中国科学技术大学
University of Science and Technology of China

GAMES 102在线课程

几何建模与处理基础

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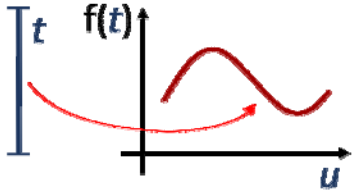
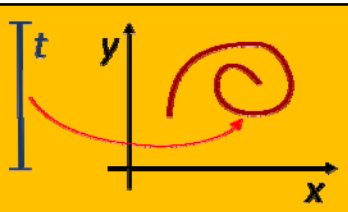
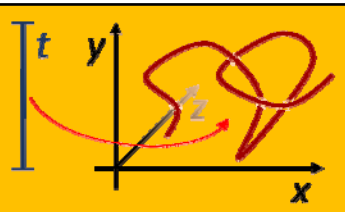
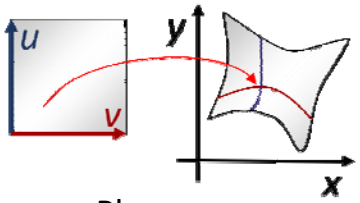
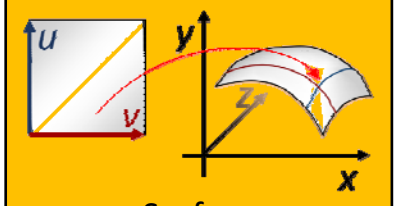
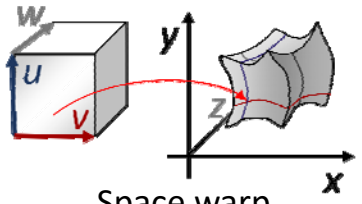
GAMES 102在线课程：几何建模与处理基础

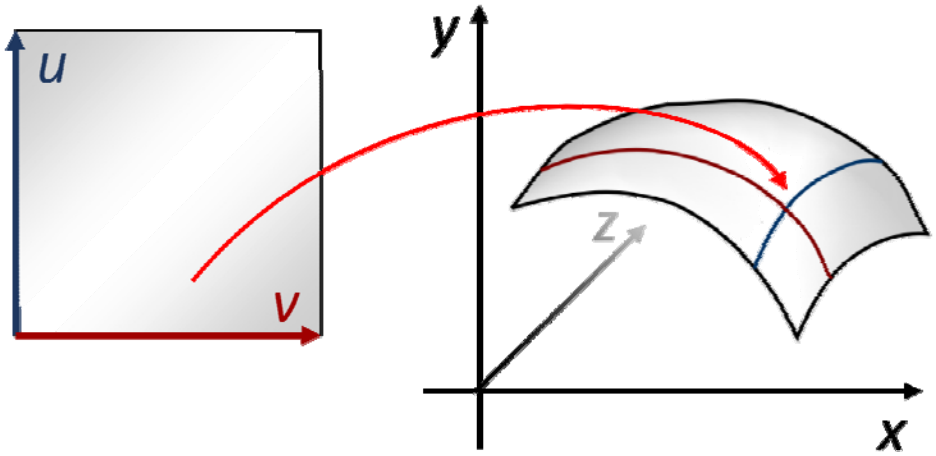
NURBS曲面

回顾：曲线曲面的不同形式

	Output: 1D	Output: 2D	Output: 3D
Input: 1D	 Function graph	 Plane curve	 Space curve
Input: 2D		 Plane warp	 Surface
Input: 3D			 Space warp

参数曲面

- 双参数(u, v)
- 参数曲面$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{pmatrix}$

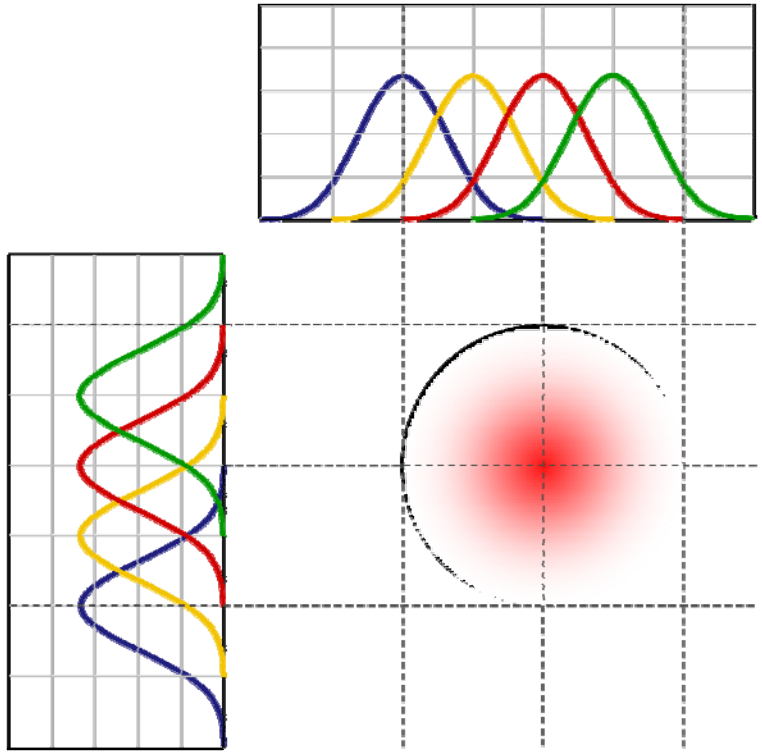


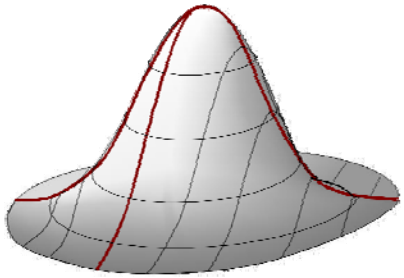
基本方法：张量积

每个参数的基函数两两相乘得到二元函数的基函数

	$b_1(u)$	$b_2(u)$	$b_3(u)$	$b_4(u)$
$b_1(v)$	$b_1(v)b_1(u)$	$b_1(v)b_2(u)$	$b_1(v)b_3(u)$	$b_1(v)b_4(u)$
$b_2(v)$	$b_2(v)b_1(u)$	$b_2(v)b_2(u)$	$b_2(v)b_3(u)$	$b_2(v)b_4(u)$
$b_3(v)$	$b_3(v)b_1(u)$	$b_3(v)b_2(u)$	$b_3(v)b_3(u)$	$b_3(v)b_4(u)$
$b_4(v)$	$b_4(v)b_1(u)$	$b_4(v)b_2(u)$	$b_4(v)b_3(u)$	$b_4(v)b_4(u)$

