中国科学技术大学
University of Science and Technology of China

GAMES 102在线课程

几何建模与处理基础

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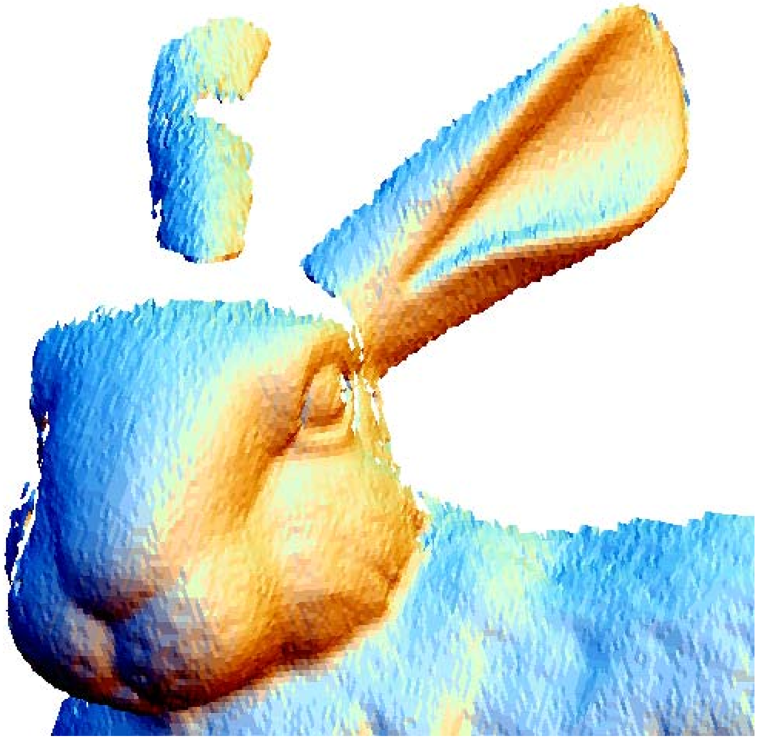
GAMES 102在线课程：几何建模与处理基础

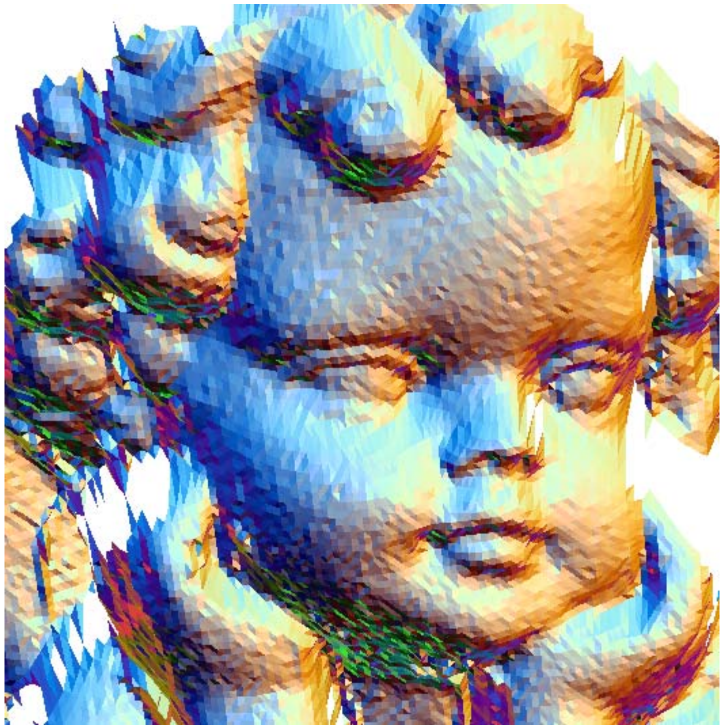
曲面去噪

网格曲面上的噪声

Meshes obtained from real world objects are often noisy.

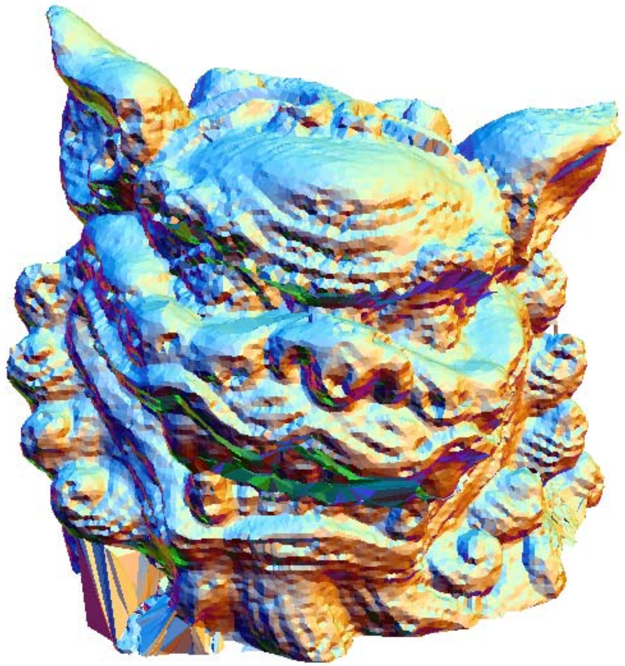


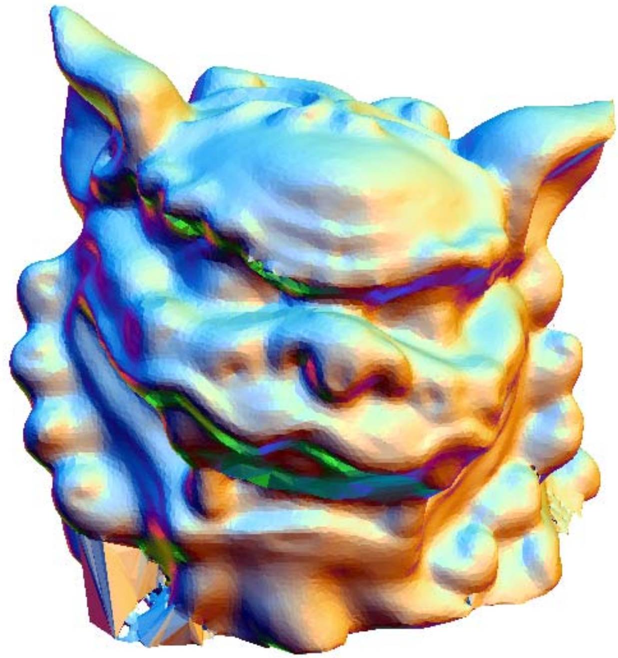
Stanford Bunny



Angel model

Mesh (surface) Denoising



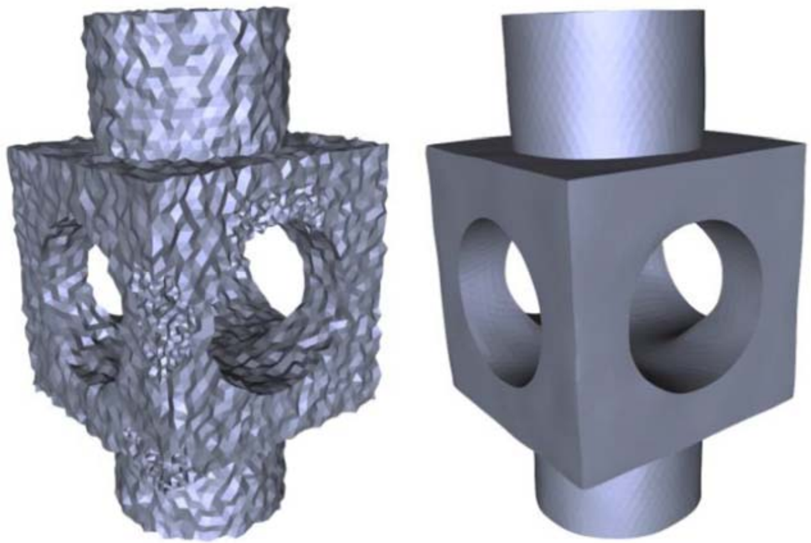


- Mesh denoising
- Mesh smoothing
- Mesh filtering
- Mesh improvement
- Surface fairing (*)

Image denoising

What is noise?

- High-frequent tiny parts
- Small bumps on the surface
- High curvature parts
- High fairing energy parts
- …

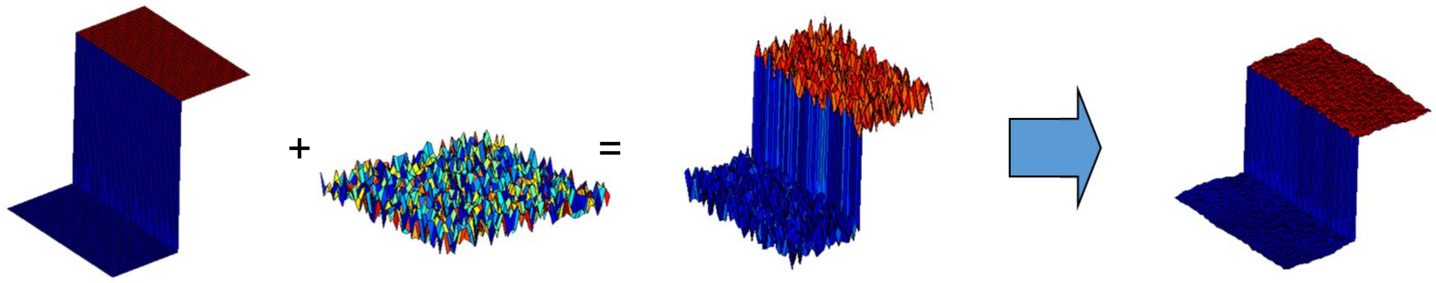


noise or feature?

No Precise Mathematical Definition!

Denoising / Smoothing [From Wiki]

- In statistics and image processing, to smooth a data set is to create an approximating function that attempts to capture important patterns in the data, while leaving out noise or other fine-scale structures/rapid phenomena.
  - Eliminate high frequency
  - Preserve global features



Smoothing / Denoising Problem

- Input: M （含噪声的网格曲面）
- Output: M^0 （无噪声的网格曲面）

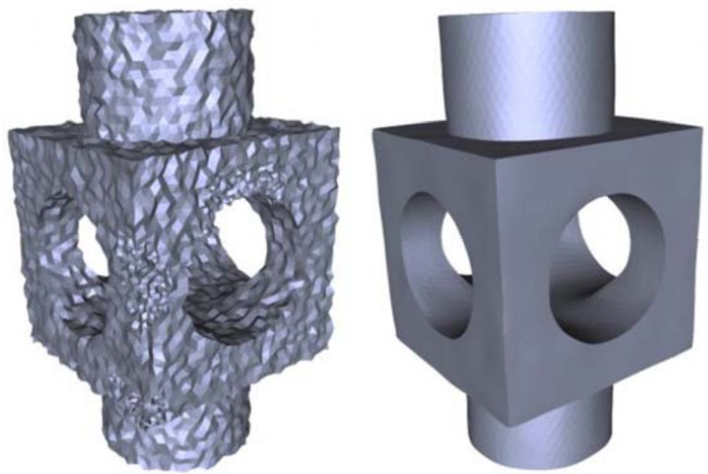
- Denoising model:

$$M = M^0 + \varepsilon$$

- Challenges
  - Both the ideal mesh M^0 and the noise ε are unknown
  - “ill-posed” problem!

