant change only in the isocurvature amplitude $\alpha$ ($\beta$ kept fixed).

![Image](54x160 to 296x394)

FIG. 2 (color online). Fisher contours for $\Lambda$CDM model plus a contribution of initial isocurvature fluctuation with fiducials amplitude of $\alpha = 0.06$, correlation phase of $\beta = 0$ and scalar spectral index of $n_{iso} = 0.982$ for the axion scenario. The red and blue contours represent 95.4% and 68% CL respectively for the unlensed CMB + SDSS (dashed lines) and for the lensed CMB + SDSS (solid lines) (see Table III).
TABLE V. Marginalized errors for ΛCDM model plus a contribution of initial isocurvature fluctuation with fiducials amplitude of $\alpha = 0.003$, correlation phase of $\beta = -1$ and scalar spectral index of $n_{\text{iso}} = 0.984$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CMB alone Planck</th>
<th>CMB alone Planck</th>
<th>Planck + P($k$) alone SDSS</th>
<th>CMB + P($k$) Planck + SDSS</th>
<th>CMB + P($k$) Planck + SDSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T + P</td>
<td>T + P + lens</td>
<td>P($k$) alone SDSS</td>
<td>T + P</td>
<td>T + P + lens</td>
</tr>
<tr>
<td>$h$</td>
<td>0.055</td>
<td>0.036</td>
<td>0.33</td>
<td>0.0094</td>
<td>0.0084</td>
</tr>
<tr>
<td>$h^2 \Omega_b$</td>
<td>0.00012</td>
<td>0.00011</td>
<td>0.029</td>
<td>0.00011</td>
<td>0.00011</td>
</tr>
<tr>
<td>$h^2 \Omega_c$</td>
<td>0.0011</td>
<td>0.00090</td>
<td>0.096</td>
<td>0.00093</td>
<td>0.00079</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.0030</td>
<td>0.0028</td>
<td>0.26</td>
<td>0.0029</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.029</td>
<td>0.030</td>
<td>17.63</td>
<td>0.028</td>
<td>0.029</td>
</tr>
<tr>
<td>$\beta$</td>
<td>4.98</td>
<td>5.19</td>
<td>3084.60</td>
<td>4.95</td>
<td>5.10</td>
</tr>
<tr>
<td>$w$</td>
<td>0.13</td>
<td>0.089</td>
<td>7.48</td>
<td>0.016</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Percentage of the parameters' fiducial values for each error above

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CMB alone Planck</th>
<th>CMB alone Planck</th>
<th>Planck + P($k$) alone SDSS</th>
<th>CMB + P($k$) Planck + SDSS</th>
<th>CMB + P($k$) Planck + SDSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T + P</td>
<td>T + P + lens</td>
<td>P($k$) alone SDSS</td>
<td>T + P</td>
<td>T + P + lens</td>
</tr>
<tr>
<td>$h$</td>
<td>7.38%</td>
<td>4.83%</td>
<td>44.29%</td>
<td>1.26%</td>
<td>1.13%</td>
</tr>
<tr>
<td>$h^2 \Omega_b$</td>
<td>0.52%</td>
<td>0.48%</td>
<td>126.47%</td>
<td>0.48%</td>
<td>0.48%</td>
</tr>
<tr>
<td>$h^2 \Omega_c$</td>
<td>1.04%</td>
<td>0.85%</td>
<td>90.73%</td>
<td>0.88%</td>
<td>0.75%</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.30%</td>
<td>0.28%</td>
<td>26%</td>
<td>0.29%</td>
<td>0.27%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Not constrained</td>
<td>Not constrained</td>
<td>Not constrained</td>
<td>Not constrained</td>
<td>Not constrained</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Not constrained</td>
<td>Not constrained</td>
<td>Not constrained</td>
<td>495%</td>
<td>Not constrained</td>
</tr>
<tr>
<td>$w$</td>
<td>13%</td>
<td>8.9%</td>
<td>Not constrained</td>
<td>1.6%</td>
<td>1.5%</td>
</tr>
</tbody>
</table>

