Electrodynamics of dual superconducting chiral medium

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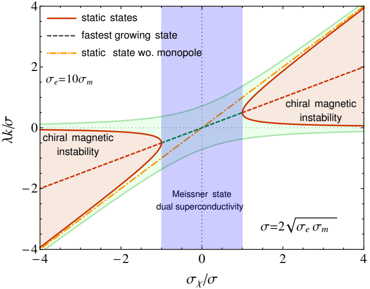
Summary

- We investigated the electromagnetism of QCD matter: described by Maxwell-Chern-Simons theory with magnetic monopoles (MCSm) and linear response: $\vec{J}_e = \sigma_e \vec{E}$, $\vec{J}_A = \sigma_\chi \vec{B}$, $\vec{J}_m = \sigma_m \vec{B}$

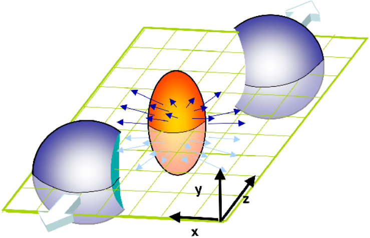


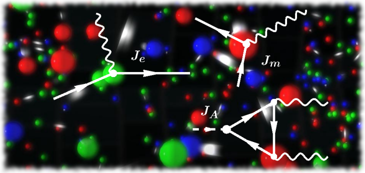
- Two regimes of the medium:

Meissner phase	chiral phase
$\sigma_\chi^2 < 4\sigma_e\sigma_m$	$\sigma_\chi^2 > 4\sigma_e\sigma_m$
dual superconductivity	chiral mag. instability

- Through the chiral evolution, which is governed chiral anomaly, the medium settles in the Meissner phase

Introduction





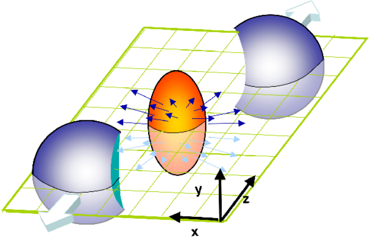
- Relativistic heavy ion collisions create some of the strongest electromagnetic fields in nature: $B \sim m_\pi^2 \approx 10^{18}$ Gauss
- Maxwell-Chern-Simons theory as an effective theory of QED×QCD

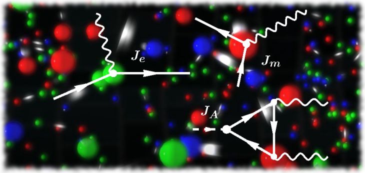
$$\vec{\nabla} \cdot \vec{E} = \rho_e + c_A \vec{\nabla}\theta \cdot \vec{B} \qquad -\vec{\nabla} \times \vec{E} = \partial_t \vec{B}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \partial_t \vec{E} + \vec{J}_e + c_A \dot{\theta} \vec{B} - c_A \vec{\nabla}\theta \times \vec{E}$$

  - electric current J_e^μ: existence of electric charges
  - axial current J_A^μ: imbalance of chiral fermions [Kharzeev '14]
- Why add monopole and magnetic current?
  - quasiparticles of non-Abelian gauge theory [review: Shnir '05]
  - CS term $\vec{E} \cdot \vec{B}$ can also be generated by the dual transformation

Introduction





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$$\vec{\nabla} \cdot \vec{B} = \rho_m \qquad \vec{\nabla} \times \vec{B} = \partial_t \vec{E} + \vec{J}_e + c_A \dot{\theta} \vec{B} - c_A \vec{\nabla}\theta \times \vec{E}$$

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Maxwell-Chern-Simons theory with monopoles

- Idealized model:
  - away from charges $\rho_e = \rho_m = 0$
  - homogeneous medium $\vec{\nabla}\theta = 0$: $\vec{J}_A = c_A \dot{\theta} \vec{B} = \sigma_\chi \vec{B}$
  - Ohm's laws: $\vec{J}_e = \sigma_e \vec{E}$, $\vec{J}_m = \sigma_m \vec{B}$

$$\vec{\nabla} \cdot \vec{E} = 0, \quad \text{(i)} \qquad -\vec{\nabla} \times \vec{E} = \partial_t \vec{B} + \sigma_m \vec{B}, \quad \text{(ii)}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \quad \text{(iii)} \qquad \vec{\nabla} \times \vec{B} = \partial_t \vec{E} + \sigma_e \vec{E} + \sigma_\chi \vec{B}. \quad \text{(iv)}$$

