On the Fixed and the Flexible Funding Mechanisms in Reward-based Crowdfunding

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Abstract

This study examines two types of crowdfunding mechanisms, namely, the fixed and the flexible funding mechanisms. Under the fixed funding mechanism, the pledges are returned to the backers if the crowdfunding project fails (All-or-Nothing), while under the flexible funding mechanism, the creators are given an opportunity to keep all the raised pledges irrespective of whether the project achieves success(Keep-it-All). According to the basic model built for these two funding mechanisms, under each mechanism, we investigate the pledging strategies of the backers as well as the expected profit of the creator. These investigations can effectively guide the decision makers during the crowdfunding activities. Subsequently, we generalize our models by considering the product qualities, and the main results are shown to be consistent with those derived in the basic model. Finally, we provide several interesting extensions of our studies on the fixed and the flexible funding mechanisms, including the following cases wherein: (i) the warm-glow takes effect, (ii) the number of backers varies, and (iii) the flexible funding mechanism is incentivized by penalties.

Keywords: OR in marketing, crowdfunding, the fixed and the flexible funding mechanisms

1. Introduction

Crowdfunding is a new external financing approach adopted by the startups to raise initial capital for their ventures from interested backers (Cassar 2004; Kuppuswamy and Bayus 2018). It offers a wide range of benefits to the entrepreneurs. For example, it can help to evaluate the market demand to avoid potential losses (Agrawal et al. 2014; Chemla and Tinn 2017), which commonly emerge as a result of the application of the traditional methods of financing. In addition, it can also attract the public attention for future development (Mollick 2014). Despite serving as a

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new way of financing, crowdfunding has experienced rapid growth during the past decade. The crowdfunding industry was estimated to be valued at more than $300 billion by 2015, and it is predicted that crowdfunding will surpass venture capital in 2020[1].

Various forms of crowdfunding have emerged such as the reward-based, donation-based, lending-based, and equity-based crowdfunding (Mollick 2014; Paschen 2017), and the main difference between these forms lies in the returns paid to the investors. For example, the investors usually get products or services from the reward-based crowdfunding, while they get equity stakes from the equity-based crowdfunding. Among all these different types of crowdfunding, the reward-based form dominates in the market. In a typical reward-based crowdfunding project, the entrepreneurs (creators) first solicit financial pledges from the investors (backers) to meet a crowdfunding target within a given time period. Subsequently, they start their businesses if they manage to raise enough funds from the backers. When the project ends, the backers are provided with products or services in return.

It is clear that the implementation of a reward-based crowdfunding project is affected in several ways. For example, the project target, one of the most important characteristics of a project, is essential to a project's success rate; however, it must be noted that it might be unlikely to meet a high target, while a low target might likely result in non-delivery (Mollick 2014). In addition, the scale of the creator's social network also has significant effects on the project performance (Zheng et al. 2014). In this study, we will focus on the impacts of refunding policies on the implementation of a reward-based project, wherein a refunding policy is simply a pre-announcement on what the creators will do with the raised funds if the project fails.

Concerning refunding policies, Indiegogo, one of the most popular reward-based crowdfunding platforms in the United States, offers two crowdfunding mechanisms to the creators to launch their projects. The first mechanism is called the fixed funding mechanism, under which the creators return the pledges to the backers if the crowdfunding project fails (All-or-Nothing), that is, the project is unable to raise enough funds from the backers on time. While in the second mechanism, referred to as the flexible funding mechanism, the creators can choose not to refund and keep all the raised funds even when the project fails (Keep-it-All)[2]. Intuitively, compared to the fixed

[1] https://www.forbes.com/sites/chancebarnett/2015/06/09/trends-show-crowdfunding-to-surpass-VC-in-2016

[2] (1) Most of the existing crowdfunding platforms choose to adopt the fixed funding mechanism. (2) Crowdfunding platforms such as Lendingclub.com and Kiva.org are adopting the flexible funding mechanism. (3) Indiegogo applies a mixed strategy where the creator can choose either the fixed or the flexible funding mechanism.

funding, the flexible funding brings more uncertainties for the backers due to the Keep-it-All policy (Strausz 2015) and lowers the pledging probability of the backers. However, as shown later, the fixed and the flexible funding mechanisms have their own strengths under certain circumstances.

In this study, we will try to compare the fixed and the flexible funding mechanisms analytically and subsequently guide the creators in their mechanism choices. The main contributions are summarized as follows.

First, our work enriches the very limited literature on the flexible funding mechanism. During the investigation, we first develop static models to describe the fixed and their flexible funding mechanisms, wherein the uncertainty generated through flexible funding is embedded as distrust of the backers on the project. By comparing the expected profit derived from each mechanism, we provide the creators with appropriate refunding strategies. The main results show that the flexible funding mechanism is preferred when the unit production cost of the project and the distrust level of the backers are low.

Second, as a generalization, we take the product quality into consideration and subsequently analyze the refunding strategies of the creators under different quality levels. The product quality is assumed to be either exogenous or endogenous. The main results are shown to be consistent with those in the basic model. Interestingly, we find that the creator always provides the backers with high-quality products under the flexible funding mechanism in the endogenous case; additionally, the creator offers high-quality products when the distrust level is high.

Third, we have successfully extended our studies by integrating the concept of warm-glow with, considering multiple backers, and introducing incentive strategies to the flexible funding mechanism. Besides the mathematical analyses, we also provide several interesting business insights behind each extension. For example, we conclude that the charitable projects usually prefer the flexible funding mechanism due to the warm-glow effect in crowdfunding.

This paper is organized as follows. The main problem and basic models are described in section 3. Section 4 introduces the product quality to the basic model, in both the exogenous and the endogenous cases. Section 5 examines several extensions. 6 concludes this study and provides several directions for future research.

2. Literature Review

The rapid growth of the crowdfunding industry has attracted the attention of researchers, and substantial work on various aspects of crowdfunding has been performed. Among all the existing literature, a majority of research has focused on the factors affecting the success rate of a crowdfunding project under the fixed funding mechanism (All-or-Nothing).

On the empirical front, researchers have studied various project characteristics that affect the success rate, including the target of the project (Mollick 2014), the duration for the backers to pledge (Mollick 2014; Cordova et al. 2015), the creator's social network (Belleflamme et al. 2013; Shane and Cable 2002), the pledge patterns of backers(Burtch et al. 2013; Fan-Osuala et al. 2018) and the geographic effect (Mollick 2014; Kang et al. 2017). Concerning the analytical side, Belleflamme et al. (2014) compare pre-ordering with profit-sharing crowdfunding and emphasize the necessity to build a community wherein backers can get additional benefits from pledging to the project. Hu et al. (2015) establish a two-period model to study the optimal product and pricing decisions in the context of crowdfunding. Du et al. (2017) establish a dynamic model to study the pledging process and show the existence of a "cascade effect" driven by the all-or-nothing nature of crowdfunding projects. They also propose several contingent stimulus strategies and show that the benefit is greatest in the middle of a project. Roma et al. (2018) study whether an entrepreneur should run a crowdfunding project when facing a need for venture capital (VC). Their works reveal that crowdfunding helps an entrepreneur to study the demand in the market, while crowdfunding failure sends a negative signal that negatively affects the entrepreneur's access to VC. Unlike these analytical works that mainly analyze the different aspects of the fixed funding mechanism, our work theoretically investigates the flexible funding mechanism and compares it with the fixed funding mechanism. We focus on the differences and the relative performance between the fixed and the flexible funding mechanisms and aim to guide the creator in their mechanism choices.

As a supplement, we need to point out that the All-or-Nothing strategy is also widely studied in other areas such as group-buying. Anand and Aron (2003) compare the grouping buying mechanism with traditional post-price mechanisms under demand uncertainty with different forms. Jing and Xie (2011) examine whether group-buying strategy offers more profit compared to the conventional selling strategy in the setting of facilitating consumer social interaction. Hu et al. (2013) studies whether the sequential group-buying mechanism leads to a higher success rate when compared to the corresponding simultaneous mechanism. Liang et al. (2014) develop a two-period model to study the group-buying mechanism in a dynamic way and show that the success rate and the customer surplus increase with an increase in the information quality. Wu et al. (2015) study the threshold effects before the target is reached and subsequently in the group-buying deals. Yan et al. (2017) study the impact of asymmetric information on the purchasing behaviors of the retailers. They develop the model with two competing retailers and show that both the informed and the uninformed retailers prefer group-buying when the discount is relatively low. Tran and Desiraju (2017) investigate whether the manufacturer wants the retailer to adopt group-buying mechanism with asymmetric information and show that the manufacturer earns more benefits under this mechanism.

Contrary to the existing literature, due to the Keep-it-All policy, it is hard to define "success" in a project with the flexible funding. In order to compare the fixed funding mechanism with the flexible one, we will shift our focus toward evaluating the expected profit earned by the creator instead of the success rate of a project. The funding mechanism with a higher expected profit is preferred by the creator.

The related literature to the flexible funding mechanism is very limited. Cumming et al. (2015) empirically find that the fixed funding ensures that the creator does not start a project with unrealistic low capital and that flexible funding is more useful when the creator can scale own business. They also show that all projects of non-profit organizations use the flexible funding. These results are consistent with our theoretical analysis. Chemla and Tinn (2017) focus on the moral hazard problem in the All-or-Nothing and Keep-it-All mechanisms and show that the All-or-Nothing mechanism is more efficient than the Keep-it-All mechanism.

Our theoretical work is closest to Chang (2016). They study pure public good projects under the fixed and flexible funding mechanisms in the common value environment wherein backers privately receive signals about the common value and subsequently decide whether to pledge or postpone purchase to the retail stage. In their work, the entrepreneur also uses crowdfunding as a tool to learn the market value of own project. Compared to their public good setting wherein backers eventually buy the product in the funding stage or the retail stage, we focus on the projects that bring new products to the market, and backers decide whether to invest based on the utility they expect to obtain from pledging to the project. Our results are more practical; for example, we offer an incentive strategy for the platform, similar to another strategy once adopted by Indiegogo, to enhance the attractions of the flexible funding by imposing a penalty on the creator when his project fails to reach the target.

3. Basic Model

In this section, we will establish basic models to describe the fixed and the flexible funding mechanisms and subsequently analyze the decision strategies of the creator and their resulting impacts on the backers.

Usually, in a reward-based crowdfunding project, first, a creator posts about own crowdfunding project on a crowdfunding platform. This post presents the detailed characteristics of the project, which include target amount K, fixed time horizon T, pledging price p for a unit product, and its unit production cost c. Along with presenting the detailed characteristics of the project, the creator is also required to announce a crowdfunding mechanism to declare the manner of returning the pledge to the investors (backers) when the project ends. After the announcement, the backer arrives at the platform and decides whether to pledge at price p based on own valuation θ of the project. Both the creator and investor aim to maximize their expected utilities (profits).

Our goal in this study is to investigate two existing crowdfunding mechanisms, that is, the fixed and flexible funding mechanisms. To strengthen our focus on the comparisons between the two mechanisms, in the basic model, we assume the following: (i) there are only two potential backers in the market, (ii) the valuations θ of the two backers are i.i.d. with a uniform distribution over interval $\left[0, 1\right]$, and the uniform distribution is common knowledge to the platform; and (iii) there is no opportunity cost. To be specific, the former two assumptions are widely applied in the group-buying and crowdfunding literature (refer to Jing and Xie 2011, Belleflamme et al. 2014, and Hu et al. 2015), and we relax the first assumption to n potential backers in Section 5.2 as an extension. The third assumption indicates that the two backers arrive at the platform simultaneously.

Now, we formally introduce the fixed and the flexible funding mechanisms as follows.

