Graphics&Geometric Computing Lab @USTC

Meshes: Definitions & Terminologies

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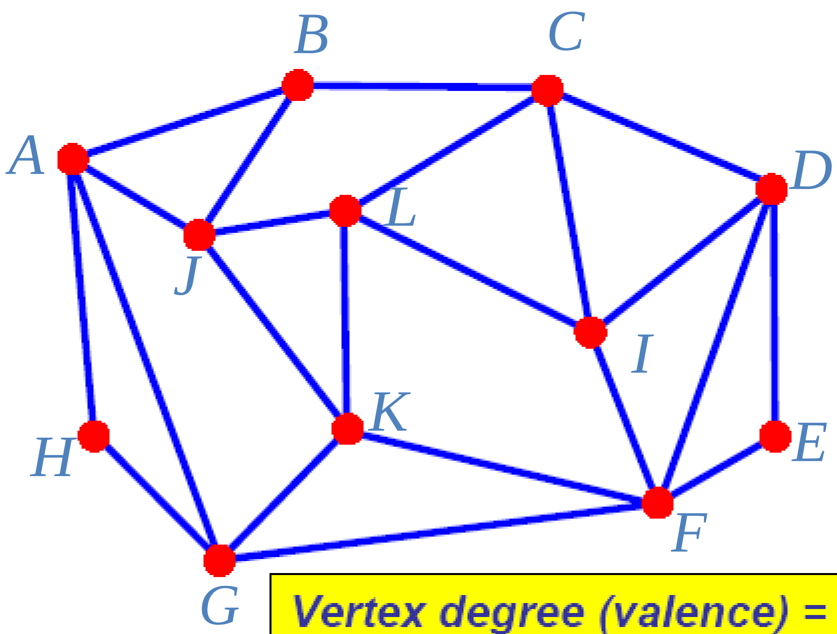
Standard Graph Definition



G = <**V**,**E**>
V = vertices =
{A,B,C,D,E,F,G,H,I,J,K,L}
E = edges =
{(A,B),(B,C),(C,D),(D,E),(E,F),(F,G),
(G,H),(H,A),(A,J),(A,G),(B,J),(K,F),
(C,L),(C,I),(D,I),(D,F),(F,I),(G,K),
(J,L),(J,K),(K,L),(L,I)}

Vertex degree (valence) = number of edges incident on vertex
deg(J) = 4, deg(H) = 2
k-regular graph = graph whose vertices all have degree *k*

Face: cycle of vertices/edges which cannot be shortened
F = faces =
{(A,H,G),(A,J,K,G),(B,A,J),(B,C,L,J),(C,I,J),(C,D,I),
(D,E,F),(D,I,F),(L,I,F,K),(L,J,K),(K,F,G)}

Connectivity

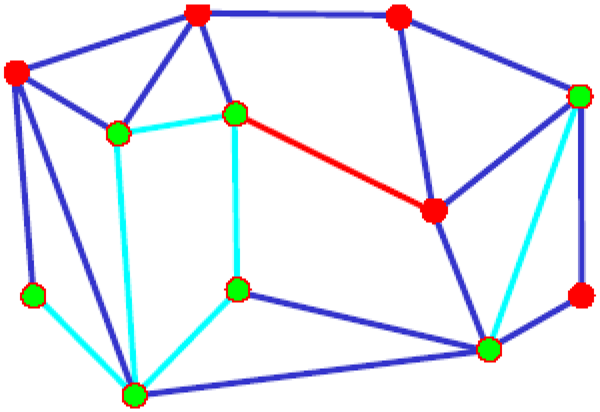
Graph is ***connected*** if there is a path of edges connecting every two vertices

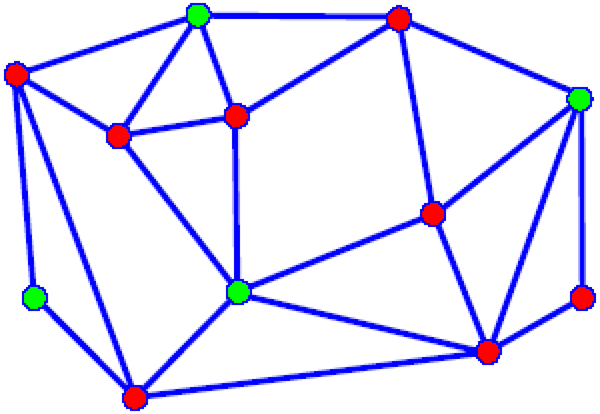
Graph is ***k-connected*** if between every two vertices there are *k* edge-disjoint paths

