Pion gravitational form factor $D_\pi(Q^2)$ from holographic light-front QCD

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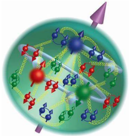
50 years of QCD

Big puzzles remain in hadron structures:

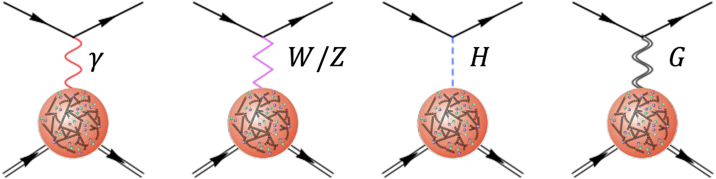
- Confinement and the strong force within hadrons
- Origin of >99% nucleon mass
- Origin of the nucleon spin



Gross and Klempt et al., 50 Years of quantum chromodynamics, Eur. Phys. J. C, 83 (2023)

Gravitational form factor D: the last global unknown

Hadronic energy-momentum tensor encodes the energy, spin and stress distributions widthin hadrons



$$\langle p', s'|T^{\mu\nu}(0)|p, s\rangle = \frac{1}{M}\overline{u}_{s'}(p')\Big[P^\mu P^\nu A(q^2) + \frac{1}{2}iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho J(q^2) + \frac{1}{4}(q^\mu q^\nu - g^{\mu\nu}q^2)D(q^2)\Big]u_s(p)$$

em: $\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\|J^\mu_{\text{em}}\|N\rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\text{C}$ $\mu = 2.792847356(23)\mu_N$
weak: PCAC	$\langle N'\|J^\mu_{\text{weak}}\|N\rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\text{grav}}\|N\rangle$	$\longrightarrow$	$m = 938.272013(23)\ \text{MeV}/c^2$ $J = \frac{1}{2}$ $\boldsymbol{D} = \boldsymbol{?}$

Physical interpretations

- Sachs densities, aka. Breit frame densities, [Sachs:1962zzc]

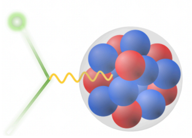
$$\mathcal{A}(r) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} A(-\vec{q}^2),$$

$$\mathcal{J}(r) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} J(-\vec{q}^2),$$

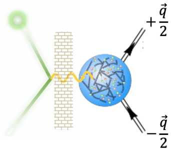
$$\mathcal{P}(r) = -\frac{1}{6M} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \vec{q}^2 D(-\vec{q}^2).$$

- Light-front densities (2D), related to the GPDs [Miller:2018ybm, Burkardt:2000za]

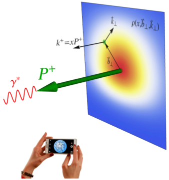
$$\mathcal{O}_{\text{LF}}(\vec{r}_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \langle P + \tfrac{1}{2}\vec{q}|\hat{O}(0_\perp)|P - \tfrac{1}{2}\vec{q}\rangle, \qquad \hat{O}(\vec{x}_\perp) = \frac{1}{2}\int \mathrm{d}x^- O(x)$$



$\lambda_\gamma \sim r_{\text{nucl}} \gg \lambda_{\text{Comp}} = M_{\text{nucl}}^{-1}$



$\lambda_\gamma \sim r_N \sim \lambda_{\text{Comp}} = M_N^{-1}$



light-front coordinates:
$V^\pm = V^0 \pm V^3,$
$\vec{V}_\perp = (V^1, V^2)$

Mechanical stability of hadrons

- Energy-momentum conservations imply: [Cotogno:2019xcl, Lorce:2019sbq]

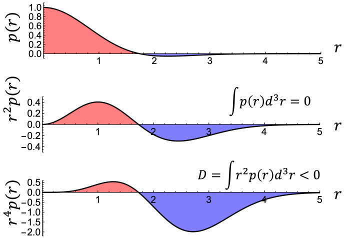
$$A(0) = 1, \quad J(0) = \frac{1}{2}, \quad \lim_{Q^2\to 0} Q^2 D(Q^2) = 0 \quad \Rightarrow \quad \int \mathrm{d}^3 r\, \mathcal{P}(r) = 0$$

the von Laue condition implies hadrons are in mechanical equilibrium [Laue:1911lrk]