Mesh smoothing

- 假定：网格顶点的数据及连接关系不变
- 问题转化为：求顶点的新位置，使得“噪声”减少！
  - 顶点进行适当的扰动或偏移



- 问题：顶点偏移的方向？

Mesh Smoothing Problem

- Input: M （含噪声的网格曲面）
- Output: M^0 （无噪声的网格曲面）

- Mesh smoothing model:

$$\boldsymbol{v} = \boldsymbol{v}^0 + \varepsilon \boldsymbol{n} \quad \text{(for all } \boldsymbol{v} \in M\text{)}$$

- Questions:
  - What is the displacement vector $\boldsymbol{n}$ for vertex $\boldsymbol{v}$?

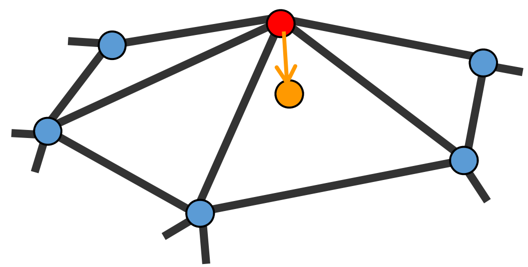
Mesh Smoothing Model

$$\boldsymbol{v} = \boldsymbol{v}^0 + \varepsilon \boldsymbol{n} \quad \text{(for all } \boldsymbol{v} \in M\text{)}$$

- Displacement vector $\boldsymbol{n}$
  - The normal of $\boldsymbol{v}^0$? -- unknown! ill-posed too!
  - The normal of $\boldsymbol{v}$: doable
- New model:

$$\boldsymbol{v}^0 = \boldsymbol{v} - \varepsilon \boldsymbol{n}$$

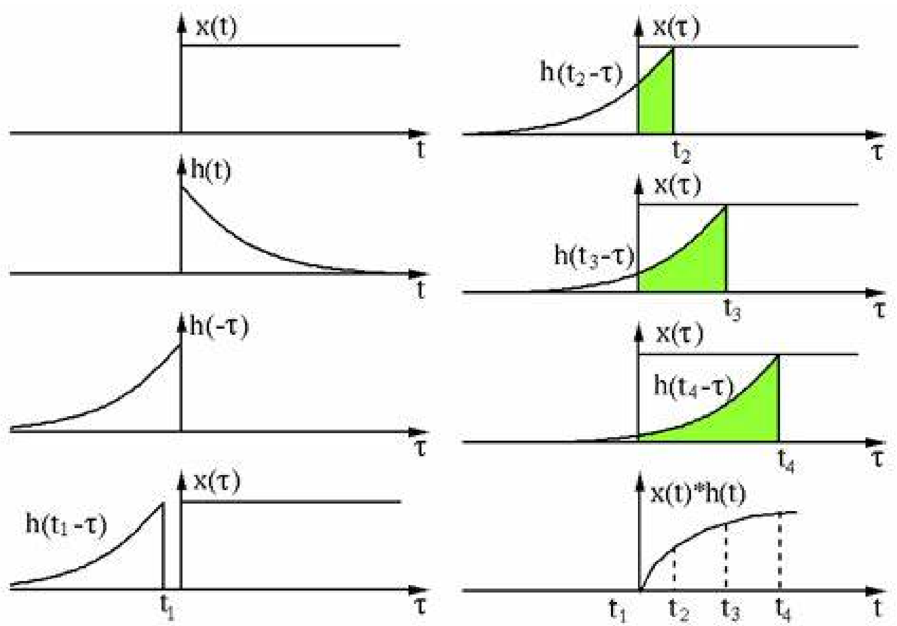
- Key: ε=?



Filtering

- Convolution $(x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

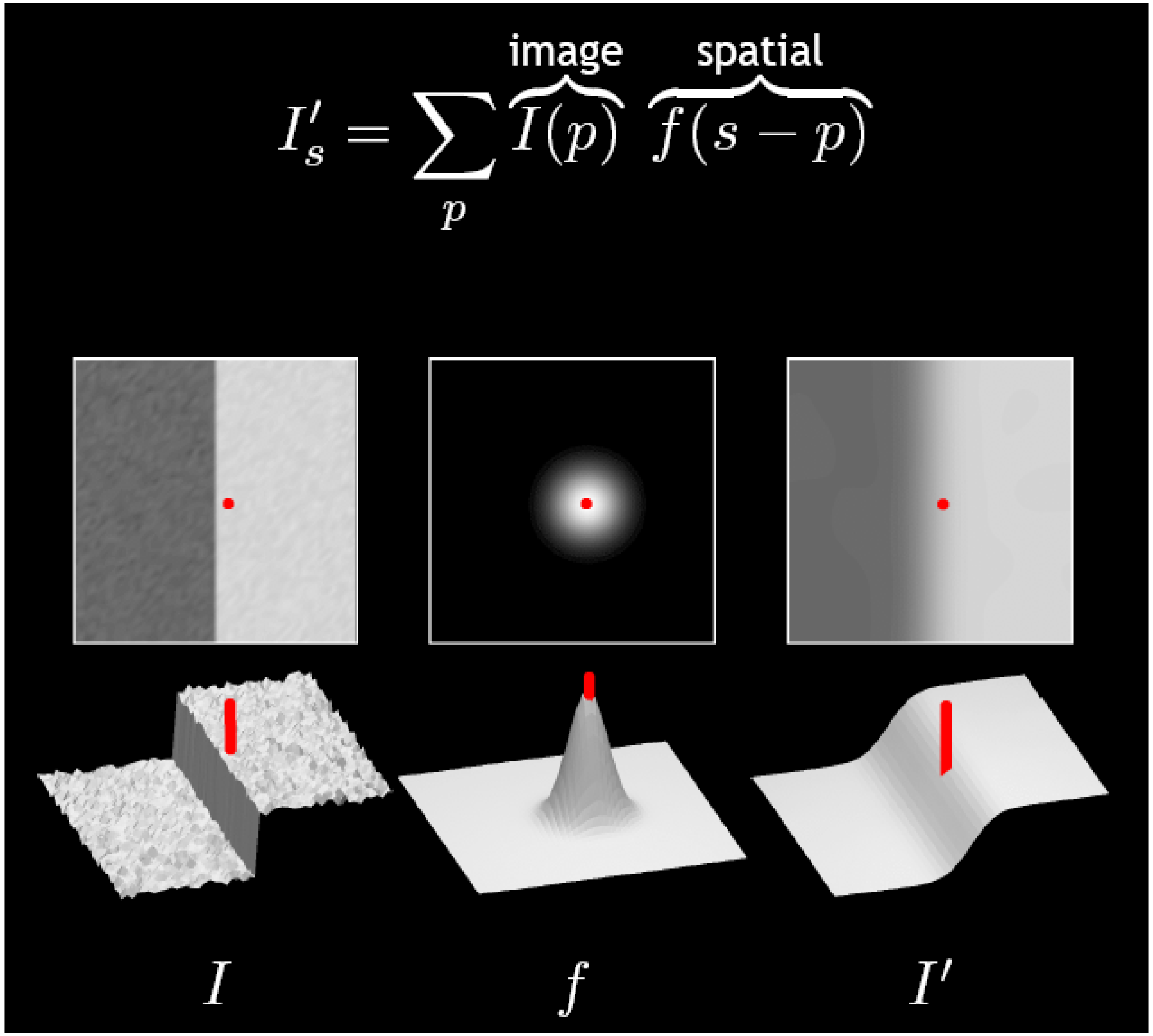
- Discrete form $(x * h)(t) \ = \sum_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)$