Graph **G'=<V',E'>** is a ***subgraph*** of graph **G=<V,E>** if **V**' is a subset of **V** and **E'** is the subset of **E** incident on **V'**

Connected component of a graph: maximal connected subgraph

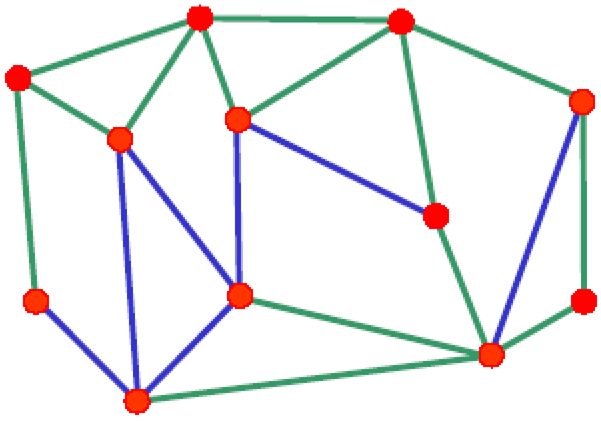
Subset **V'** of **V** is an ***independent*** set in **G** if the subgraph it induces does not contain any edges of **E**



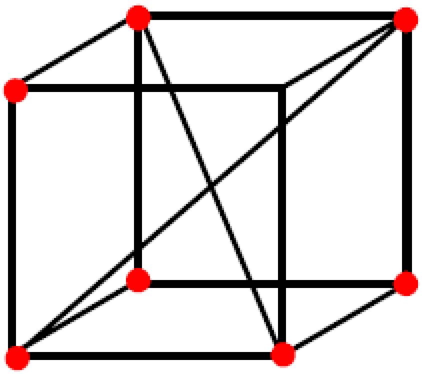


Graph Embedding

Graph is ***embedded*** in R^d if each vertex is assigned a position in R^d



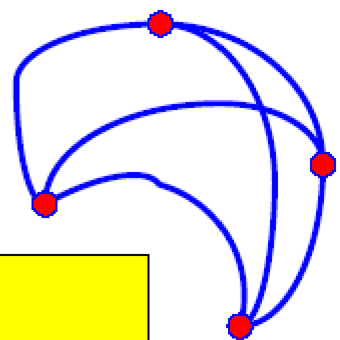
Embedding in R^2



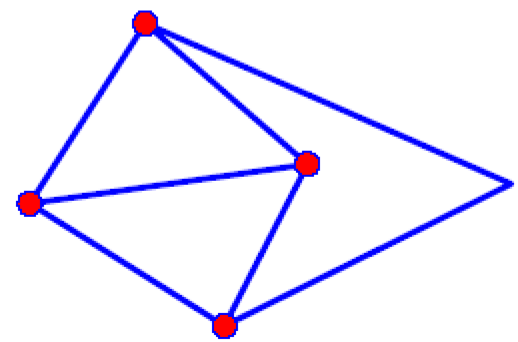
Embedding in R^3

Planar Graphs

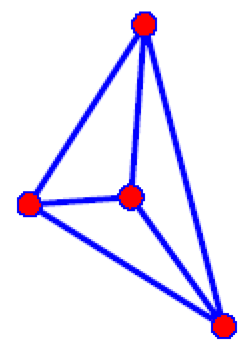
Planar Graph



Plane Graph



Straight Line Plane Graph



Planar graph: graph
whose vertices and edges can
be embedded in R^2 such that
its edges do not intersect