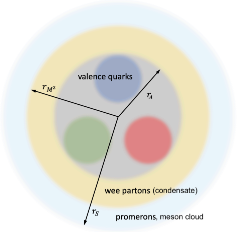
- Polyakov et al. conjectured that $D < 0$ for mechanically stable systems [Polyakov:2018zvc]

$$D = \int \mathrm{d}^3 r\, r^2 \mathcal{P}(r) \overset{???}{<} 0$$

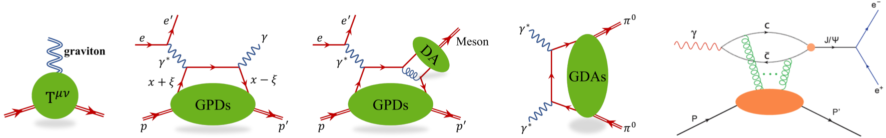
- D-term also contributes to the trace anomaly $S \equiv T^\mu_\mu = \frac{\beta(g_s)}{2g_s} G^{\mu\nu a} G^a_{\mu\nu} + O(m_q)$. In particular, a negative D gives a layered structure to the proton $r_A < r_{M^2} < r_S$



$$r^2_{M^2} = r^2_A - 3\lambda^2_C D,$$
$$r^2_S = r^2_A - \frac{9}{2}\lambda^2_C D$$



Experiments

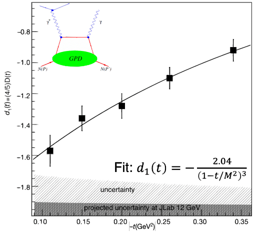


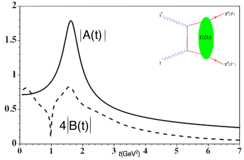
- Ji's sum rules: [Ji:1996nm, Polyakov:2002yz]

$$\int_{-1}^{1} \mathrm{d}x\, x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t),$$

$$\int_{-1}^{1} \mathrm{d}x\, x E^{q,g}(x,\xi,t) = B^{q,g}(t) - \xi^2 D^{q,g}(t),$$

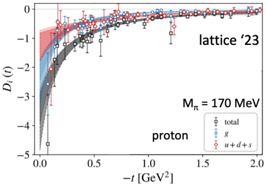
  - Deeply virtual Compton scattering (DVCS) and Deeply virtual meson production (DVMP) [Burkert:2018bqq, Burkert:2021ith]
  - Di-photon pair production [Kumano:2017lhr]
- Near threshold VM photo-production [Kharzeev:2021qkd, Duran:2022xag]
- Large uncertainties $\to$ electron-ion colliders

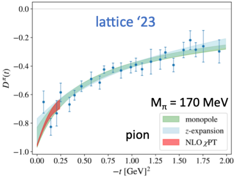


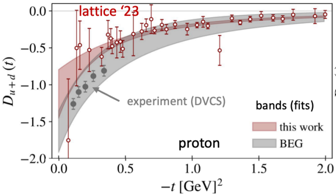


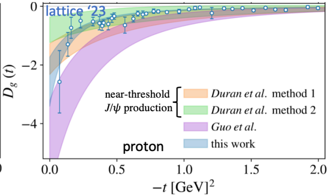
Theories

- Chiral perturbation theory: $D_\pi = -1$ in the chiral limit [Donoghue:1991qv]
- pQCD: scaling at large Q^2, e.g. $A_\pi(Q^2) \sim D_\pi(Q^2) \sim 1/Q^2$ [Tong:2021ctu, Tong:2022zax]
- Lattice QCD: considerable uncertainties & discrepancies with extracted data from experiments
  - r^π_{mech} is 2.5 times smaller than the result extraction from $\gamma\gamma \to \pi\pi$ on Belle [Hackett:2023nkr]
- QCD-like models: [Polyakov:2018zvc, Burkert:2023wzr]
  - Bag model, chiral quark model, NJL model, light-front quark model, continuum QCD, ...
  - Requires a consistent treatment of the non-perturbative dynamics
- Holographic QCD [Abidin:2009hr, Brodsky:2008pf, Mamo:2019mka, Mamo:2021tzd, Mamo:2022eui, Fujita:2022jus]