几何意义：将函数$h(t)$作为权来对$x(t)$进行加权平均（滤波）

- 将$x(t)$的局部信息进行混合平均

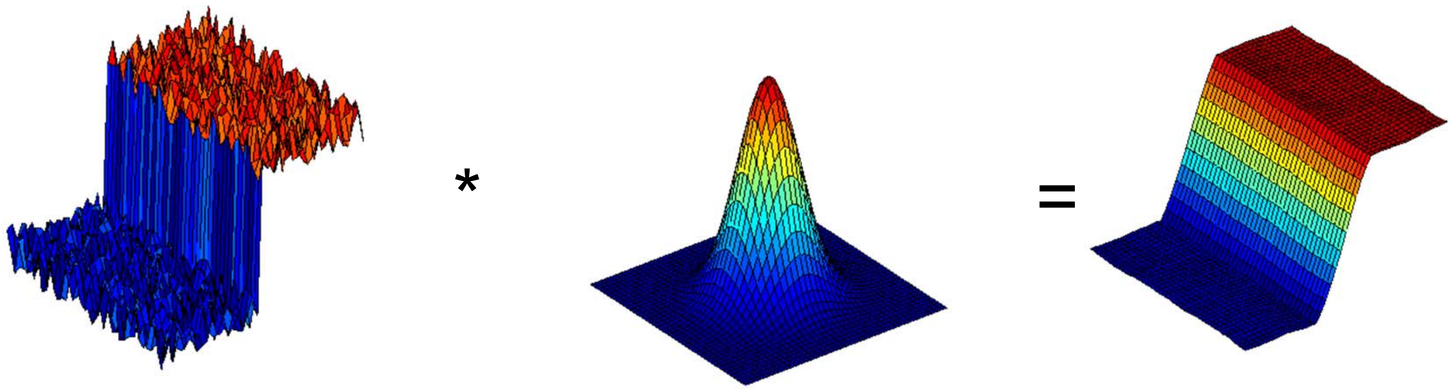
Image Filtering



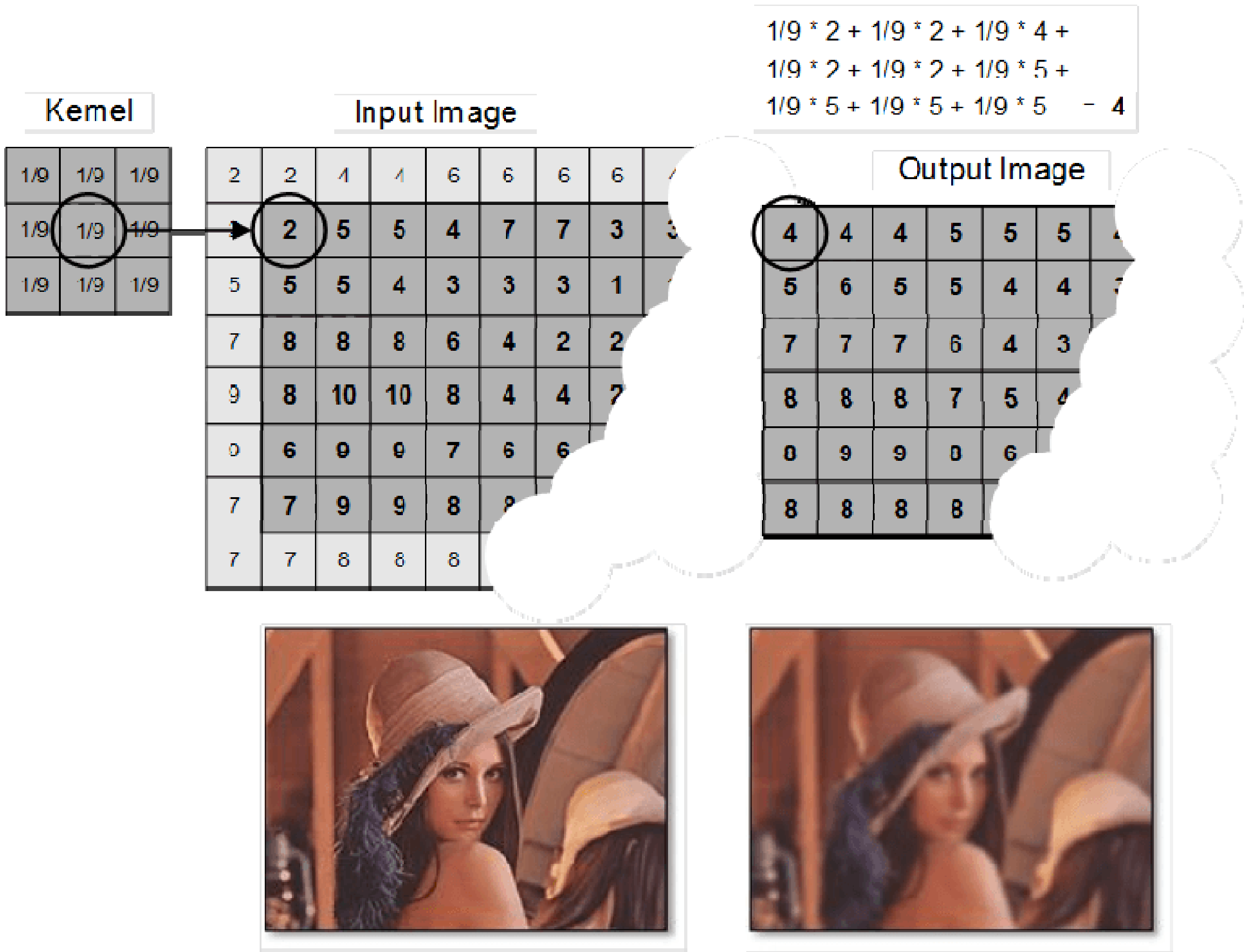
Gaussian Filtering

- 使用Gauss函数作为权函数

$$I'(u) = \sum_{p \in N(u)} e^{-\frac{\|u-p\|^2}{2\sigma^2}} I(p)$$

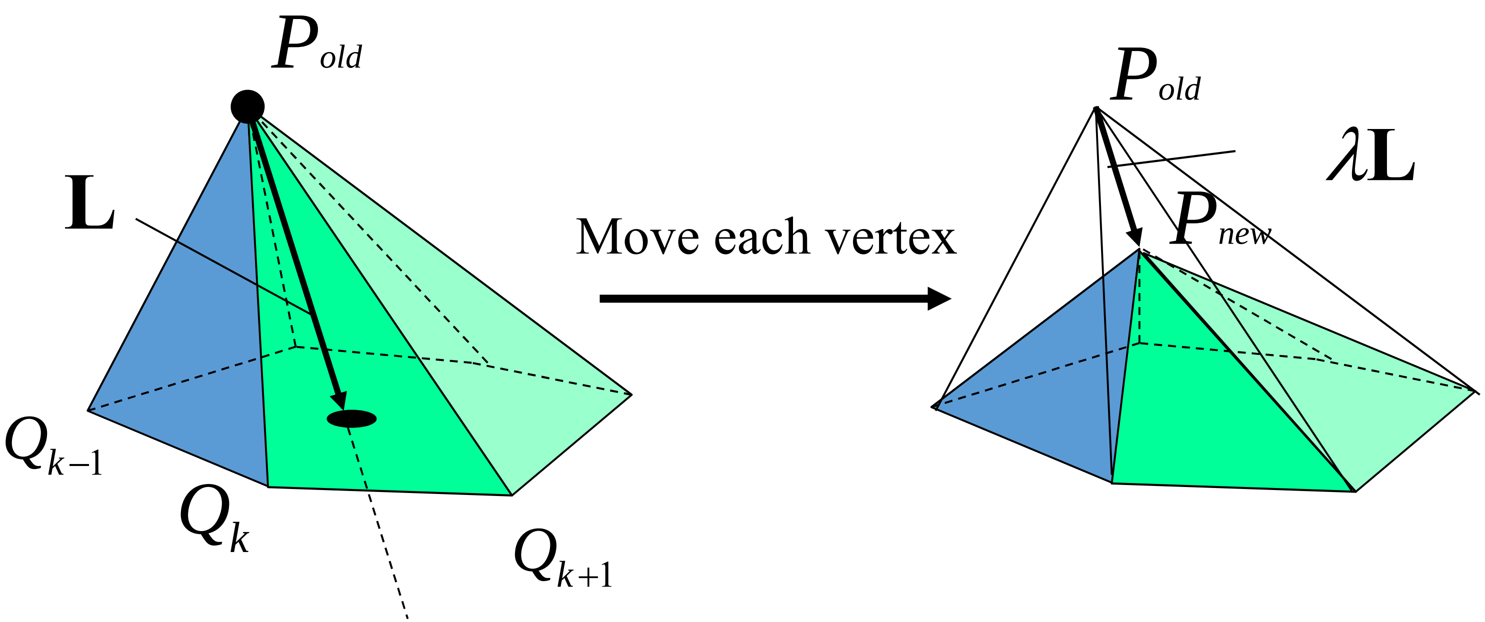


Discrete Filtering (mask)



Mesh Vertex Filtering:
Laplacian operator / Umbrella Operator

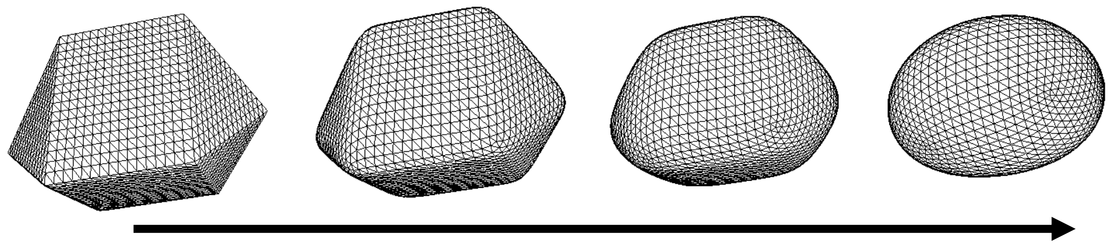
$$P_{new} \leftarrow P_{old} + \lambda \mathbf{L}(P_{old})$$



滤波对象

- Vertex
- Normal
- Curvature
- Color
- Other physical properties (texture, albedo, …)

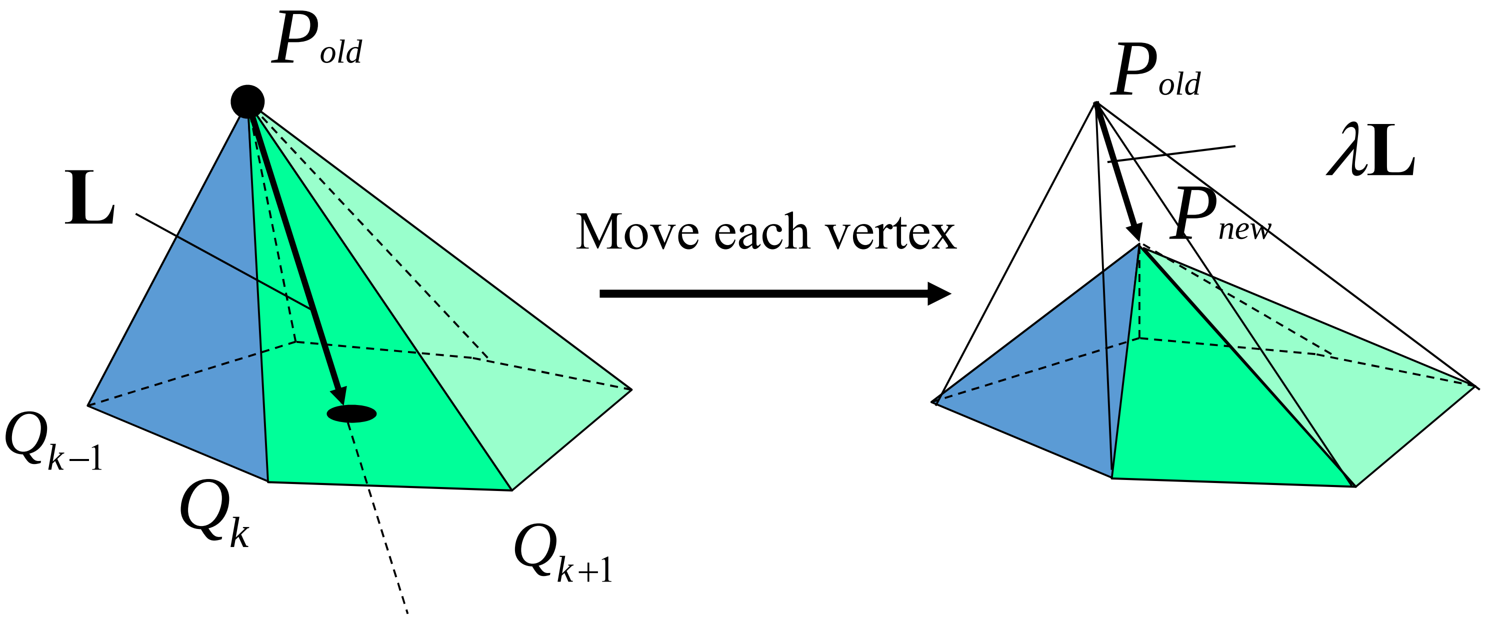
- Challenges:
    - Iteration number
    - Shrinkage