- Introduce vector potential: $\vec{\nabla} \times \vec{A} = \vec{B}$ w. Coul. gauge $\vec{\nabla} \cdot \vec{A} = 0$
  - Bianchi identity violated: $\partial_\mu \widetilde{F}^{\mu\nu} \neq 0$
  - $\vec{E} = -\partial_t \vec{A} - \sigma_m \vec{A}$ satisfies (i–ii)
  - can also work with the dual vector potential $\vec{\nabla} \times \vec{C} = \vec{E}$
- Wait, what about gauge invariance?
  - Ginzburg-Landau-Anderson-Higgs mechanism: $\psi = |\psi| e^{i\phi}$
  - unitary gauge: $\phi = 0$

Wait, are we getting superconductivity?

Superconductivity

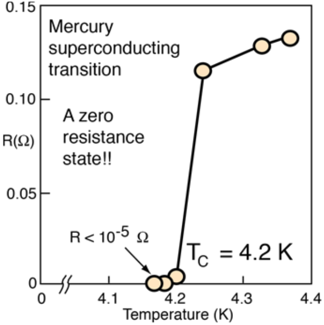
London equations: [F. London & H. London, 1935]

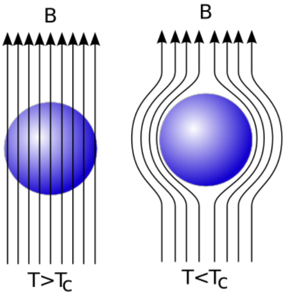
$$\partial_t \vec{J} = +\lambda^{-2}\vec{E}$$

zero resistivity

$$\vec{\nabla} \times \vec{J} = -\lambda^{-2}\vec{B}$$

Meissner effect





The second London equation + Maxwell equation: $\nabla^2\vec{B} = \lambda^{-2}\vec{B}$, where λ is known as the London penetration length.

Dual superconductivity

- Introduce the super current: $\vec{J}_s = -\sigma_e \sigma_m \vec{A}$ and the normal current $\vec{J}_n = \sigma_e \vec{E}_n$ with $\vec{E}_n = -\partial_t \vec{A}$; $\vec{J}_e = \vec{J}_s + \vec{J}_n$
- Maxwell-Chern-Simons-London equations

$$\vec{\nabla} \cdot \vec{E}_n = 0, \qquad -\vec{\nabla} \times \vec{E}_n = \partial_t \vec{B}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \times \vec{B} = \partial_t \vec{E}_n + (1 + \frac{\sigma_m}{\sigma_e})\vec{J}_n + \vec{J}_s + \vec{J}_A$$

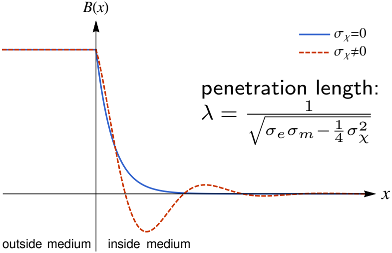
$$\vec{\nabla} \times \vec{J}_s = -\sigma_e \sigma_m \vec{B}, \qquad \partial_t \vec{J}_s = \sigma_e \sigma_m \vec{E}_n$$

- Dual Meissner effect

$$\nabla^2 \vec{B} = \sigma_e \sigma_m \vec{B} + \sigma_\chi \vec{\nabla} \times \vec{B}, \quad \nabla^2 \vec{E} = \sigma_e \sigma_m \vec{E} + \sigma_\chi \vec{\nabla} \times \vec{E}$$