Fixed Funding. Under the fixed funding, a project succeeds only if both backers pledge; otherwise, the raised funds are returned to the backers (All-or-Nothing). Subsequently, the pledging utility of a backer can be denoted as $u_i = \theta - p$. In this case, each backer will pledge if and only if $u_i \geq 0$, that is, $\theta \in [p, 1]$. By noting that θ follows a uniform distribution over $[0, 1]$, we have that the pledging probability of each backer is $(1 - p)$ and the success rate of the whole crowdfunding project is $(1-p)^2$. Since the unit profit of a product is $(p-c)$, the expected profit of the creator under the fixed funding can be expressed as

$$\pi_i = (1-p)^2 \times \big[2(p-c)\big],\ 0 \le c \le p \le 1. \tag{1}$$

Flexible Funding. Under the flexible funding, a creator can keep the raised funds even if the project fails, that is, if the target K is not achieved in the end (Keep-it-All). In this case, the backers will not trust the creator owing to the latter's incentive to abandon the project and escape with the former's pledges, and this distrust, in turn, would lower the valuations of the backers on the project. By denoting the distrust level as σ, we can express the pledging utility of a backer as $u_e = \theta(1-\sigma) - p$. In this case, each backer will pledge if and only if $u_e \ge 0$, that is, $\theta \in \big[p/(1-\sigma), 1\big]$. Similar to the fixed case, we have that the pledging probability of each backer is $(1 - p/(1-\sigma))$, and the expected profit of the creator earned from each pledged backer is $(1-p/(1-\sigma))(p-c)$. Therefore, the total expected profit of the creator under the flexible funding is simply

$$\pi_e = 2 \times \left[\left(1 - \frac{p}{1-\sigma}\right)(p-c)\right],\ 0 \le c \le p \le 1,\ 0 \le \sigma \le 1-c. \tag{2}$$

Without the loss of generality, we assume that $0 \le c \le 1$ and $0 \le \sigma \le 1-c$, under which the flexible funding project is profitable, that is, $\pi_e \ge 0$. We can further divide the expected profit π_e into the following two parts: the part when both the backers pledge (BBP) and the part when only one backer pledges (OBP), that is,

$$\pi_e = \left(1 - \frac{p}{1-\sigma}\right)^2 \big[2(p-c)\big] + 2\frac{p}{1-\sigma}\left(1 - \frac{p}{1-\sigma}\right)(p-c). \tag{3}$$

From (3), we can see that the BBP part of π_e can be regarded as a generalization of π_i, wherein the σ does not have to be necessarily 0, while the OBP part is an additional profit due to the feature of Keep-it-All in the flexible funding mechanism.

When facing the fixed and flexible funding mechanisms, a creator can choose either one to start the crowdfunding project and subsequently decide the optimal pledging price p to maximize own expected profit. The following Lemma 1 summarizes the optimal pledging prices and the corresponding expected profits of the creator under each funding mechanism.

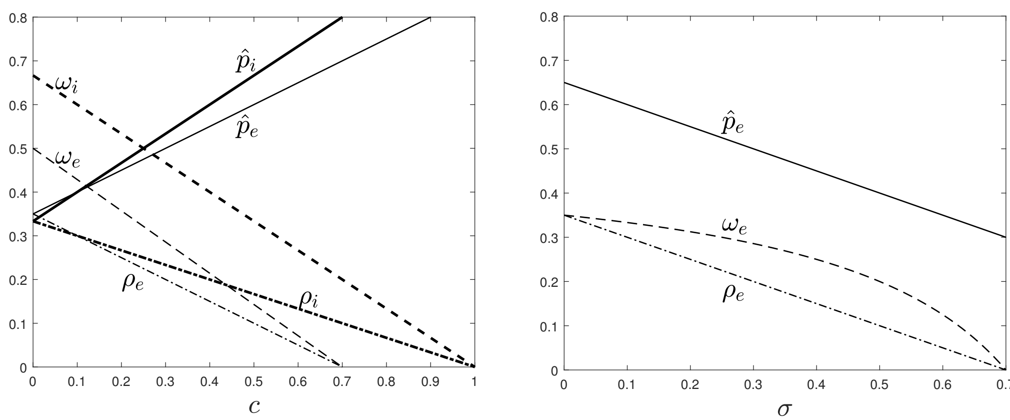
Lemma 1. *Denote the optimal pledging price and the resulting expected profit of the creator as* $\hat{p}$

and $\hat{\pi}$, respectively. We have that[3]

$$\hat{p}_i = \frac{1+2c}{3}, \ \hat{\pi}_i = \frac{8(1-c)^3}{27}, \ \hat{p}_e = \frac{1+c-\sigma}{2}, \ \textit{and} \ \hat{\pi}_e = \frac{(1-c-\sigma)^2}{2(1-\sigma)}.$$

As we can see from Lemma 1, the optimal pledging prices and the expected profits of the creator are decided by the unit production cost c and the distrust level σ. According to the expressions of $\hat{p}$, we can derive the resulting pledging probability ω of each backer and the unit profit ρ earned by the creator from each pledged backer. Subsequently, we study their monotonicities and convexities in c and σ; the main results are shown in Corollary 1.

Corollary 1. *Given the optimal pledging price $\hat{p}$, we have that the unit profit $\rho = \hat{p} - c$ and the pledging probability $\omega = 1 - \hat{p}/(1-\sigma)$ ($\sigma = 0$ in the fixed funding); subsequently,*
(i) *$\hat{p}_i$ and $\hat{p}_e$ are linearly increasing in c; $\hat{p}_e$ is linearly decreasing in σ;*
(ii) *ρ_i and ρ_e are linearly decreasing in c; ρ_e is linearly decreasing in σ; the unit profit gap $\Delta\rho = \rho_i - \rho_e$ is linearly increasing in c; and*
(iii) *ω_i and ω_e are linearly decreasing in c; and ω_e is decreasing and concave in σ.*



(a) $\sigma = 0.3$ (b) $c = 0.3$

Figure 1: Monotonicities and Convexities in c and σ

The main results of Corollary 1 are intuitive; for example, when the unit production cost c increases, the resulting optimal pledging prices will also increase, while the pledging probabilities will decrease; when the distrust level σ is high, the creator will tend to lower the pledging price

[3] In the remainder of this study, subscripts i and e represent the situations of adopting the fixed and flexible mechanisms, respectively.

to enhance the pledging probabilities of the backers. We would like to elaborate some additional factors as follows. First, $\hat{p}$ increases in c while the growth rate is smaller than the unit production cost; thus, the unit profit $\rho = \hat{p} - c$ decreases in c. In addition, the growth rate of $\hat{p}_i$ is larger than the growth rate of $\hat{p}_e$ in c; consequently, $\Delta\rho$ ($\Delta\rho = \rho_i - \rho_e$) increases in c. Second, ω_e decreases in σ, that is, the negative impact of the distrust dominates, even though the creator lowers the pledging price. One can refer to Figure 1 for a better understanding of the Corollary 1.

From Lemma 1 and Corollary 1, we know that the distrust level σ and the unit production cost c jointly determine the optimal pledging price $\hat{p}$ and the resulting expected profit $\hat{\pi}$. To compare the two mechanisms, we adopt the control variates method by first fixing the value of σ and subsequently observing the changes of $\hat{p}$ and $\hat{\pi}$ according to the change in c. The detailed results are shown in Proposition 1.

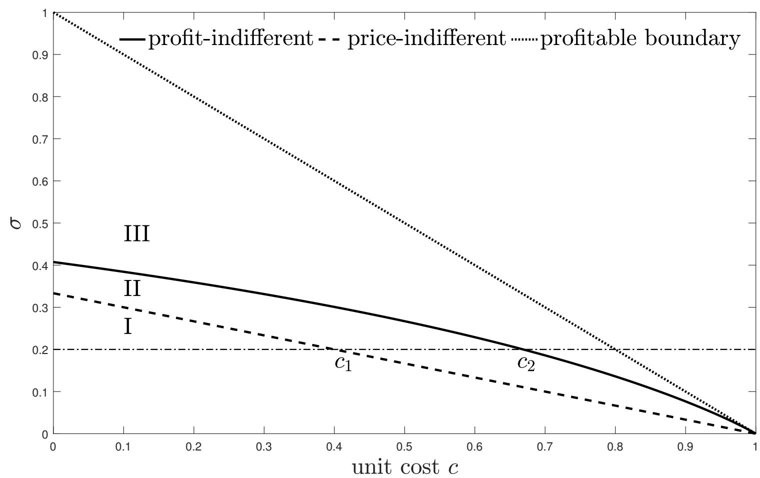
Proposition 1. *For any given σ, there exists two indifferent unit production cost thresholds c_1 and c_2, with $c_1 \leq c_2$, such that*
(i) $\hat{p}_i \leq \hat{p}_e$ *if* $c \leq c_1$*; and,* $\hat{p}_i > \hat{p}_e$ *if* $c > c_1$*; and* (ii) $\hat{\pi}_i \leq \hat{\pi}_e$ *if* $c \leq c_2$*; and,* $\hat{\pi}_i > \hat{\pi}_e$ *if* $c > c_2$.

Proposition 1 indicates that, for a given distrust level σ, when c is low, the optimal pledging price and the expected profit under the fixed funding would be lower than those under the flexible funding; additionally, when c is high, the results are opposite. Particularly, when c is in the middle of c_1 and c_2, the flexible funding would lead to a lower pledging price, while its expected profit would be higher when compared to the fixed funding. We refer to c_1 and c_2 as the price-indifferent and the profit-indifferent thresholds of the unit production cost, respectively, in the fixed and the flexible funding mechanisms.

Corollary 2. *For any given σ, we have that*
(i) *if* $\frac{11}{27} \leq \sigma \leq 1$*, then* $c_1 = c_2 = 0$*, and* $\hat{\pi}_i \geq \hat{\pi}_e$ *always holds, that is, the fixed funding is preferred;*
(ii) *if* $\frac{1}{3} \leq \sigma < \frac{11}{27}$*, then* $0 = c_1 < c_2 < 1$*, and* $\hat{p}_i > \hat{p}_e$ *always holds; and*
(ii) *if* $0 \leq \sigma < \frac{1}{3}$*, then* $0 < c_1 < c_2 \leq 1$*. Particularly, if* $\sigma = 0$*, then* $c_1 = c_2 = 1$*, and* $\hat{\pi}_i \leq \hat{\pi}_e$ *always holds, that is, the flexible funding is preferred.*

Corollary 2 shows that: (i) when the distrust level is high, the creator will always choose the fixed funding mechanism to maximize own expected profit; (ii) when the distrust level is medium, either the fixed or the flexible funding would be preferred by the creator; and (iii) when the backers totally trust the creator, the creator would consider the flexible funding mechanism as optimal.

To better study the joint impacts of c and σ on the mechanism choice of the creators, by increasing σ from 0 to 1, we derive the price-indifferent and the profit-indifferent curves for the fixed and flexible funding mechanisms. The curves are shown in Figure 2.



	$\hat{\pi}_e > \hat{\pi}_i$	$\hat{\pi}_e < \hat{\pi}_i$
$\hat{p}_e > \hat{p}_i$	I	–
$\hat{p}_e < \hat{p}_i$	II	III

Figure 2: Optimal Strategy with Different Values of c and σ

In Figure 2, the two-dimensional space formed by c and σ is separated into three parts by the price-indifferent and profit-indifferent curves. Specifically, the part I represents the subspace wherein $\hat{p}_e$ and $\hat{\pi}_e$ are both higher; part II represents the subspace wherein $\hat{p}_e$ is lower, while $\hat{\pi}_e$ is higher; and part III represents the subspace wherein $\hat{p}_i$ and $\hat{\pi}_i$ are both higher. It must be noted that the area above the profitable boundary is associated with inequality $\sigma > 1 - c$, which represents the area wherein the flexible funding is unprofitable, and the fixed funding is the only choice for the creator. The following are some deep insights gained from the three parts introduced earlier.

First, part I suggests that the flexible funding is popular in low-distrust projects. This can explain why the flexible funding is widely used in charitable activities on platforms such as Indiegogo. For such charitable crowdfunding projects, the backers often pay for moral satisfaction and ignore the potential fraud. Typically, the rewards of charitable activities comprise inexpensive souvenirs, such as T-shirts, which incur minimum production costs but have high memorable value.

Second, part II suggests that the distrust does not always damage the interest of backers under the flexible funding. Specifically, there exists a subspace wherein the creator can get a higher expected profit and the backers can get the products at a lower price, under the flexible funding. The creator is aware that the distrust of the backers will affect their valuation and lower the pledging probability; hence, the creator will lower the pledging price to enhance the pledging probabilities of the backers.