1. Vertex Filtering

1.1 Laplacian Smoothing

$$P_{new} \leftarrow P_{old} + \lambda \mathbf{L}(P_{old})$$



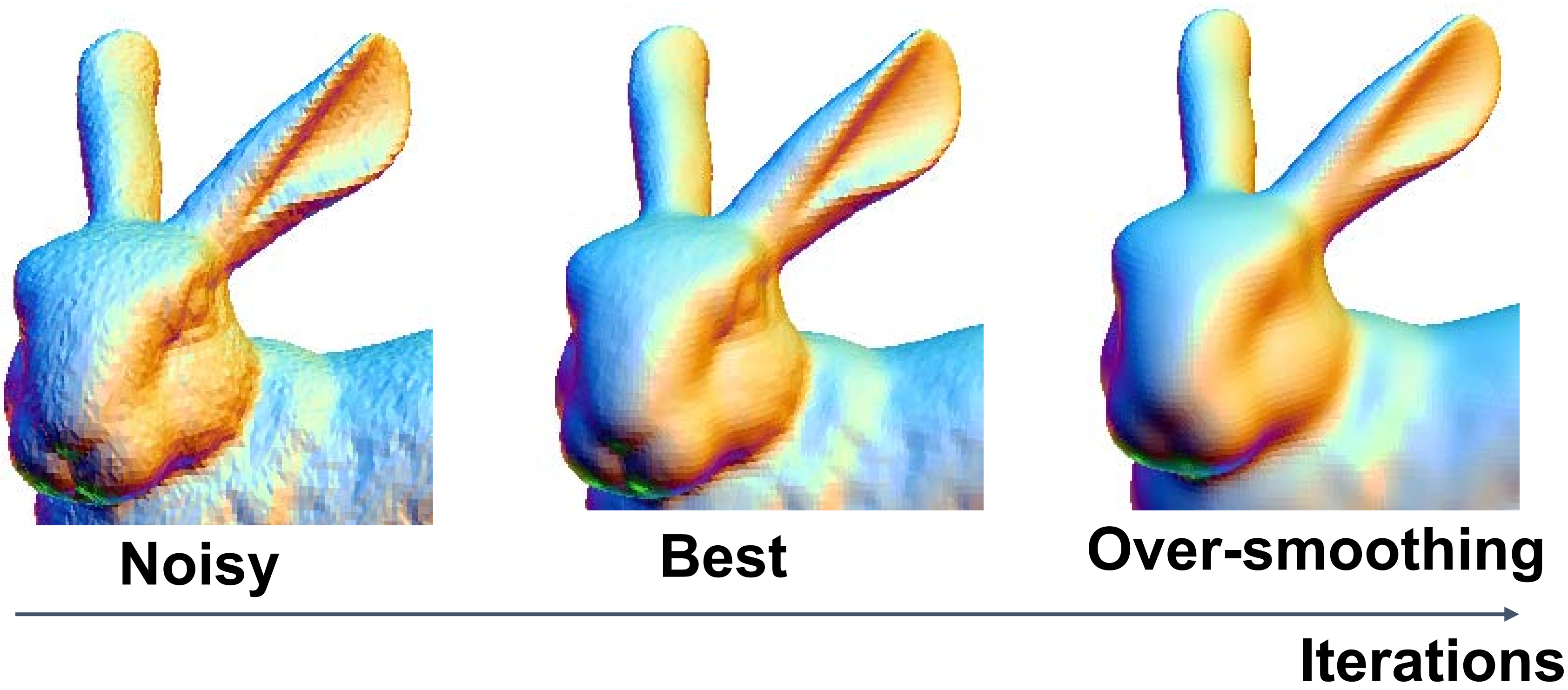
Laplacian Smoothing

$$P^{new} = P^{old} + \lambda L\left(P^{old}\right)$$

- Equivalent to box filter in signal processing
- Apply to all vertices on mesh
- Typically repeat several times
- Can describe as energy minimization
  - Energy = sum of squared edge lengths in mesh
  - Parameter $\lambda > 0$ controls convergence "speed"

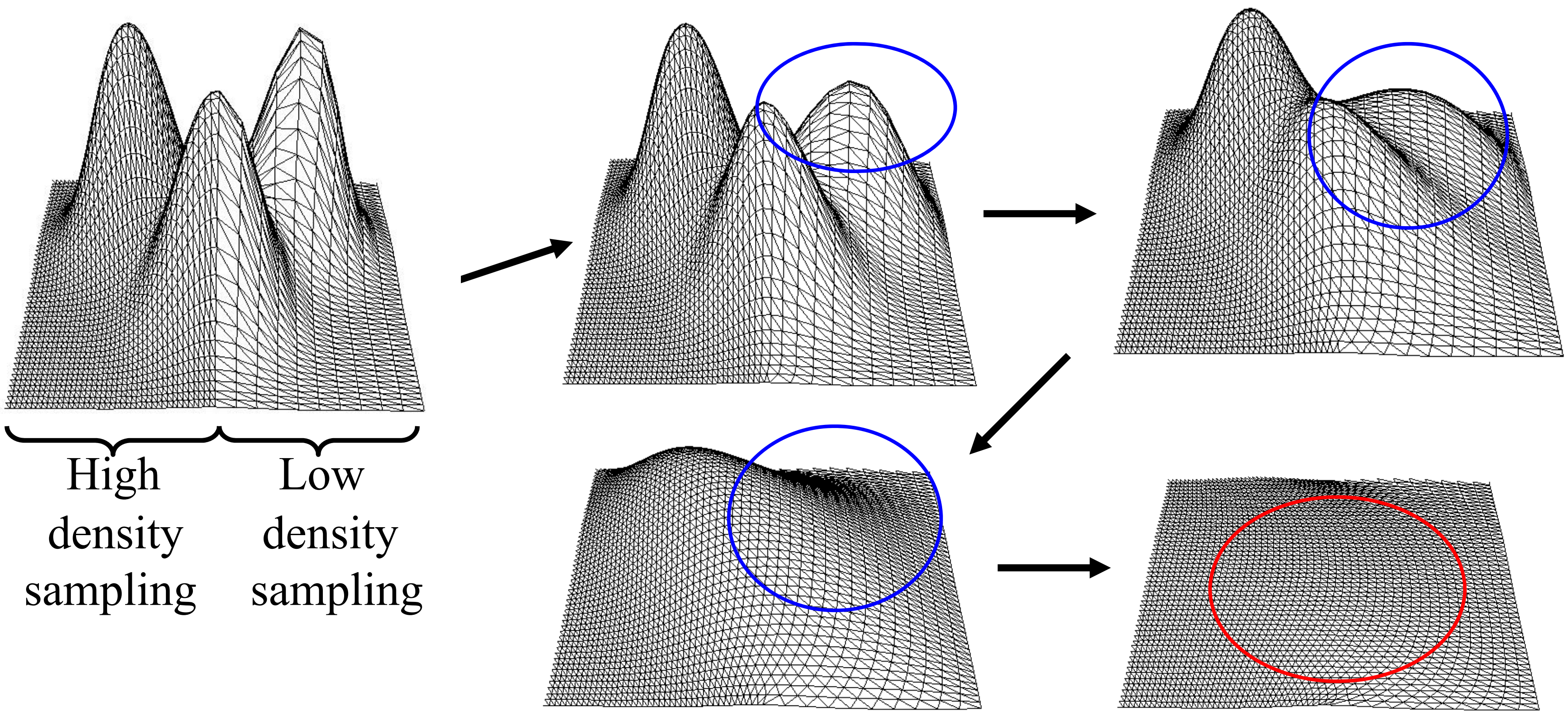
Problem of Over-smoothing

How to find appropriate λ and number of iterations?



Shrinkage Problem

- Increases mesh regularity
- Develops unnatural deformations



Improved Laplacian

- Laplacian

$$P^{new} = P^{old} + \lambda L\left(P^{old}\right)$$

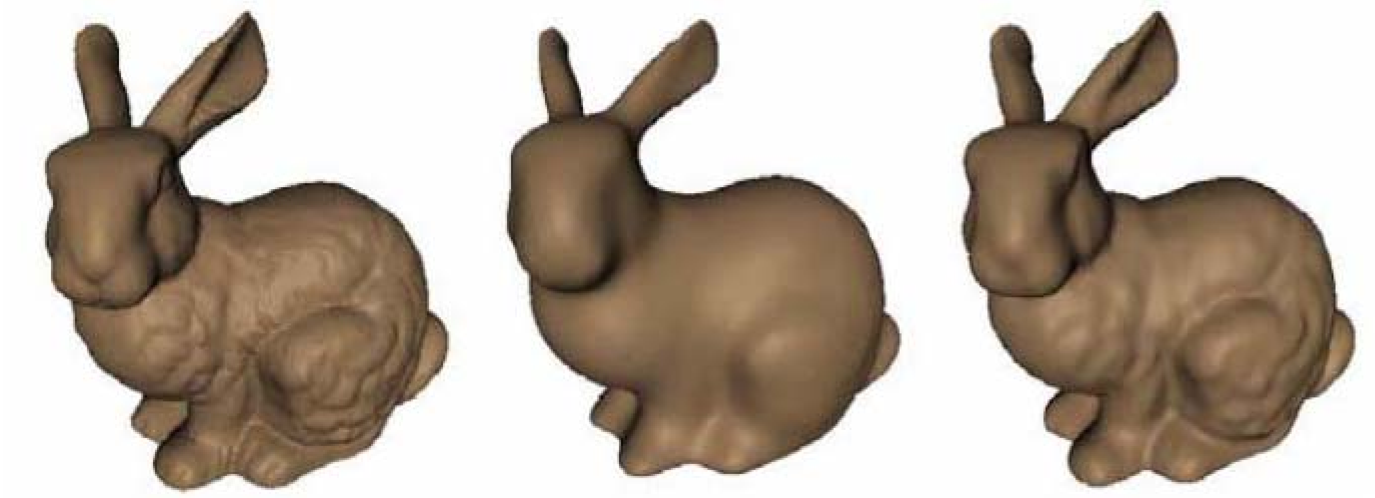
- Taubin'95
  - Laplacian + Expansion

$$P^{new} = P^{old} - (\mu - \lambda) L\left(P^{old}\right) - \mu\lambda L^2\left(P^{old}\right), \mu > \lambda > 0$$

- Bilaplacian
  - Special case of Taubin's

$$P^{new} = P^{old} + \lambda L^2\left(P^{old}\right)$$

Comparison



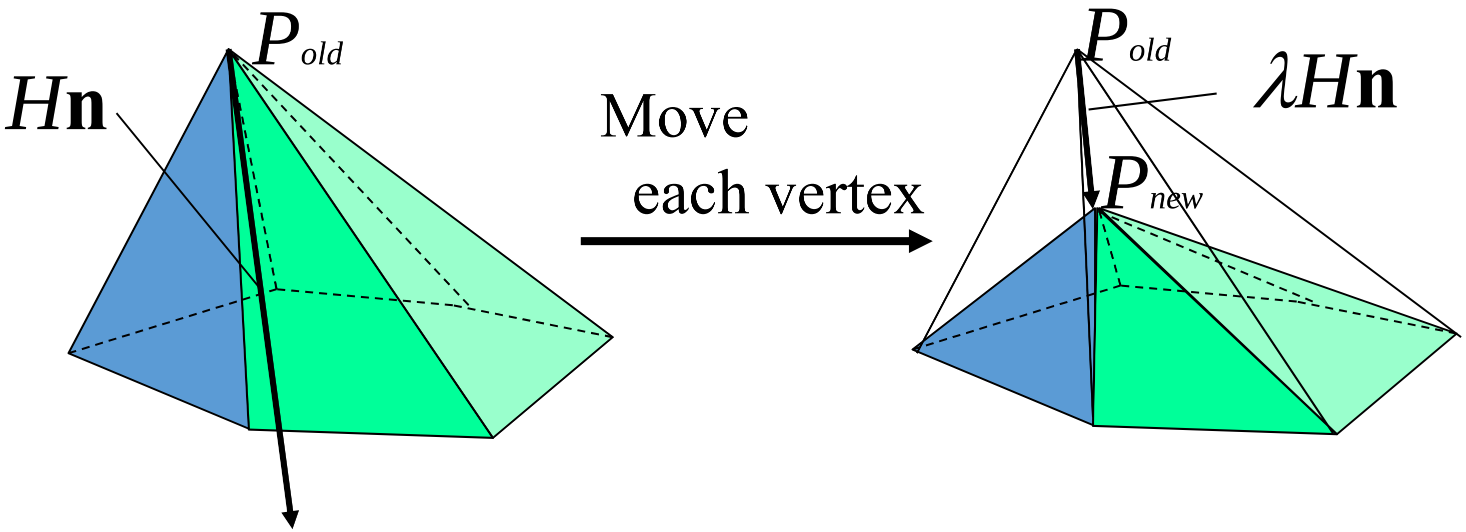
- Drawbacks
  - Slow
  - No stopping criteria

1.2 Mean Curvature Flow

$$P_{new} \leftarrow P_{old} + \lambda \boxed{H(P_{old})} \boxed{\mathbf{n}(P_{old})}$$