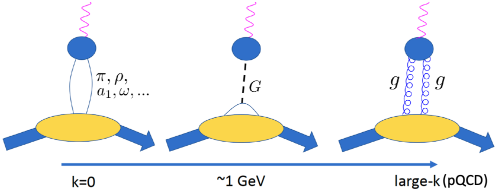


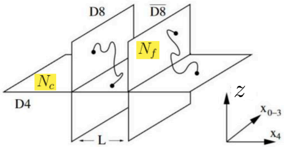
GFFs in holographic QCD

- $A(q^2)$ were obtained by coupling to gravitation waves (GWs) in AdS$_5$; however, $D(q^2)$ is not fully constrained since GW can only couple to the traceless part of EMT [Abidin:2008hn, Abidin:2008ku, Abidin:2009hr]
- Mamo et al. showed in AdS/QCD $D(q^2) \propto A(q^2)$ with $D(0)$ undetermined; Further speculated that finite-N_c corrections lift the degeneracy between scalar (0^{++}) and tensor (2^{++}) glueballs and lead to,

$$D_N(q^2) = \frac{4M_N^2}{3q^2}\Big[A(q^2) - A_S(q^2)\Big]$$

  where, A_S is the scalar GFF associated with the trace [Mamo:2019mka, Mamo:2021krl, Mamo:2022eui]
- Fujita et al. extracted $D_N(q^2)$ from the Sakai-Sugimoto model (a top-down model in 10D) [Fujita:2022jus]
  - Large q^2 scaling different from pQCD prediction
  - Predicted $D(0) = -0.140(22)$ resulting from cancellation between $U(1)$ and $SU(2)$ fields





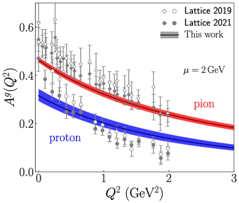
Holographic light-front QCD

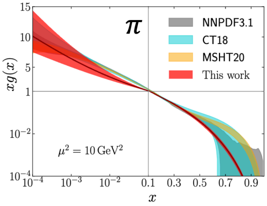
- The D-term involves non-minimal coupling terms in gravitational EFT, [Donoghue:1994dn]

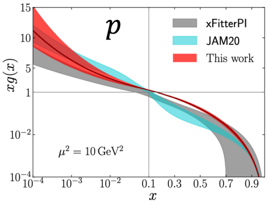
$$S_D = -\frac{D}{4}\int \mathrm{d}^d x\, \sqrt{-g} R \phi^2$$

  - Need constraints from both the QCD side and the gravity side
- Light-front holography: correspondence between semi-classical LFQCD and AdS/QCD in 5D
  - HLFQCD allows us to impose constraints from both the QCD and the gravity sides
  - Further insights: super-conformal algebra, Veneziano amplitudes, parton counting rules and GPD sum rules

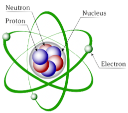
[Review: Brodsky:2014yha]





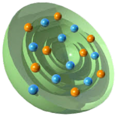


Semiclassical QCD



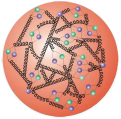
Non-relativistic, weakly coupling

Bohr Model



Non-relativistic, strongly coupling

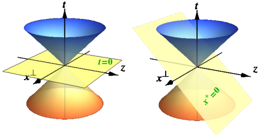
Shell Model



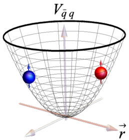
Relativistic, strongly coupling

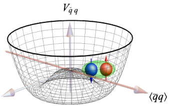
?

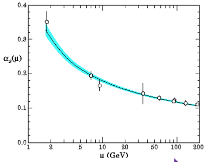
$$\left[-\nabla^2_{\zeta\perp} + V^{\text{eff}}_{q\bar{q}}(\vec{\zeta}_\perp)\right]\varphi_h(\vec{\zeta}_\perp) = M_h^2 \varphi_h(\vec{\zeta}_\perp)$$



where $\vec{\zeta}_\perp = \sqrt{x(1-x)}\vec{r}_\perp$







$\Lambda_{\text{QCD}} \sim 0.3$ GeV $\qquad \Lambda_\chi \sim 1$ GeV $\qquad \mu \gg 1$ GeV

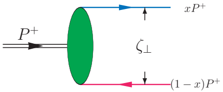
Soft-wall AdS/QCD and light-front holography