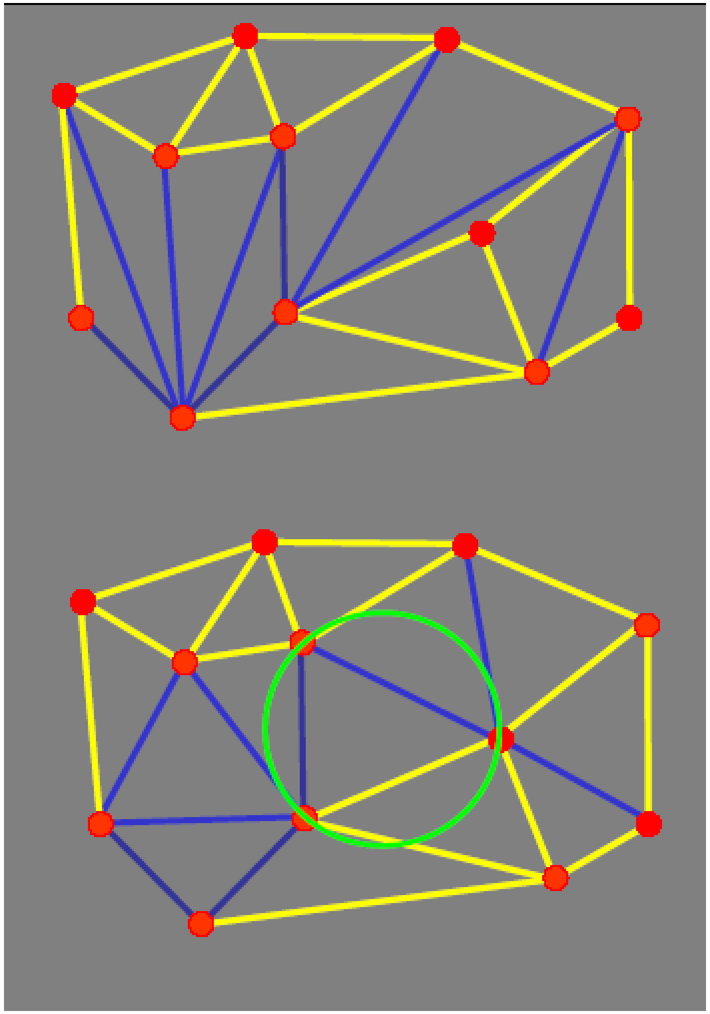
Every planar graph can be drawn
as a ***straight-line plane graph***

Triangulation

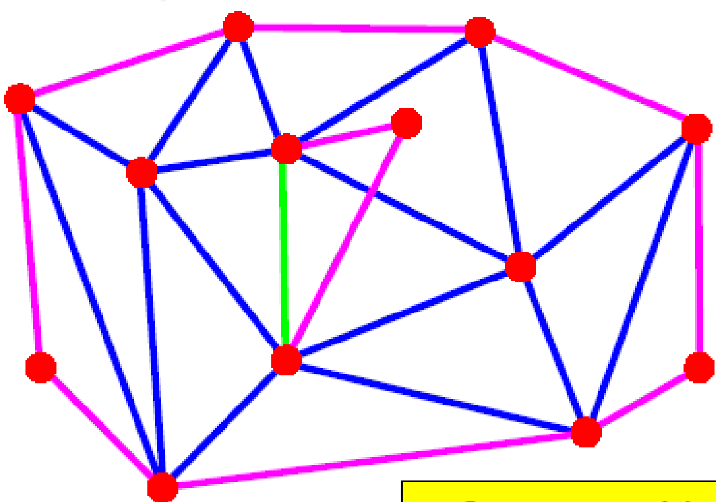
Triangulation: straight line plane graph all of whose faces are triangles

Delaunay triangulation of a set of points: unique set of triangles such that the circumcircle of any triangle does not contain any other point

Delaunay triangulation avoids long and skinny triangles



Meshes

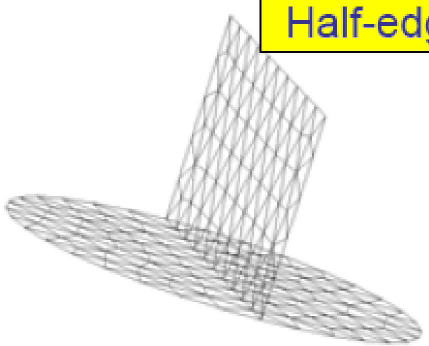


Mesh: straight-line graph embedded in R^3

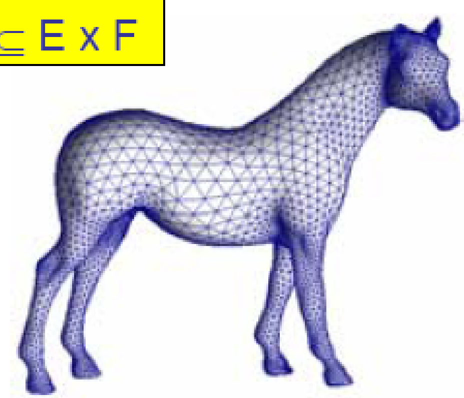
Boundary edge: adjacent to exactly *one* face
Regular edge: adjacent to exactly *two* faces
Singular edge: adjacent to more than two faces