In the stationary limit $\vec{E}_n = 0$, $\vec{J}_e \to \vec{J}_s$

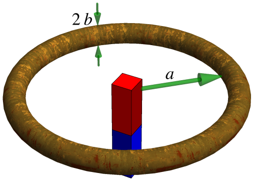
- Chandrasekhar-Kendall modes: $k = \frac{\lambda \sigma_\chi}{2} \pm \sqrt{\frac{1}{4}\sigma_\chi^2 - \sigma_e \sigma_m} = k_\chi + i/\lambda$
- $\sigma_\chi^2 < 4\sigma_e \sigma_m$: $B_z(x) = B_0 \cos(k_\chi x) \exp(-x/\lambda)$
- $\sigma_\chi^2 > 4\sigma_e \sigma_m$: no Meissner effect



penetration length: $\lambda = \frac{1}{\sqrt{\sigma_e \sigma_m - \frac{1}{4}\sigma_\chi^2}}$

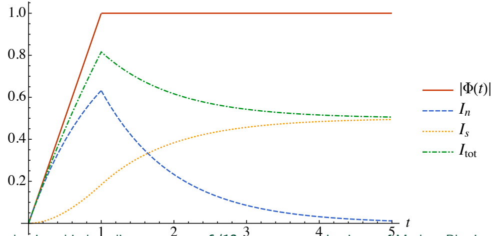
Persistent current

Can we have both the finite conductivity and a persistent current? Let's consider a realistic problem – a dual superconducting ring ($\sigma_\chi = 0$):



circuit law:

$$\vec{\nabla} \times \vec{E}_n + \partial_t \vec{B} = 0,$$
$$\partial_t \vec{J}_s = \sigma_e \sigma_m \vec{E}_n,$$
$$\vec{\nabla} \times \vec{B} = \partial_t \vec{E}_n + (1 + \frac{\sigma_m}{\sigma_e}) \vec{J}_n + \vec{J}_s$$
$$\vec{J}_e = \vec{J}_n + \vec{J}_s$$



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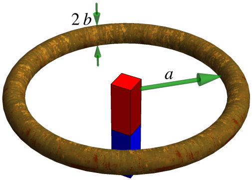

circuit law:

$$I_n R + \dot{\Phi}_{\text{tot}} = 0,$$
$$\partial_t \vec{J}_s = \sigma_e \sigma_m \vec{E}_n,$$
$$\vec{\nabla} \times \vec{B} = \partial_t \vec{E}_n + (1 + \frac{\sigma_m}{\sigma_e}) \vec{J}_n + \vec{J}_s$$
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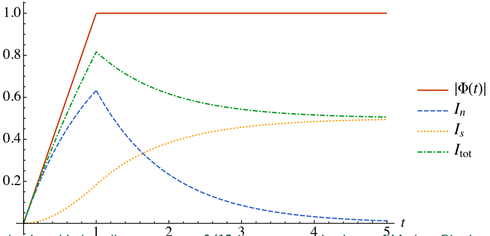

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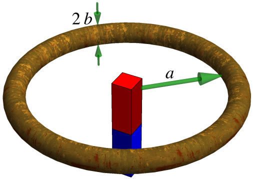

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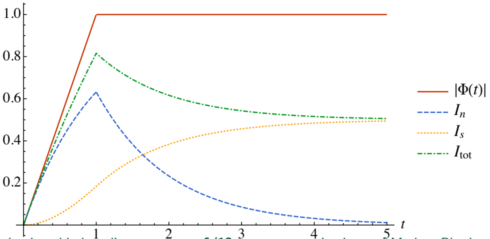
$$\dot{I}_s = \sigma_m I_n,$$

$$\vec{\nabla} \times \vec{B} = \partial_t \vec{E}_n + (1 + \frac{\sigma_m}{\sigma_e})\vec{J}_n + \vec{J}_s$$

$$\vec{J}_e = \vec{J}_n + \vec{J}_s$$



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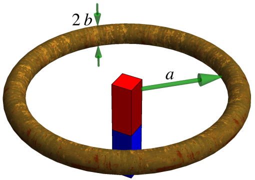
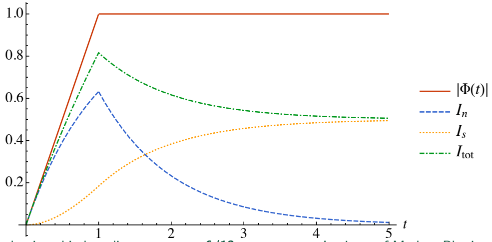