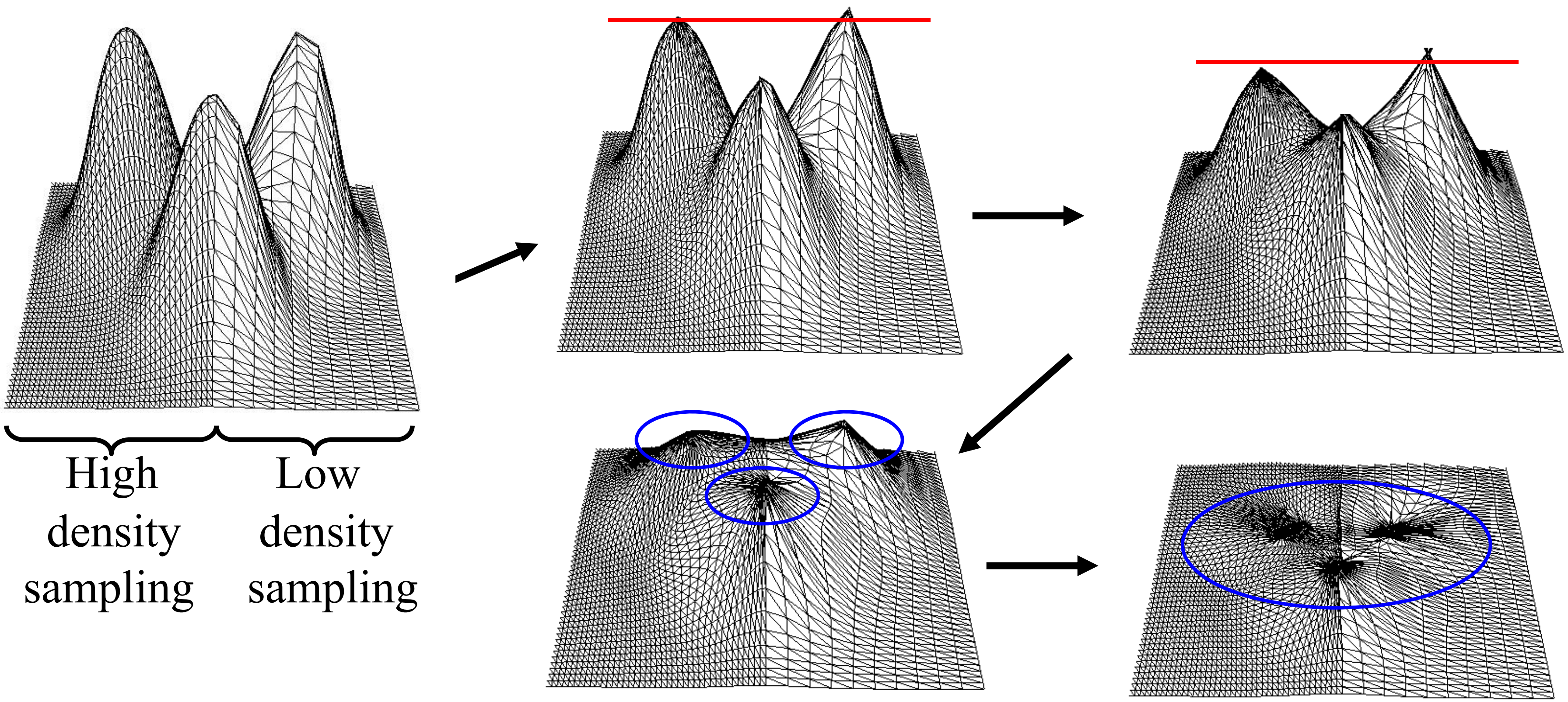
Speed = discrete mean curvature

Direction = normal



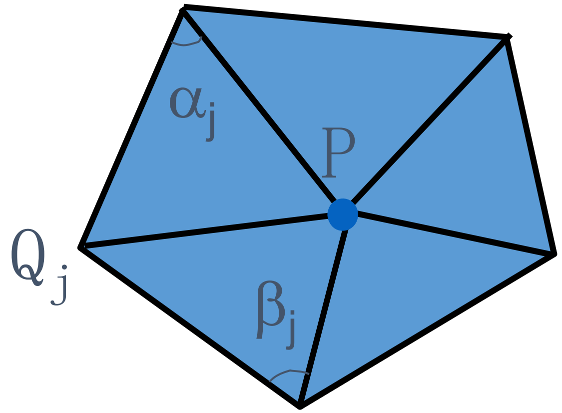
Mean Curvature Filtering

- Increases mesh irregularity.
- Doesn't develop unnatural deformations



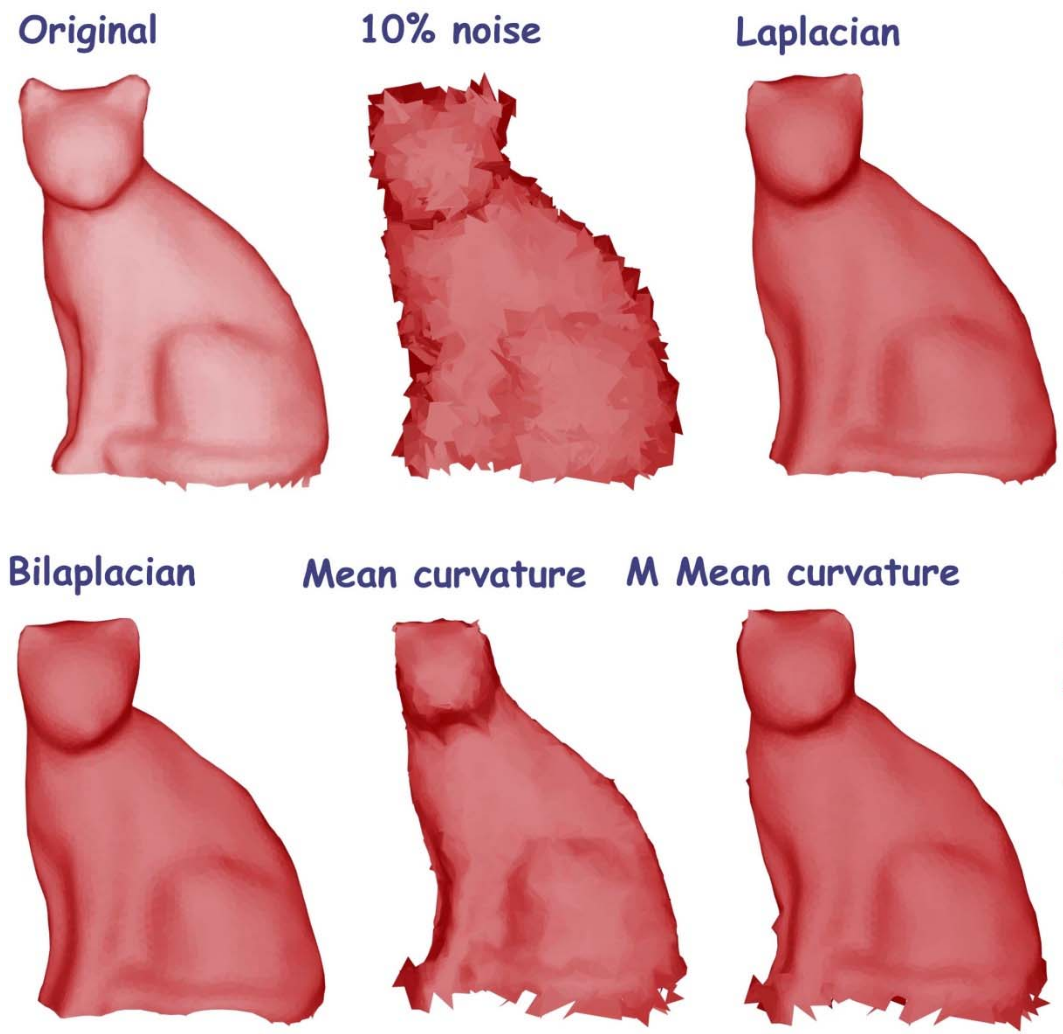
Discrete Mean Curvature

$$H\mathbf{n} = \frac{\nabla_P \mathrm{A}}{2\mathrm{A}}$$



$$H\mathbf{n} = \frac{1}{4\mathrm{A}} \sum_j (\cot \alpha_j + \cot \beta_j)(\mathbf{P} - \mathbf{Q}_j)$$

