Shape Descriptors

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Most of slides are from Thomas Funkhouser
Outline

- Why shape descriptors?
- How do we represent shapes?
- Conclusion
Goal

• Find 3D models with similar shape
Goal

• Shape Descriptor:
  – *Structured* abstraction of a 3D model
  – Capturing *salient* shape information
Shape Descriptors

• Shape Descriptors
  – Fixed dimensional vector
  – Independent of model representation
  – Easy to match
Shape Descriptors

• Representation:
  – What can you represent?
  – What are you representing?

• Matching:
  – How do you align?
  – Part or whole matching?
Shape Descriptors

• Representation:
  – What can you represent?
  – What are you representing?

• Matching:
  – How do you align?
  – Part or whole matching?

Point Clouds  Polygon Soups  Closed Meshes  Genus-0 Meshes

Shape Spectrum
Shape Descriptors

• Representation:
  – What can you represent?
  – What are you representing?

• Matching:
  – How do you align?
  – Part or whole matching?

Is the descriptor invertible?

What is represented by the difference in descriptors?
Shape Descriptors

• Representation:
  – What can you represent?
  – What are you representing?

• Matching:
  – How do you align?
  – Part or whole matching?

How do you represent models across the space of transformations that don’t change the shape?
Shape Descriptors

- **Representation:**
  - What can you represent?
  - What are you representing?

- **Matching:**
  - How do you align?
  - Part or whole matching?

Can you match part of a shape to the whole shape?
Outline

● Why shape descriptors?
● How do we represent shapes?
  – Volumetric Representations
  – Surface Representations
  – View-Based Representations
● Conclusion
Volumetric Representations

Represent models by the volume that they occupy:

- Rasterize the models into a binary voxel grid
  - A voxel has value 1 if it is inside the model
  - A voxel has value 0 if it is outside
Volumetric Representations

• Compare models by measuring the overlaps of their volumes
  – Similarity is measured by the size of the intersection
Volumetric Representations

• Properties:
  – Invertible
  – 3D array of information
  – Comparison gives the measure of overlap

• Limitations:
  – Models need to be water-tight

Point Clouds
Polygon Soups
Closed Meshes
Genus-0 Meshes

Shape Spectrum
Outline

- Why shape descriptors?
- How do we represent shapes?
  - Volumetric Representations
  - Surface Representations
    - Spherical Parameterization
    - Extended Gaussian Image
    - Shape Histograms (Sectors + Shells)
    - Gaussian EDT
  - View-Based Representations
- Conclusion
Spherical Parameterization

- Create a 1-to-1 mapping between points on the surface of the model and points on the surface of the sphere.
- Compare two models by comparing the distances between two points on the models that map to the same point on the sphere.
Spherical Parameterization

• Properties:
  – Invertible
  – 2D array of information
  – Comparison gives the distance between surfaces

• Limitations:
  – Models need to be genus-0

Point Clouds  Polygon Soups  Closed Meshes  Genus-0 Meshes

Shape Spectrum
Extended Gaussian Image

- Represent a model by a spherical function by binning surface normals

[Horn, 1984]
Extended Gaussian Image

[Horn, 1984]

• Properties:
  – Invertible for convex shapes
  – 2D array of information
  – Can be defined for most models

Point Clouds  Polygon Soups  Closed Meshes  Genus-0 Meshes

Shape Spectrum
Extended Gaussian Image

• Properties:
  – Invertible for convex shapes
  – 2D array of information
  – Can be defined for most models

• Limitations:
  – Too much information is lost
  – Normals are sensitive to noise

[Horn, 1984]
Extended Gaussian Image

[Horn, 1984]

• Properties:
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• Limitations:
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Retrieval Results

- Princeton Shape Benchmark
  - ~900 models, 90 classes

14 biplanes 50 human bipeds 7 dogs 17 fish
16 swords 6 skulls 15 desk chairs 13 electric guitars

http://www.shape.cs.princeton.edu/benchmark/
Shape Histograms

[Ankerst et al., 1999]

• Shape descriptor stores a histogram of how much surface resides at different bins in space

Model

Shape Histogram (Sectors + Shells)
Boundary Voxel Representation

• Represent a model as the (anti-aliased) rasterization of its surface into a regular grid:
  – A voxel has value 1 (or area of intersection) if it intersects the boundary
  – A voxel has value 0 if it doesn’t intersect
Boundary Voxel Representation

• Properties:
  – Invertible
  – 3D array of information
  – Can be defined for any model
Retrieval Results

Precision

Recall

- Shape Histograms (3D)
- Extended Gaussian Image (2D)
- D2 (1D)
- Random
Histogram Representations

• Challenge:
  – Histogram comparisons measure overlap, not proximity.
Convolving with a Gaussian

- The value at a point is obtained by summing Gaussians distributed over the surface of the model.
  - Distributes the surface into adjacent bins
  - Blurs the model, loses high frequency information

![Diagram of convolving with a Gaussian](image)
Gaussian EDT

• The value at a point is obtained by summing the Gaussian of the closest point on the model surface.
  ✓ Distributes the surface into adjacent bins
  ✓ Maintains high-frequency information

[Kazhdan et al., 2003]
Gaussian EDT

[Kazhdan et al., 2003]

• Properties:
  – Invertible
  – 3D array of information
  – Can be defined for any model
  – Difference measures proximity between surfaces

Point Clouds
Polygon Soups
Closed Meshes
Genus-0 Meshes

Shape Spectrum
Retrieval Results

Precision vs. Recall plot showing different retrieval methods:
- Gaussian EDT (3D)
- Shape Histograms (3D)
- Extended Gaussian Image (2D)
- D2 (1D)
- Random
Outline

- Why shape descriptors?
- How do we represent shapes?
  - Volumetric Representations
  - Surface Representations
  - View-Based Representations
    - Spherical Extent Function
    - Light Field Descriptor
- Conclusion
Spherical Extent Function

• For every view direction, store the distance to the first point a viewer would see when looking at the origin.