TABLE VI. The same as Table V, but in this case $\beta = -1$ will be kept fixed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CMB alone Planck</th>
<th>CMB alone Planck</th>
<th>Planck + P($k$) alone SDSS</th>
<th>CMB + P($k$) Planck + SDSS</th>
<th>CMB + P($k$) Planck + SDSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T + P</td>
<td>T + P + lens</td>
<td>P($k$) alone SDSS</td>
<td>T + P</td>
<td>T + P + lens</td>
</tr>
<tr>
<td>$h$</td>
<td>0.055</td>
<td>0.0345</td>
<td>0.31</td>
<td>0.0092</td>
<td>0.0083</td>
</tr>
<tr>
<td>$h^2 \Omega_b$</td>
<td>0.00012</td>
<td>0.00011</td>
<td>0.027</td>
<td>0.00011</td>
<td>0.00011</td>
</tr>
<tr>
<td>$h^2 \Omega_c$</td>
<td>0.0010</td>
<td>0.00088</td>
<td>0.092</td>
<td>0.00091</td>
<td>0.00077</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.0027</td>
<td>0.0025</td>
<td>0.24</td>
<td>0.0026</td>
<td>0.0024</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0019</td>
<td>0.0012</td>
<td>0.17</td>
<td>0.00047</td>
<td>0.00045</td>
</tr>
<tr>
<td>$w$</td>
<td>0.14</td>
<td>0.088</td>
<td>7.47</td>
<td>0.016</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Percentage of the parameters' fiducial values for each error above

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CMB alone Planck</th>
<th>CMB alone Planck</th>
<th>Planck + P($k$) alone SDSS</th>
<th>CMB + P($k$) Planck + SDSS</th>
<th>CMB + P($k$) Planck + SDSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T + P</td>
<td>T + P + lens</td>
<td>P($k$) alone SDSS</td>
<td>T + P</td>
<td>T + P + lens</td>
</tr>
<tr>
<td>$h$</td>
<td>7.38%</td>
<td>4.63%</td>
<td>41.61%</td>
<td>1.23%</td>
<td>1.11%</td>
</tr>
<tr>
<td>$h^2 \Omega_b$</td>
<td>0.52%</td>
<td>0.48%</td>
<td>117.75%</td>
<td>0.48%</td>
<td>0.48%</td>
</tr>
<tr>
<td>$h^2 \Omega_c$</td>
<td>0.94%</td>
<td>0.83%</td>
<td>87.90%</td>
<td>0.86%</td>
<td>0.72%</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.27%</td>
<td>0.25%</td>
<td>24%</td>
<td>0.26%</td>
<td>0.24%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>63.33%</td>
<td>40%</td>
<td>Not constrained</td>
<td>15.67%</td>
<td>15%</td>
</tr>
<tr>
<td>$w$</td>
<td>14%</td>
<td>8.8%</td>
<td>Not constrained</td>
<td>1.6%</td>
<td>1.5%</td>
</tr>
</tbody>
</table>
In this case, we found that $\alpha < 0.066$ (95% CL) against $\alpha < 0.062$ (95% CL) for $n_{\text{iso}} = 2.7$ and $\alpha < 0.1$ (95% CL) for $n_{\text{iso}} = 0.982$ of all them considering Planck only with CMB lensing information. As expected, $\alpha$ is better constrained for higher $n_{\text{iso}}$ values.

In the last scenario, the isocurvature primordial fluctuations are generated by the decay of the curvaton. Our fiducial parameters’ values are set to be $h = 0.745$, $\Omega_c = 0.02293h^2$, $\Omega_{\Lambda} = 0.1058h^2$, $n_{\text{ad}} = 0.984$ ($n_{\text{iso}} = 0.984$ fixed, as generally predicted by curvaton scenarios [12]), $\alpha = 0.003$, $\beta = -1$ and $w = -1$. Unlike the first two cases, none of the isocurvature parameters can be constrained if $\beta$ is allowed to vary, as it can be seen in Table V. Nevertheless, the upper limit found for $\alpha$ ($\beta$ fixed) is improved, with Planck only, $\alpha < 0.0068$ (95% CL), compared to the one found for WMAP 7-year data only, $\alpha < 0.011$ (95% CL). An even better constraint is reached considering the CMB lensing effect in our analysis (see Table VI and Fig. 3), improving the Planck limit to $\alpha < 0.0054$ (95% CL). For Planck (including CMB lensing) + SDSS, $\alpha < 0.0039$ (95% CL), against $\alpha < 0.0037$ (95% CL) for WMAP + BAO + SN. Therefore, using other cosmological probes such as BAO and SN to Planck with lensing information + SDSS, the error bars in the isocurvature amplitude can become even smaller, allowing for a very limited isocurvature contribution in the primordial fluctuations when it is completely anticorrelated with the adiabatic component.

VI. DISCUSSION AND CONCLUSIONS

In this paper we studied a possible contribution of isocurvature initial perturbations in the pure adiabatic fluctuations scenario from the well tested $\Lambda$CDM model. Using the Fisher formalism we obtained the best constraints possible for the isocurvature parameters using CMB and galaxy distribution information.