Third, part III indicates that the backers will not pledge to a project with a high distrust level, regardless of the unit production cost. For such high-distrust projects, the fixed funding mechanism is considered better for creators to raise capital. A relevant phenomenon in crowdfunding is that most of the platforms adopt fixed funding as the only funding mechanism (e.g., Kickstarter, GoFundMe, and Teespring). This can be attribute to the fact that the backers have low trust on these platforms as well as the creators because of asymmetric information. After a platform is recognized by the public, the flexible funding mechanism can become an alternative mechanism.

4. Product Quality

As discussed in Section 1, crowdfunding is mainly used for start-ups to launch their new products. Usually, such products are new to the market, and most of them are unavailable in other channels and will not be produced again. In this case, the backers care about the product quality when making pledging decisions.

In this section, we take the product quality into consideration to compare the fixed and the flexible funding mechanisms. In Section 4.1, we focus on the case of exogenous quality, while in Section 4.2, we shift our focus to the endogenous case.

4.1. Exogenous Quality

Suppose that the creator offers a product with an exogenous quality q. By following some existing literature (refer to Guo and Zhang 2012; Hu et al. 2015), we assume that the unit production cost with quality q is $q^2/2$. Additionally, we let $0 \leq q \leq 2$ to ensure that the unit production cost is within $\left[0, 1\right]$, which is consistent with our parameter setting of c in the previous section.

We formally introduce the fixed and the flexible funding mechanisms with exogenous product quality as follows.

Fixed Funding with Exogenous Product Quality. Similar to the basic case studied in Section 3, the pledging utility of a backer can be denoted as $u_i^{ex} = \theta q - p$. Since θ uniformly distributes over $\big[0,1\big]$, we have that the pledging probability of each backer is $(1-p/q)$ and the success rate of the project is $\big(1-p/q\big)^2$. By noting that the unit profit is now $\big(p-q^2/2\big)$, we can express the expected profit of the creator under the fixed funding mechanism with exogenous product quality as

$$\pi_i^{ex} = \Big(1-\frac{p}{q}\Big)^2 \times \big[2(p-\frac{q^2}{2})\big],\ 0 \le q \le 2. \tag{4}$$

Flexible Funding with Exogenous Product Quality. In the flexible funding mechanism with exogenous product quality, the distrust level σ defined in the basic model can be regarded by the backers as a signal of low product quality. Compared to the fixed funding mechanism wherein the All-or-Nothing strategy motivates a creator to improve product quality to attract backers, the incentive is relatively lower in case of the Keep-it-All strategy. Sunbsequently, by integrating the quality factors σ and q, we can express the pledging utility of each backer as $u_e^{ex} = \theta(q-\sigma)-p$. In this case, each backer will pledge if and only if $u_e \ge 0$,that is, $\theta \in \big[p/(q-\sigma),1\big]$, and the backer's pledging probability is $(1-p/(q-\sigma))$. Therefore, the expected profit of the creator under the flexible funding with exogenous product quality would be

$$\pi_e^{ex} = 2 \times \Big[\Big(1-\frac{p}{q-\sigma}\Big)(p-\frac{q^2}{2})\Big],\ 0 \le q \le 2,\ 0 \le \sigma \le q-\frac{q^2}{2}. \tag{5}$$

Similar to what we have done in the basic model, we still assume that $0 \le \sigma \le q - q^2/2$, under which the flexible funding is profitable for the creator, and we can still divide π_e into the BBP and OBP parts, that is,

$$\pi_e^{ex} = \Big(1-\frac{p}{q-\sigma}\Big)^2 \times [2(p-\frac{q^2}{2})] + 2\frac{p}{q-\sigma}\Big(1-\frac{p}{q-\sigma}\Big)(p-\frac{q^2}{2}). \tag{6}$$

When facing the fixed and flexible funding mechanisms, the creator can choose either of the mechanisms to launch own project and decide the optimal pledging price p to maximize own expected profit when the product quality level is exogenous. The following Lemma 2 summarizes the optimal pledging prices and the corresponding expected profits under each mechanism.

Lemma 2. *Denote the optimal pledging price and the resulting expected profit of the creator in the exogenous case as* $\hat{p}^{ex}$ *and* $\hat{\pi}^{ex}$*, respectively. Subsequently, we have that*

$$\hat{p}_i^{ex} = \frac{q(1+q)}{3},\ \hat{\pi}_i^{ex} = \frac{q(2-q)^3}{27},\ \hat{p}_e^{ex} = \frac{2q+q^2-2\sigma}{4}\ \textit{and}\ \hat{\pi}_e^{ex} = \frac{(2q-q^2-2\sigma)^2}{8(q-\sigma)}.$$

As we can see from the Lemma 2, when the exogenous product quality is considered, the optimal pledging prices and the expected profits of the creator are jointly decided by the product quality q and the distrust level σ. According to the expression of $\hat{p}^{ex}$, we can derive the resulting pledging probability ω^{ex} and the unit profit ρ^{ex} earned from each backer. Subsequently, we can study their monotonicities and convexities in q and σ. The main results are shown in Corollary 3.

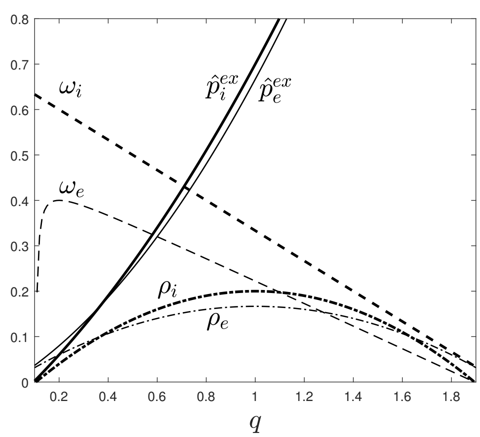
Corollary 3. *Given the optimal pledging price $\hat{p}^{ex}$, we have $\rho^{ex} = \hat{p}^{ex} - q^2/2$ and $\omega^{ex} = 1 - \hat{p}^{ex}/(q-\sigma)$ ($\sigma = 0$ in the fixed funding), subsequently,*

(i) *$\hat{p}_i^{ex}$ and $\hat{p}_e^{ex}$ are increasing and convex in q; $\hat{p}_e^{ex}$ is linearly decreasing in σ;*

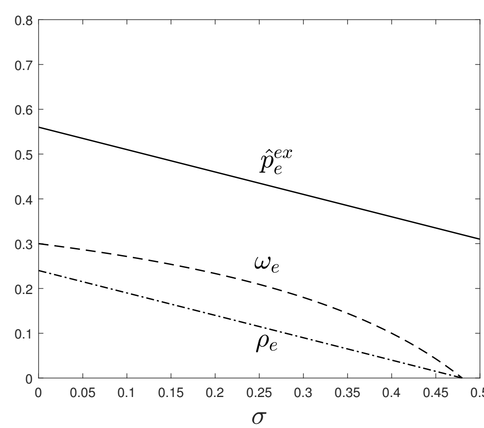
(ii) *ρ_e^{ex}, ρ_i^{ex} and $\rho_e^{ex} - \rho_i^{ex}$ are concave and are unimodal functions of q; ρ_e^{ex} is linearly decreasing in σ; and*

(iii) *ω_i^{ex} is linearly decreasing in q, and ω_e^{ex} is a concave and unimodal function of q if $\sigma > 0$, or linearly decreasing in q if $\sigma = 0$; ω_e^{ex} is decreasing and concave in σ.*

Compared to the basic model, by noting that the unit production cost is now $q^2/2$ instead of c and the distrust level σ is integrated with the quality level q, we can analytically explain the above results as in Corollary 1. For example, the optimal pledging price is convex and increasing in q, while it is linearly increasing in c; this is because $c = q^2/2$. In Figure 3, we show two numerical examples wherein $\sigma = 0.3$ and $q = 0.8$, respectively, for a better understanding of Corollary 3.



(a) $\sigma = 0.1$



(b) $q = 0.8$

Figure 3: Monotonicities and convexities in q and σ

To study the joint impacts of the distrust level σ and the product quality q on the optimal pledging price $\hat{p}^{ex}$ and the resulting expected profit $\hat{\pi}^{ex}$, we first adopt the control variates method by letting $q = \widetilde{q}$. Subsequently, there exists a price-indifferent threshold of distrust $\widetilde{\sigma}_1$ $\big($i.e., $\hat{p}_i^{ex}(\widetilde{q}) = \hat{p}_e^{ex}(\widetilde{q}, \widetilde{\sigma}_1)\big)$ and a profit-indifferent threshold of distrust $\widetilde{\sigma}_2$ $\big($i.e., $\hat{p}_i^{ex}(\widetilde{q}) = \hat{p}_e^{ex}(\widetilde{q}, \widetilde{\sigma}_2)\big)$. By increasing $\widetilde{q}$ from 0 to 2, we drove the price-indifferent and the profit-indifferent curves for the fixed and flexible funding mechanisms. Denote the upper bounds of $\widetilde{\sigma}_1$ and $\widetilde{\sigma}_2$ as σ_1 and σ_2 ($\sigma_1 < \sigma_2$), respectively, then we have Proposition 2.

Proposition 2. *For any given σ, we have that*
(i) *if $\sigma > \sigma_1$, then $\hat{p}_e^{ex} < \hat{p}_i^{ex}$; if $\sigma > \sigma_2$, then $\hat{\pi}_e^{ex} < \hat{\pi}_i^{ex}$;*
(ii) *if $\sigma < \sigma_1$, then there would exist two indifferent product quality thresholds q_1^l and q_1^h such that $\hat{p}_e^{ex} > \hat{p}_i^{ex}$ if $q_1^l < q < q_1^h$, and $\hat{p}_e^{ex} < \hat{p}_i^{ex}$ otherwise; and*
(iii) *if $\sigma < \sigma_2$, then there would exist two indifferent product quality thresholds q_2^l and q_2^h such that $\hat{\pi}_e^{ex} > \hat{\pi}_i^{ex}$ if $q_2^l < q < q_2^h$, and $\hat{\pi}_e^{ex} < \hat{\pi}_i^{ex}$ otherwise.*

Proposition 2 indicates that, for a given distrust level, σ, when the product quality q is medium, the optimal pledging price and the expected profit, under the flexible funding mechanism, would be higher than those under the fixed funding mechanism; additionally, when q is low or high, the results would be opposite. Particularly, when σ is in the middle of σ_1 and σ_2, there exists an interval of q wherein the flexible funding results in a lower pledging price, while its expected profit is higher when compared to the fixed funding.

To get more insights from Proposition 2 and Corollary 3, we show the indifferent curves in Figure 4 wherein the vertical axis represents the distrust level σ and the horizontal axis represents the product quality level q. Part I, II, and III in Figure 4 are consistent with Figure 2.

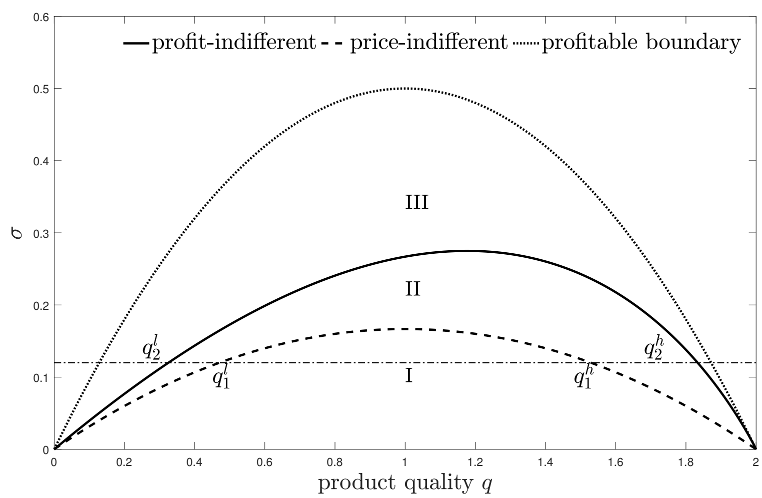


Figure 4: Optimal Strategy with Different Values of q and σ

4.2. Endogenous Quality

It is usual that a reward-based crowdfunding project can choose to offer several types of products with different prices to the backers. In this section, we treat the product quality as an endogenous decision for the creator.

