第四章：放射性与核衰变

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- Introduction
- α **decay**
- β decay
- γ decay

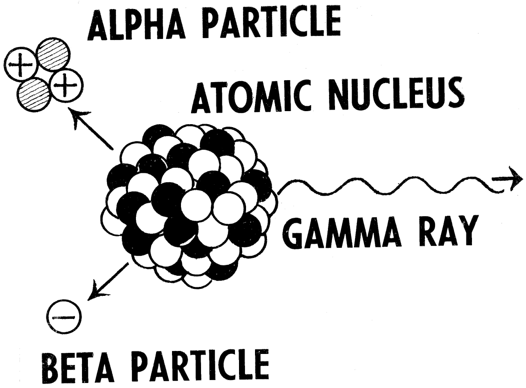
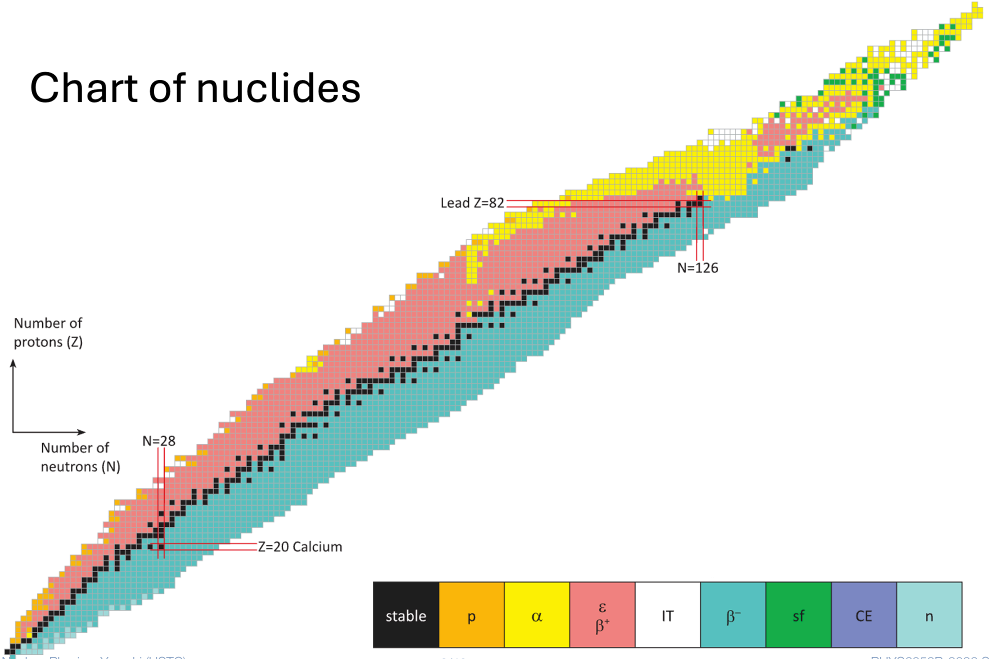


Chart of nuclides



History of α decay

- Rutherford showed in 1899 that uranium minerals emit at least two kinds of radiations having very different penetrating powers. He named the softer component α rays and harder or more penetrating component β rays.
- During the subsequent 15 years, Rutherford and his students carried a series of experiments on properties of α rays.
- In 1903, Rutherford measured the charge to mass ratio of α rays, which is about 1/4000 of e/m_e for electrons.
- Rutherford and Royds showed in 1909 that the α particles are actually ionized Helium. Rutherford and Geiger showed that an α particle carries the charge $2e$. These experiments provide the most accurate measurement of the electron's charge prior to the oil-drop experiments by Millikan.

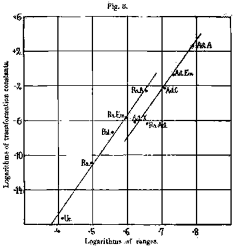
History of α decay

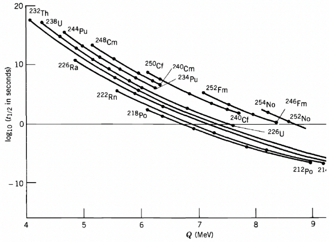
- In 1911, Geiger and Nuttall proposed an empirical relation between the decay constant λ and R the range of alpha particles in air:

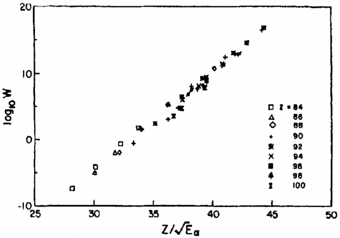
$$\log \lambda = a + b \log R$$

- In 1912, Richard Swinne suggested to use the α-particle velocity v_α instead of $\log R$
- With more data, Hans Bethe in 1937 gave the modern version of Geiger-Nuttall law:

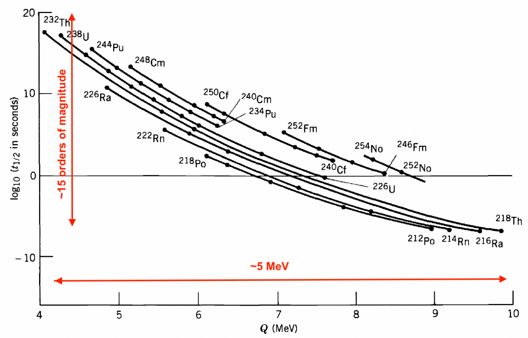
$$\log_{10} T_{1/2} = \frac{A(Z)}{\sqrt{E_\alpha}} + B(Z)$$







Note the range of the decay half-lives $T_{1/2}$ vs the range of the alpha energy E_α