张量积函数





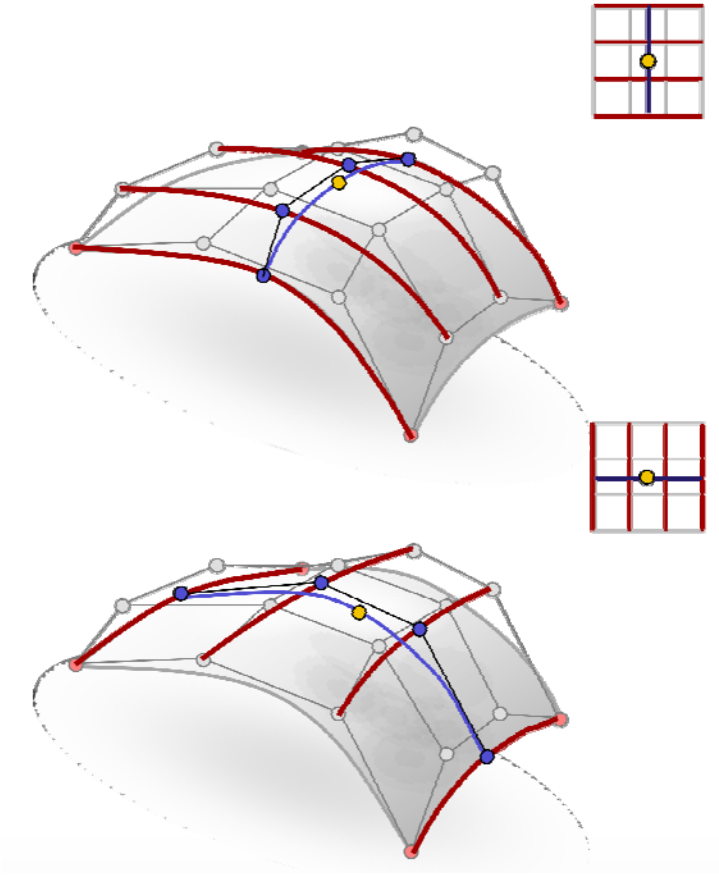
张量积曲面

$$\boldsymbol{f}(u,v) = \sum_{i=1}^{n}\sum_{j=1}^{n} b_i(u)b_j(v)\boldsymbol{p}_{i,j}$$

$$= \sum_{i=1}^{n} b_i(u)\sum_{j=1}^{n} b_j(v)\boldsymbol{p}_{i,j}$$

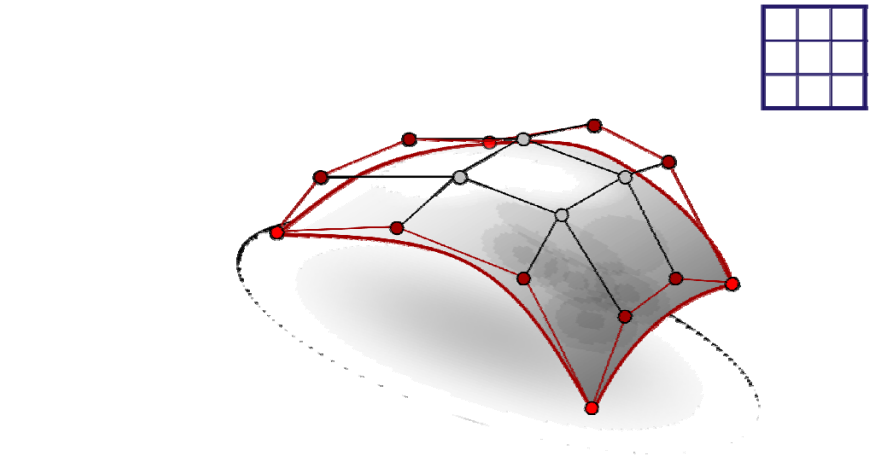
$$= \sum_{j=1}^{n} b_j(v)\sum_{i=1}^{n} b_i(u)\boldsymbol{p}_{i,j}$$

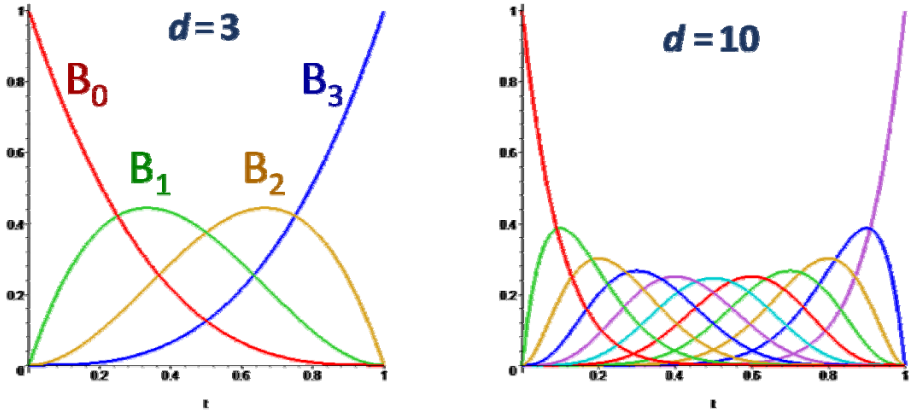
- “曲线的曲线”
- 两个参数的顺序无关



Bezier曲面

$$\boldsymbol{f}(u,v) = \sum_{i=0}^{\boldsymbol{d}} \sum_{j=0}^{\boldsymbol{d}} B_i^{(d)}(u) B_j^{(d)}(v) \boldsymbol{p}_{i,j}$$