Comparisons



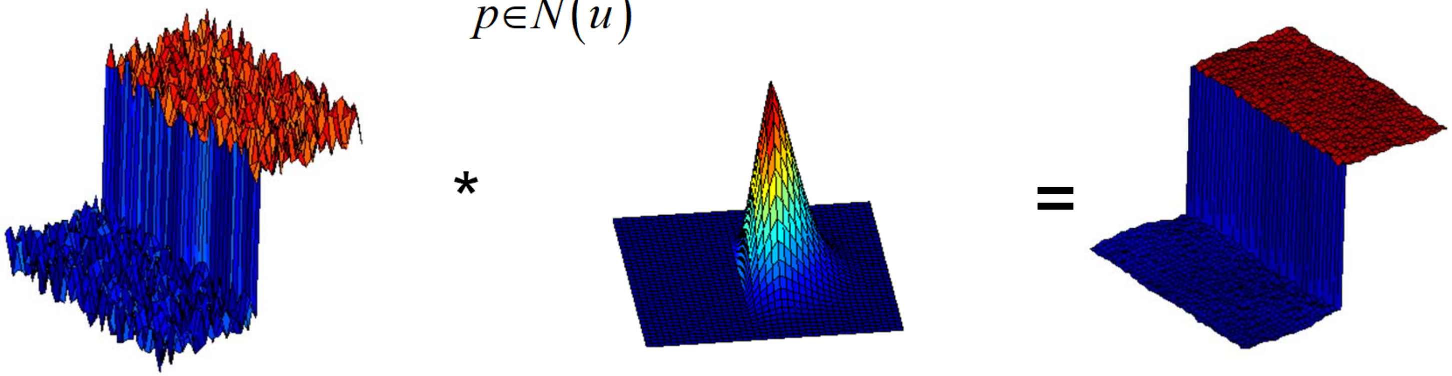
1.3 Bilateral filtering

Denoise

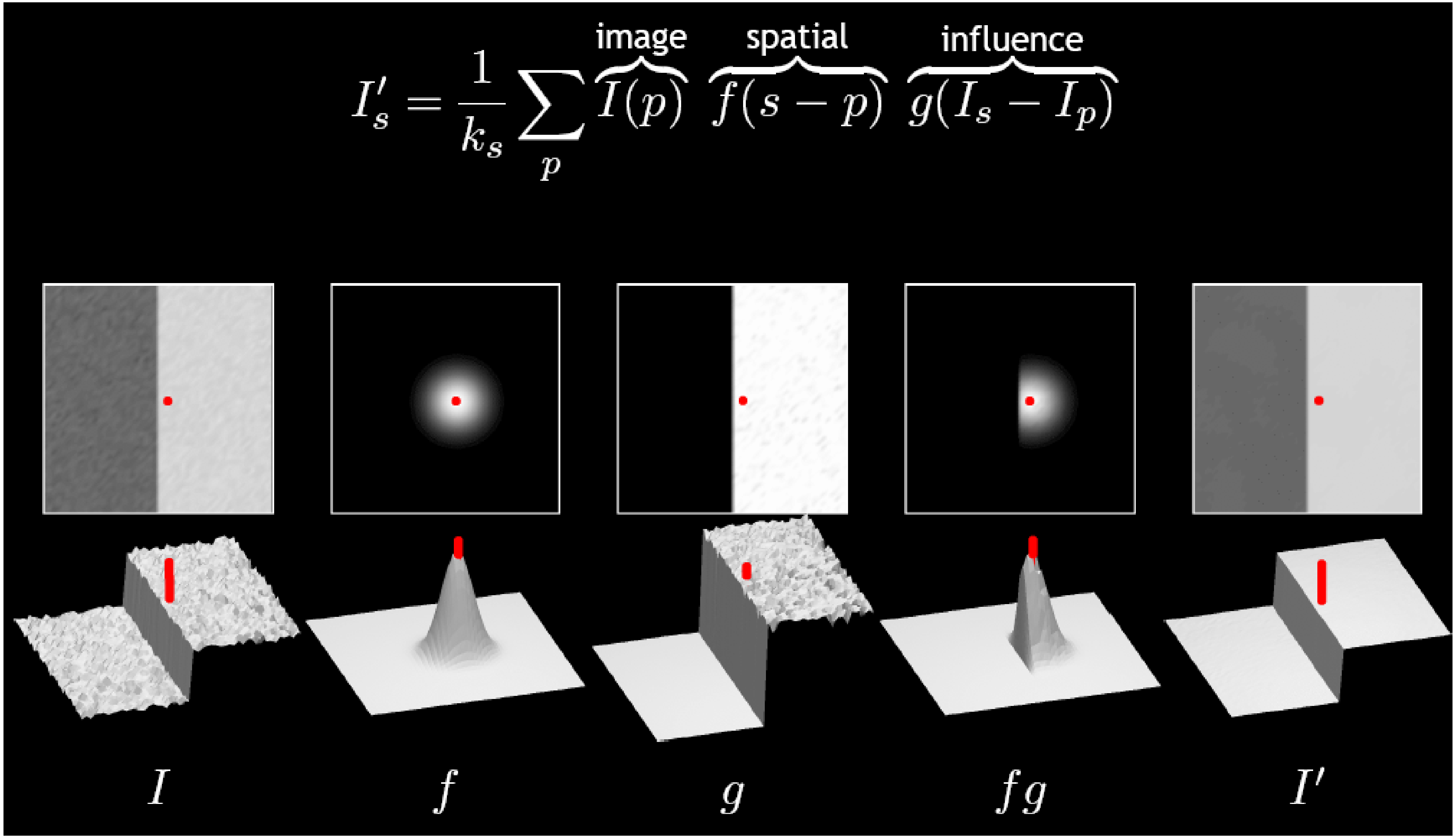
Feature preserving

$$I'(u) = \frac{\sum_{p\in N(u)} e^{-\frac{\|u-p\|^2}{2\sigma_c^2}} e^{-\frac{|I(u)-I(p)|}{2\sigma_s^2}} I(p)}{\sum_{p\in N(u)} e^{-\frac{\|u-p\|^2}{2\sigma_c^2}} e^{-\frac{|I(u)-I(p)|}{2\sigma_s^2}}}$$

Normalization

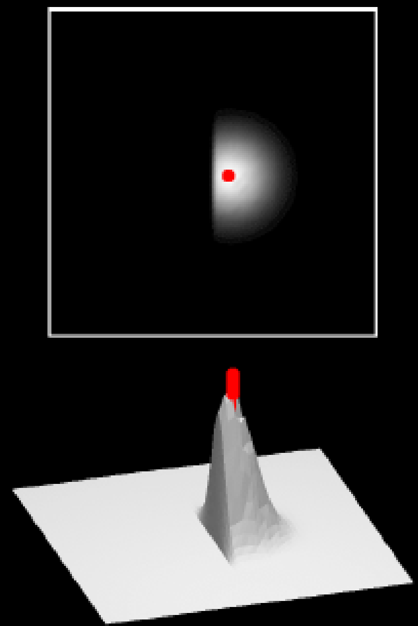


Bilateral filtering



$$I'_s = \frac{1}{k_s} \sum_p \overbrace{I(p)}^{\text{image}} \; \overbrace{f(s-p)}^{\text{spatial}} \; \overbrace{g(I_s - I_p)}^{\text{influence}}$$

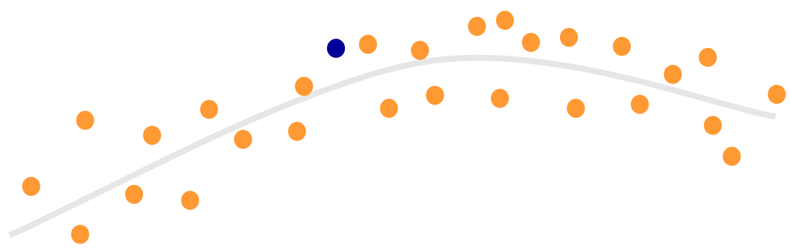
$$k_s = \sum_p f(s-p)\, g(I_s - I_p)$$



Bilateral filtering of meshes

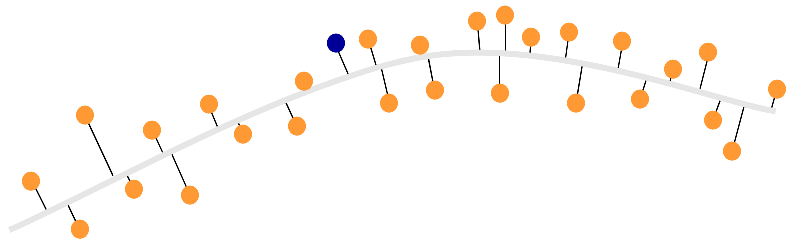
[Siggraph 2003]

Bilateral filtering of meshes



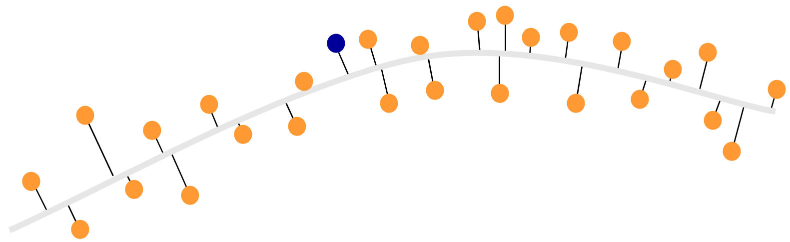
Bilateral filtering of meshes

- Height above surface is equivalent to the gray level values in images



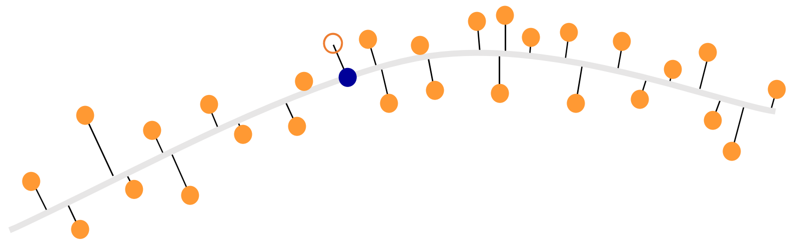
Bilateral filtering of meshes

- Height above surface is equivalent to the gray level values in images
- Apply the bilateral filter to heights



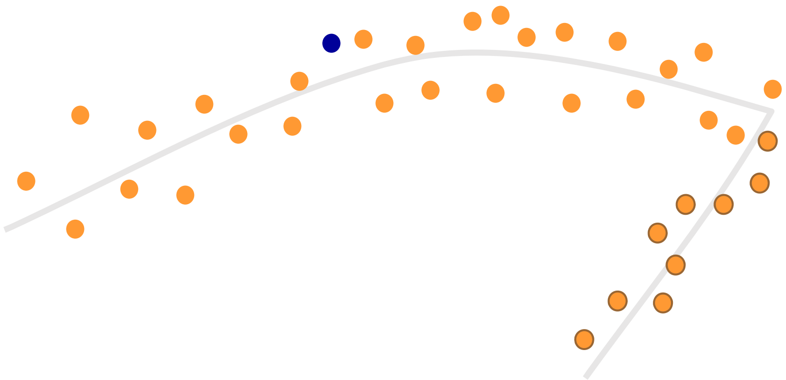
Bilateral filtering of meshes

- Height above surface is equivalent to the gray level values in images
- Apply the bilateral filter to heights
- Move the vertex to its new height



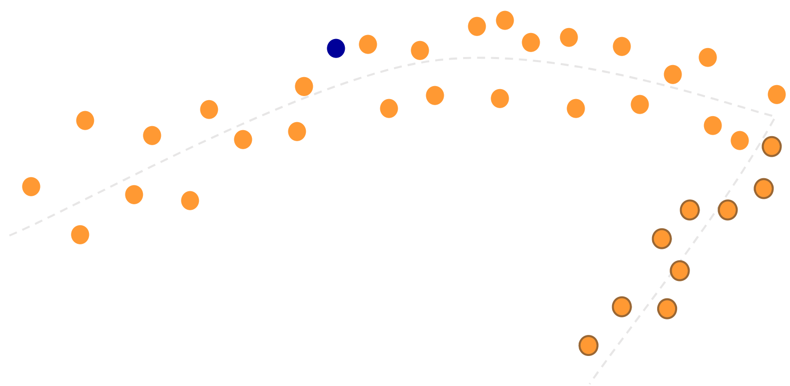
Bilateral filtering of meshes

- Height above surface is equivalent to the gray level values in images
- Apply the bilateral filter to heights
- Move the vertex to its new height
- In practice:
  - Sharp features



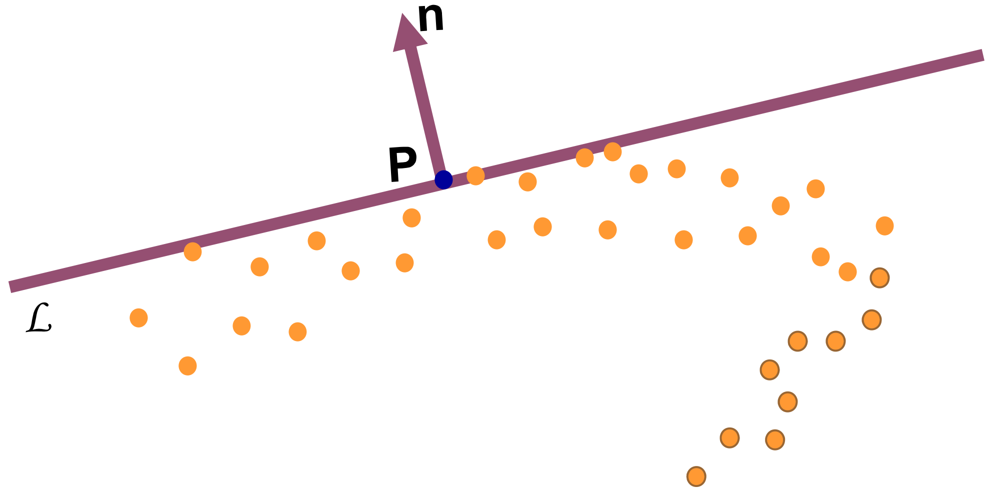
Bilateral filtering of meshes

- Height above surface is equivalent to the gray level values in images
- Apply the bilateral filter to heights
- Move the vertex to its new height
- In practice:
  - Sharp features
  - The noise-free surface is unknown



Solution

- A plane that passes through the point is the estimator to the smooth surface
- Plane $\mathcal{L}$=(**p**,**n**)