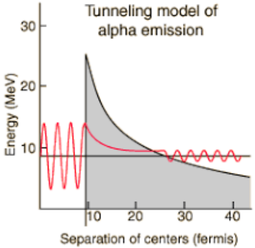
History of α decay

- Relation between R and v:

$$\mathrm{d}E/\mathrm{d}x \propto 1/E$$

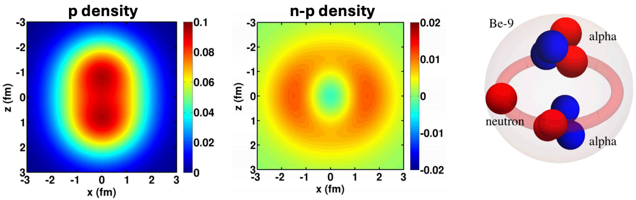
$$\Rightarrow \quad R = \int \frac{1}{\mathrm{d}E/\mathrm{d}x}\mathrm{d}E \propto E^2 \propto v_\alpha^4$$

- In 1928, Gamow, and independently Condon and Gurney, proposed that α decay is a quantum tunneling process, which successfully explained the Geiger-Nuttall law and become a cornerstone of the new-born Quantum Mechanics



Alpha decay

- The α decay is one of the most important decays for heavy nuclei. Especially the decay chains of naturally occurring nuclei involve only the α decay from strong interaction.
- The binding energy per nucleon for a Helium-4 nucleus or an α particle is much larger than its neighbors (much more stable), so it should be present in the heavy nuclei as clusters.
- In the binding energy formula of nuclei, the Coulomb term increases as $Z^2 A^{-1/3} \propto A^{5/3}$ while the volume term increases as A. So for heavy nuclei, the Coulomb repulsion effects increase rapidly and match or even exceed the volume effects. This makes the nuclei unstable for cluster emission. The α decay is the most frequently occurring cluster decay.



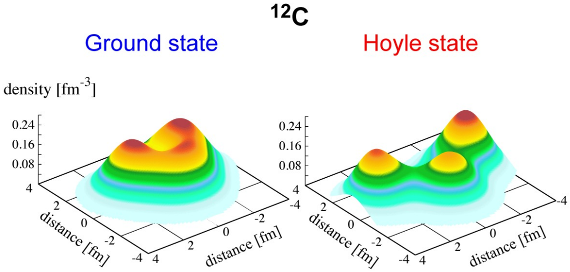


Figure: α-clustering in Be-9 and in C-12

Kinematics of alpha decay

$$ {}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}\mathrm{He} $$

Energy and momentum conservation:

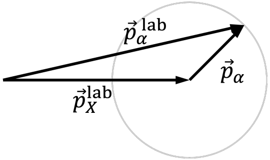
$$ \vec{p}_X = \vec{p}_Y + \vec{p}_\alpha, $$
$$ m_X c^2 + \frac{p_X^2}{2m_X} = m_Y c^2 + \frac{p_Y^2}{2m_Y} + m_\alpha c^2 + \frac{p_\alpha^2}{2m_\alpha} $$

The Q-value:

$$ Q = (m_X - m_Y - m_\alpha)c^2 = (M_X - M_Y - M_\alpha)\underbrace{m_u c^2}_{1\ \text{GeV}} \sim 10\,\mathrm{MeV} $$

In the c.m. frame (parent nucleus rest frame),

$$ E_\alpha = \frac{M_Y}{M_\alpha + M_Y} Q \approx \frac{A-4}{A} Q $$



Decay energy

For the decay to take place, the decay energy (Q-value) must be positive:

$$Q = (m_X - m_Y - m_\alpha)c^2 = B(Z-2, A-4) + B(2,4) - B(Z,A) > 0 \tag{1}$$

For example, consider the decay of ${}^{210}_{84}\mathrm{Po}$ within different channels. The Q-values are listed as follows

possible decay channel	Q [MeV]	possible decay channel	Q [MeV]
${}^{209}_{84}\mathrm{Po} + n$	-7.6	${}^{205}_{82}\mathrm{Pb} + {}^{5}\mathrm{He}$	-3.5
${}^{209}_{83}\mathrm{Bi} + {}^{1}\mathrm{H}$	-4.96	${}^{204}_{82}\mathrm{Pb} + {}^{6}\mathrm{He}$	-8.3
${}^{208}_{83}\mathrm{Bi} + {}^{2}\mathrm{H}$	-10.15	${}^{204}_{81}\mathrm{Tl} + {}^{6}\mathrm{Li}$	-5.7
${}^{207}_{83}\mathrm{Bi} + {}^{3}\mathrm{H}$	-10.85	${}^{203}_{81}\mathrm{Tl} + {}^{7}\mathrm{Li}$	-5.03
${}^{206}_{82}\mathrm{Pb} + {}^{4}\mathrm{He}$	+5.4		

In reality, ${}^{210}_{84}\mathrm{Po}$ goes through α decay, ${}^{210}_{84}\mathrm{Po} \to {}^{206}_{82}\mathrm{Pb} + {}^{4}\mathrm{He}$

Decay energy

- Decay energy:

$$Q = (m_X - m_Y - m_\alpha)c^2 = B(Z-2, A-4) + B(2,4) - B(Z, A) > 0 \tag{2}$$

- Recall the Weizsäcker's mass formula:

$$B(Z, A) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - a_{sym} I^2 A + s \cdot a_P A^{-1/2}$$

where, $I = (N - Z)/A = (A - 2Z)/A$, $a_V = 15.75\,\text{MeV}$, $a_S = 17.8\,\text{MeV}$, $a_C = 0.71\,\text{MeV}$, $a_{sym} = 23.3\,\text{MeV}$, $a_P = 12\,\text{MeV}$, and

$$s = \begin{cases} +1, & \text{even-even nuclei,} \\ 0, & \text{odd } A, \\ -1, & \text{odd-odd nuclei} \end{cases}$$