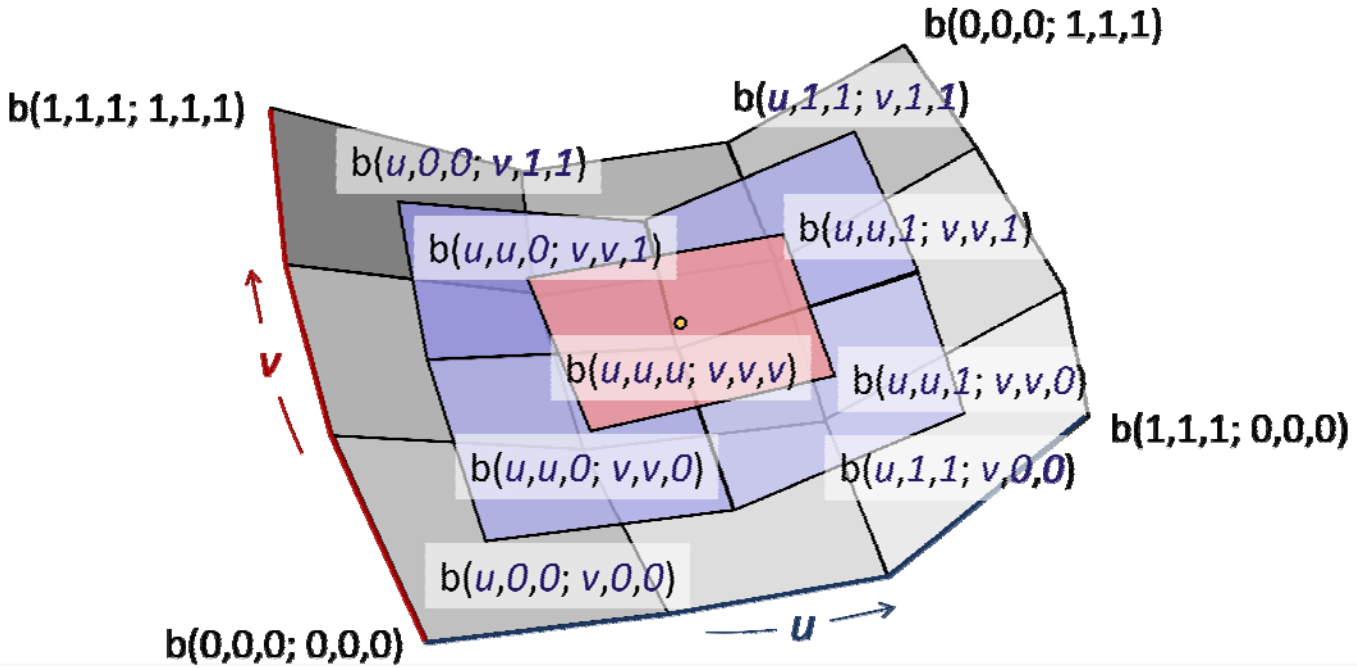




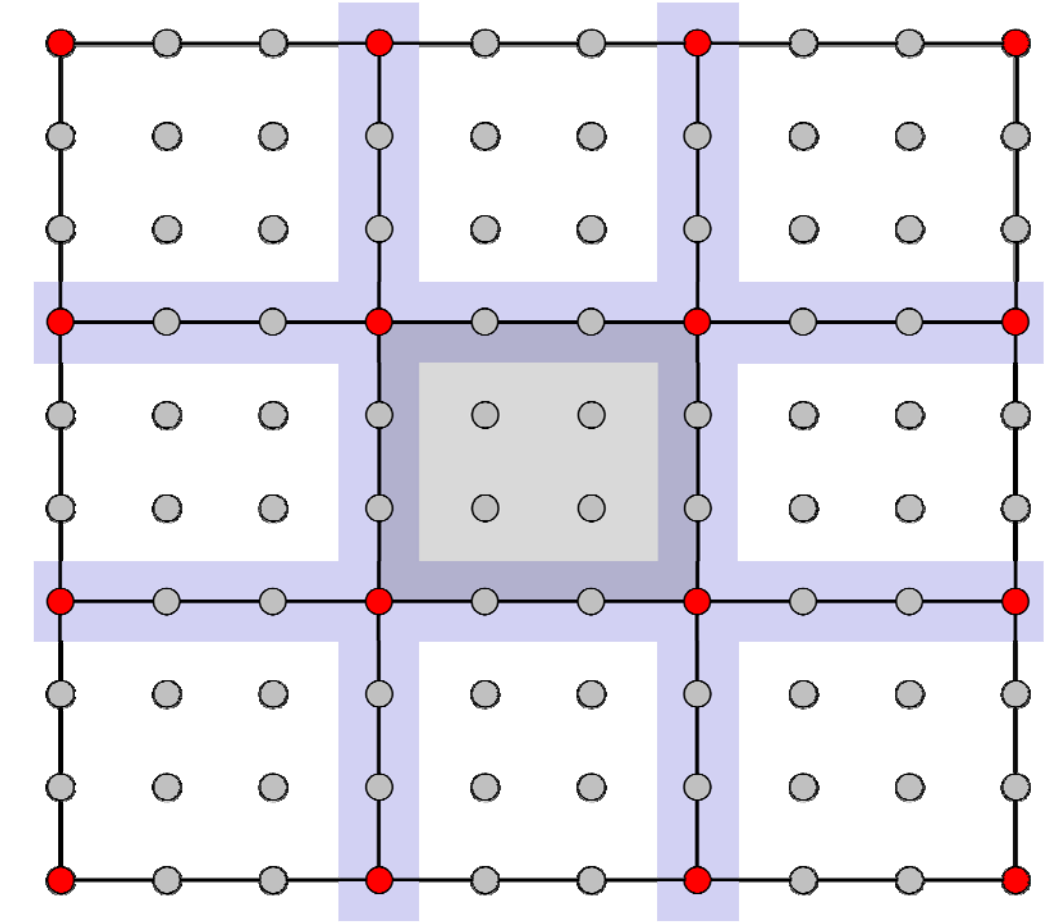
张量积曲面的性质

- 类似于曲线情形，性质取决于基函数的性质
  - 先沿一个方向做，然后再沿另一个方向做（方向顺序无关）
- Bezier曲面片具有类似的良好性质
  - 边界插值
  - 凸包
  - 变差缩减
  - 几何作图法
  - …