Closed mesh: mesh with no boundary edges
Manifold mesh: mesh with no singular edges

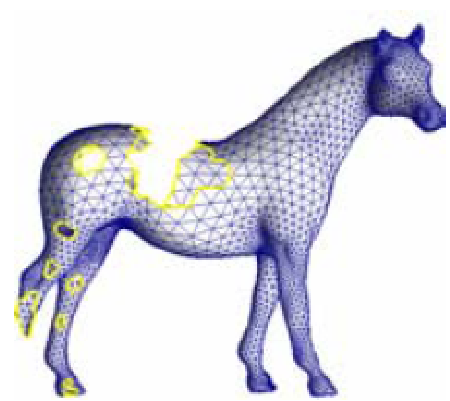
Corners $\subseteq$ V x F
Half-edges $\subseteq$ E x F



Non-Manifold

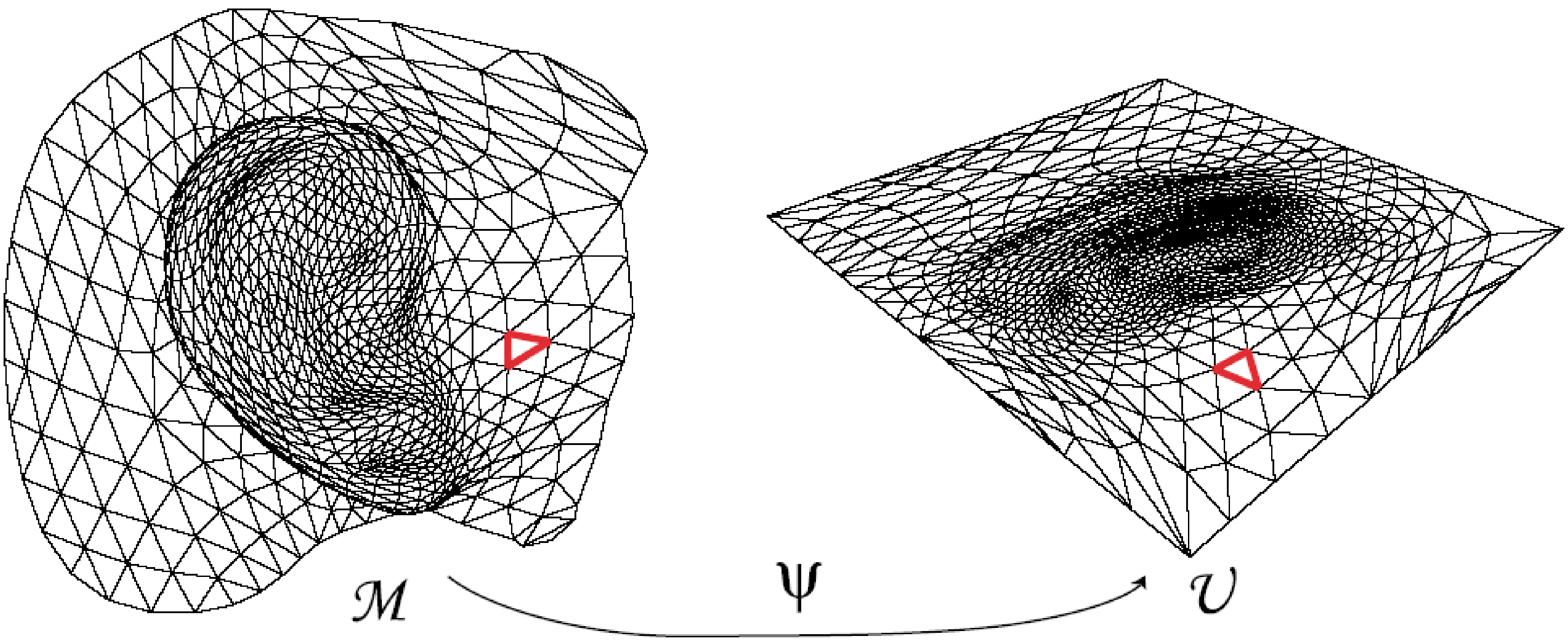


Closed Manifold

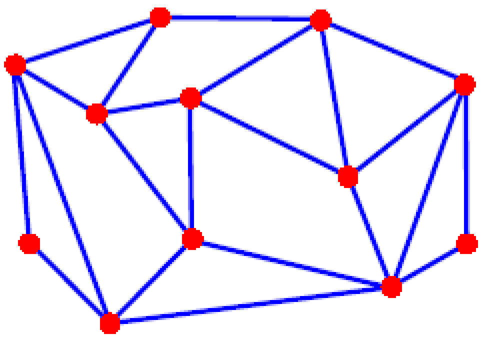


Open Manifold

Planar Graphs and Meshes



Topology



v =12
f = 14
e = 25
c = 1
g = 0
b = 1