AdS/QCD is a bottom-up approach to holographic QCD based on semiclassical field theory in 5D anti-de Sitter space (AdS$_5$), [Maldacena:1997re, Polchinski:2000uf, Erlich:2005qh]

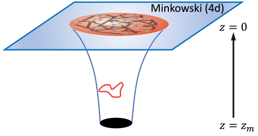
$$S = \int \mathrm{d}^5 x e^{-\Phi(x)} \sqrt{-g} \Big\{ \big|DX\big|^2 + m_5^2 |X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \Big\}$$

- Soft-wall AdS/QCD introduced a dilaton $\Phi(z)$ to break the conformal symmetry in IR
- Karch et al. adopted $\Phi(z) = \kappa^2 z^2$ to reproduce the Regge trajectory $M_n^2 \propto n$, where $\kappa = 0.388\,\mathrm{GeV}$ is fixed by fitting to the ρ mass [Karch:2006pv]
- Improved soft-wall AdS/QCD [e.g., Gherghetta:2009ac, Sui:2009xe, Li:2012ay, Li:2013oda, Cui:2013xva]

[Review: Brodsky:2014yha]



semiclassical LFQCD	$\leftrightarrow$	semiclassical field theory in AdS$_5$
$\zeta_\perp = \sqrt{x(1-x)} r_\perp$	$\leftrightarrow$	fifth coordinate z,
LF amplitude	$\leftrightarrow$	string amplitude
confining potential $V_{q\bar{q}}^{\text{eff}}$	$\leftrightarrow$	dilation field Φ
$L^2 - (J-2)^2$	$\leftrightarrow$	$(\mu R)^2$
form factors	$\leftrightarrow$	form factors



Pion electromagnetic form factor

- Drell-Yan-West formula: [Brodsky:2007hb]

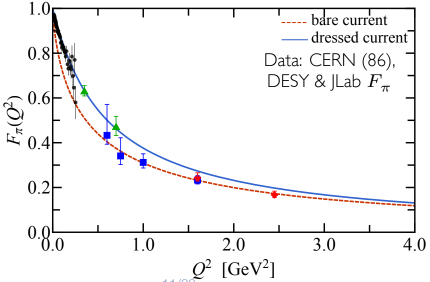
$$F_\pi(q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \big|\varphi_\pi(\zeta_\perp)\big|^2 \zeta_\perp Q K_1(\zeta_\perp Q) + \text{higher Fock sector contributions}$$

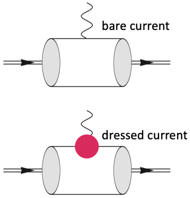
- Electromagnetic coupling in AdS_5: [Grigoryan:2007wn, Abidin:2009hr]

$$S_{\text{int}} = e_5 \int \mathrm{d}^5 x \sqrt{g} g^{NM} \Phi^*(x) i \overleftrightarrow{\nabla}_N \Phi(x) A_M(x) \quad \Rightarrow \quad F_\pi(q^2) = \int z \mathrm{d}z \big|\varphi_\pi(z)\big|^2 V(q^2, z)$$

where, $V(q^2, z)$ is the bulk-to-boundary propagator of the 5D EM field $A_N(x)$. In soft-wall model,

$$V(q^2, z) = \Gamma(1 - \frac{q^2}{4\kappa^2}) U(-\frac{q^2}{4\kappa^2}, 0; \kappa^2 z^2) \overset{Q^2\to\infty}{=} zQK_1(zQ)\Big[1 + O(\frac{1}{Q})\Big]$$





Pion gravitational form factor A

- Brodsky-Hwang-Ma-Schmidt formula: [Brodsky:2008pf]

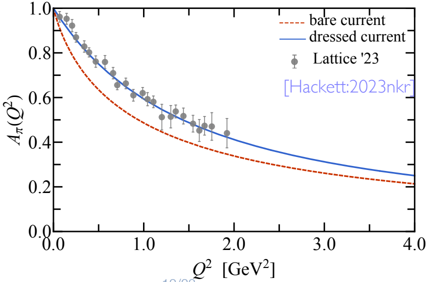
$$A_\pi(q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp |\varphi_\pi(\zeta_\perp)|^2 \frac{1}{2}\zeta_\perp^2 Q^2 K_2(\zeta_\perp Q) + \text{higher Fock sector contributions}$$