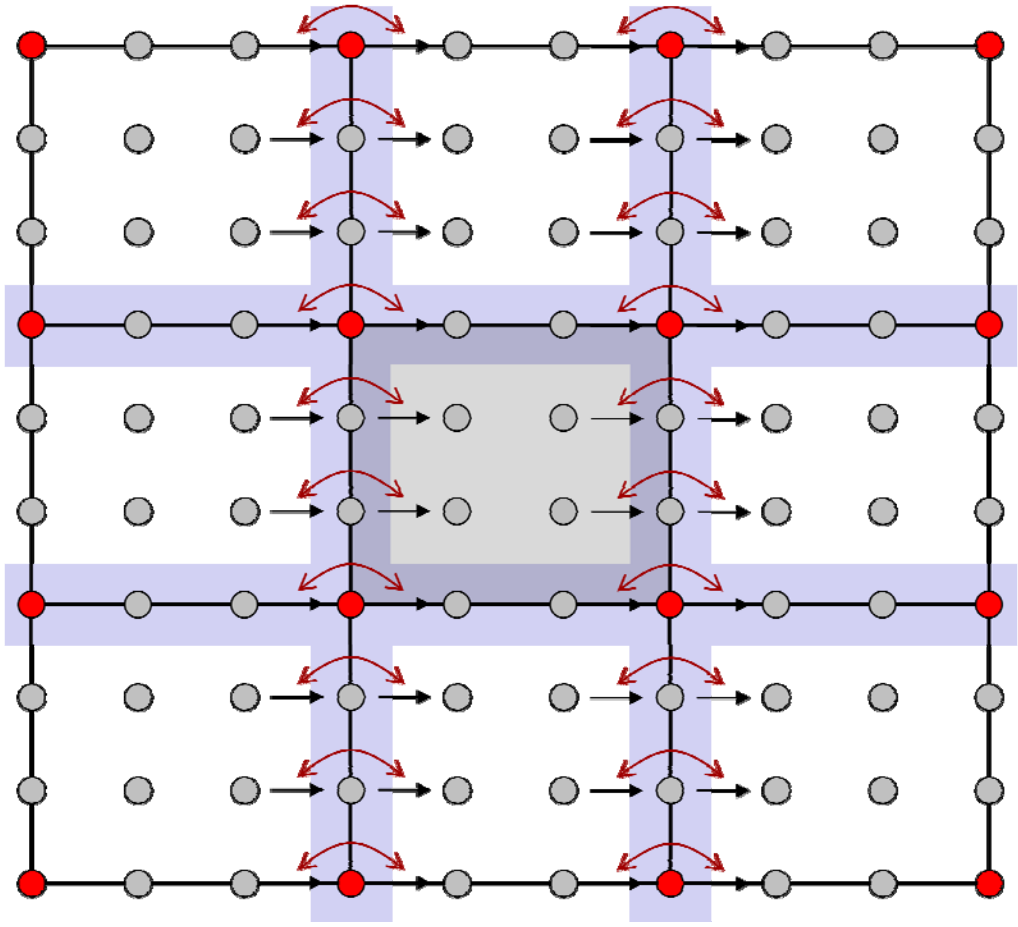
de Casteljau作图法



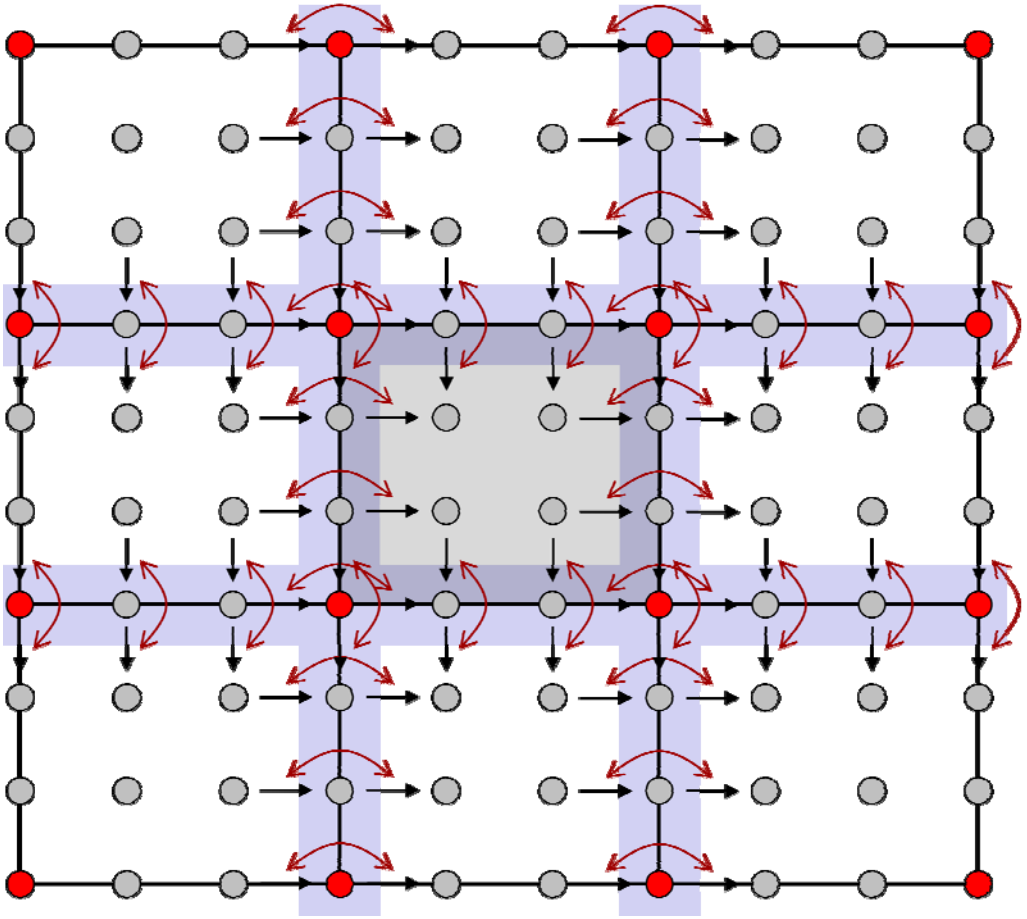
Bezier曲面片之间的拼接连续性



C^1 Continuity

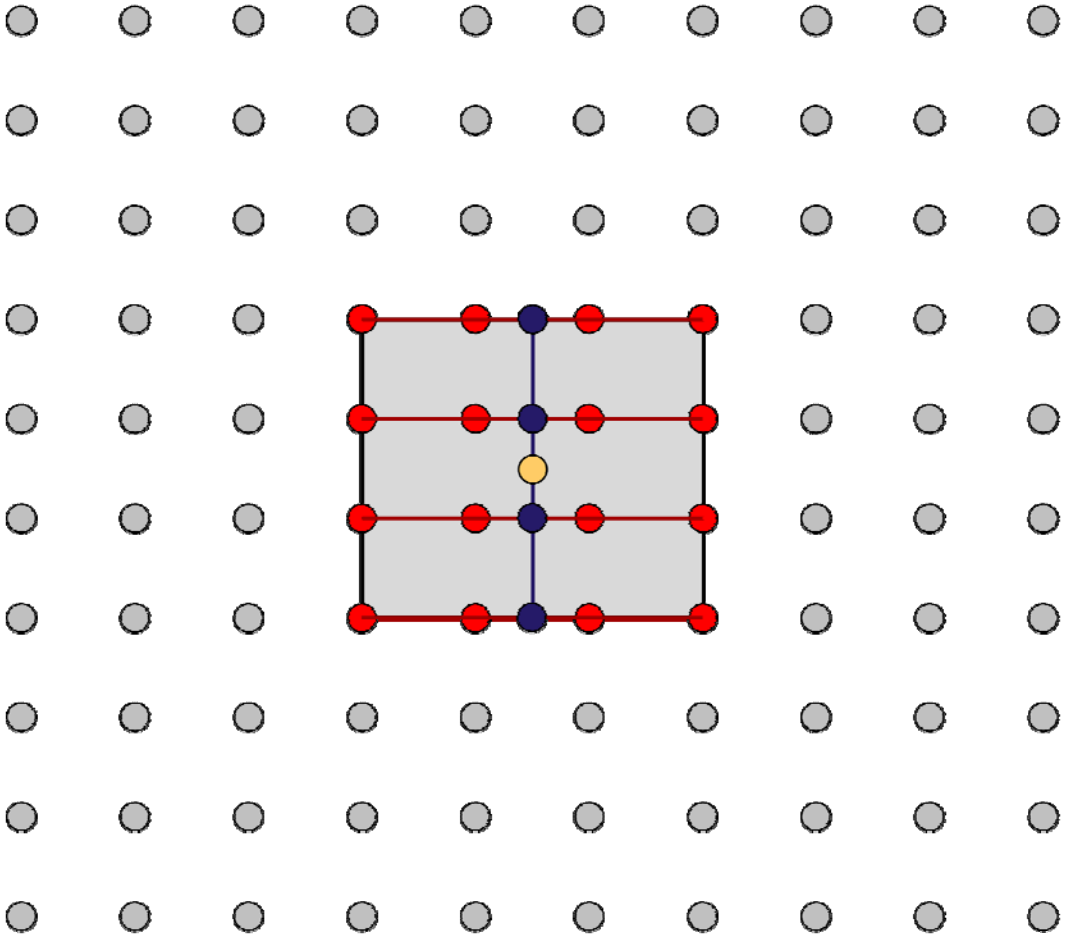


C^1 Continuity