circuit law:

$$I_n R + \dot{\Phi}_{\text{tot}} = 0,$$
$$\dot{I}_s = \sigma_m I_n,$$
$$\Phi_{\text{int}} = L\left[(1 + \tfrac{\sigma_m}{\sigma_e}) I_n + I_s\right]$$
$$\vec{J}_e = \vec{J}_n + \vec{J}_s$$

self inductance $L = 4\pi\left[\ln(8b/a) - 7/4\right] + O(b^2/a^2)$ (for a ring).



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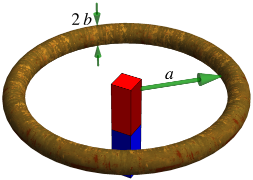

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$$\Phi_{\text{int}} = L\left[(1 + \tfrac{\sigma_m}{\sigma_e}) I_n + I_s\right]$$

$$I_{\text{tot}} = I_n + I_s,\ \Phi_{\text{tot}} = \Phi_{\text{ext}} + \Phi_{\text{int}}$$

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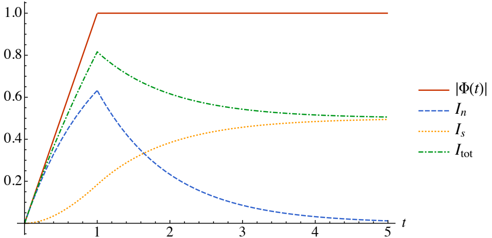


Dispersion relation

$$(\partial_t^2 - \nabla^2)\vec{A} + (\sigma_e + \sigma_m)\partial_t \vec{A} + \sigma_e \sigma_m \vec{A} - \sigma_\chi(t)\vec{\nabla} \times \vec{A}.$$

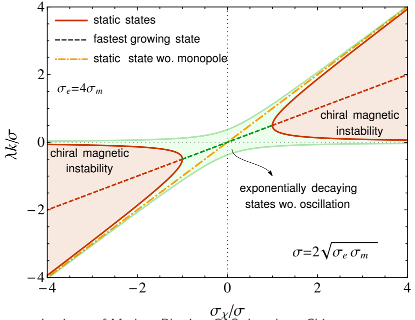
- $\vec{E}$, $\vec{B}$ satisfy the same equation
- Expand $\vec{A}$ with CK states – circularly polarized planewaves

$$\omega_{\lambda k}(t) = -\frac{i}{2}(\sigma_e + \sigma_m) \pm \sqrt{k^2 - \lambda\sigma_\chi(t)k - \tfrac{1}{4}(\sigma_e - \sigma_m)^2}$$

- $\sigma_\chi = c_A\dot{\theta}$ has non-trivial time dependence dictated by chiral anomaly
- Full time evolution has to take into account the evolution of σ_χ

If $\sigma_\chi^2 > 4\sigma_e\sigma_m$, there exist unstable modes w. $\operatorname{Im}\omega_{\lambda k} > 0$

For example, the fastest growing state $k_\star = \frac{\lambda\sigma_\chi}{2}$: $A_\star \sim e^{\frac{1}{2}\Gamma_\star t}$ where $\Gamma_\star = \sqrt{\sigma_\chi^2 + (\sigma_e - \sigma_m)^2} - (\sigma_e + \sigma_m) > 0$



Chiral anomaly

- Magnetic helicity: $\mathcal{H}_{\rm em} = \int \mathrm{d}^3x\, \vec{A}\cdot\vec{B}$ topological quantity
- Chiral anomaly equation governs the evolution of chiral conductivity:

$$\partial_\mu J^\mu_A = c_A \vec{E}\cdot\vec{B} \quad \Rightarrow \quad \partial_t(\beta^{-1}\sigma_\chi + \mathcal{H}_{\rm em}) = 0$$

- Without monopole, the total helicity is conserved [Tuchin '17]
  - chiral magnetic instability = helicity transfer from medium to field
  - chiral anomaly terminates the growth of magnetic field ($\sigma_\infty = 0$)