Solution

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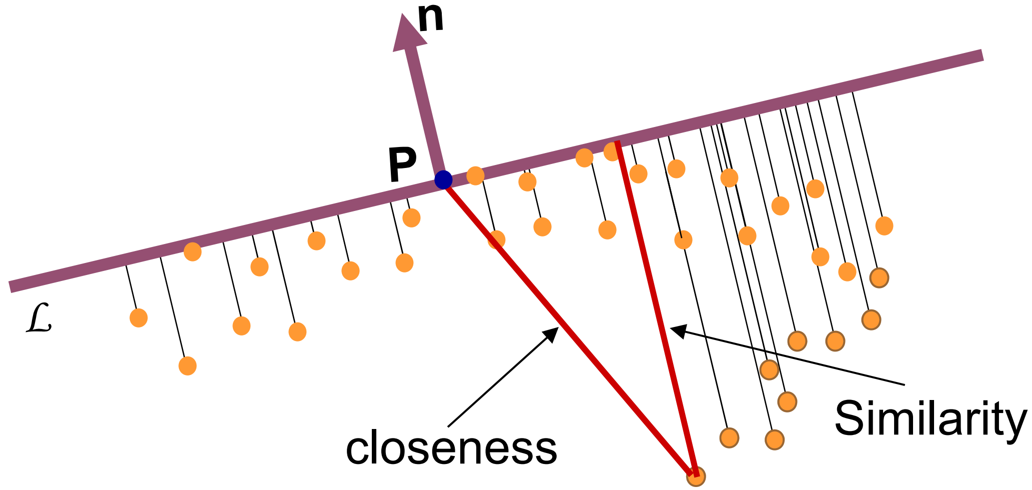
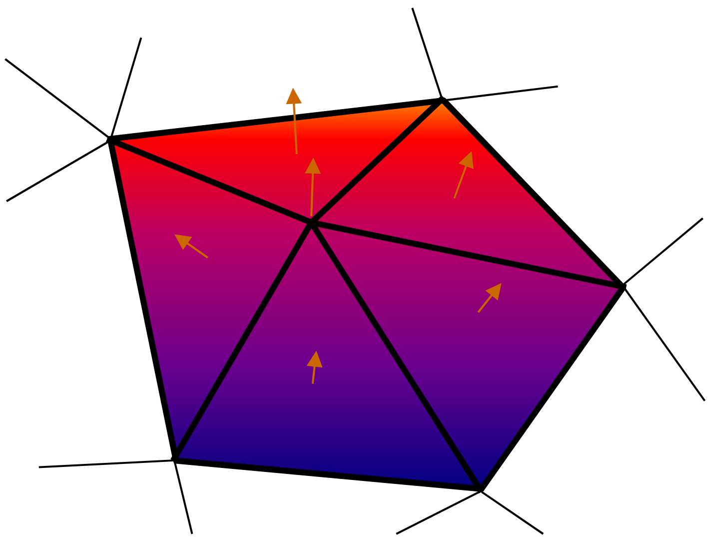


Computing the plane

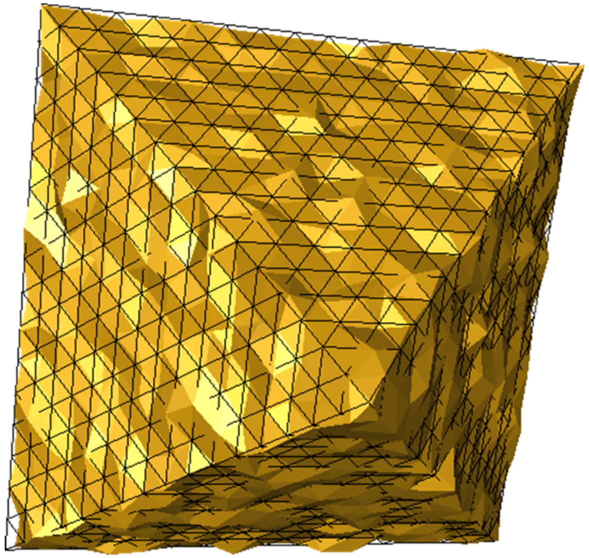
- The approximating plane should be:
  - A good approximation to the surface
  - Preserve features
- Average of the normal to faces in the 1-ring neighborhood



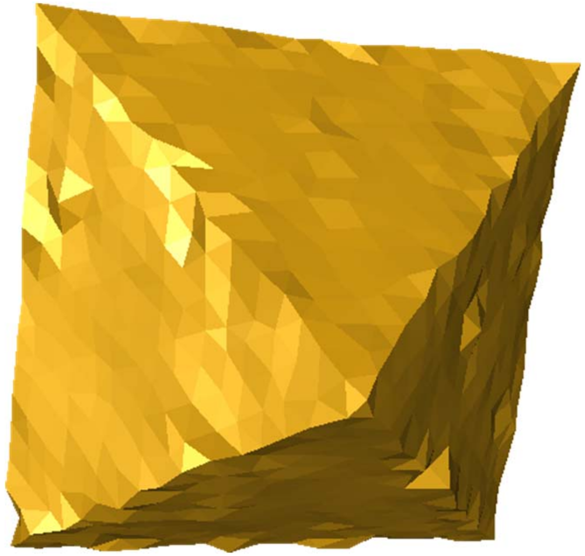
Parameters

- The two parameters to the weight function: σ_c, σ_s
  - Interactively select a point ***p*** and the neighborhood radius ρ
  - $\sigma_c = {}^1/_2\, \rho$
  - σ_s = stdv(Nbhd(***p***, ρ))
- Number of Iterations