其他曲面的定义完全同

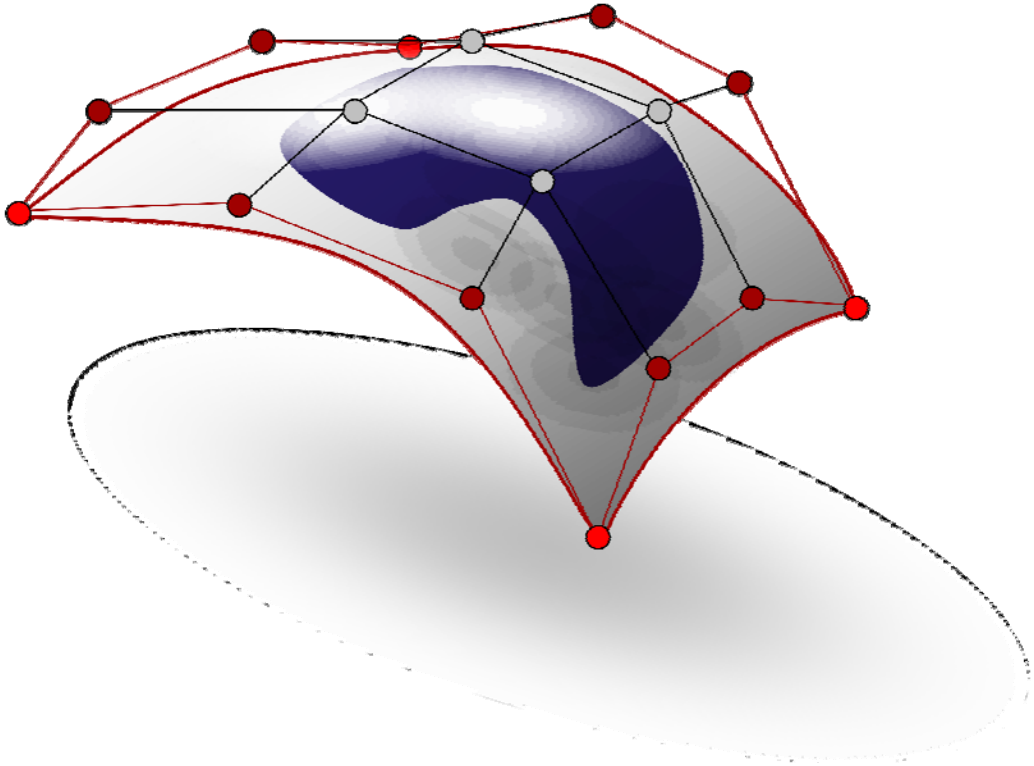
- B样条曲面
- 有理曲面
- NURBS曲面

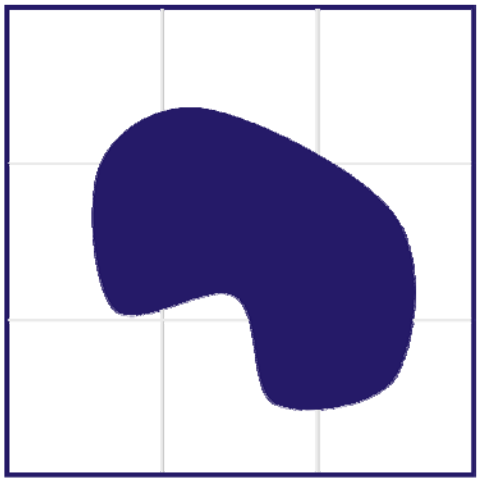


Trimmed NURBS曲面

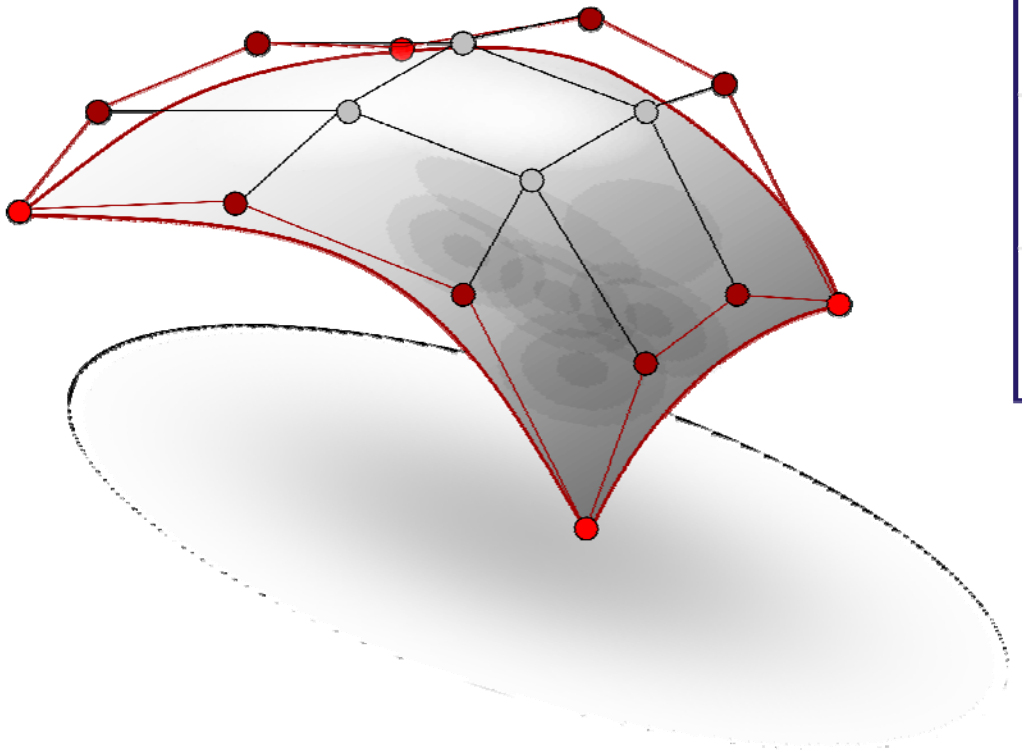
表达带“洞”或非矩形边界的曲面