The main goal of this work has been to quantify how CMB lensing information can provide better constraints in the cosmological parameters, especially in the ones related to the isocurvature contribution. Moreover, we saw that CMB lensing information broke the parameter degeneracy between the isocurvature parameters $\alpha$ and $\beta$ for one of the three studied scenarios.

In all tested inflationary scenarios, the CMB lensing information improves the constraints of all chosen parameters, including the ones related to the isocurvature mode. If we consider Planck information alone (with $\beta$ not allowed to vary) the smallest improvement obtained on $\alpha$’s standard deviation is in the axion type inflation for $n_{\text{ad}} \neq n_{\text{iso}}$ with a difference of 0.17% of its fiducial value between the lensed and unlensed analysis. This improvement gets bigger for the scenarios considered by WMAP reaching almost 9% (axion type with $n_{\text{ad}} = n_{\text{iso}}$) and 25% (curvaton type) (see the lower part where $\beta$ is kept fixed in Tables II, IV, and VI).

Moreover, if CMB lensing can be measured, it would be possible to distinguish between the axion models with $n_{\text{iso}} = 0.982$ and $n_{\text{iso}} = 2.7$ for instance. The effect of CMB lensing is bigger for higher $n_{\text{iso}}$ values, as can be seen in the comparison of Figs. 1 and 2. We can better visualize this lensing effect on $n_{\text{iso}}$ by analyzing the power spectra derivatives in respect to $\alpha$ and $\beta$ in Fig. 4.

When the combined Planck + SDSS forecast is done, the improvement with the use of lensing information is not so significant for any of the scenarios. This is due to the poor ability of SDSS to constrain the parameters compared to Planck, especially when CMB lensing information is included. For a CMB experiment alone, or combined with any other precise experiments on galaxy distribution, lensing is important extra information in the attempt to know how well observations can constrain the presence of isocurvature contributions to the primordial fluctuations. An interesting forecast would include future galaxy surveys, such as EUCLID, combined with planck CMB information including the lensing effects.

ACKNOWLEDGMENTS

The authors would like to thank Julien Lesgourgues for useful discussions.
APPENDIX: ELEMENTS OF THE COVARIANCE MATRIX AND LENSING CORRECTIONS

The elements of the covariance matrix in the unlensed case are

\[ \begin{align*}
    \Xi_{TTTT} &= (C_i^{TT} + N_i^{TT})^2, \\
    \Xi_{EEEE} &= (C_i^{EE} + N_i^{PP})^2, \\
    \Xi_{BBBB} &= (C_i^{BB} + N_i^{PP})^2, \\
    \Xi_{TITE} &= (C_i^{TE})^2 + (C_i^{TT} + N_i^{TT}) \times (C_i^{EE} + N_i^{PP}), \\
    \Xi_{TTEE} &= (C_i^{TE})^2, \\
    \Xi_{TTTE} &= C_i^{TT} (C_i^{TT} + N_i^{TT}), \\
    \Xi_{EEET} &= C_i^{TE} (C_i^{EE} + N_i^{PP}), \\
    \Xi_{TTBB} &= \Xi_{EEBB} = \Xi_{TEBB} = 0.
\end{align*} \]

In these equations, \( N_i^{TT} \) and \( N_i^{PP} \) are the Gaussian random detector noises for temperature and polarization, respectively, whose expression is written using the window function, \( B_i^2 = \exp[-l(l+1)\theta_{\text{beam}}^2/8 \ln 2] \), and the inverse square of the detector noise level for temperature and polarization, \( w_T \) and \( w_P \). The full width half maximum, \( \theta_{\text{beam}} \), is used in radians, and \( w = (\theta_{\text{beam}} \sigma)^{-2} \) is the weight given to each considered Planck channel [37]. The experimental specifications can be checked in Table VII:

\[ N_i^{TT} = \left[ (w_T B_i^2)_{100} + (w_T B_i^2)_{143} + (w_T B_i^2)_{217} + (w_T B_i^2)_{353} \right]^{-1}, \]

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>( \theta_{\text{beam}} )</th>
<th>( \sigma_T(\mu K - \text{arc}) )</th>
<th>( \sigma_P(\mu K - \text{arc}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>9.5'</td>
<td>6.82</td>
<td>10.9120</td>
</tr>
<tr>
<td>143</td>
<td>7.1'</td>
<td>6.0016</td>
<td>11.4576</td>
</tr>
<tr>
<td>217</td>
<td>5.0'</td>
<td>13.0944</td>
<td>26.7644</td>
</tr>
<tr>
<td>353</td>
<td>5.0'</td>
<td>40.1016</td>
<td>81.2944</td>
</tr>
</tbody>
</table>