From Lemma 2, we know that, for any given product quality level, q, the respective expected profits under the fixed and flexible funding mechanisms are

$$\hat{\pi}_i^{ex} = \frac{q(2-q)^3}{27}, \text{ and } \hat{\pi}_e^{ex} = \frac{(2q-q^2-2\sigma)^2}{8(q-\sigma)}, \ 0 < q < 2, \ 0 < \sigma < 1/2,$$

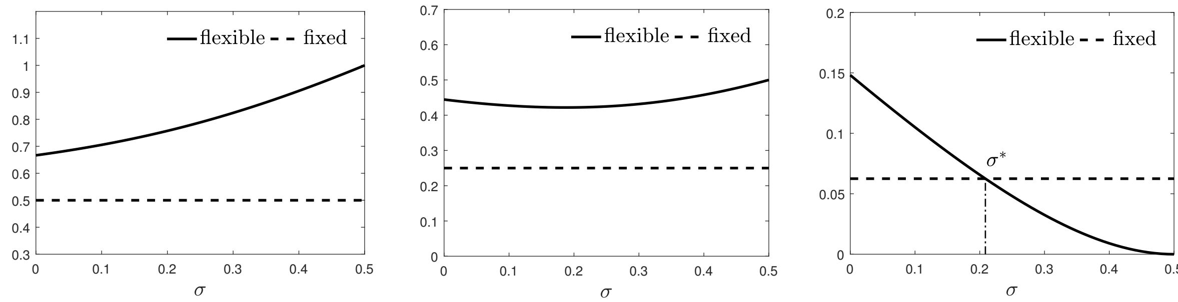
, where we let $0 < \sigma < 1/2$ to ensure that the flexible funding mechanism is associated with a positive expected profit. Since the quality level is endogenous, the distrust level σ would be the unique exogenous factor left. Thus, the creator would decide the product quality and the pledging price in order to maximize the expected profit under either of the funding mechanisms based on the value of σ. We summarize the main results in Lemma 3.

Lemma 3. *Denote the optimal product quality, pledging price, and expected profit as $\hat{q}^{en}$, $\hat{p}^{en}$, $\hat{\pi}^{en}$, respectively. In the endogenous case, we have that*
(i) *under the fixed funding, $\hat{q}_i^{en} = 1/2$, $\hat{p}_i^{en} = 1/4$, and $\hat{\pi}_i^{en} = 1/16$; and*
(ii) *under the flexible funding, $\hat{q}_e^{en} = \left(1 + 2\sigma + \sqrt{1 - 2\sigma + 4\sigma^2}\right)/3$, $\hat{p}_e^{en} = (2q_e^{en} + (q_e^{en})^2 - 2\sigma)/4$ and $\hat{\pi}_e^{en} = (2q_e^{en} - (q_e^{en})^2 - 2\sigma)^2 / \big(8(q_e^{en} - \sigma)\big)$. Specifically, $\hat{q}_e^{en}$ is increasing and convex in σ, $\hat{p}_e^{en}$ is convex in σ, and $\hat{\pi}_e^{en}$ is convex and decreasing in σ.*

According to Lemma 3, we can compare the fixed and the flexible funding mechanisms adopted by the creator. The analytical results are shown in the Proposition 3 and the illustrative figures are shown in Figure 5.

Proposition 3. *When the product quality is endogenous, we have*
(i) *$\hat{q}_i^{en} < \hat{q}_e^{en}$;* (ii) *$\hat{p}_i^{en} < \hat{p}_e^{en}$; and* (iii) *there exists a unique σ^* such that $\hat{\pi}_e^{en} > \hat{\pi}_i^{en}$ if $\sigma < \sigma^*$, and $\hat{\pi}_e^{en} < \hat{\pi}_i^{en}$ if $\sigma > \sigma^*$.*

Proposition 3 and Figure 5 show that the creator will always provide the backers with higher-quality products under the flexible funding, even if the resulting pledging price is higher due to a larger unit production cost. In some sense, this indicates that the flexible funding mechanism with distrust can benefit the backers by facilitating the delivery of high-quality products. In terms



(a) Product Quality (b) Pledge Price (c) Expected Profit

Figure 5: Product quality, pledging price, and expected profit with different values of σ

of the expected profits earned from each funding mechanism, the results are consistent with those in Proposition 1, which claim that the fixed and flexible funding mechanisms each has its own strengths in certain situations.

5. Model Extensions

In this section, we discuss several extensions to our models to get a deeper understanding of the differences between the fixed and flexible funding mechanisms. To strengthen our focus on each extension, in each subsection, we analyze the impacts of different factors based on the model studied in Section 4.1 wherein the product quality is exogenous.

5.1. Warm Glow

In the previous two sections, it is assumed that the backers are self-interested. In such cases, if a backer's valuation, coupled with the product quality and the distrust level, of the project is larger than its pledging price, then the backer will pledge. However, in practice, there may exist some non-economic effects that may stimulate the backers to pledge, including the well-known warm-glow effect. To the best of our knowledge, the concept of warm-glow was first proposed by Becker (1974); subsequently, Andreoni (1989, 1990) used the concept to explain why individuals are motivated to pledge voluntarily for the charitable activities.

In simple words, warm-glow provides individuals (backers) with additional utilities while pledging, and the additional utilities are usually assumed to be proportional to the pledging price (Andreoni 1990; Hu et al. 2015). Thus, if the project succeeds, then we can reformulate the pledging utilities of a backer under the fixed and flexible funding mechanisms as $u_i^{wg} = \theta q - p + \lambda p$ and

$u_e^{wg} = \theta(q-\sigma) - p + \lambda p$, respectively, wherein λ represents the magnitude of the warm-glow effect. Subsequently, we can derive the respectively expected profits of the creator under the fixed and flexible mechanisms shown as

$$\pi_i^{wg} = \Big(1 - \frac{p(1-\lambda)}{q}\Big)^2 \times \big[2(p - \frac{q^2}{2})\big] \text{ and } \pi_e^{wg} = 2\Big(1 - \frac{p(1-\lambda)}{q-\sigma}\Big)\Big(p - \frac{q^2}{2}\Big).$$

As we can see from the expressions of u_i^{wg} and u_e^{wg}, in view of the backers, the warm-glow effect can be simply regarded as a discount on the pledging price, that is, a deduction of the pledging price from p to $\big(1-\lambda\big)p$ while the unit profit earned from each pledged backer is still $p - q^2/2$. In this regard, it is intuitive that, compared to the original case, the optimal pledging price $\hat{p}^{wg}$ posted by the creator, the unit profit ρ^{ex} earned from each backer, the pledging probability ω^{wg}, and the expected profit $\hat{\pi}^{wg}$ of the creator will witness an increase. We summarize the main results in Lemma 4.

Lemma 4. *With the warm-glow effect, we have* (i) $\hat{p}_i^{wg}$ *and* $\hat{p}_e^{wg}$ *are increasing and convex in* λ*;* (ii) ρ_i^{wg} *and* ρ_e^{wg} *are increasing and convex in* λ*;* (iii) ω_i^{wg} *and* ω_e^{wg} *are linearly increasing in* λ*; and* (iv) $\hat{\pi}_i^{wg}$ *and* $\hat{\pi}_e^{wg}$ *are increasing and convex in* λ*;*

Similar to Sections 3 and 4, we can also study the monoticities and convexities of $\hat{p}^{wg}$, ρ^{wg} and ω^{wg} in q and σ. The detailed analyses are omitted for simplification, and one can refer to Appendix for gaining further understanding. It would be sufficient to state that the results in Corollary 3 still hold with the warm-glow effect.

Lemma 5. *With the warm-glow effect, for any given* q*, both* $\widetilde{\sigma}_1$ *and* $\widetilde{\sigma}_2$ *increase in* λ*.*

As mentioned above, the warm-glow effect can be regarded as a discount on the pledging price from the perspective of the backers. Therefore, a backer will have a higher tolerance on the distrust level when making pledging decision than before. In addition, as the warm-glow effect grows, the price-indifferent and the profit-indifferent thresholds of distrust increases, that is, both $\widetilde{\sigma}_1$ and $\widetilde{\sigma}_2$ increase in λ.

Lemma 5 also helps to explain why the flexible funding is popular for funding charitable activities. Owing to the existence of the warm-glow effect in charitable activities, the price-indifferent threshold of distrust $\widetilde{\sigma}_1$ increases, and the subspace (Part I in Figure 2 or Figure 4) expands to the point where the flexible funding dominates the fixed funding in terms of the expected profit. Particularly, when λ is close to unit one, we have Corollary 4 shown as follows.

Corollary 4. *When λ is close to unit one, we have that*

$$\lim_{\lambda \to 1} \frac{\hat{p}_i^{wg}}{\hat{p}_e^{wg}} = \frac{2q}{3(q-\sigma)}, \quad and \quad \lim_{\lambda \to 1} \frac{\hat{\pi}_i^{wg}}{\hat{\pi}_e^{wg}} = \frac{16q}{27(q-\sigma)}.$$

If the distrust level is very low (equal to 0) in projects such as charitable activities with thorough altruism backers, the optimal pledging price set by the creator in the flexible funding would be 50% higher than the one in fixed funding. As a result, the creator can earn an additional 68.75% by adopting the flexible funding mechanism.

5.2. Multiple Backers

We now consider the case wherein there are n ($n > 2$) potential backers in the market, and a project will succeed if all of the n backers choose to pledge. Subsequently, we can derive the respective expected profits of the creator under the fixed and flexible funding mechanisms as follow:

$$\pi_i^N = \left(1 - \frac{p}{q}\right)^n \times \left[n\left(p - \frac{q^2}{2}\right)\right], \ \pi_e^N = n\left(1 - \frac{p}{q - \sigma(n)}\right)\left(p - \frac{q^2}{2}\right),$$

where $\sigma(n)$ is an increasing concave function in n to capture the increasing distrust level of the backers in the flexible funding mechanism since the project is more unlikely to succeed when more backers are involved. In fact, under the same individual pledging probabilities of the backers, the success rate of the project decreases exponentially in n.

According to the expressions of π_i^N and π_e^N, we have the Proposition 4 showing how the number of potential backers will affect the optimal pledging price $\hat{p}(n)$ and the optimal expected profit $\hat{\pi}(n)$.

Proposition 4. *When there are n potential backers in the market, we have that*

(i) $\hat{p}_i^N(n)$ *and* $\hat{p}_e^N(n)$ *are decreasing in n;*

(ii) $\hat{\pi}_i^N(n)$ *is concave and unimodal in n, and the optimal n^* is given by equality*

$$2 - q = 2e^{-\frac{1}{n^*}}\left(1 + \frac{1}{n^*}\right), \tag{7}$$

where $\hat{p}_i^N(n)$, $\hat{\pi}_i^N(n)$, $\hat{p}_e^N(n)$ and $\hat{\pi}_e^N(n)$ are equal to

$$\frac{q(2+nq)}{2(1+n)}, \ \left(\frac{2n - nq}{2n+2}\right)^{n+1}, \ \frac{2q + q^2 - 2\sigma(n)}{4}, \ and \ \frac{n\big(2q - 2\sigma(n) - q^2\big)}{16\big(q - \sigma(n)\big)}, \text{ respectively.}$$

As mentioned above, when n increases, the success rate of the whole project decreases even with the same individual pledging probability of each backer. Hence, to offset such a reduction of success rate, the creator tends to decrease the pledging prices to pursue higher expected profits. The second part of Proposition 4 shows the monotonicity of the expected profits in n under the fixed funding; in other words, when n is relatively small, more backers will bring more profit for the creator, while the results are opposite when n is large. The optimal number of backers is n^* to the creator.

We need to point out that, since there is no explicit expression of $\sigma(n)$, it is hard to study the monotonicity of $\hat{\pi}_e^N(n)$ as well as the analytical comparisons of the fixed and the flexible funding mechanisms.