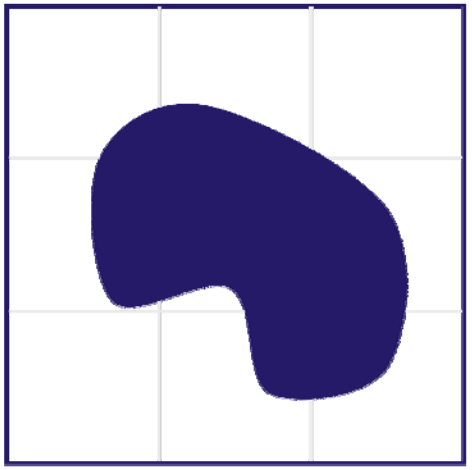
- 曲面上的曲线：使用参数域上的NURBS曲线来定义，然后复合得到曲面上的曲线



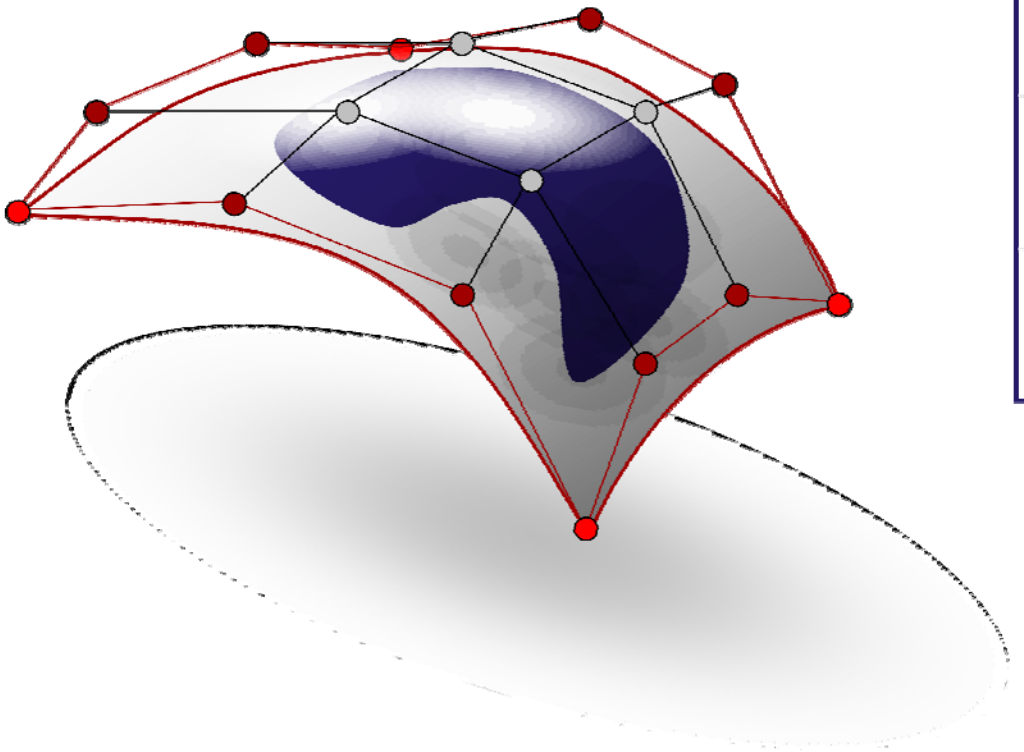


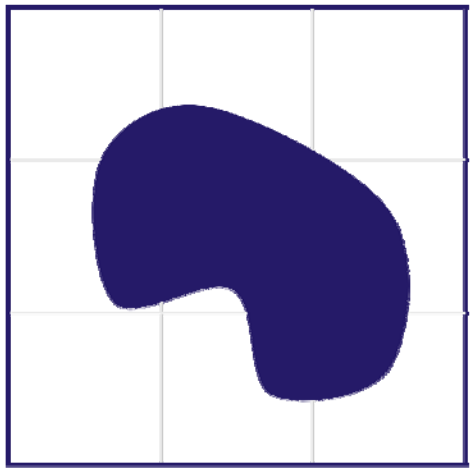
Curves-on-Surfaces (CONS)