Decay energy

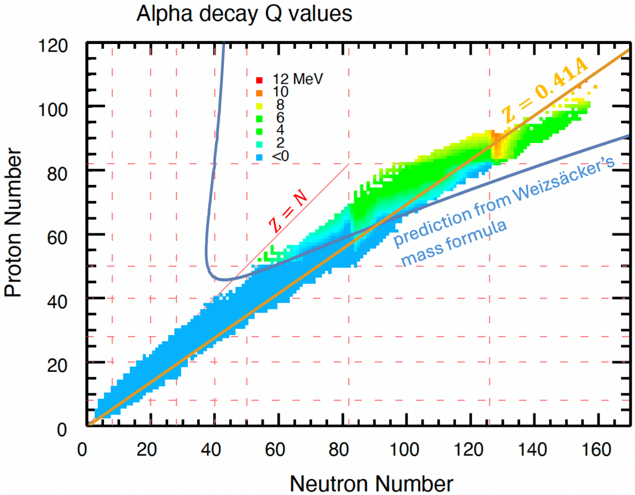
For heavy nuclei, we neglect the pairing term, and approximate the finite difference by the derivative,

$$\begin{aligned}
Q &\approx -2\frac{\partial B}{\partial Z} - 4\frac{\partial B}{\partial A} + B(2,4) \\
&\approx 4a_C Z A^{-1/3} - 8a_{sym}(A-2Z)/A \\
&\quad -4\Big\{a_V - \frac{2}{3}a_S A^{-1/3} - 2a_{sym}(A-2Z)/A \\
&\quad + a_{sym}(A-2Z)^2/A^2 + \frac{1}{3}a_C Z^2 A^{-4/3}\Big\} + 4\times 7.074\,\mathrm{MeV} \\
&\approx 4a_C Z A^{-1/3} - \frac{4}{3}a_C Z^2 A^{-4/3} - 4a_{sym}(A-2Z)^2/A^2 \\
&\quad + \frac{8}{3}a_S A^{-1/3} - 4a_V + 28.3\,\mathrm{MeV}
\end{aligned} \tag{3}$$

For heavy nuclei, $Z \approx 0.41A$, we find that Q is positive for $A \gtrsim 154$ ($Z \gtrsim 63$), ^{174}Gd.

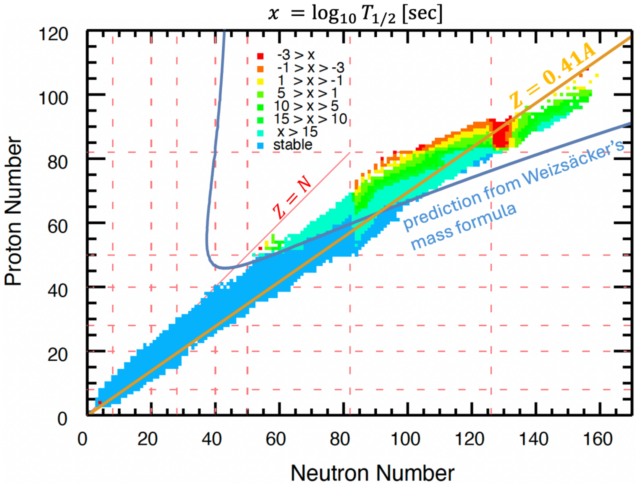
Decay energy

Figure: Experimental values for the alpha decay Q values. The blue curve is the prediction from Weizsäcker's mass formula.



Decay energy

Figure: Experimental values for the alpha decay half-life. The blue curve is the prediction from Weizsäcker's mass formula.

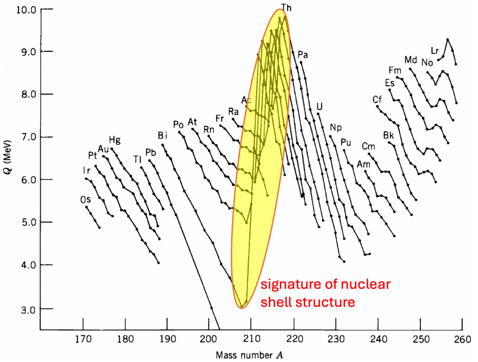


Decay energy

We look at the derivative of Q with respect to A at fixed Z,

$$\left.\frac{\partial Q}{\partial A}\right|_Z \approx -\frac{4}{3}a_C\frac{Z}{A^{4/3}}\left(1-\frac{4Z}{3A}\right)-16\frac{Z}{A^2}a_{sym}\left(1-\frac{2Z}{A}\right)-\frac{8}{9}a_S A^{-4/3} \tag{4}$$

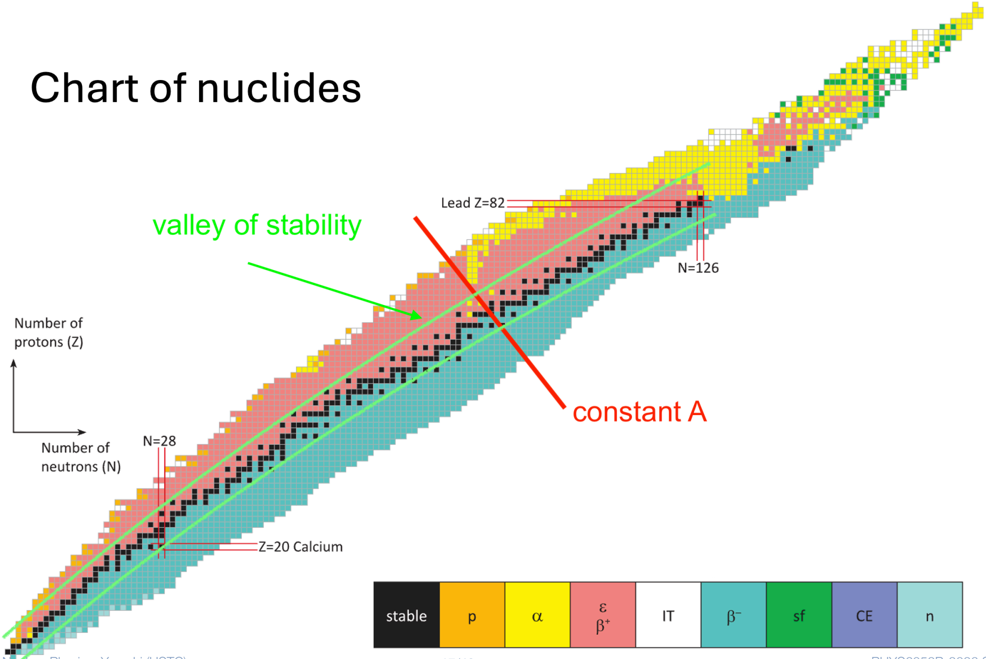
since $\frac{4Z}{3A}\leq\frac{2Z}{A}\leq 1$, $\left.\frac{\partial Q}{\partial A}\right|_Z$ is always negative, i.e. Q decreases with an increasing A at fixed Z.