We now investigate the limiting case wherein there is an infinite number of backers. The results are shown in Corollary 5.

Corollary 5. *When the number of backers n goes to infinity, we have that*
(i) *under the fixed funding,*

$$\lim_{n\to\infty} \hat{p}_i^N(n) = \frac{q^2}{2}, \quad \lim_{n\to\infty} \hat{\pi}_i^N(n) = 0;$$

(ii) *under the flexible funding, by letting $\sigma(1) = 0$ and $\lim_{n\to\infty} \sigma(n) = \sigma^N$,*

$$\lim_{n\to\infty} \hat{p}_e^N(n) = \max\left\{\frac{q^2}{2},\ \frac{2q + q^2 - 2\sigma^N}{4}\right\}, \quad \lim_{n\to\infty} \hat{\pi}_e^N(n) = \max\left\{0,\ \frac{n\big(2q - 2\sigma^N - q^2\big)^2}{16\big(q - \sigma^N\big)}\right\}.$$

Specifically, if $\sigma^N \geq q - q^2/2$, then $\lim_{n\to\infty} \hat{\pi}_e^N(n) = 0$; otherwise, if $\sigma^N < q - q^2/2$, the expected profit is positive.

Corollary 5 shows that, under the fixed funding, when there is an infinite number of backers, the expected profit of the creator is zero and each backer is exactly charged with $q^2/2$ that covers the unit production cost. Under the flexible funding, there exists a threshold $q - q^2/2$ of the limiting distrust level above which the flexible funding is not profitable. We want to elaborate that the threshold $q - q^2/2$ is consistent with that in Section 4.1 because the expected profit earned from each pledged backer does not change when the number of backers varies[4]. Hence, the threshold is still $q - q^2/2$.

[4]This is not the case with the fixed funding mechanism because its success rate decreases with the number of backers. Hence, the expected profit earned from each pledged backer also witnesses a decline.

5.3. An Incentive Strategy for Flexible Funding

We know that one "drawback" of the flexible funding, from the perspective of backers, is that the creator can keep all of the raised pledges even when the project fails; this is because a failed project is likely to result in non-delivery. Concerning the platform, to enhance the attractions of the flexible funding, one can impose a penalty on the creator who posts a project with flexible funding, but the project meets with failure. In this case, the trust of backers on creator will be strengthened, and the former will get more attracted to a project with the flexible funding. Thus, even when a project with flexible funding is threatened due to a penalty, its expected profit may, in turn, increase and the creators may get more incentives to choose the flexible funding.

By denoting the penalty cost of failure as C and the associating distrust level function of the backers as $\sigma(C)$, we can express the expected profit of the creator under the flexible funding as

$$\pi_e^{is} = 2\Big(1 - \frac{p}{q-\sigma(C)}\Big)^2\Big(p - \frac{q^2}{2}\Big) + 2\frac{p}{q-\sigma(C)}\Big(1 - \frac{p}{q-\sigma(C)}\Big)\Big(p - \frac{q^2}{2} - C\Big), \tag{8}$$

where

$$0 \le q \le 2,\ 0 \le C + \sigma(C) \le q - \frac{q^2}{2}.$$

It must be noted that $C + \sigma(C)$ is assumed to be less than $q - \frac{q^2}{2}$ to ensure that the flexible funding is profitable. It is natural that $\sigma(C)$ is decreasing in C; additionally, if $\sigma(C)$ is assumed to be convex in C, then we can characterize the optimal penalty to maximize the expected profit of the creator under the flexible funding by Proposition 5.

Proposition 5. *Suppose that $\sigma(C)$ is decreasing and convex in C, let $\bar{C} = q - \frac{q^2}{2} - \sigma(\bar{C})$, $C^0 = \{C : \sigma(C) = 0\}$ and $C^{-1} = \{C : \sigma(C) = -1\}$, then the optimal C^* of (8) is*
(i) $C^* = \min\{C^0,\ \bar{C}\}$, *if* $\sigma'(C^0) < -1$; (ii) $C^* = 0$, *if* $\sigma'(0) > -1$; *and* (iii) $C^* = \min\{C^{-1},\ \bar{C}\}$, *if* $C^* \ge 0$ *and* $\sigma(C^*) \ge 0$.

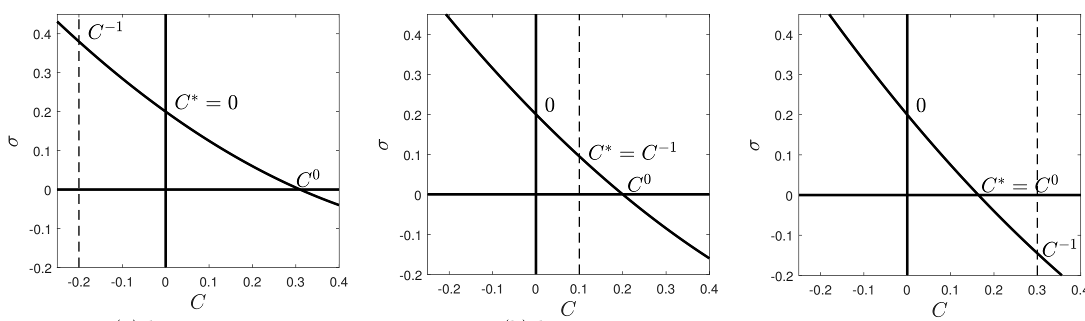
Simply putting it, Proposition 5 indicates that, when $\sigma(C)$ is decreasing and convex in C, the optimal penalty C^* for (8) always associates with a non-negative point $\big(C^*, \sigma(C^*)\big)$ on curve $\sigma(C)$ at which the slope is as close to -1 as possible. We now apply the well-known polynomial form to express the distrust level of the backers, that is,

$$\sigma(C) = aC^2 + bC + \sigma_0 \text{ with } a \ge 0,\ b \le 0,\ \sigma_0 \ge 0,\ \text{and } \sigma_0 - b^2/(4a) \le 0, \tag{9}$$

to get a deeper understanding of Proposition 5. In (9), $\sigma(C)$ is clearly decreasing and convex, in C in the first quadrant, where $C \ge 0$ and $\sigma(C) \ge 0$

Lemma 6. *According to the expression of $\sigma(C)$ shown in (9), we have that*
(i) *when $a = 0$, if $b > -1$, then $C^* = 0$; if $b < -1$, then $C^* = \min\{-\sigma_0/b,\ \bar{C}\}$; otherwise, if $b = -1$, then C^* is indifferent in $\left[0, \sigma_0\right]$;*
(ii) *when $a > 0$, if $b \geq -1$, then $C^* = 0$; if $b \leq -\sqrt{4a\sigma_0 + 1}$, then $C^* = \min\left\{(-b - \sqrt{b^2 - 4a\sigma_0})/(2a),\ \bar{C}\right\}$; otherwise, if $-\sqrt{4a\sigma_0 + 1} < b < -1$, then $C^* = \min\{(-1-b)/(2a),\ \bar{C}\}$.*

The results of Lemma 6 coincide with those in Proposition 5. Particularly, when $\sigma(C)$ decreases linearly in C: (i) if the decreasing speed of the distrust level is slow, then the platform will not impose any penalties; otherwise, (ii) if the decreasing speed is high, then the platform will set the penalty as high as possible until the resulting distrust level is 0. When $\sigma(C)$ is in a quadratic form, for any given $a > 0$, the absolute value of b would decide how steep $\sigma(C)$ is within the first quadrant. Intuitively, when $|b|$ is small, then the curve would be relatively flat. Hence, the optimal penalty is achieved at $C^* = 0$ such that the resulting slope at point $\left(0, \sigma_0\right)$ is the closest to -1; however, when $|b|$ is large, the curve is steep, and the optimal point is $\left(C^0, 0\right)$ with a slope closest to -1. One can refer to a numerical example shown in Figure 6, where $a = 0.5,\ \sigma_0 = 0.2,\ q = 1$, for a better understanding of Lemma 6.



(a) $b = -0.8$ (b) $b = -1.1$ (c) $b = -1.3$

Figure 6: Optimal C^* with Different Values of b

In fact, Indiegogo once adopted a similar incentive strategy. Under the flexible funding mechanism, they set the service fee at 4% of the raised pledges for a successful project while increasing it to 9% if a project failed. However, they abandoned this strategy not long after. This might be attributed to the fact that the associating distrust function $\sigma(C)$ is so flat that the effects of penalty are minor (Figure 6(a)).

6. Conclusion

In this study, we investigated two types of crowdfunding mechanisms: the fixed and flexible funding mechanisms and analyzed the mechanism choice and the corresponding operational decisions of the creator under different conditions. We stressed on the uncertainty brought by the flexible funding mechanism and the resulting distrust of the backers and subsequently studied their impacts on the creator. Due to the nature of the Keep-it-All policy, we focused on the expected profit instead of the success rate when we compared the two mechanisms.

Our theoretical analysis contributes to several novel insights. First, the analyses on the basic model suggest that the flexible funding mechanism yields more profit for the creator when the unit production cost and the distrust level are both low. Our model offers a possible explanation why the flexible funding mechanism is widely adopted in charitable projects on the Indiegogo crowdfunding platform. This explanation is reinforced when we take altruistic reasons into consideration. Second, when we generalize our basic model by considering the product quality level, we show that the creator will always offer higher-quality products in the endogenous case. To be more specific, the offered product quality increases in the distrust level. Such results suggest that the distrust level might benefit the backers in terms of higher-quality products. Third, this study also offers implications for the crowdfunding platforms. The platforms need to reinforce the market regulations and select more reliable entrepreneurs to post their ideas. Platforms are also supposed to adopt some strategies to motivate the creator like charging an extra penalty cost when the project fails under the flexible funding mechanism. Generally, this strategy is effective only when the decreasing speed of the distrust level is high.

Although this study offers some insights into the two crowdfunding mechanisms, there still exist some limitations in our model. For example, in reality, backers enter a project in a different sequence with different valuations. The uniform distribution assumption can be replaced with a more general form. Second, there always exists an opportunity cost when a buyer decides to invest in this project. Such opportunity cost consists of the following two parts—the monetary cost incurred because the backer could have invested the same amount elsewhere and the psychological frustration when the backer fails to get the desired product. The role of opportunity cost will be an interesting extension to crowdfunding for future research. Finally, how to stimulate the creator under the flexible funding mechanism would be another interesting direction for deeper research.

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References

Agrawal, A., Catalini, C., Goldfarb, A., 2014. Some simple economics of crowdfunding. Innovation Policy and the Economy 14 (1), 63–97.

Anand, K. S., Aron, R., 2003. Group buying on the web: A comparison of price-discovery mechanisms. Management Science 49 (11), 1546–1562.

Andreoni, J., 1989. Giving with impure altruism: Applications to charity and ricardian equivalence. Journal of political Economy 97 (6), 1447–1458.

Andreoni, J., 1990. Impure altruism and donations to public goods: A theory of warm-glow giving. The Economic Journal 100 (401), 464.

Becker, G. S., 1974. A theory of social interactions. Journal of political economy 82 (6), 1063–1093.

Belleflamme, P., Lambert, T., Schwienbacher, A., 2013. Individual crowdfunding practices. Venture Capital 15 (4), 313–333.

Belleflamme, P., Lambert, T., Schwienbacher, A., 2014. Crowdfunding: Tapping the right crowd. Journal of business venturing 29 (5), 585–609.

Burtch, G., Ghose, A., Wattal, S., 2013. An empirical examination of the antecedents and consequences of contribution patterns in crowd-funded markets. Information Systems Research 24 (3), 499–519.

Cassar, G., 2004. The financing of business start-ups. Journal of Business Venturing 19 (2), 261–283.

Chang, J.-W., 2016. The economics of crowdfunding.Ssrn.com/abstract=2827354.

Chemla, G., Tinn, K., 2017. Learning through crowdfunding.

Cordova, A., Dolci, J., Gianfrate, G., 2015. The determinants of crowdfunding success: Evidence from technology projects. Procedia - Social and Behavioral Sciences 181, 115–124.

Cumming, D., Leboeuf, G., Schwienbacher, A., 2015. Crowdfunding models: Keep-it-all vs. all-or-nothing.