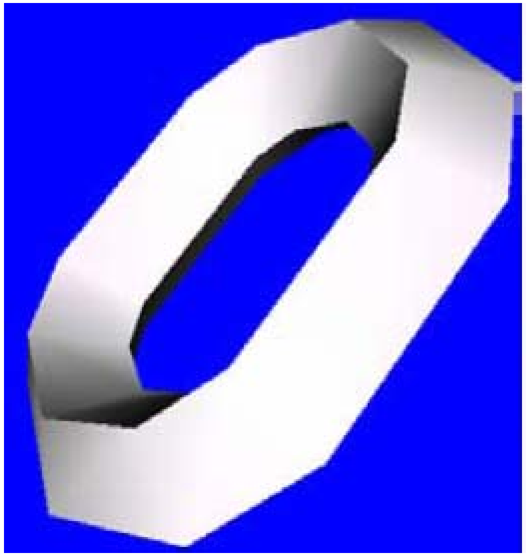
Genus of graph: *half* of *maximal* number of closed paths that do *not* disconnect the graph (number of "holes")

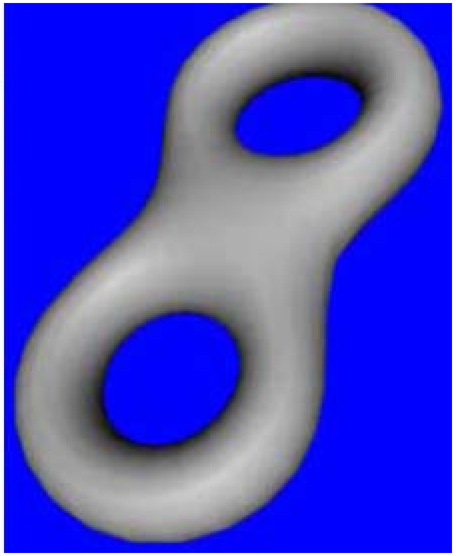
Genus(sphere)= 0
Genus(torus) = 1

Euler-Poincare Formula

$$v+f-e = 2(c-g)-b$$

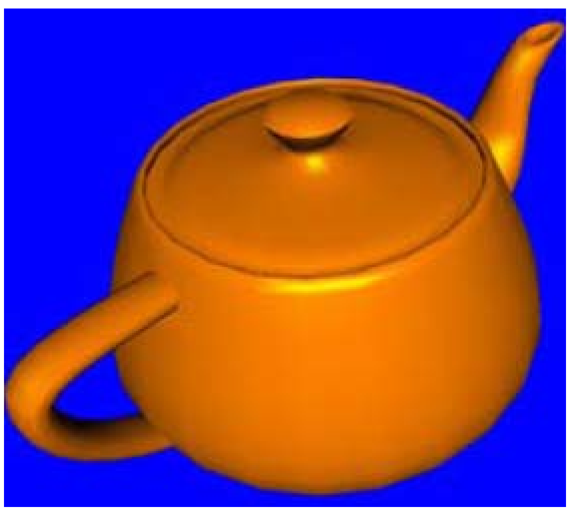
v = # vertices c = # conn. comp
f = # faces g = genus
e = # edges b = # boundaries