Homework 8

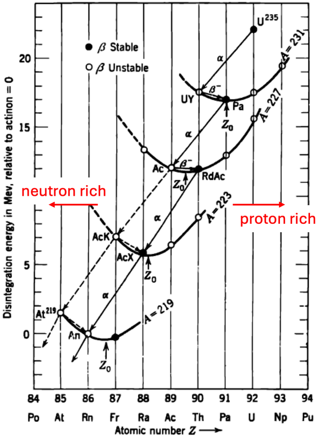
Similar to the last figure, try to use IAEA nuclear data base nudat to list the latest data of E_0 or Q values for the alpha decay of all nuclides (all isotops for each nuclide) starting from Bi (Z=83) to U (Z=92) [i,e, all nuclides for Z=83,84,85,86,87,88,89,90,91,92]. Only consider alpha decays from the ground state to the ground state.

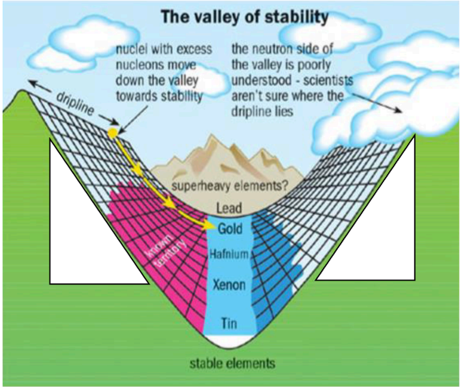
Chart of nuclides



Isobaric mass parabola

As we can see from the nuclear chart, there is a valley of stability, which is revealed as a isobaric mass parabola. Namely, for fixed A, there is a minimum in nuclear mass as a function of Z





Isobaric mass parabola

$$m(A,Z) = Zm_p + (A-Z)m_n - a_V A + a_S A^{2/3} + a_C Z^2 A^{-1/3} + a_{sym}(A-2Z)^2 A^{-1} - s a_P A^{-1/2}$$

- For fixed A, the nuclear mass m is a quadratic function of Z, i.e. a parabola
- The parabola is concave, since

$$\frac{\partial^2 m}{\partial Z^2} = 2a_C A^{-1/3} + 8a_{sym} A^{-1} = 1.42 A^{-1/3} + 186.4 A^{-1} > 0$$

- The location of the minimum:

$$\left.\frac{\partial m}{\partial Z}\right|_A = 0 \quad \Rightarrow \quad m_p - m_n + 2a_C Z A^{-1/3} - 4a_{sym} + 8a_{sym} Z A^{-1} = 0$$

$$\Rightarrow \quad Z_0 = A\frac{m_n - m_p + 4a_{sym}}{2a_C A^{2/3} + 8a_{sym}} = A\frac{94.5}{186.4 + 1.42 A^{2/3}}$$

Nuclear mass as function of Z

The minimum is located at

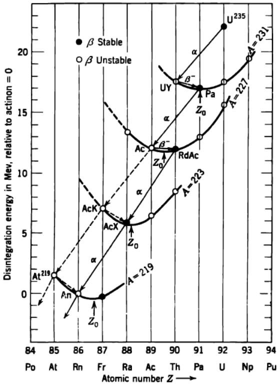
$$\begin{aligned} Z_0 &= A\frac{m_n - m_p + 4a_{sym}}{2a_C A^{2/3} + 8a_{sym}} \\ &= A\frac{94.5}{186.4 + 1.42A^{2/3}} \end{aligned} \tag{5}$$

which is an increasing function of A. This is a minimum since $\partial^2 m_X / \partial Z^2 > 0$ (prefactor of Z^2 is positive)

$$\begin{aligned} \frac{\partial^2 m_X}{\partial Z^2} &= 2a_C A^{-1/3} + 8a_{sym} A^{-1} \\ &= 1.42A^{-1/3} + 186.4A^{-1} > 0 \end{aligned} \tag{6}$$

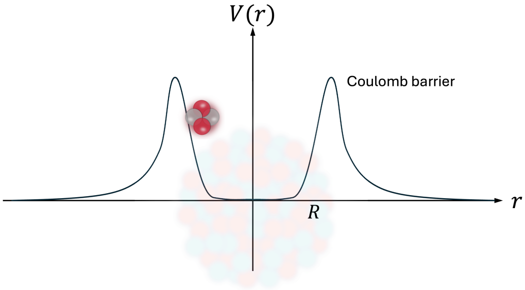
Decay energy

Figure: Mass-energy parabolas.