Du, L., Hu, M., Wu, J., 2017. Contingent stimulus in crowdfundingSsrn.com/abstract=2925962.

Fan-Osuala, O., Zantedeschi, D., Jank, W., 2018. Using past contribution patterns to forecast fundraising outcomes in crowdfunding. International Journal of Forecasting 34 (1), 30–44.

Guo, L., Zhang, J., 2012. Consumer deliberation and product line design. Marketing Science 31 (6), 995–1007.

Hu, M., Li, X., Shi, M., 2015. Product and pricing decisions in crowdfunding. Marketing Science 34 (3), 331–345.

Hu, M., Shi, M., Wu, J., 2013. Simultaneous vs. sequential group-buying mechanisms. Management Science 59 (12), 2805–2822.

Jing, X., Xie, J., 2011. Group buying: A new mechanism for selling through social interactions. Management Science 57 (8), 1354–1372.

Kang, L., Jiang, Q., Tan, C.-H., 2017. Remarkable advocates: An investigation of geographic distance and social capital for crowdfunding. Information & Management 54 (3), 336–348.

Kuppuswamy, V., Bayus, B. L., 2018. Crowdfunding creative ideas: The dynamics of project backers in kickstarter. In: Cumming D., Hornuf L. (eds) The Economics of Crowdfunding. Palgrave Macmillan, Cham.

Liang, X., Ma, L., Xie, L., Yan, H., 2014. The informational aspect of the group-buying mechanism. European Journal of Operational Research 234 (1), 331–340.

Mollick, E., 2014. The dynamics of crowdfunding: An exploratory study. Journal of business venturing 29 (1), 1–16.

Paschen, J., 2017. Choose wisely: Crowdfunding through the stages of the startup life cycle. Business Horizons 60 (2), 179–188.

Roma, P., Gal-Or, E., Chen, R. R., 2018. Reward-based crowdfunding campaigns: Informational value and access to venture capital. Information Systems Research.

Shane, S., Cable, D., 2002. Network ties, reputation, and the financing of new ventures. Management Science 48 (3), 364–381.

Strausz, R., 2015. Crowdfunding, demand uncertainty, and moral hazard: A mechanism design approach.

Tran, T., Desiraju, R., 2017. Group-buying and channel coordination under asymmetric information. European Journal of Operational Research 256 (1), 68–75.

Wu, J., Shi, M., Hu, M., 2015. Threshold effects in online group buying. Management Science 61 (9), 2025–2040.

Yan, Y., Zhao, R., Lan, Y., 2017. Asymmetric retailers with different moving sequences: Group buying vs. individual purchasing. European Journal of Operational Research 261 (3), 903–917.

Zheng, H., Li, D., Wu, J., Xu, Y., 2014. The role of multidimensional social capital in crowdfunding: A comparative study in china and us. Information & Management 51 (4), 488–496.

7. Appendix

Proof of Lemma 1.

Taking the derivative of (2) and (3), with respect to p yields

$$\frac{d\pi_i}{dp} = 2(1-p)(1+2c-3p),\ \frac{d\pi_e}{dp} = 2(1 - \frac{2p-c}{1-\sigma}). \tag{10}$$

By setting (10) to 0, we have that

$$\hat{p}_i = \frac{(1+2c)}{3},\ \hat{p}_e = \frac{(1+c-\sigma)}{2}. \tag{11}$$

According to (2), (3), and (11), the corresponding maximized profits under the fixed and the flexible funding mechanisms, respectively, are

$$\hat{\pi}_i = \Big(1 - \frac{1+2c}{3}\Big)^2\Big(2\frac{1+2c}{3} - 2c\Big) = \frac{8(1-c)^3}{27},\ \hat{\pi}_e = \frac{(1-c-\sigma)^2}{2(1-\sigma)}. \tag{12}$$

Proof of Corollary 1.

Corollary 1(i) is obvious from the expression of $\hat{p}_i$, $\hat{p}_e$. It must be noted that $\sigma < 1 - c$ from the constraint that $\hat{p}_e \geq c$.

By (11), we have that

$$\rho_i = \frac{(1+2c)}{3} - c = \frac{(1-c)}{3}, \ \rho_e = \frac{(1-c-\sigma)^2}{2(1-\sigma)} - c = \frac{(1-c-\sigma)}{2} \ \rho_i - \rho_e = \frac{(c-1+3\sigma)}{6}; \quad (13)$$

$$\omega_i = 1 - \frac{1+2c}{3} = \frac{2-2c}{3}, \ \omega_e = 1 - \frac{(1-c-\sigma)}{(2-2\sigma)} = \frac{(1-c-\sigma)}{(2-2\sigma)}. \quad (14)$$

By taking the first and second order derivative of ω_e with respect to σ, we have

$$\frac{d\omega_e}{d\sigma} = \frac{-c}{2}\frac{1}{(1-\sigma)^2} < 0, \ \frac{d^2\omega_e}{d\sigma^2} = \frac{c}{(1-\sigma)^3} > 0. \quad (15)$$

Corollary 1(ii) and (iii) are obvious from (13),(14) and (15).

Proof of Proposition 1.

According to (11), denote $\Delta p = \hat{p}_i - \hat{p}_e$; subsequently, by setting $\Delta p = 0$, we have that $c_1 = 1 - 3\sigma$. By (12), denote $\Delta\pi = \hat{\pi}_i - \hat{\pi}_e$; subsequently, π is a one variable cubic equation of c. The discriminant of this function is

$$\Delta = \frac{-729(1-2\sigma)^2\sigma^3}{4096(1-\sigma)^3} < 0, \quad (16)$$

where $0 < \sigma < 1 - c$. Therefore, the three roots of π are all real. Take π as a function of σ and set $\pi = 0$; subsequently, the profit-indifferent threshold of the distrust is

$$\sigma_\pi = \frac{19 - 3c - 24c^2 + 8c^3}{27} - 4\sqrt{4 + 3c - 21c^2 + c^3 + 33c^4 - 24c^5 + 4c^6}. \quad (17)$$

Subsequently, we prove that (17) decreases in c. By taking derivative of (17) with respect to c, we have

$$\frac{d\sigma_\pi}{dc} = \frac{6(1-c)^2(-1+12c+24c^2-8c^3)}{27\sqrt{(4-c)(1-c)^3(1+2c)^2}} - \frac{3+48c-24c^2}{27}. \quad (18)$$

By letting $L = \left[6(1-c)^2(-1+12c+24c^2-8c^3)\right]^2 - \left[(3+48c-24c^2) * \sqrt{(4-c)(1-c)^3(1+2c)^2}\right]^2$ and by noting that $3 + 48c - 24c^2 > 0$ and $6(1-c)^2(-1+12c+24c^2-8c^3) > 0$ when $0 < c < 1$, proving $\frac{d\sigma}{dc} < 0$ is equivalent to proving $L < 0$.

$$L = 2187c + 2187c^2 - 10935c^3 - 2187c^4 + 17496c^5 - 8748c^6 = -2178(1-c)^3(1+2c)^2 < 0.$$

Therefore, c is a decreasing function of σ_π. In other words, for a given σ, the profit-indifferent threshold of unit cost, denoted by c_2, satisfies (17).

Subsequently, we show that $\sigma_1(c) < \sigma_2(c)$ because this is equivalent to proving $c_1 < c_2$. Let

$$\begin{aligned} A &= \sigma_2(c) - \sigma_1(c) \\ &= (19 - 3c - 24c^2 + 8c^3)/27 - 4\sqrt{4 + 3c - 21c^2 + c^3 + 33c^4 - 24c^5 + 4c^6} - (1 - 3c) \\ &= \frac{2}{27}[(5 + 3c - 12c^2 + 4c^3) - 2\sqrt{(4 - c)(1 - c)^3(1 + 2c)^2}]. \end{aligned}$$

It must be noted that $5 + 3c - 12c^2 + 4c^3 = (1 - c)(5 - 2c)(1 + 2c) > 0$ and $(5 + 3c - 12c^2 + 4c^3)^2 - 4(4 - c)(1 - c)^3(1 + 2c)^2 = 9(1 - c)^2(1 + 2c)^2 > 0$; therefore, $A \geq 0$.

Proof of Corollary 2.

By (17), the profit-indifferent threshold of distrust decreases in c. Hence, the upper bound is achieved when $c = 0$ and the upper bound is $11/27$. Similarly, the upper bound of the price-indifferent threshold of distrust is $1/3$. Hence, Corollary 2 is obvious from the discussion above.

Proof of Lemma 2.

Taking derivative of (4) and (5) with respect to p yields

$$\frac{d\hat{\pi}_i^{ex}}{dp} = \frac{2(p - q)(3p - q - q^2)}{q^2}, \; \frac{d\hat{\pi}_e^{ex}}{dp} = \frac{2q + q^2 - 2\sigma - 4p}{q - \sigma}. \tag{19}$$

By setting (19) to 0, we have that

$$\hat{p}_i^{ex} = \frac{q(1 + q)}{3}, \; \hat{p}_e^{ex} = \frac{(2q + q^2 - 2\sigma)}{4} \tag{20}$$

According to(4), (5), and(19), the corresponding maximized profits under the fixed and the flexible funding mechanisms, respectively, are

$$\hat{\pi}_i^{ex} = q(2 - q)^3/27, \; \hat{\pi}_e^{ex} = \frac{(2q - 2\sigma - q^2)^2}{8(q - \sigma)} \tag{21}$$

Proof of Corollary 3.

According to (20), we have that

$$\rho_i^{ex} = \frac{q(1 + q)}{3} - \frac{q^2}{2} = \frac{(2q - q^2)}{6}; \; \rho_e^{ex} = \frac{(2q + q^2 - 2\sigma)}{4} - \frac{q^2}{2} = \frac{2q - q^2 - 2\sigma}{4}; \tag{22}$$

$$\omega_i^{ex} = 1 - \frac{q(1 + q)}{3q} = \frac{(2 - q)}{3}; \; \omega_e^{ex} = 1 - \frac{2q + q^2 - 2\sigma}{4q - 4\sigma} = \frac{2q - q^2 - 2\sigma}{4q - 4\sigma}; \tag{23}$$

Hence, by taking the first and second order derivative of (20), the first order derivative of (22) and (23) with respect to q, we have that

$$\frac{d\hat{p}_i^{ex}}{dq} = \frac{1+2q}{3} > 0,\ \frac{d^2\hat{p}_i^{ex}}{dq^2} = \frac{2}{3} > 0;\ \frac{d\hat{p}_e^{ex}}{dq} = \frac{2+2q}{4} > 0,\ \frac{d^2\hat{p}_e^{ex}}{dq^2} = \frac{1}{2} > 0.$$

$$\frac{d\rho_i^{ex}}{dq} = \frac{2-2q}{6};\ \frac{d\rho_e^{ex}}{dq} = \frac{2-2q}{4};\ \frac{d\omega_i^{ex}}{dq} = \frac{-1}{3} < 0,;\ \frac{d\omega_e^{ex}}{dq} = \frac{q(2\sigma-q)}{4(q-\sigma)^2}.$$

Taking the first and second order derivative of ω_e^{ex}, the yield is

$$\frac{d\omega_e^{ex}}{d\sigma} = \frac{q^2}{4(q-\sigma)} > 0;\ \frac{d^2\omega_e^{ex}}{d\sigma^2} = \frac{q^2}{2(q-\sigma)^3} > 0.$$

Corollary 3 is obvious from the discussion above.

Proof of Proposition 2.

To make sure that the pledging price and expected profit under the flexible funding are positive, we assume $\sigma \le q - q^2/2$. Let $\hat{p}_i = p_e^*$, we have the price-indifferent threshold of distrust $\widetilde{\sigma}_1 = (2q-q^2)/6$ with $0 < q < 2$. The upper bound of $\widetilde{\sigma}_1$ is $1/6$ when $q = 1$. Therefore, $\hat{\pi}_e^{ex} < \hat{\pi}_i^{ex}$ when $\sigma > \sigma_1 = 1/6$. When $0 < \sigma < \sigma_1$, letting $\sigma = (2q-q^2)/6$ leads to the following two real roots: $q_1^l = 1 - \sqrt{1-6\sigma}$ and $q_1^h = 1 + \sqrt{1-6\sigma}$. Hence, $\hat{p}_e^{ex} > \hat{p}_i^{ex}$ if $0 < q_1^l < q < q_1^h < 2$; $\hat{p}_e^{ex} < \hat{p}_i^{ex}$ if $0 < q < q_1^l$ or $q_1^h < q < 2$.