$\sigma_m = 0$	
instability exists for all σ_χ	
$\sigma_\chi \to 0$	
$\mathcal{H}_{\rm em} \to$ const	
$A \to \infty$, $B \to 0$	

[Hirono '15]

magnetic fields with definite $\mathcal{H}_{\rm em}$

Chiral anomaly

- Magnetic helicity: $\mathcal{H}_{\rm em} = \int \mathrm{d}^3x\, \vec{A}\cdot\vec{B}$ topological quantity
- Chiral anomaly equation governs the evolution of chiral conductivity:

$$\partial_\mu J_A^\mu = c_A \vec{E}\cdot\vec{B} \quad \Rightarrow \quad \partial_t(\beta^{-1}\sigma_\chi + \mathcal{H}_{\rm em}) = -2\sigma_m \mathcal{H}_{\rm em}$$

- Without monopole, the total helicity is conserved [Tuchin '17]
  - chiral magnetic instability = helicity transfer from medium to field
  - chiral anomaly terminates the growth of magnetic field ($\sigma_\infty = 0$)
- With monopole, J_m dissipates helicity (though it conserves energy): $\mathcal{H}_{\rm em} \to 0$; but there is no constraint on σ_∞

$\sigma_m = 0$	$\sigma_m \neq 0$
instability exists for all σ_χ	instability exists for $\sigma_\chi^2 \geq 4\sigma_e\sigma_m$
$\sigma_\chi \to 0$	$\sigma_\chi \to \sigma_\infty \overset{?}{<} 2\sqrt{\sigma_e\sigma_m}$
$\mathcal{H}_{\rm em} \to$ const	$\mathcal{H}_{\rm em} \to 0$
$A \to \infty$, $B \to 0$	$A \to 0$, $B \to 0$

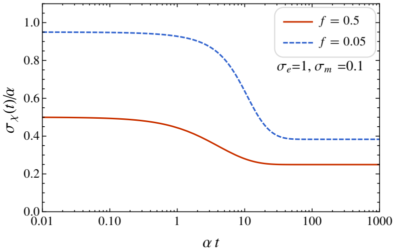
final state is always dual superconducting (see next slide for proof)

Analytic solutions

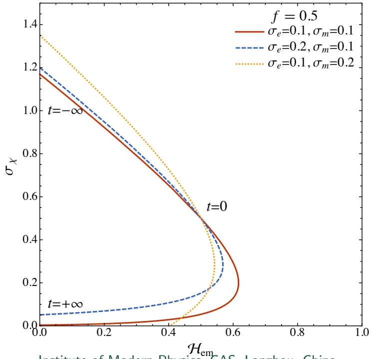
- Ansätze:
  - adiabatic approximation: $\dot{\omega}_{\lambda k} \approx 0$
  - fastest growing state approximation: only consider the FGS: $k = \frac{\lambda}{2}\sigma_\chi$

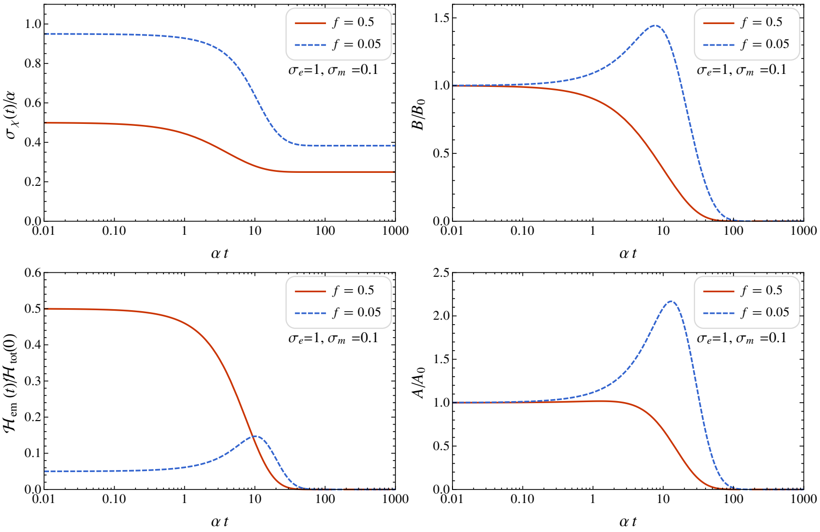
$$\dot{\sigma}_\chi = -\mathcal{H}_{\rm em}(\sigma_\chi)\left[\sqrt{(\sigma_e - \sigma_m)^2 + \sigma_\chi^2} - (\sigma_e - \sigma_m)\right]$$