Quantum theory of alpha decay

- The physical picture of α decay was established by George Gamow and others in 1928
- Gamow postulated that α moving in the potential well of the combination of an attractive strong nuclear force and a repulsive Coulomb force
- The strong force is short-ranged and its effective range is $R = R_d + R_\alpha$, where $R_d = 1.2A^{1/3}$ is the radius of the daughter nucleus and $R_\alpha = 2.15\,\mathrm{fm}$ is the radius of the α particle
- Beyond R, the Coulomb force $V_C(r) = Z_\alpha Z_d e^2/r$ is dominant; hence forming a potential barrier



Quantum theory of alpha decay

- In principle, α particles with positive total energy E_α can travel to the infinity which results the α decay of the parent nucleus
- The height of the barrier is,

$$E_B = \frac{Z_\alpha Z_d e^2}{R_d + R_\alpha}$$

- Take $^{212}\mathrm{Po}$ as an example.

$$E_B = 26\,\mathrm{MeV}$$

  And the energy of the α particle is $E_\alpha = 2\,\mathrm{MeV} \ll E_B$
- Classically, if $E_\alpha < E_B$, the α particle can never pass through the Coulomb barrier

Quantum tunneling

- Gamow employed the then new-born quantum mechanics and proposed that α particle can penetrate the barrier through quantum tunneling.
- The probability to tunnel through the barrier in WKB approximation is,

$$P = \exp\left\{ -2\int_{R_1}^{R_2} dr \sqrt{2\mu[V(r) - E_\alpha]} \right\}$$

where $R_{1,2}$ are the radii of the classical turning points, $\mu = m_\alpha m_d/(m_d + m_\alpha)$ is the reduced mass.

WKB approximation

- Time-independent Schrödinger equation:

$$\Big(-\frac{\hbar^2}{2\mu}\nabla^2+V\Big)\psi(x)=E\psi(x)$$

- Quantum Hamilton-Jacobi theory (Riccati equation):

$$\psi(\vec{r})=e^{\frac{i}{\hbar}S(\vec{r})}\quad\Rightarrow\quad -\frac{i\hbar}{2\mu}\nabla^2 S(\vec{r})+\frac{1}{2\mu}[\nabla S(\vec{r})]^2+V=E$$

- Eikonal expansion:

$$S=S_0+\hbar S_1+\hbar^2 S_2+\cdots$$

- The leading order term gives the classical Hamilton-Jacobi equation:

$$\frac{1}{2\mu}\big(\nabla S_0\big)^2+V(r)=E$$

Therefore, terms in high-order of $\hbar$ represents quantum effects. Semi-classical approximation takes the leading order S_0 only.

WKB approximation

- Note that the problem is in 3D. The angular parts are simple spherical harmonics.

$$\psi(\vec{r}) = r^{-1}\phi(r)Y_\ell^m(\hat{r})$$

  $\phi(r)$ behaves like a 1D wave function

- Radial equation,

$$\Big[-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial r^2} + V_{\text{eff}}(r)\Big]\phi(r) = E\phi(r)$$

  where $V_{\text{eff}}(r) = V(r) + \ell(\ell+1)/2\mu r^2$

- Semi-classical approximation:

$$\phi(r) = e^{\frac{i}{\hbar}W(r)} \quad \Rightarrow \quad \frac{1}{2\mu}(W_0')^2 + V_{\text{eff}}(r) = E$$

WKB approximation

- Semi-classical tunneling solution in radial direction,

$$W_0(r) = i \int_0^r dr' \sqrt{2\mu[V_{\text{eff}}(r') - E]}$$

- Wave function,

$$\psi(r) = \frac{1}{r} e^{iW_0(r)} = \exp\left\{ - \int_0^r dr' \sqrt{2\mu[V_{\text{eff}}(r') - E]} \right\}$$

- Tunneling probability:

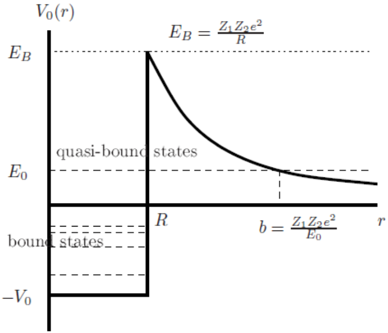
$$P_{1\to2} = \frac{\left|\psi(r_2)\right|^2}{\left|\psi(r_1)\right|^2} = \exp\left\{ -2 \int_{r_1}^{r_2} dr \sqrt{2\mu[V_{\text{eff}}(r) - E]} \right\} \equiv \exp\{-2G\}$$

G is known as the Gamow factor.

Square nuclear potential

As a concrete model, consider a simple square well as the nuclear potential,

$$V(r) = \begin{cases} -V_0, & r < R \\ \frac{Z_1 Z_2 e^2}{r}, & r > R \end{cases} \quad (7)$$



Gamow factor

- Classical turning points: $R_1 = R$ and $R_2 = R_c \equiv Z_d Z_\alpha e^2/E_\alpha$ for $\ell = 0$ state
- Gamow factor:

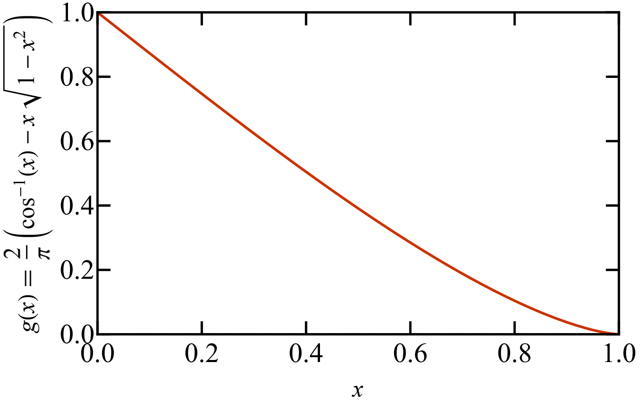
$$G \equiv \int_{R_1}^{R_2} dr \sqrt{2\mu[\frac{Z_\alpha Z_d e^2}{r} - E_\alpha]} = Z_\alpha Z_d \alpha_{\text{em}} \sqrt{\frac{2\mu}{E_\alpha}} \int_{R/R_c}^{1} dy \sqrt{1/y - 1},$$