Following the same logic, by letting $\hat{\pi}_i = \hat{\pi}_e$, we have the profit-indifferent threshold $\widetilde{\sigma}_2 = \frac{1}{54}q(2-q)(19+8q-2q^2) - \frac{1}{2}q(1+q)(2-q)\sqrt{(8-q)(2-q)}$ when $0 < q < 2$. We show that this function is a concave and unimodal function. Taking derivative of $\widetilde{\sigma}_2$ with respect to q, we have

$$\frac{d\widetilde{\sigma}_2}{dq} = \frac{19-22q+4q^2}{27} - \frac{(2-q)(16+9q-30q^2+4q^3)}{27\sqrt{(8-q)(2-q)}}.$$

It must be noted that $\frac{d\widetilde{\sigma}_2}{dq}|_{q=0} = 11/27 > 0$, $\frac{d\widetilde{\sigma}_2}{dq}|_{q=2} < 0$. Hence, we only need to prove that $\frac{d\widetilde{\sigma}_2}{dq}$ decreases in q. We have

$$\frac{d^2\widetilde{\sigma}_2}{dq^2} = \frac{-1}{9(8-q)^2\sqrt{(8-q)(2-q)}}(A+B)$$

where

$$A = 32 - 359q + 297q^2 - 64q^3 + 4q^4,\ B = (8+95q-44q^2+4q^3)\sqrt{(8-q)(2-q)}.$$

We have $B \ge 0$ when $0 < q < 2$. Let $B^2 - A^2 = 729q(64-41q+4q^2)$, where the three real roots are $q = 0, q = 1.92, andq = 8.329$. Therefore, $A + B > 0$ when $0 < q < 1.92$. Moreover,

$32-359q+297q^2-64q^3+4q^4 > 32-359q+297q^2-64q^3+3q^3*2 = 32-359q+297q^2-56q^3 > 32-359q+297q^2-56q^2*2 = 32-359q+185q^2 > 32-359q+185q*1.9 = 32-7.5q > 0$ when $1.9 < q < 2$. Therefore, $A+B>0$ when $0<q<2$, which means $\frac{d^2\widetilde{\sigma}_2}{dq^2} \le 0$ when $0<q<2$.

Subsequently, we prove that $\widetilde{\sigma}_1 < \widetilde{\sigma}_2$, denote

$$L = \widetilde{\sigma}_2 - \widetilde{\sigma}_1 = \frac{1}{54}q(2-q)(19+8q-2q^2) - \frac{1}{2}q(1+q)(2-q)\sqrt{(8-q)(2-q)} - (2q-q^2)/6.$$

denote $AA = (38q-3q^2-12q^3 = 2q^4)/54 - (2q-q^2)/6$, $BB = 27(64q^2+24q^3-84q^4+2q^5+33q^6-12q^7+q^8)^{1/2}$, where AA and BB are both positive, It must be noted that $AA = q(10+3q-6q^2+q^3) = (q(1+q)(2-q)(5-q))/27 > 0$, and $AA^2 - BB^2 = (2-q)^2(1+q)^2q^2/81 > 0$. Therefore, $\widetilde{\sigma}_2 > \widetilde{\sigma}_1$ when $0<q<2$.

Suppose $\widetilde{\sigma}_2$ reaches its maximization of σ_2 at $q=q^*$. It must be noted that $\widetilde{\sigma}_2|_{q=0} = \widetilde{\sigma}_2|_{q=2} = 0$. Therefore, when $\sigma < \sigma_2$, there exists q_2^l and q_2^h to ensure that $\hat{\pi}_e^{ex} > \hat{\pi}_i^{ex}$ if $0 < q_2^l < q < q_2^h < 2$, $\hat{\pi}_e^{ex} < \hat{\pi}_i^{ex}$ if $0 < q < q_2^l(\sigma)$, or $2 > q > q_2^h(\sigma)$. This completes the proof.

Proof of Lemma 3.

Taking derivative of (21) with respect to q yields

$$\frac{d\hat{\pi}_i^{ex}}{dq} = \frac{-2(2-q)^2(2q-1)}{27}, \quad \frac{d\hat{\pi}_e^{ex}}{dq} = \frac{(-2q+q^2+2\sigma)(3q^2+2\sigma-2q(1+2\sigma))}{8(q-\sigma)^2}. \tag{24}$$

According to (24), (20), and (21), we have that

$$\hat{q}_i^{en} = \frac{1}{2}, \ \hat{p}_i^{en} = \frac{1}{4}, \ \hat{\pi}_i^{en} = \frac{1}{16} \tag{25}$$

$$\hat{q}_e^{en} = \frac{1+2\sigma+\sqrt{1-2\sigma+4\sigma^2}}{3} \tag{26}$$

$$\hat{p}_e^{en} = \frac{1}{9}\big(2+\sqrt{4+8\sigma(-1+2\sigma)} + \sigma\big(-1+2\sigma+\sqrt{1-2\sigma+4\sigma^2}\big)\big) \tag{27}$$

$$\hat{\pi}_e^{en} = \frac{2\left(\sqrt{4\sigma^2-2\sigma+1} - \sigma\left(\sqrt{4\sigma^2-2\sigma+1}+2\sigma+2\right)+1\right)^2}{27\left(\sqrt{4\sigma^2-2\sigma+1}-\sigma+1\right)} \tag{28}$$

Taking derivative of (26) yields

$$\frac{d\hat{q}_e^{en}}{d\sigma} = \frac{1}{3}\left(\frac{8\sigma-2}{2\sqrt{4\sigma^2-2\sigma+1}}+2\right).$$

By noting

$$\frac{d^2\hat{q}_e^{en}}{d\sigma^2} = \frac{1}{(4\sigma^2-2\sigma+1)^{3/2}} > 0, \text{ and } \frac{d\hat{q}_e^{en}}{d\sigma}|_{\sigma=0} = \frac{1}{3} > 0,$$

$\hat{q}_e^{en}$ is increasing and convex in σ. Taking the first and second order derivative of (27) yields

$$\frac{d\hat{p}_e^{en}}{d\sigma} = \frac{8\sigma^2+(4\sigma-1)\sqrt{\sigma^2-2\sigma+1}+5\sigma-1}{9\sqrt{4\sigma^2-2\sigma+1}};$$

$$\frac{d^2\hat{p}_e^{en}}{d\sigma^2} = \frac{32\sigma^3+(16\sigma^2-8\sigma+4)\sqrt{4\sigma^2-2\sigma+1}\sigma^2-24\sigma^2+15\sigma+4}{9\left(4\sigma^2-2\sigma+1\right)^{3/2}}.$$

To prove $\frac{d^2\hat{p}_e^{en}}{d\sigma^2} > 0$ is equivalent to proving

$$U = 32\sigma^3+(16\sigma^2-8\sigma+4)\sqrt{4\sigma^2-2\sigma+1}\sigma^2-24\sigma^2+15\sigma+4 > 0.$$

By noting that

$$\frac{dU}{d\sigma} = 3\left(-4\sqrt{4\sigma^2-2\sigma+1}+16\sigma\left(\sqrt{4\sigma^2-2\sigma+1}+2\sigma-1\right)+5\right),$$

$$32\sigma^2-16\sigma+5>0 \text{ and } \left(32\sigma^2-16\sigma+5\right)^2-(16\sigma-4)^2\left(4\sigma^2-2\sigma+1\right)=9>0;$$

hence,

$$\frac{d^2\hat{p}_e^{en}}{d\sigma^2} \text{ increases in } \sigma, \text{ and } \frac{d^2\hat{p}_e^{en}}{d\sigma^2}|_{\sigma=0}=0,\ \frac{d\hat{p}_e^{en}}{d\sigma}|_{\sigma=0}=-\frac{2}{9},\ \frac{d\hat{p}_e^{en}}{d\sigma}|_{\sigma=0.5}=\frac{1}{2}.$$

Hence, $\hat{p}_e^{en}$ is convex.

Taking the first and second order derivative of (28), the yield is

$$\frac{d\hat{\pi}_e^{en}}{d\sigma} = \frac{2}{9}\left(-\sqrt{4\sigma^2-2\sigma+1}+4\sigma\left(\sqrt{4\sigma^2-2\sigma+1}+2\sigma-1\right)-1\right);$$

$$\frac{d^2\hat{\pi}_e^{en}}{d\sigma^2} = \frac{2}{9}\left(\frac{8(4\sigma^2-2\sigma+1)-3}{\sqrt{4\sigma^2-2\sigma+1}}+16\sigma-4\right).$$

By noting that

$$8\left(4\sigma^2-2\sigma+1\right)-3>0,\ \textit{and}\ \left(8\left(4\sigma^2-2\sigma+1\right)-3\right)^2-(16\sigma-4)^2\left(4\sigma^2-2\sigma+1\right)=9>0.$$

Hence,

$$\frac{d^2\hat{\pi}_e^{en}}{d\sigma^2}>0,\ \textit{and}\ \frac{d\hat{\pi}_e^{en}}{d\sigma}|_{\sigma=0.5}=0,\ \text{that is, }\ \frac{d\hat{\pi}_e^{en}}{d\sigma}\le 0,$$

where $0<\sigma<0.5$. Hence, $\hat{\pi}_e^{en}$ is convex and decreasing in σ.

Proof of Proposition 3.

The maximized expected profit under the fixed funding is $\hat{\pi}_i^{en} = 1/16$, and corresponding profit under the flexible funding is

$$\hat{\pi}_e^{en} = \frac{2(1-2\sigma-2\sigma^2+(1-\sigma)\sqrt{1-2\sigma+4\sigma^2})^2}{27(1-\sigma+\sqrt{1-2\sigma+4\sigma^2}}.$$

Denote $\hat{\pi}^{en} = \hat{\pi}_e^{en} - \hat{\pi}_i^{en}$, $\hat{\pi}^{en}|_{(\sigma=0)} = 0.0856 > 0$, $\hat{\pi}^{en}|_{(\sigma=0.5)} = -0.0625 > 0$, and $\hat{\pi}^{en}$ decreases in σ. Taking derivative of $\hat{\pi}^{en}$ with respect to σ, we have

$$\frac{d\hat{\pi}^{en}}{d\sigma} = \frac{2AB}{9\sqrt{1-2\sigma+4\sigma^2}(1-\sigma+\sqrt{1-2\sigma+4\sigma^2})^2}.$$

where $A = -1+2\sigma+2\sigma^2-(1-\sigma)\sqrt{1-2\sigma+4\sigma^2}$ and $B = 2-3\sigma+6\sigma^2+4\sigma^3+(2-\sigma+2\sigma^2)\sqrt{1-2\sigma+4\sigma^2}$. It must be noted that $2-3\sigma > 0$ and $2-\sigma+2\sigma^2$, when $0 < \sigma < 0.5$; therefore $B > 0$. Subsequently, we show that $A < 0$. It must be noted that $A|_{\sigma=0} = 0$,

$$\frac{dA}{d\sigma} = \frac{2-7\sigma+8\sigma^2+(2+4\sigma)\sqrt{1-2\sigma+4\sigma^2}}{\sqrt{1-2\sigma+4\sigma^2}}.$$

Additionally, $2-7\sigma+8\sigma^2 > 0$ when $0 < \sigma < 1/2$. Therefore, $A < 0$, $\frac{d\hat{\pi}^{en}}{d\sigma} < 0$. Thus, there exists a unique σ^* satisfying $\hat{\pi}^{en}|_{(\sigma=\sigma^*)} = 0$.

Proof of Lemma 4.