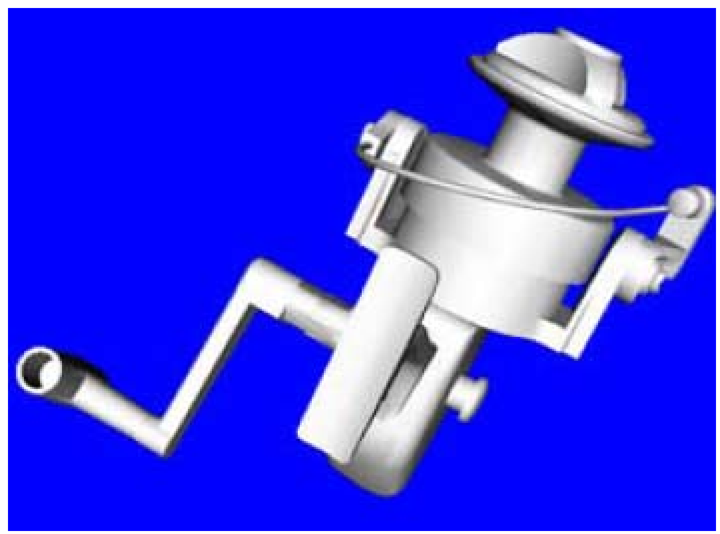




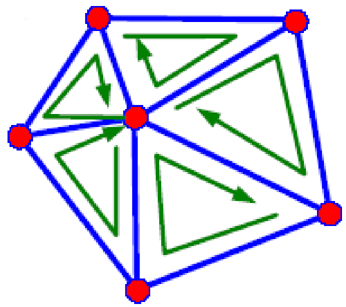
Topology Quiz

What can you say about the genus of these meshes ?





Orientability



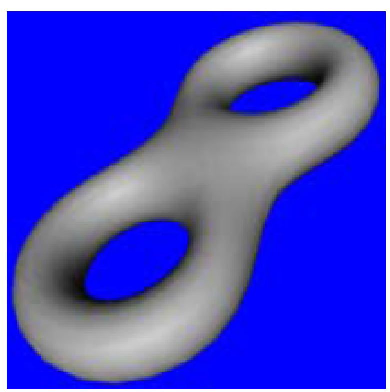
Oriented
F={(L,J,B),(B,C,L),(L,C,I), (I,K,L),(L,K,J)}
Not Oriented
F={(B,J,L),(B,C,L),(L,C,I), (L,I,K),(L,K,J)}

Orientation of a face is clockwise or anticlockwise order in which its vertices and edges are listed

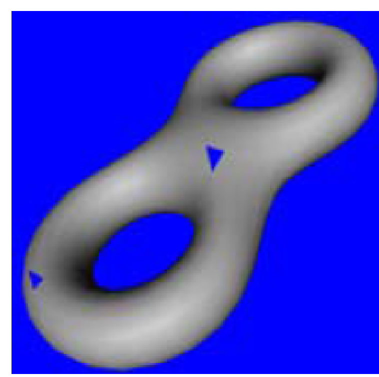
This defines the direction of face ***normal***

Straight line graph is ***orientable*** if orientations of its faces can be chosen so that each edge is oriented in *both* directions