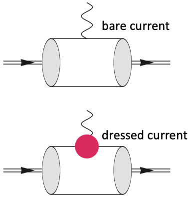
- Gravitational coupling in AdS$_5$: $g_{NM} \to g_{NM} + \delta g_{NM}$ [Abidin:2008hn]

$$A_\pi(q^2) = \int z\mathrm{d}z |\varphi_\pi(z)|^2 H(q^2, z)$$

where, $H(q^2, z)$ is the bulk-to-boundary propagator of the 5D gravitational field. In soft-wall model,

$$H(q^2, z) = \Gamma(2 - \frac{q^2}{8\kappa^2}) U(-\frac{q^2}{8\kappa^2}, -1; 2\kappa^2 z^2) \overset{Q^2\to\infty}{=} \frac{1}{2} z^2 Q^2 K_2(zQ)\Big[1 + O(\frac{1}{Q})\Big]$$





GFF D in LFQCD

Given the effective $q\bar{q}$ interaction, how to compute $D(Q^2)$?

$$U_{\text{sw}}(\zeta_\perp) = \kappa^4\zeta_\perp^2 + 2\kappa^2(J-1)$$

- Light-front quark-diquark model [Chakrabarti:2020kdc]

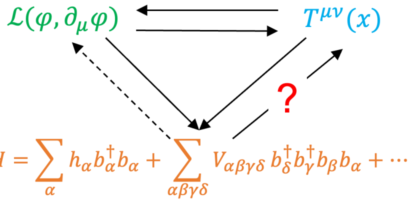
$$t^{\alpha\beta}(q_\perp^2) = \langle P - \tfrac{q}{2}|T^{\alpha\beta}|P + \tfrac{q}{2}\rangle$$

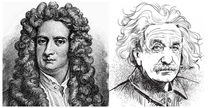
  - Adopted soft-wall light-front wave functions (LFWFs) for and the free EMT operator $T_0^{\mu\nu}$
  - $D_N(Q^2)$ is extracted from spin-flip hadronic matrix elements $\sim T^{11} + T^{22}, T^{+-}$ etc.
  - Problems: in violation of the von Laue condition, absence of the interaction
- von Laue condition is equivalent to light-front energy conservation
  - $T^{\alpha\beta}$ should be consistent with the Hamiltonian H
  - We should adopt T^{+-}, which is the density of the light-front Hamiltonian $P^- = \int \mathrm{d}^3x\, T^{+-}(x)$,

$$P^\mu|p\rangle = p^\mu|p\rangle \;\Rightarrow\; \langle P - \tfrac{q}{2}|T^{+-}|P + \tfrac{q}{2}\rangle = 2p^+p^- \;\Rightarrow\; \lim_{q_\perp^2\to 0} q_\perp^2 D(q_\perp^2) = 0$$

T^{+-} from the effective Hamiltonian P^-

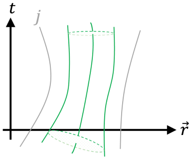
Is it possible to obtain local one-body densities of systems described by an effective Hamiltonian?





Everything gravitates

They gravitate through energy-momentum tensor



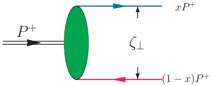
- In non-relativistic QMBT, operators can be localized with the position operator:

$$O \quad \longrightarrow \quad \sum_i O_i \delta^3(r - r_i)$$

- Unfortunately, there is no consistent position operator in relativistic quantum theory
- Exception: particles can be localized on the transverse plane tangential to the light cone, which suffices to specify the hadronic one-body densities (OBDs)

$$O \quad \longrightarrow \quad \sum_i O_i \delta^2(r_\perp - r_{i\perp})$$

Example: effective $q\bar{q}$ interaction $U_{q\bar{q}}$



- Second quantization:

$$\underline{P}^-_{q\bar{q}} = \frac{1}{2\pi}\int\frac{\mathrm{d}x}{2x(1-x)}\int \mathrm{d}^2r_{1\perp}\mathrm{d}^2r_{2\perp}U_{q\bar{q}}(x,\vec{r}_{1\perp}-\vec{r}_{2\perp}) \times b^\dagger(x,\vec{r}_{1\perp})d^\dagger(1-x,\vec{r}_{2\perp})d(1-x,\vec{r}_{2\perp})b(x,\vec{r}_{1\perp})$$