$$= Z_\alpha Z_d \alpha_{\text{em}} \sqrt{\frac{2\mu}{E_\alpha}} \Big[\arccos \sqrt{\frac{R}{R_c}} - \sqrt{\frac{R}{R_c}} \sqrt{1 - \frac{R}{R_c}} \Big]$$

$$= Z_\alpha Z_d \alpha_{\text{em}} \sqrt{\frac{2\mu}{E_\alpha}} \frac{\pi}{2} g\Big(\sqrt{\frac{R}{R_c}}\Big)$$

$$= \frac{1}{2} \sqrt{\frac{E_G}{E_\alpha}} g\Big(\sqrt{\frac{R}{R_c}}\Big)$$

where, $g(x) = \frac{2}{\pi}\Big[\arccos x - x\sqrt{1 - x^2}\Big]$, $E_G = \Big(\frac{2\pi Z_1 Z_2 e^2}{\hbar c}\Big)^2 \frac{\mu c^2}{2}$ is known as the Gamow energy.



For $x \ll 1$, i.e. $R \ll R_c$,

$$g(x) \approx 1 - \frac{4}{\pi} x$$

Gamow factor

For $R \ll R_c$,

$$
\begin{aligned}
G &\approx \frac{1}{2}\sqrt{\frac{E_G}{E_\alpha}}\left\{1 - \frac{4}{\pi}\sqrt{\frac{R}{R_c}}\right\} \\
&\approx \sqrt{2}\pi\alpha_{\text{em}}Z\sqrt{\frac{m_\alpha}{E_\alpha}} - 4\sqrt{\alpha_{\text{em}}Z}\sqrt{m_\alpha R} \\
&= 3.97\frac{Z}{\sqrt{E_\alpha}} - 2.98\sqrt{ZR}
\end{aligned}
$$

where, R is in unit fm and E_α is in MeV. We have used the fine structure constant $\alpha_{\text{em}} = e^2 = 1/137$. Obtaining the last line, we have used $Z_\alpha = 2$, $Z_d = Z$, $m_\alpha = 3750$ MeV.

Gamow factor

- Gamow further realized that the probability of an α tunneling out is the same as it tunneling in
- Gamow factor gives the probability for two nuclei to overcome the Coulomb barrier in order to undergo nuclear reaction.

$$P(E) = e^{-\sqrt{\frac{E_G}{E}}}$$

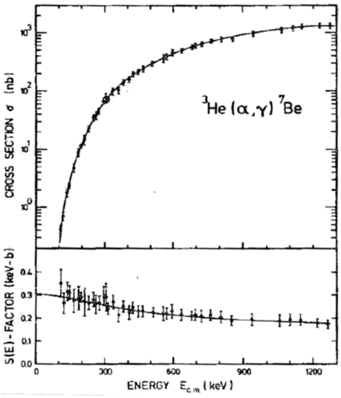
where, the Gamow energy $E_G = 2\mu c^2(\pi\alpha Z_1 Z_2)^2$

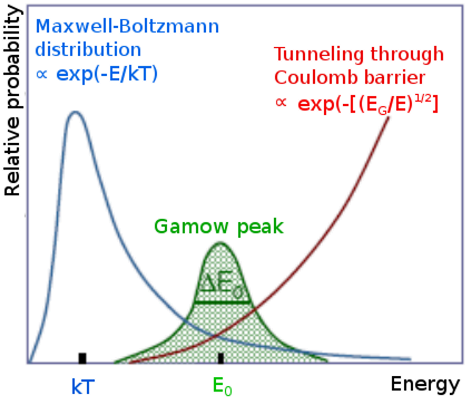
- In nuclear physics, we often introduce the S-factor, aka. astronomical factor, to rescale away the Coulomb repulsion effect from the cross section:

$$\sigma(E) = \frac{e^{-\sqrt{\frac{E_G}{E}}}}{E} S(E)$$

here, σ is the cross section, E is the center-of-mass energy

- In comparison to cross section, S factor is more smooth wrt E





Thermal reactivity (astrophysics, fusion, ...):

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu (kT)^3}} \int dE\, S(E) e^{-\frac{E}{kT} - \sqrt{\frac{E_G}{E}}}$$

Decay time

- The decay constant of of the nucleus subject to α-decay is the number of decay per unit time, which equals ν the frequency that the α particle hits the barrier multiplied by $P = \exp(-2G)$ the probability that the α particle penetrates the barrier:

$$\lambda = P\nu$$

- It takes $\Delta t = 2R/v$ for the α particle to transverse the nucleus. Then the frequency ν is simply $\nu = 1/\Delta t$ and the decay constant,

$$\lambda = \frac{v}{2R} P = \frac{v}{2R} e^{-2G}$$

- The velocity of the α particle is,

$$v_\alpha = \sqrt{2E_\alpha m_\alpha}$$

- Then, the life-time of the radioactive nucleus is,

$$\tau = \lambda^{-1} = 2R\sqrt{\frac{m_\alpha}{2E_\alpha}} e^{2G}$$

Geiger-Nuttall law

We can take logarithm of above and obtain,

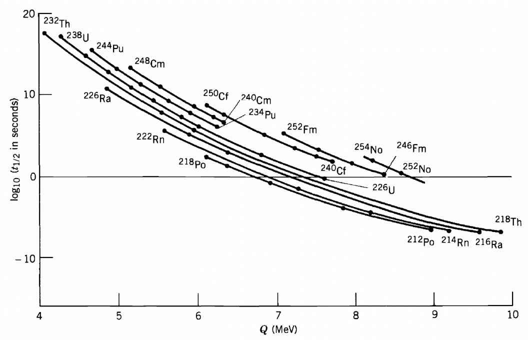
$$\ln \tau = a_1 + a_2 \frac{1}{\sqrt{E_\alpha}}, \tag{8}$$

where $a_1 = -49.4 + \ln \frac{A^{1/3}}{\sqrt{E_\alpha}} - 3.26\sqrt{ZA^{1/3}}$ and $a_2 = 3.97Z$. Hence, the Gamow model reproduces the Geiger-Nuttall law

$$\log_{10} T_{1/2} = \frac{A(Z)}{\sqrt{E_\alpha}} + B(Z) \tag{9}$$

and further predicts

$$\begin{aligned} A(Z) &= 1.72Z \\ B(Z) &= -21.3 + \log_{10} \frac{A^{1/3}}{\sqrt{E_\alpha}} - 1.42\sqrt{ZA^{1/3}} \end{aligned}$$