Mobius strip or Klein bottle
- not orientable



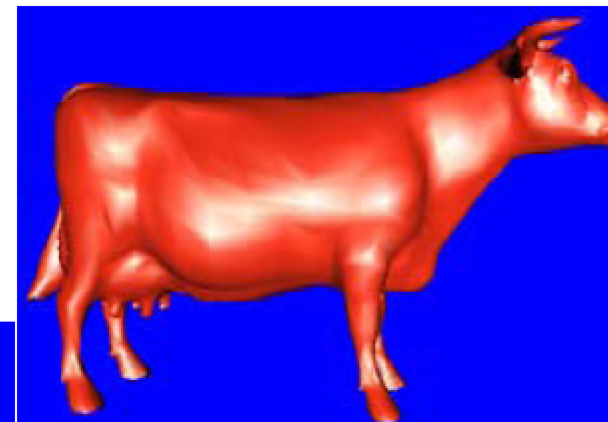
Not Backface Culled

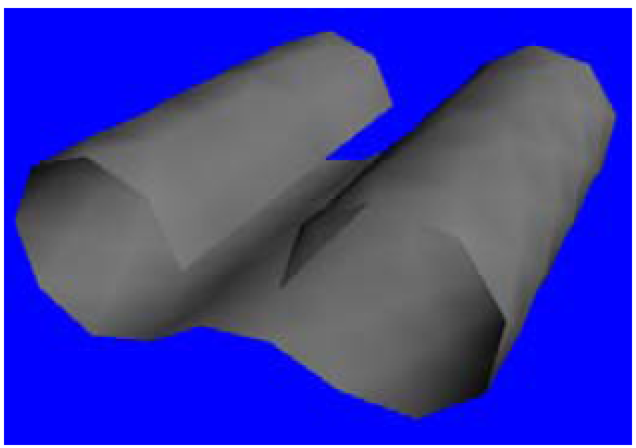


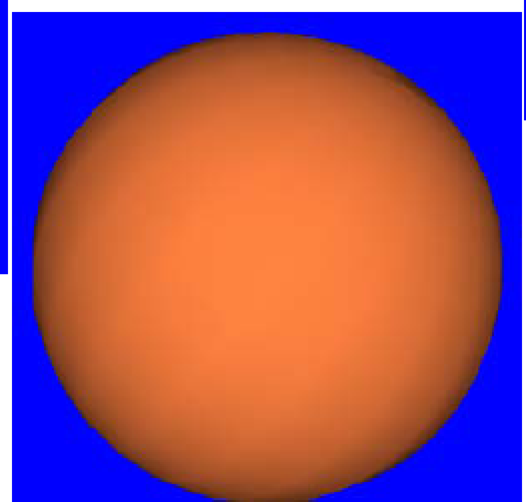
Backface Culled

Developablility

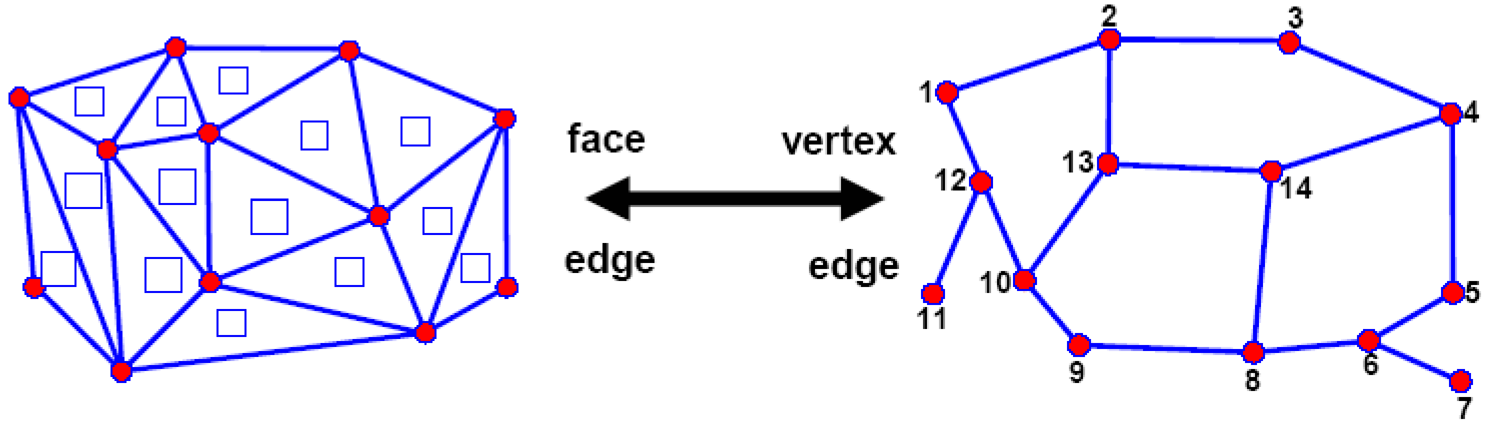
Mesh is ***developable*** if it may be embedded in R^2 without distortion







Duality



- Delaunay Triangulation vs. Voronoi Graph

Q&A