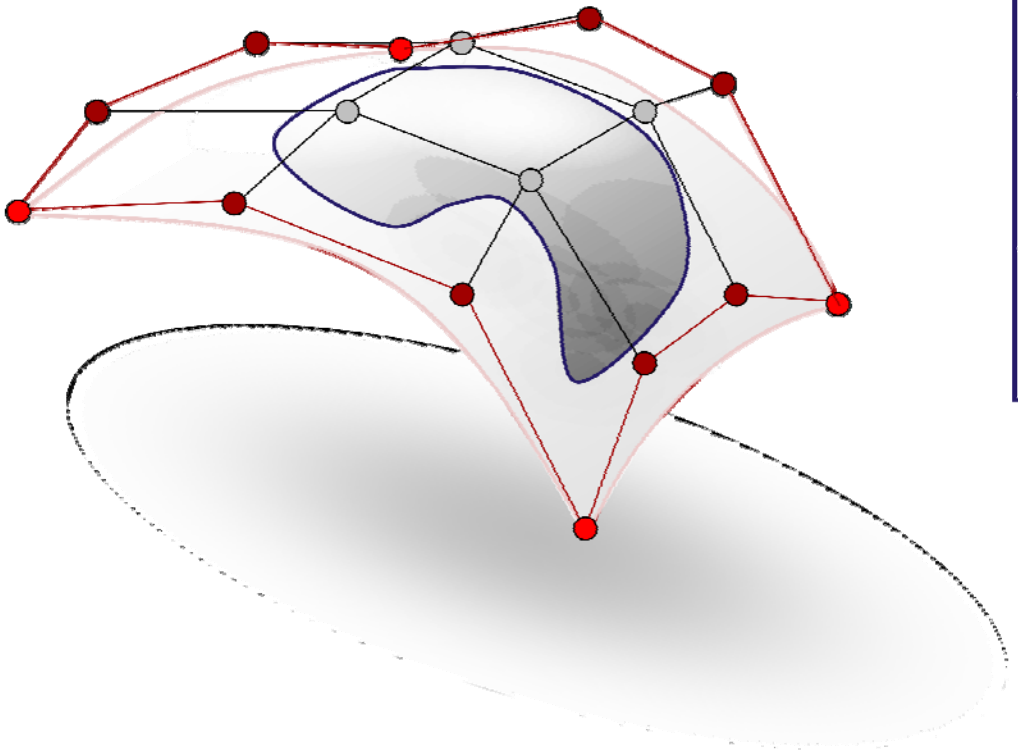


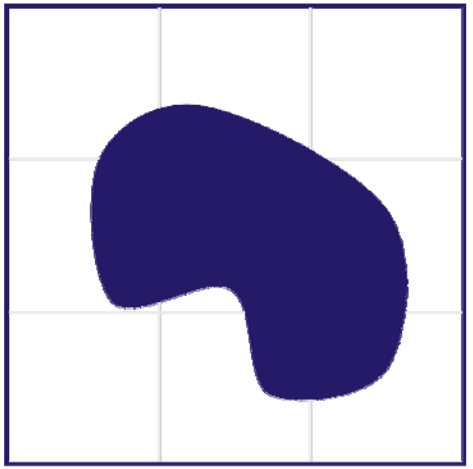
Curves-on-Surfaces (CONS)





Curves-on-Surfaces (CONS)