- Final state: $\dot{\sigma}_\chi = 0 \;\Rightarrow\; \mathcal{H}_{\rm em}(\sigma_\infty) = 0$
- σ_∞ is determined by the details of the evolution. But $\mathcal{H}_{\rm em}$ peaks at $2\sqrt{\sigma_e\sigma_m} \Rightarrow \sigma_\infty^2 \leq 4\sigma_e\sigma_m$



$f = \mathcal{H}_{\rm em}/\mathcal{H}_{\rm tot}$ is the initial fraction of helicity stored in the magnetic field



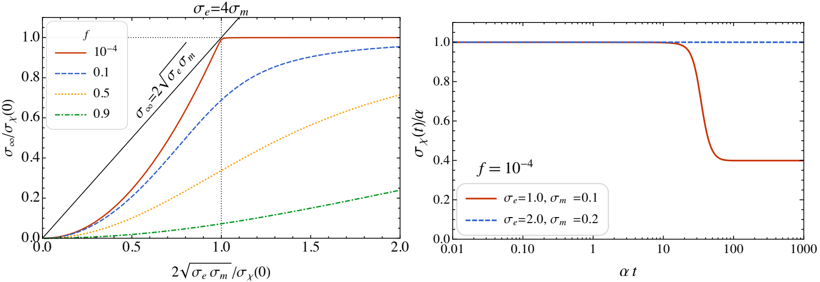


$f = 0.5$
$f = 0.05$
σ_e=1, σ_m =0.1
$\sigma_\chi(t)/\alpha$
B/B_0
$\mathcal{H}_{em}(t)/\mathcal{H}_{tot}(0)$
A/A_0
$\alpha\, t$

Stability of the chiral medium

Under what condition is the chiral medium stable with respect to helicity fluctuation, if there is no electromagnetic field initially?

- In an unstable chiral medium, a small fluctuation will grow exponentially. In other words, if you disturb an unstable chiral medium, it will blow up.
- Chiral medium is always unstable without magnetic monopole — chiral magnetic instability
- Stability condition (for $\mathcal{H}_{\text{em}} = 0$ at $t = 0$):
  $\sigma_\chi(t) = \sigma_\chi(0)$ and $\mathcal{H}'_{\text{em}}(\sigma_\chi) < 0 \;\Rightarrow\; \sigma_\chi^2 < 4\sigma_e\sigma_m$

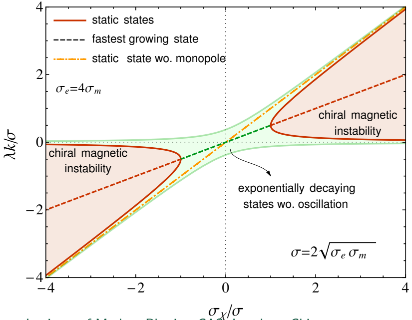


Conclusion and outlook

- We investigated the Maxwell-Chern-Simons theory with magnetic monopoles
- We discovered there exist two distinct regimes:
  - $\sigma_\chi^2 < 4\sigma_e\sigma_m$ is dual superconducting phase featuring dual Meissner effect and stable chiral medium
  - $\sigma_\chi^2 > 4\sigma_e\sigma_m$ is a chiral phase featuring chiral magnetic instability
- In presence of monopoles, the chiral medium undergoes an inverse cascade before settling to the superconducting phase

Questions to be answered:

- What is the microscopic origin of the dual superconductivity?
- What is the implication to quark-gluon plasma physics?
- Can we observe these phenomena in condensed matter systems, e.g. spin ice?
- What is the role of Dirac quantization?



Y. Li and K. Tuchin, Phys. Lett. B **776** (2018) 270; arXiv:1708.08536

Thank you!

Questions?