[Vranic et al. 2002]
Spherical Extent Function

• A model is represented by its star-shaped envelope:
  – The minimal surface containing the model with the property that the center sees every point on the surface
  – Transforms arbitrary genus models to genus-0 surfaces
Spherical Extent Function

• A model is represented by its star-shaped envelope:
  – The minimal surface containing the model with the property that the center sees every point on the surface
  – Transforms arbitrary genus models to genus-0 surfaces
Spherical Extent Function

• Properties:
  – Invertible for star-shaped models
  – 2D array of information
  – Can be defined for most models
Spherical Extent Function

• Properties:
  – Can be defined for most models
  – Invertible for star-shaped models
  – 2D array of information

• Limitations:
  – Distance only measures angular proximity
Light Field Descriptor

[Chen et al. 2003]

• For every view direction, store the image the viewer would see when looking at the origin.
Light Field Descriptor

- Hybrid boundary/volume representation
Light Field Descriptor

• Properties:
  – Represents the visual hull of the model
  – 4D array of information
  – Can be defined for most models
Light Field Descriptor

• Properties:
  – Can be defined for most models
  – Invertible for star-shaped models
  – 4D array of information
  – Similarity = sum of area and contour similarities
    • There is a well defined interior
    • Can parameterize contours in 2D

Area Comparison

Contour Comparison
Retrieval Results

![Precision vs Recall Plot]

- Light Field Descriptor (4D)
- Spherical Extent Function (2D)
- Gaussian EDT (3D)
- Shape Histograms (3D)
- Extended Gaussian Image (2D)
- D2 (1D)
- Random
高斯不变量描述子

[曹，2011]

• 离散计算

\[ \text{GCMI}_n = (\text{GCM}_0)^{n-1} (\text{GCM}_n) = (\text{GCM}_0)^{n-1} \iint_S K^n \rho ds \approx (\text{GCM}_0)^{n-1} \rho \sum_{i=1}^{N} K_i^n \Delta S_i \]

• 描述子

\[ \overrightarrow{GIV} = (\text{GCMI}_1, \text{GCMI}_2, \cdots, \text{GCMI}_n) \]
谱矩不变量

- 基于Diffusion距离
  \[ NSMI_{d_m} = (SMI_{d_m})(SMI_{d_0})^{\frac{m-2}{2}} \]
  \[ t \in \{ [\frac{1}{\lambda_i}], [\frac{1}{\lambda_{i+1}}], [\frac{1}{\lambda_{i+2}}], \ldots, [\frac{1}{\lambda_N}] \} \]

- 基于Commute-time距离
  \[ NSMI_{c_m} = \frac{SMI_{c_m}}{SMI_{c_0}} \]

- 基于Biharmonic距离
  \[ NSMI_{b_m} = \frac{SMI_{b_m}}{(MID_{b_0})^{\frac{m}{2}} + 1} \]
离散计算及描述子

• **M**阶谱矩不变量离散形式

\[
SMI_m = \int \int \int \int_D (x, y)^m \rho(x)\rho(y) \, ds_y \, ds_x \approx \sum_{i=1}^{N} \sum_{j=1}^{N} D_{ij} \Delta s_{y_j} \Delta s_{x_i} \rho(x) \rho(y)
\]

• 描述子

\[
\overrightarrow{MIV} = (NSMI_1, NSMI_2, \cdots, NSMI_n)
\]
特征函数矩不变量描述子

- 特征函数矩

\[ EM_{(i_k)}^{(m_k)} = \int \int M \prod_{k=1}^{K} (\phi_k(x))^m_k \, ds \approx \sum_{j=0}^{n-1} \prod_{k=1}^{K} (v_{i_k,j})^{m_k} \Delta s_j \]

- 特征函数矩不变量

\[ EMI_{(i_k)}^{(m_k)} = S^{\frac{1}{2}} \sum_{k=1}^{K} m_k ^{n-1} \prod_{j=0}^{K} (v_{i_k,j})^{m_k} \Delta s_j \]

- 描述子

\[ \overrightarrow{EIV} = (EMI_1, EMI_2, EMI_3, \ldots, EMI_n) \]
Conclusion

– Extended Gaussian Image
  • Differential properties are not always stable
– Gaussian Euclidean Distance Transform
  • Distributes surface across space without blurring
– Spherical Extent Function
  • Represents arbitrary genus shape by a genus-0 model
– Light Field Descriptors
  • 2D matching allows for volumetric comparisons and silhouette parameterizations
Conclusion

• In designing a shape descriptor, you want to consider:
  – What kind of models can be represented?
  – What kind of shape metric is defined?
Discussion