In Fig. 4 (color online), CMB power spectra derivatives in respect to the isocurvature parameters \( \alpha \) (purple) and \( \beta \) (red). The dashed darker lines are related to the unlensed power spectra’s derivative and the solid lighter ones are related to the lensed power spectra’s derivative for the axion scenario of \( \alpha = 0.06, \beta = 0 \). On the left column the scalar spectral index is \( n_{\text{iso}} = 2.7 \) and on the right column the scalar spectral index is \( n_{\text{iso}} = 0.982 \).
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\[ N_{i}^{PP} = [(w_{p}B_{p}^{2})_{100} + (w_{p}B_{p}^{2})_{143} + (w_{p}B_{p}^{2})_{217} + (w_{p}B_{p}^{2})_{353}]^{-1}. \] (A10)

Here we used four channels, 100, 143, 217 and 353 GHz of the Planck experiment, as can be seen from the equations (A9) and (A10).

 Corrections for the lensed case are [42]

\[ \xi_{TTTT} = (\hat{C}_{i}^{TT} + N_{i}^{TT})^2 - \frac{2(\hat{C}_{i}^{TT})(\hat{C}_{i}^{TT})}{(\hat{C}_{i}^{EE} + N_{i}^{PP})(\hat{C}_{i}^{dd} + N_{i}^{dd})^2}, \] (A11)

\[ \xi_{EEEE} = (\hat{C}_{i}^{EE} + N_{i}^{PP})^2, \] (A12)

\[ \xi_{BBBB} = (\hat{C}_{i}^{BB} + N_{i}^{PP})^2, \] (A13)

\[ \xi_{TTEE} = \frac{1}{2}[(\hat{C}_{i}^{TE})^2 + (\hat{C}_{i}^{TT} + N_{i}^{TT})(\hat{C}_{i}^{EE} + N_{i}^{PP})] - \frac{(\hat{C}_{i}^{EE} + N_{i}^{PP})(\hat{C}_{i}^{TT})^2}{2(\hat{C}_{i}^{dd} + N_{i}^{dd})}, \] (A14)

\[ \xi_{TTdd} = \frac{1}{2}[(\hat{C}_{i}^{dd} + N_{i}^{dd})(\hat{C}_{i}^{TE})^2] - \frac{(\hat{C}_{i}^{EE} + N_{i}^{EE})(\hat{C}_{i}^{TT})^2}{2(\hat{C}_{i}^{dd} + N_{i}^{dd})}, \] (A15)

\[ \xi_{dddd} = (\hat{C}_{i}^{dd} + N_{i}^{dd})^2, \] (A16)

\[ \xi_{TTEE} = (\hat{C}_{i}^{TE})^2, \] (A17)

\[ \xi_{TTTE} = \hat{C}_{i}^{TE}[\hat{C}_{i}^{TT} + N_{i}^{TT}] - \frac{(\hat{C}_{i}^{dd})^2}{(\hat{C}_{i}^{dd} + N_{i}^{dd})}. \] (A18)

where \( N_{i}^{dd} \) is the optimal quadratic estimator. Here we consider the TT quadratic estimator since it provides the best estimator for the Planck experiment (for a review, see [29] and [30]) noise of the deflection field and it can be written in the form [43]:

\[ N_{i}^{dd} = \sum_{i \leq j} \frac{(C_{i}^{TT} F_{i,j} + C_{i}^{TT} F_{j,i})^2}{2(C_{i}^{TT} + N_{i}^{TT})(C_{i}^{TT} + N_{i}^{TT})}, \]

\[ F_{i,j} = \sqrt{\frac{(2l_1 + 1)(2l_1 + 1)(2l_2 + 1)}{4\pi} \begin{pmatrix} l_1 & l & l_2 \\ 0 & 0 & 2 \end{pmatrix}} \times \frac{1}{2} [l(l + 1) + l(l + 1) - l_1(l_1 + 1)]. \] (A23)

Note that the Fisher matrix analysis approximates the likelihood as a Gaussian function; however, the likelihood function could in general be non-Gaussian. Nonetheless, as stated in [42], CMB lensing information gives a more Gaussian-like function, breaking some parameters’ degeneracies, and consequently providing a better error estimation.

FORECASTING ISOCURVATURE MODELS WITH CMB . . .

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