- Localize the interaction operator on the light front:

$$\underline{T}^{+-}_{q\bar{q}}(r_\perp) = \frac{1}{2\pi}\int\frac{\mathrm{d}x}{2x(1-x)}\int \mathrm{d}^2r_{1\perp}\mathrm{d}^2r_{2\perp}U_{q\bar{q}}(x,\vec{r}_{1\perp}-\vec{r}_{2\perp}) \times b^\dagger(x,\vec{r}_{1\perp})d^\dagger(1-x,\vec{r}_{2\perp})d(1-x,\vec{r}_{2\perp})b(x,\vec{r}_{1\perp})\frac{1}{2}\Big\{\delta^2(r_\perp-r_{1\perp})+\delta^2(r_\perp-r_{2\perp})\Big\}$$

- Extracting the OBD of the interaction:

$$t^{+-}_{q\bar{q}}(q^2_\perp) = \frac{1}{2\pi}\int\frac{\mathrm{d}x}{2x(1-x)}\int \mathrm{d}^2r_\perp\big|\psi(x,\vec{r}_\perp)\big|^2U_{q\bar{q}}(x,r_\perp)\frac{1}{2}\Big\{e^{i(1-x)\vec{q}_\perp\cdot\vec{r}_\perp}+e^{-ix\vec{q}_\perp\cdot\vec{r}_\perp}\Big\}$$

Free energy-momentum tensor

The LFWF representation of the free EMT is, [Cao:2023ohj]

$$t_0^{+-}(Q^2, \mu) = \sum_n \int^{\mu} \big[\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}\big]_n \widetilde{\psi}_n^*(\{x_i, \vec{r}_{i\perp}\}) \sum_j e^{i\vec{r}_{j\perp}\cdot\vec{q}_\perp} \frac{-\nabla_{j\perp}^2 + m_j^2 - \frac{1}{4}q_\perp^2}{x_j} \widetilde{\psi}_n(\{x_i, \vec{r}_{i\perp}\})$$

where, $Q^2 = q_\perp^2$. For the pion,

$$t_0^{+-}(Q^2) = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\zeta_\perp \varphi_\pi^*(\zeta_\perp)\Big\{e^{i\sqrt{\frac{1-x}{x}}\vec{q}_\perp\cdot\vec{\zeta}_\perp}\Big[-(1-x)\nabla_{\perp\zeta}^2 - \frac{q_\perp^2}{4x}\Big] + e^{-i\sqrt{\frac{x}{1-x}}\vec{q}_\perp\cdot\vec{\zeta}_\perp}\Big[-x\nabla_{\perp\zeta}^2 - \frac{q_\perp^2}{4(1-x)}\Big]\Big\}\varphi_\pi(\zeta_\perp)$$

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possible divergence?

$$\int_0^1 \frac{dx}{x} J_0\left(\sqrt{\frac{1-x}{x}} zQ\right) = 2K_0(zQ)$$

Pion gravitational form factor D

[arXiv:2312.02543]

$$t^{+-}(Q^2) = t_0^{+-}(Q^2) + t_{\text{int}}^{+-}(Q^2)$$
$$= (2M_\pi^2 + 2P_\perp^2 + \tfrac{1}{2}Q^2)A_\pi(Q^2) + Q^2 D_\pi(Q^2)$$
$$\Rightarrow \; D_\pi(Q^2) = \int z\mathrm{d}z|\varphi_\pi(z)|^2\Big\{\frac{z^2Q^2}{4}K_2(zQ) - 2K_0(zQ) - \frac{2U(z)}{Q^2}\Big[zQK_1(zQ) - \frac{z^2Q^2}{2}K_2(zQ)\Big]\Big\}$$

where, $K_\nu(z)$ are the modified Bessel function of the second kind

- Similar to the $A_\pi(Q^2)$
- Forward limit,

$$\lim_{Q^2\to 0} D_\pi(Q^2) = -\infty, \qquad \lim_{Q^2\to 0} Q^2 D_\pi(Q^2) = 0$$