Taking the derivative of the expected profit functions under the fixed and the flexible funding, respectively, we have

$$\frac{d\hat{\pi}_i^{wg}}{d\sigma} = \frac{2(q-p(1-\lambda))(q(1+q-q\lambda)-3p(1-\lambda))}{q^2}.$$

$$\frac{d\hat{\pi}_e^{wg}}{d\sigma} = \frac{2q-2\sigma-(1-\lambda)(4p-q^2)}{q-\sigma}.$$

Therefore, the optimal prices are

$$\hat{p}_i^{wg} = \frac{q}{3-3\lambda} + \frac{q^2}{3},\ \hat{p}_e^{wg} = \frac{2(q-\sigma)}{4(1-\lambda)} + \frac{q^2}{4}, \tag{29}$$

with the maximized expected profits

$$\hat{\pi}_i^{wg} = \frac{q(2-q(1-\lambda))^3}{27(1-\lambda)},\ \hat{\pi}_e^{wg} = \frac{(2(q-\sigma)-q^2(1-\lambda))^2}{8(1-\sigma)(1-\lambda)}. \tag{30}$$

Hence, by taking the first and second order derivative of (29) to λ, we have that

$$\frac{d\hat{p}_i^{wg}}{d\lambda} = \frac{q}{3(1-\lambda)^2} > 0,\ \frac{d^2\hat{p}_i^{wg}}{d\lambda^2} = \frac{2q}{3(1-\lambda)^3} > 0;\ \frac{d\hat{p}_e^{wg}}{d\lambda} = \frac{2(q-\sigma)}{4(1-\lambda)^2} > 0,\ \frac{d^2\hat{p}_3^{wg}}{d\lambda^2} = \frac{q-\sigma}{(1-\lambda)^3} > 0.$$

Hence, $\hat{p}_i^{wg}$ and $\hat{p}_e^{wg}$ are increasing and convex in λ. According to (29), we also have

$$\rho_i^{wg} = \hat{p}_i^{wg} - \frac{q^2}{2},\ \rho_e^{wg} = \hat{p}_e^{wg} - \frac{q^2}{2},$$

; hence, the monoticities and convexities are the same with $\hat{p}_i^{wg}$ and $\hat{p}_e^{wg}$. By noting that

$$\omega_i^{wg} = \frac{2-q+q\lambda}{3},\ \omega_e^{ewg} = \frac{\lambda q^2 - q^2}{4(q-\sigma)} + \frac{1}{2}.$$

Lemma 4 is obvious from the discussion above.

Proof of Lemma 5.

Let $\hat{p}_i^{wg} = \hat{p}_e^{wg}$ and $\hat{\pi}_i^{wg} = \hat{\pi}_e^{wg}$, we have $\widetilde{\sigma}_1^{wg} = q(2-q+q\lambda)/6$ and

$$\widetilde{\sigma}_2^{wg} = \frac{q(2-q+q\lambda)}{54}(19+8q-2q^2-8q\lambda+4q^2\lambda-2q^2\lambda^2-2(1+q-q\lambda)\sqrt{(8-q+q\lambda)(2-q+q\lambda)}).$$

It is easy to see that $\widetilde{\sigma}_1^{wg}$ increases in λ. Taking the derivative of $\widetilde{\sigma}_1^{wg}$ with respect to λ, it is also easy to show that

$$\frac{d\widetilde{\sigma}_1^{wg}}{d\lambda} = \frac{q^2}{18}(A + \frac{B}{C}),$$

where $A = 1 + 2q(1-\lambda)(4-q+q\lambda) > 0,\ B = 2(2-q+q\lambda)(1-7q+q^2+7q\lambda-2q^2\lambda+q^2\lambda^2)$ and $C = \sqrt{(8-q+q\lambda)(2-q+q\lambda)}$. It must be noted that $A > 0$, $C > 0$, and $(AC)^2 - B^2 = 243(1-\lambda)(2-q+q\lambda) > 0$. Therefore, $\widetilde{\sigma}_2^{wg}$ increases in λ.

Proof of Corollary 4.

$$\lim_{\lambda\to 1} \frac{\hat{p}_i^{wg}}{\hat{p}_e^{wg}} = \frac{q(1+q(1-\lambda))4(1-\lambda)}{3(1-\lambda)(2(q-\sigma)+q^2(1-\lambda))} = \frac{2q}{3(q-\sigma)}.$$

$$\lim_{\lambda\to 1} \frac{\hat{\pi}_i^{wg}}{\hat{\pi}_e^{wg}} = \lim_{\lambda\to 1} \frac{q(2-q(1-\lambda))^3 8(q-\sigma)(1-\lambda)}{27(1-\lambda)(2(q-\sigma)-q^2(1-\lambda))^2} = \frac{16q}{27(q-\sigma)}.$$

Proof of Proposition 4.

Taking derivative of π_i with respect to p, we have $\hat{p}_i^N = \frac{(2+nq)q}{2(1+n)}$, and the corresponding maximized profit

$$\hat{\pi}_i^N = q\Big(\frac{n(2-q)}{2(n+1)}\Big)^{n+1}.$$

Taking derivative of $\hat{\pi}_i^N$ with respect to n, we have:

$$\frac{d\hat{\pi}^N}{dn} = \pi_i^N * \Big(\frac{1}{n} - ln(1+\frac{1}{n}) - ln(\frac{2}{2-q})\Big).$$

It must be noted that $ln(\frac{2}{2-q})$ is positive; additionally, if we denote $x=\frac{1}{n}$, $y=x-ln(1+\frac{1}{n})$ is an increasing function of x and has no limitation. We have $y|_{x=0}=0$; therefore, there exists a x^*, satisfying $y(x)<ln(\frac{2}{2-q})$ if $x<x^*$, $y(x), ln(\frac{2}{2-q})$ if $x>x^*$. Additionally, $n^*=1/x^*$, satisfying $\frac{1}{n^*}-ln(1+\frac{1}{n^*})-ln(\frac{2}{2-q})$, which is equation (7).

Additionally, under the flexible funding, the optimal price is $\hat{p}_e^N=\frac{2q+q^2-2\sigma(n)}{4}$, with corresponding maximized expected profit

$$\hat{\pi}_e^N=\frac{n(2q-q^2-2\sigma(n))^2}{16(q-\sigma(n))}.$$

Proof of Corollary 5.

The first part of Corollary 5(i) is obvious. To prove the second part, we have

$$\hat{\pi}_i^N=q\Big(\frac{n(2-q)}{2(n+1)}\Big)^{1+n}=q\Big(\frac{1}{2}\Big)^{n+1}\Big(\frac{n-\frac{q}{2}}{n+1}\Big)^{n+1}<q\Big(\frac{1}{2}\Big)^{n+1}.$$

. It must be noted that

$$\frac{n-\frac{q}{2}}{n+1}<1, \lim_{n\to\infty}\Big(\frac{1}{2}\Big)^{n+1}=0, \text{ hence, } \lim_{n\to\infty}\hat{\pi}_i^N=0.$$

Under the flexible funding, $\sigma(n)$ is a non-decreasing function, and $0<\sigma<q$. Therefore, σ has a limitation. Suppose $\lim_{n\to\infty}\sigma(n)=\sigma^N$, if $\sigma^N=q-q/2$, let $x(n)=q-q^2/2-\sigma(n)$, we have $\lim_{n\to\infty}x(n)=0$. Therefore,

$$\lim_{n\to\infty}\hat{\pi}_e==\lim_{n\to\infty}\frac{1}{4(x(n)+\frac{q^2}{2})}\frac{n}{\frac{1}{x^2(n)}}=\frac{1}{2q^2}*\lim_{n\to\infty}\frac{1}{\frac{-2}{x^3(n)}}=0.$$

Proof of Lemma 6.

With this strategy, the creator's expected profit is

$$\hat{\pi}_e^{is}=2(1-\frac{p}{q-\sigma(C)})^2(p-\frac{q^2}{2})+2\frac{p}{q-\sigma(C)}(1-\frac{p}{q-\sigma(C)})(p-\frac{q^2}{2}-C).$$

Taking derivative of $\hat{\pi}_e^{is}$ with respect to p, we have the optimal pledging price and corresponding maximized expected profit, respectively,

$$\hat{p}_e^{is}=\frac{(q-\sigma)(2q+q^2-2\sigma-2C)}{4(q-\sigma-C)},\ \hat{\pi}_e^{is}=\frac{(2q-q^2-\sigma-C)^2}{8(q-\sigma-C)}.$$

Taking the derivative of $\hat{\pi}_e^{is}$ with respect to C, we have

$$\frac{d\hat{\pi}_e^{is}}{dC}=\frac{-(1+\sigma^{'}(C))}{2}+\frac{q^4(1+\sigma^{'}(C))}{8(q-\sigma(C)-C)^2}=(1+\sigma^{'}(C))\Big(\frac{q^4-4(q-\sigma(C)-C)^2}{8(q-\sigma(C)-C)^2}\Big).$$

From the expression of p_e^{is}, we must have $p_e^{is}/(q-\sigma) < 1$, that is, $2q - 2\sigma(C) - 2C > q^2$, $q^4 - 4(q-\sigma(C)-C)^2 < 0$.

If $\sigma^{'} < -1$, $\frac{d\hat{\pi}}{dC} \geq 0$. To maximize $\hat{\pi}$, we must choose the largest C, that is, satisfying $\sigma(C^*) = 0$;

If $0 < \sigma^{'} < -1$, $\frac{d\hat{\pi}}{dC} \leq 0$. To maximize $\hat{\pi}$, we must choose the smallest C, that is, satisfying $C^* = 0$;

Otherwise, taking derivative of $\frac{d\hat{\pi}_e^{is}}{dC}$ with respect to C, we have

$$\frac{d^2\hat{\pi}_e^{is}}{dC^2} = \frac{-\sigma^{''}(C)}{2} + \frac{q^4\sigma^{''}(C)(q-\sigma-C)^2 + 2q^4(q-\sigma(C)-C)c^2(1+\sigma^{'}(C))^2}{8(q-\sigma(C)-C)^4}.$$

Let $\frac{d\hat{\pi}_e^{is}}{dC} = 0$, we have $1+\sigma^{'}(C) = 0$ or $2q - 2\sigma(C) - 2C = q^2$. However, substituting the later root into $\frac{d^2\hat{\pi}_e^{is}}{dC^2}$, we have $\frac{d^2\hat{\pi}_e^{is}}{dC^2} \geq 0$. Therefore, the root we are looking for satisfies $1+\sigma^{'}(C) = 0$. It must be noted that if we substitute this root into $\frac{d^2\hat{\pi}_e^{is}}{dC^2}$, we have:

$$\frac{d^2\hat{\pi}_e^{is}}{dC^2} = \sigma^{''}(C) * \Big(\frac{q^4 - 4(q-\sigma(C)-C)^2}{8(q-\sigma(C)-C)^2}\Big).$$

Therefore, a concave function of σ assures $\frac{d^2\hat{\pi}_e^{is}}{dC^2} \leq 0$.

Proof of Lemma 6.

(i): When $a = 0$ and $b < -1$, this is a special case in Lemma 6(i), then C^* satisfies $b * C^* + \sigma_0 = 0$, then, $C^* = -\frac{\sigma_0}{b}$. When $-1 < b < 0$, this is a special case in Lemma 6(ii), then $C^* = 0$. If $b = -1$, $\frac{d\hat{\pi}_e}{dC} = 0$, then $\hat{\pi}_e$ does not change with different C.

(ii): In this case, $C^{-1} = (-1-b)/(2a)$ and $C^0 = (-b - \sqrt{b^2 - 4a\sigma_0})/(2a)$.

Hence, $\sigma^{'} < -1$ is equivalent to $C^0 < C^{-1}$, that is, $b \leq -\sqrt{4a\sigma_0 + 1}$. Subsequently, $C^* = C^0$.

$\sigma^{'} > -1$ is equivalent to $C^{-1} < 0$, that is, $b \leq -1$. Subsequently, $C^* = 0$.

Otherwise, when $0 < C^{-1} < C^0$, that is, $-\sqrt{4a\sigma_0+1} < b < -1$. Subsequently, $C^* = C^{-1} = \frac{-1-b}{2a}$.

It must be noted that $C + \sigma(C) < q + q^2/2$; subsequently, $C^* < \bar{C}$. This completes the proof.