The inverse relationship between decay half-life and decay energy, known as the Geiger-Nuttall law. Only even-Z, even-A nuclei are shown. The solid lines connect the data points.

Geiger-Nuttall law

Let us estimate the half life of $^{210}\mathrm{Po}$ from α decay,

$$
\begin{aligned}
T_{1/2}(^{210}\mathrm{Po}) &= \frac{\ln 2}{\lambda} = \ln 2 \times 3.5 \times 10^{-22} A^{1/3} E_k^{-1/2} e^G \\
&\approx \ln 2 \times 3.5 \times 10^{-22} \times 210^{1/3} \times (5.4)^{-1/2} \times \exp(3.97 \times 84/\sqrt{5.4} - 2.98 \times \sqrt{84 \times 9}) \\
&\approx 6.2 \times 10^{-22} \times \exp(61.5705) \\
&\approx 4.06\,\mathrm{days} \\
T_{1/2}(^{212}\mathrm{Po}) &\approx \ln 2 \times 3.5 \times 10^{-22} \times 212^{1/3} \times (9.03)^{-1/2} \\
&\quad \times \exp(3.97 \times 84/\sqrt{9.03} - 2.98 \times \sqrt{84 \times 9}) \\
&\approx 4.81 \times 10^{-22} \times \exp(29) \\
&\approx 1.89 \times 10^{-9}\ \mathrm{s}
\end{aligned}
$$

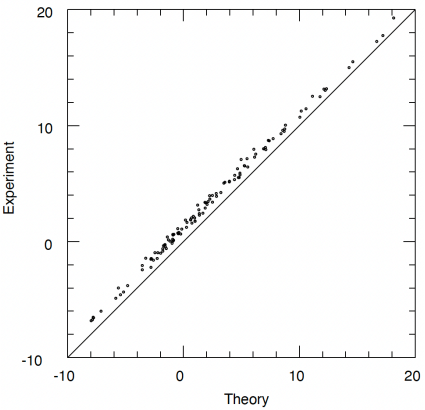
We can compare with the data, $T_{1/2}(^{210}\mathrm{Po}) = 138.4\,\mathrm{days}$ and $T_{1/2}(^{212}\mathrm{Po}) = 3 \times 10^{-7}$ s.

Geiger-Nuttall law

Table: Calculated α-Decay Half-lives for Th Isotopes

A	Q (MeV)	$t_{1/2}$ (s)	
		Measured	Calculated
220	8.95	10^{-5}	3.3×10^{-7}
222	8.13	2.8×10^{-3}	6.3×10^{-5}
224	7.31	1.04	3.3×10^{-2}
226	6.45	1854	6.0×10^{1}
228	5.52	6.0×10^{7}	2.4×10^{6}
230	4.77	2.5×10^{12}	1.0×10^{11}
232	4.08	4.4×10^{17}	2.6×10^{16}

Geiger-Nuttall law



Geiger-Nuttall law

- While Gamow model provides a qualitative picture of α decay, it has considerable deviation from the experimental data.
- An improved fit for the Geiger-Nuttall law is,

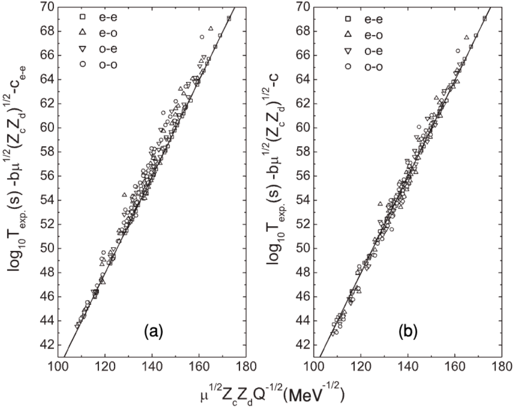
$$\log_{10} T_{1/2}\,(\mathrm{sec}) = -51.37 + 9.54 \frac{Z_d^{0.6}}{\sqrt{E_\alpha}\,(\mathrm{MeV})}$$

- Gamow model can be extended to the non-zero spin nuclei with the addition of the centrifugal barrier $\ell(\ell+1)/2\mu r^2$ to the potential
- An important factor for decay with non-zero angular momentum is the parity conservation. If a decay violates the parity conservation, it will be suppressed and go through other channels, e.g. excited states
- The shape of heavy nuclei has a very substantial influence on the predicted half-lives since nuclei with $A \gtrsim 230$ are strongly deformed.

Geiger-Nuttall law for cluster decay

$$\log_{10} T_{1/2} = c + b\sqrt{\mu}\sqrt{Z_c Z_d} + a\sqrt{\mu}\frac{Z_c Z_d}{\sqrt{Q}}$$

where, $\mu = M_c M_d/(M_c + M_d)$.



Microscopic interpretation of α decay

- Gamow model does not contain the structure information of the parent nucleus
- Modern theoretical understanding starts from the scattering theory and Gamow model emerges as part of the approximate solution to the scattering matrix

$$\lambda_\ell = 2P_\ell(R)\frac{1}{2\mu R}g_\ell^2(R)$$

where, R is the point that separate the internal and external wave functions.