- Large Q^2 scaling consistent with pQCD prediction:

$$D_\pi(Q^2) \sim \frac{1}{Q^2}$$

- In hard-wall model, the potential $U(z) = 0$; in soft-wall model, the potential term has a small but non-vanishing contribution

$D_\pi(Q^2)$ with holographic currents

- The above result adopts the bare currents, which will be dressed in AdS/QCD
- Dressing the currents also improves the IR behavior of $D(Q^2)$
- Straightforward to identify: $zQK_1(zQ) \to V(Q^2, z)$ as the vector current, $\frac{1}{2}z^2Q^2K_2(zQ) \to H(Q^2, z)$ as the tensor current
- What about $2K_0(zQ)$? It can be identified as a scalar current [Colangelo:2007pt, Colangelo:2008us]
    - Bulk-to-boundary propagators of scalar mesons $\Delta = 3$ and scalar glueballs $\Delta = 4$:

$$S_{\Delta=3} = z\Gamma(a+\tfrac{3}{2})U(a+\tfrac{1}{2}, 0, \xi), \qquad M_n^2 = 2\kappa^2(2n+3), \quad \text{scalar mesons}$$
$$S_{\Delta=4} = \Gamma(a+2)U(a, -1, \xi), \qquad M_n^2 = 2\kappa^2(2n+4), \quad \text{scalar glueballs}$$

    where, $a = Q^2/(4\kappa^2)$, and $\xi = \kappa^2 z^2$

    - Assume mixing scalar mesons and scalar glueballs. The mixing coefficients are determined by matching to chiral limit $D_\pi(0) = -1$ and large-Q^2 scaling

$$S(Q^2, z) = c_1 S_{\Delta=3}(Q^2, z) + c_2 S_{\Delta=4}(Q^2, z)$$

$D_\pi(Q^2)$ with holographic currents

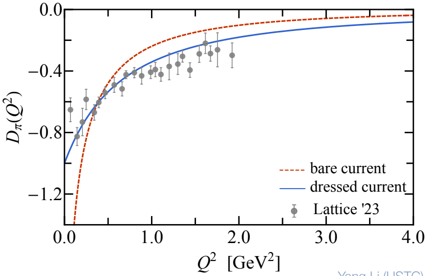
$$D_\pi(Q^2) = \int z\mathrm{d}z|\varphi_\pi(z)|^2\Big\{\frac{1}{2}H(Q^2,z) - 2S(Q^2,z) - \frac{2U(z)}{Q^2}\Big[V(Q^2,z) - H(Q^2,z)\Big]\Big\}$$

- Large Q^2 scaling consistent with pQCD prediction
- $D_\pi(0) = -1$ is finite: contributions from scalar and tensor glueballs cancel out; scalar mesons dominates
- U-term consists of a vector and a tensor current: $V(Q^2,z) - H(Q^2,z)$
  - The ρ pole $M_\rho^2 = 4\kappa^2$ cancels out
  - From pQCD counting rule, U-term consists of a twist-3 ($|q\bar{q}g\rangle$) contribution subtracted by a twist-2 ($|q\bar{q}\rangle$) contribution

scalar meson	$D_\pi^{\Delta=3}(0) = -0.83,$
scalar glueball	$D_\pi^{\Delta=4}(0) = -0.5,$
tensor glueball	$D_\pi^{T}(0) = +0.5,$
residual gluon	$D_\pi^{g}(0) = -0.17$

HLFQCD: r_D = 0.60 fm, r_A = 0.39 fm
Lattice '23: r_D = 0.61(7) fm, r_A = 0.41(1) fm

[Hackett:2023nkr]

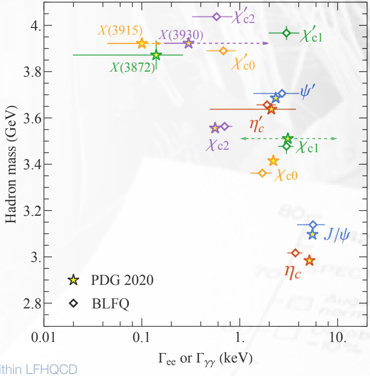


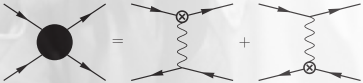
Charmonium

[Xu et al., in preparation]