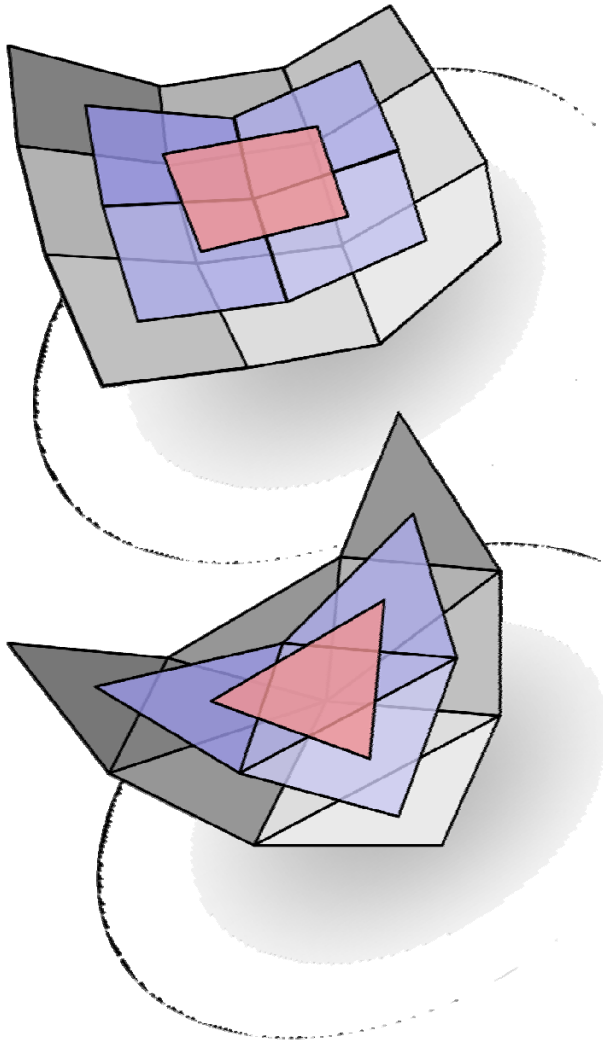




三角域上的Bezier曲面片

三角域的Bernstein-Bezier曲面片

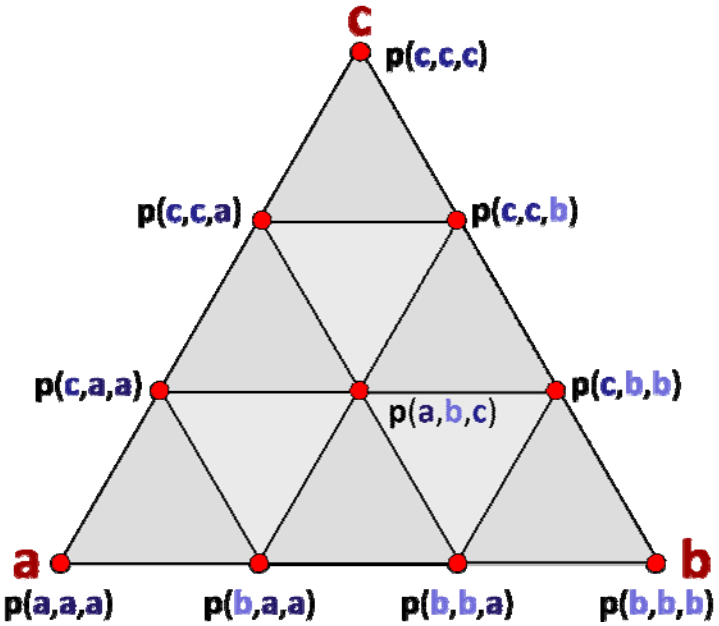
- 矩形域有时不方便
- 使用三角域来定义曲面片

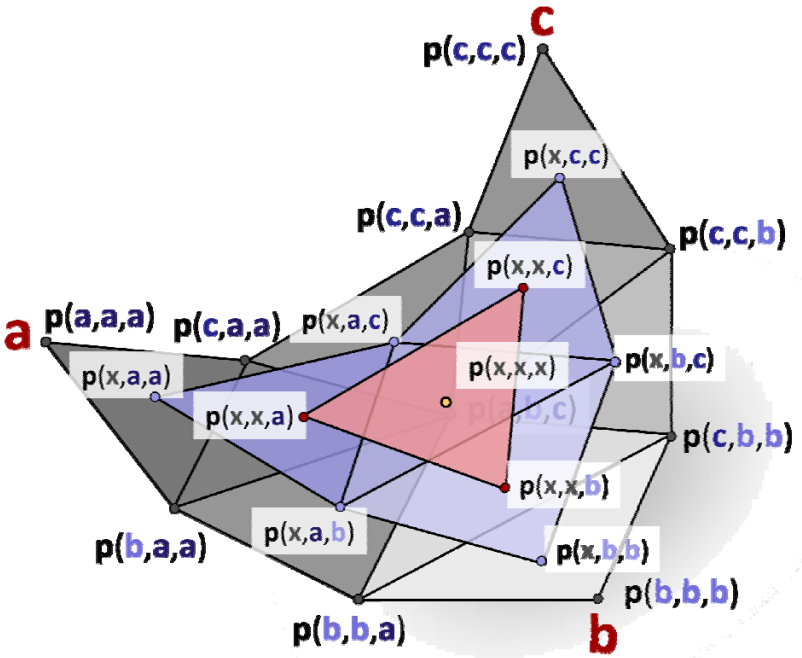


三角Bezier曲面片

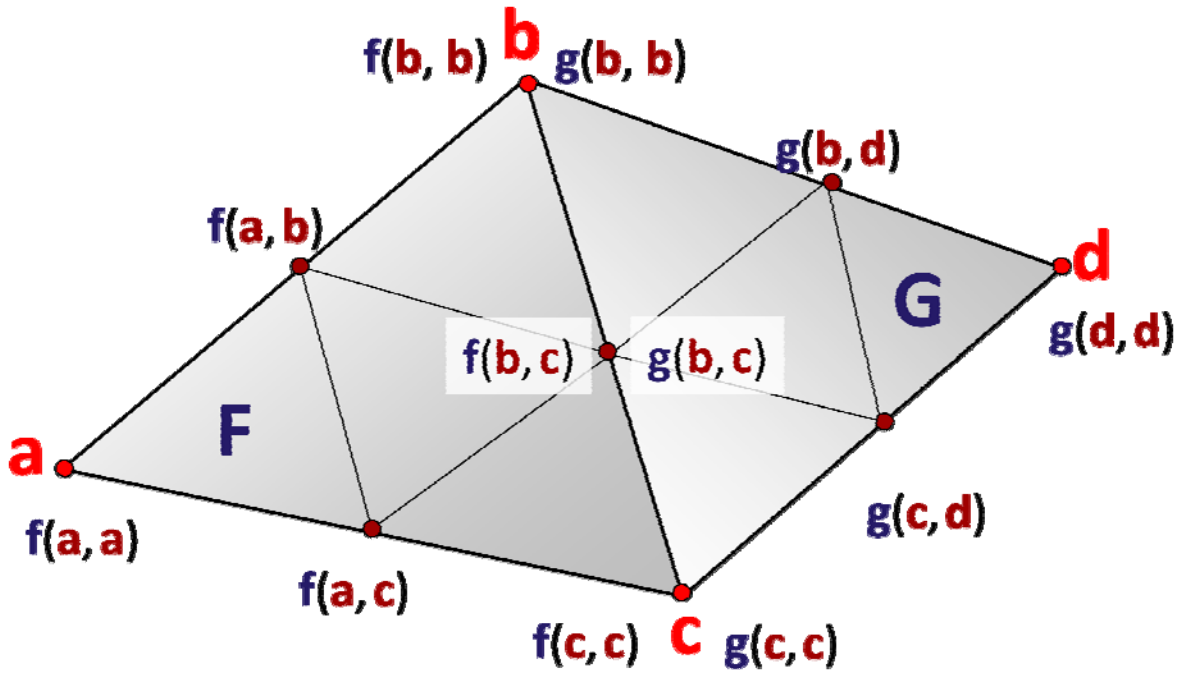
$$F(\boldsymbol{x}) = \sum_{\substack{i+j+k=n \\ i,j,k\geq 0}} \frac{n!}{i!\,j!\,k!}\alpha^i\beta^j\gamma^k\boldsymbol{p_{i,j,k}}$$

$$\boldsymbol{x} = \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma\boldsymbol{c},\ \alpha+\beta+\gamma = 1$$





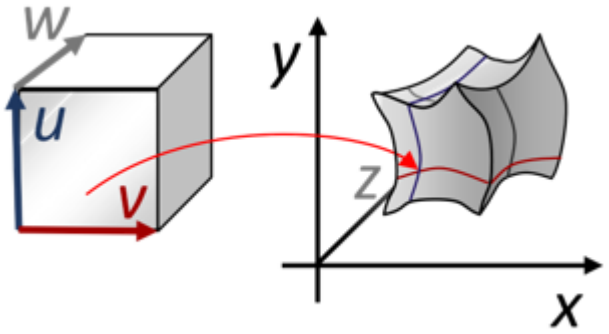
连续性



总结

- 张量积曲面
  - 两个独立方向的“曲线的曲线”
  - 性质大都类同于曲线的性质
  - 表达、公式形式比曲线情形复杂
  - 特殊问题：角点的光滑性

- 张量积体（三参数）
  - Bezier体



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谢　谢　！