  - $P_\ell(R)$ is the probability of an α particle with angular momentum ℓ penetrates the Coulomb barrier starting from R, and can be well approximated by WKB ansatz.
  - $g_\ell(R)$ is the α-particle formation amplitude,

$$g_l(r) = r\int d\hat{r}Y_l^*(\hat{r})g(\vec{r}), \qquad g(\vec{r}) = \langle\mathcal{A}\Big\{\Phi^{(A)}\Phi^{(a)}\delta(\vec{r}-\vec{r}_{Aa})\Big\}|\Psi\rangle$$

- The nuclear many-body wave function Ψ encodes the full information of the system
- Recent review: 1810.07745

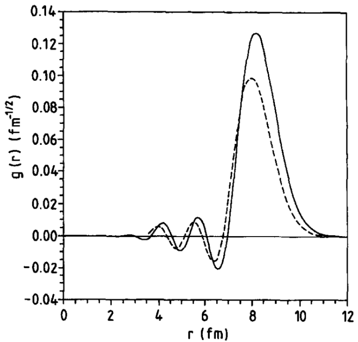


Figure: Radial α formation amplitudes in the g.s. of $^{212}\mathrm{Po}$ produced by two nuclear shell models.

Fine structure of α spectra

The Gamow model assumes that the kinetic energies are the same for all α particles. In reality, the α particles may have several different kinetic energies, originating from decay into different excited states of the daughter nucleus. The excited states then may decay into the ground state via γ decays.

Figure: Fine structure of α spectra for ${}^{212}_{83}\mathrm{Bi} \to {}^{208}_{81}\mathrm{Tl} + \alpha$ obtained from magnetic spectrometer. The horizontal axes gives the magnetic field for about 40 cm of curvature.

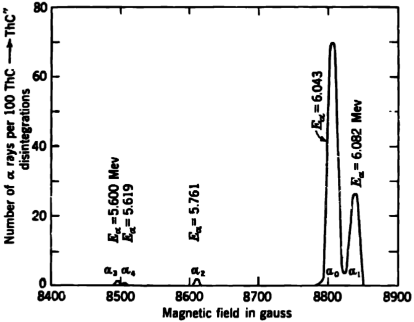


Table: Energies of α particles for ${}^{212}_{83}\mathrm{Bi} \rightarrow {}^{208}_{81}\mathrm{Tl} + \alpha$. The disintegration energy E_0 or Q-value is $Q = E_0 = E_\alpha + E_R = \frac{A}{A-4} E_\alpha$, where E_R is the energy of the daughter nucleus.

group	branching ratio	E_α	E_0 (Q-value)	$\Delta E = E_{\alpha i} - E_{\alpha 1}$
α_1	0.272	6.082	6.199	0
α_0	0.698	6.043	6.159	0.040
α_2	0.0180	5.761	5.871	0.328
α_4	0.0016	5.619	5.727	0.472
α_3	0.0110	5.600	5.707	0.492

Figure: Energy-level diagram (scheme plot) of ${}^{212}_{83}\mathrm{Bi} \to {}^{208}_{81}\mathrm{Tl} + \alpha$.

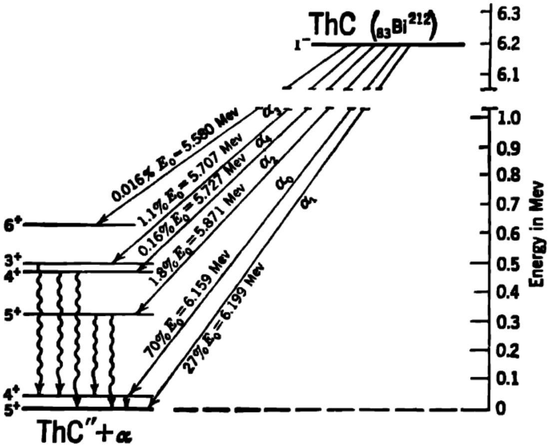


Figure: Long range α rays for ${}^{212}_{84}\mathrm{Po} \to {}^{208}_{82}\mathrm{Pb} + \alpha$: emitted from excited levels of ${}^{212}_{84}\mathrm{Po}$, the branching ratio is 10^{-4}.

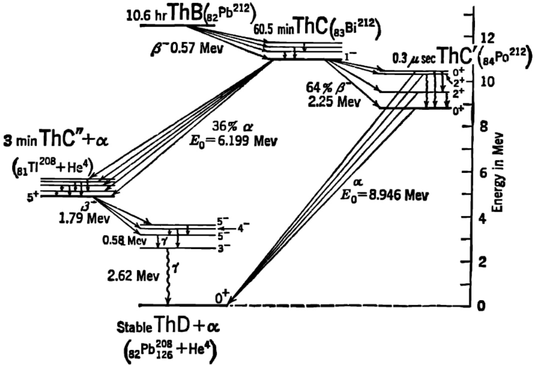


Figure: Long range α rays for ${}^{212}_{84}\mathrm{Po} \to {}^{208}_{82}\mathrm{Pb} + \alpha$.

TABLE 1.3. THE LONG-RANGE α-RAY SPECTRUM OF ThC′ → ThD ($_{84}\mathrm{Po}^{212} \to {}_{82}\mathrm{Pb}^{208}_{126} + \mathrm{He}^4$) ACCORDING TO RYTZ (R54)

Group	Relative abundance	α-Ray energy E_α, Mev	Disintegration energy E_0, Mev	Excitation energy in parent ThC′, Mev
Normal: α_0	10^6	8.776	8.946	0
Long-range: α_2	35	9.489	9.671	0 725
Long-range: α_3	20	10.417	10 617	1.671
Long-range: α_1	170	10 536	10 739	1.793