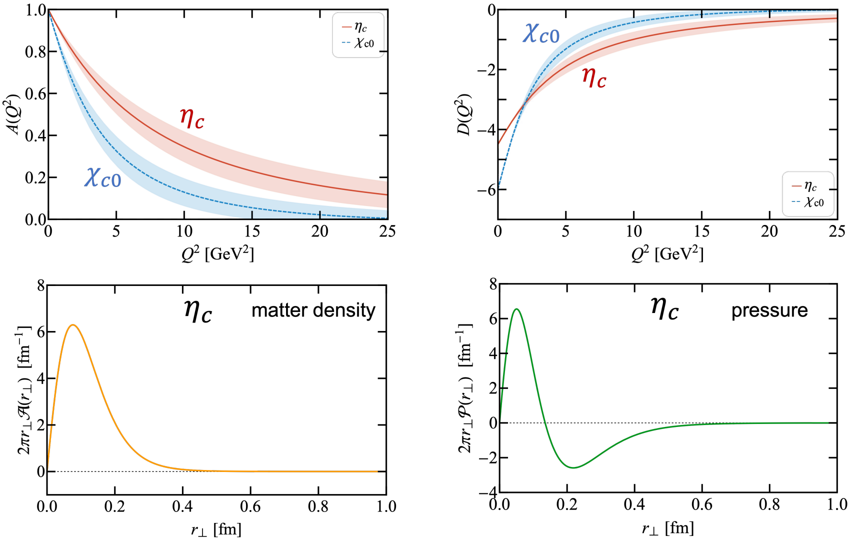
$$H = H_{\text{sw}} - \frac{C_F 4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_1'\gamma_\mu u_1 \bar{v}_2\gamma_\nu v_2' d^{\mu\nu}$$

- For charmonium, the one-gluon-exchange interaction is important for the short-distance physics
- Remarkable agreement with experiments for spectrum, radiative widths and transition form factors
- Parameter-free predictions for hadronic observables and for charmonium productions in DIS





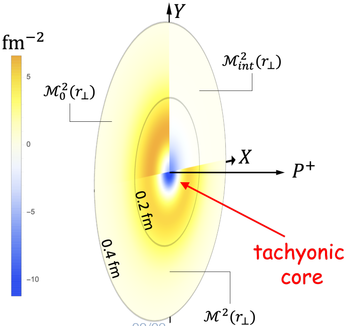
Charmonium gravitational form factors



Distribution of the invariant mass squared

$$\mathcal{P}^-(r_\perp) = \frac{1}{2P^+}\int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp}\langle P-\tfrac{q}{2}|T^{+-}(0)|P+\tfrac{q}{2}\rangle$$
$$= \frac{P_\perp^2 \mathcal{A}(r_\perp) + \mathcal{M}^2(r_\perp)}{P^+}$$

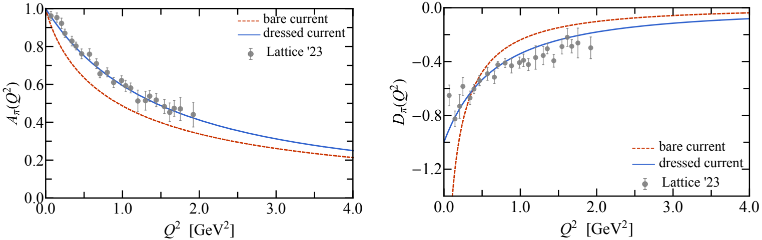
- Recall, light-front energy $P^- = (P_\perp^2 + M^2)/P^+$. We can identify $\mathcal{M}^2(r_\perp)$ as the distribution of the invariant mass squared
- It consists of a free part and an interacting part: $\mathcal{M}^2(r_\perp) = \mathcal{M}_0^2(r_\perp) + \mathcal{M}_{\text{int}}^2(r_\perp)$



Summary

[arXiv:2312.02543]

- The gravitational form factors emerge as a vital tool to unravel the internal structure of hadrons
- We computed the gravitational form factors of the pion using holographic light-front QCD
- By matching to the holographic currents, we find contributions from scalar mesons, glueballs as well as residual gluons. The obtained GFFs are in good agreement with recent Lattice QCD simulations



Thank you!