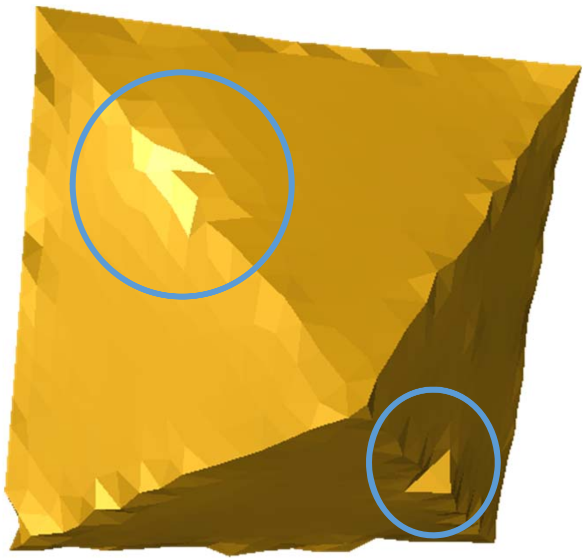
Results



Source

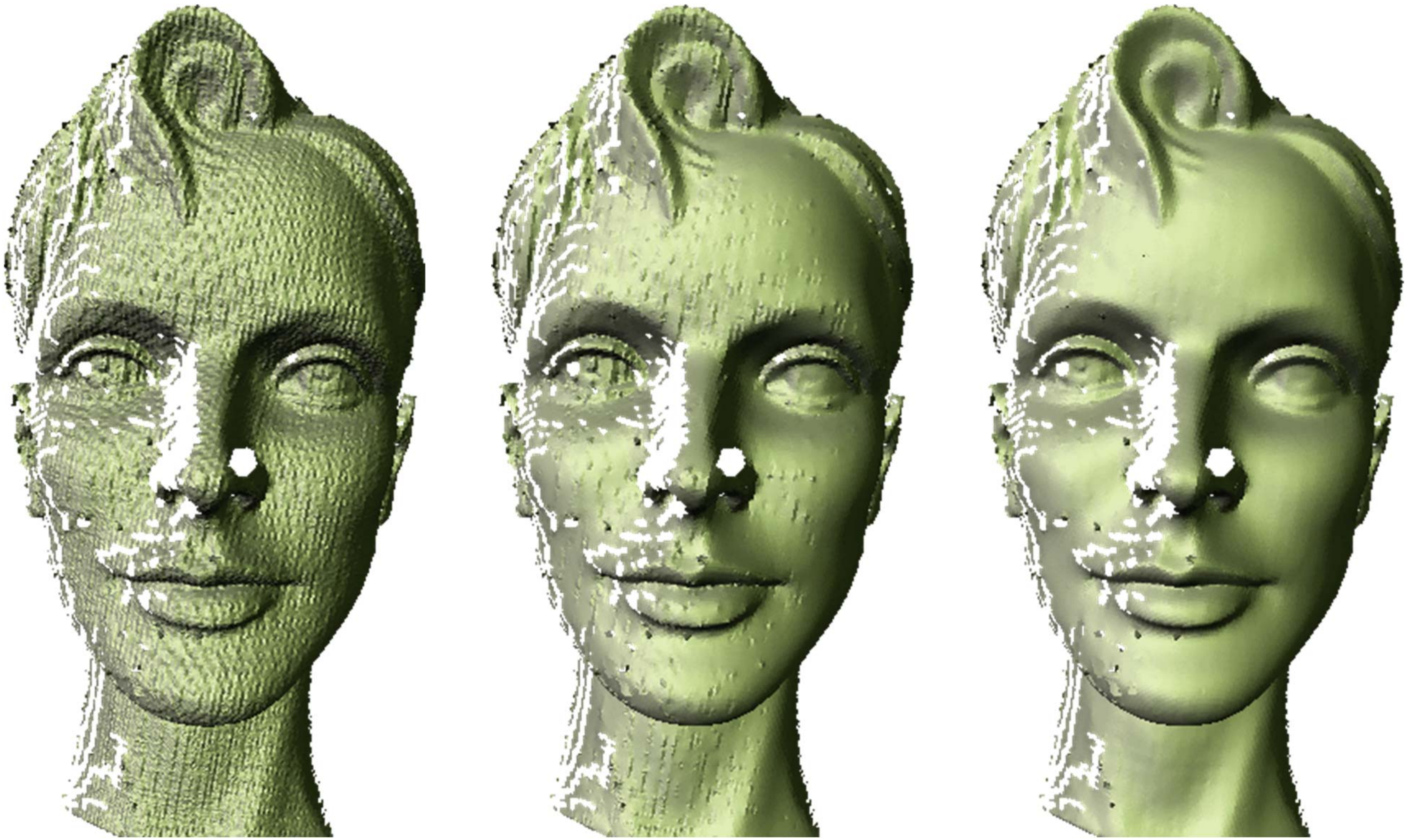


Two iterations



Five iterations

Experimental Result



1.4 Implicit Mesh Evolutions

Shape evolution $$\frac{\partial P}{\partial t} = \mathbf{F}(P)$$

$$M_{n+1} = M_n + \lambda \mathsf{L}(M_n) \quad \text{explicit scheme}$$

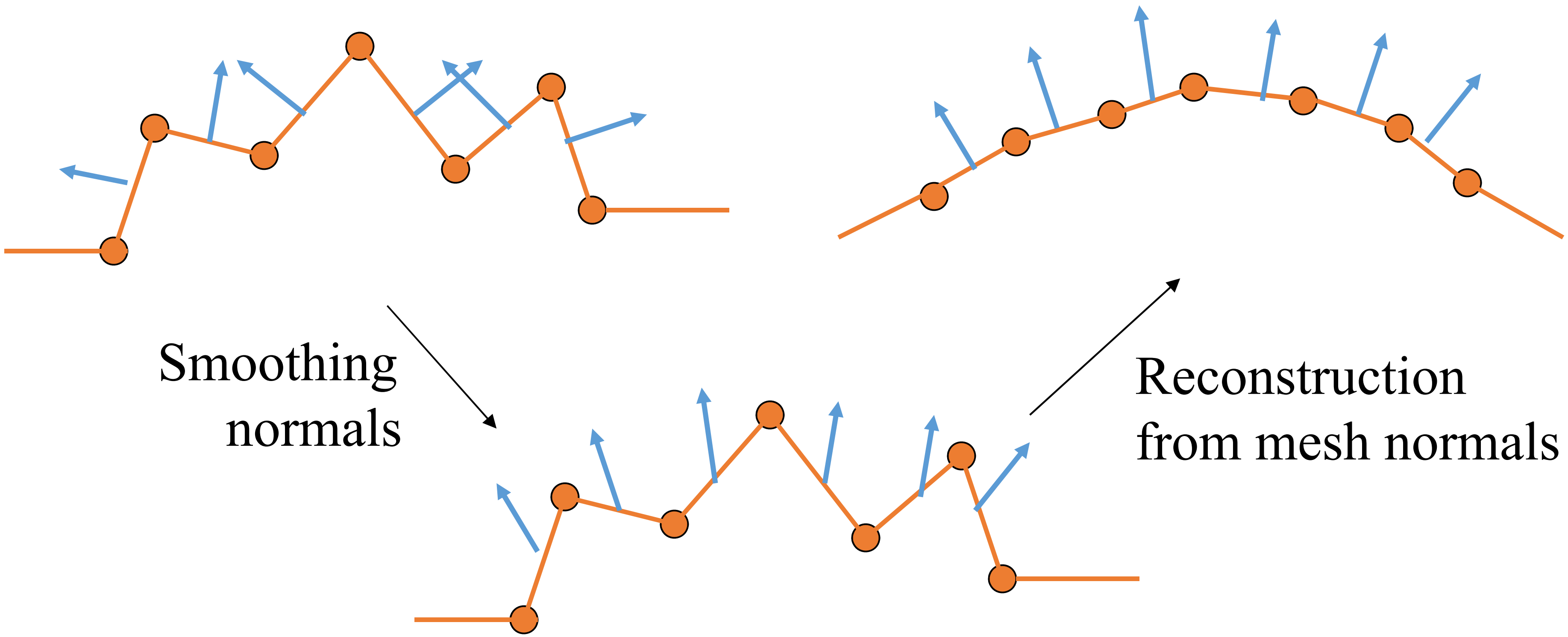
$$M_{n+1} = M_n + \lambda \mathsf{L}(M_{n+1}) \quad \text{implicit scheme}$$

$$\Rightarrow \quad (I - \lambda \mathsf{L})\, M_{n+1} = M_n$$

2. Normal Filtering

Normal Filtering

- 先对法向进行滤波：可使用顶点滤波的任何方法
- 根据滤波后的法向重建网格顶点



由法向重建顶点

- 输入：滤波后的法向量场
- 输出：重建网格顶点，使得其法向量接近输入
- 优化方法：

Vertex Updating:

$$\begin{cases} \boldsymbol{n}_f^T \cdot (\boldsymbol{x}_j - \boldsymbol{x}_i) = 0 \\ \boldsymbol{n}_f^T \cdot (\boldsymbol{x}_k - \boldsymbol{x}_j) = 0 \\ \boldsymbol{n}_f^T \cdot (\boldsymbol{x}_i - \boldsymbol{x}_k) = 0 \end{cases}$$

求解线性方程组

Energy:

$$E = \sum_{f_k} \sum_{i,j \in f_k} \left(\boldsymbol{n}_k^T \cdot (\boldsymbol{x}_j - \boldsymbol{x}_i) \right)^2$$

See more in [Zhang et al. Guided Mesh Normal Filtering. PG 2015.]

3. Global Smoothing

Liu et al. Non-Iterative Approach for Global Mesh Optimization. CAD 2007.

Smoothing Formulation

- Find a smoothed surface with minimum fairing energy

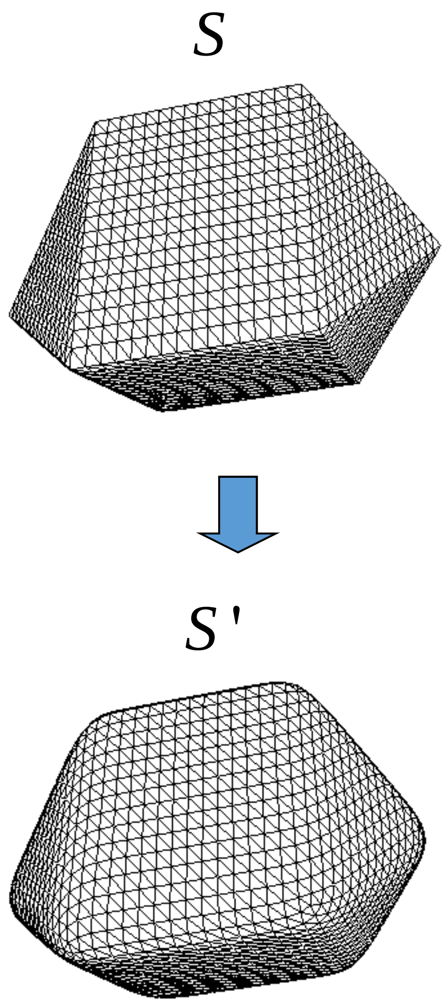
$$\min_{\mathcal{S}'} E(\mathcal{S}'),$$

- Fairing energy:

$$E(\mathcal{S}') = \underbrace{\alpha \int_{\Omega} \Psi(\mathcal{S}') du dv}_{\text{Smoothness constraint}} + \underbrace{\beta \int_{\Omega} (\mathcal{S}' - \mathcal{S})^2 du dv}_{\text{Data fidelity}},$$

$$\int_{\Omega} \Psi(\mathcal{S}') du dv = \int_{\Omega} (\boldsymbol{F}_u^2 + \boldsymbol{F}_v^2) du dv,$$

$$\int_{\Omega} \Psi(\mathcal{S}') du dv = \int_{\Omega} (\boldsymbol{F}_{uu}^2 + 2\boldsymbol{F}_{uv}^2 + \boldsymbol{F}_{vv}^2) du dv.$$



Smoothing Problem

- A global optimization problem
  - Minimize smoothness energy within some tolerance

$$\min_{S'} \quad E(S')$$

  - A mathematical model

$$E(S') = \alpha \int_{\Omega} \Psi(S') dudv + \beta \int_{\Omega} (S' - S) dudv$$

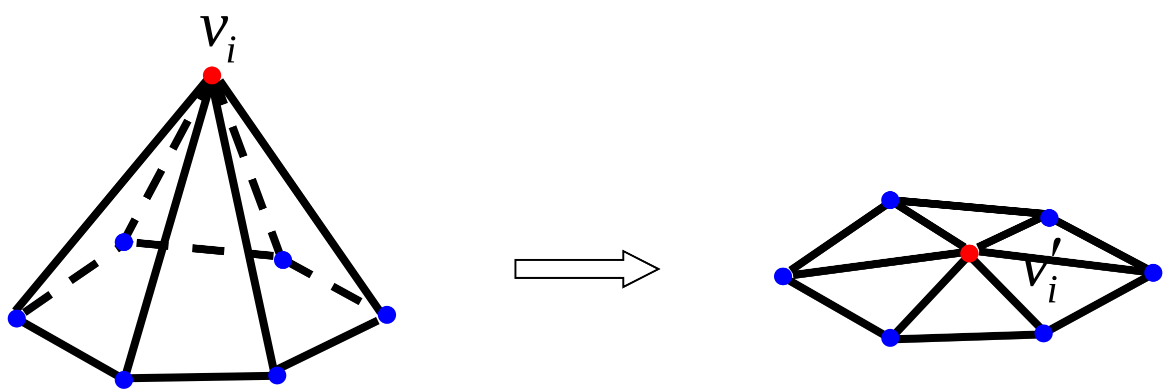
- **Smoothness term**

  membrane, thin-plate…

- **Fidelity term**

Local Lapacian Fairness

- Local discrete Laplacian smoothing operator

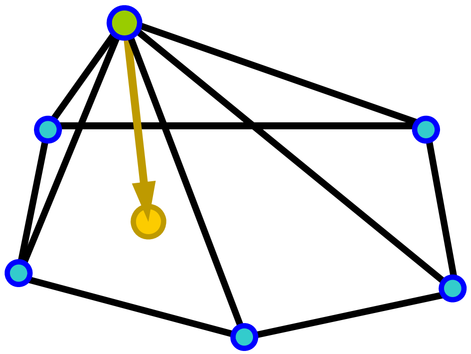


$$L(v_i) = v_i - \sum_{j \in N(i)} \omega_{ij} v_j = 0$$

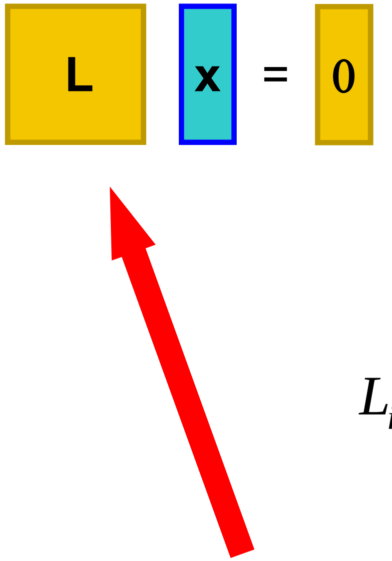
$\delta_{\text{cotangent}}$: $w_{ij} = \cot \alpha_{ij} + \cot \beta_{ij}$

Laplacian of Mesh

- Discrete Laplacians



$$L(v_i) = v_i - \sum_{j \in N(i)} \omega_{ij} v_j = 0$$

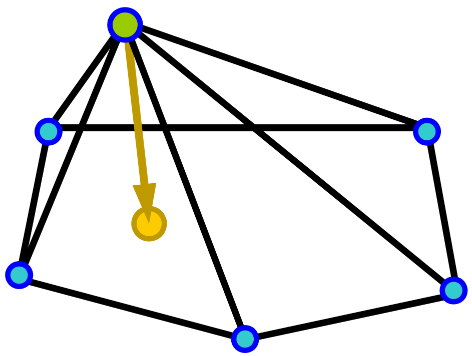


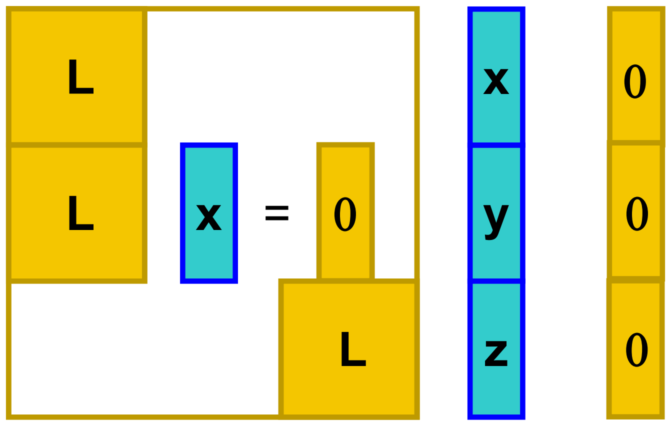
$$L_{ij} = \begin{cases} 1, & i = j, \\ -\omega_{ij}, & (i, j) \in E, \\ 0, & \text{other.} \end{cases}$$

- Laplacian of the mesh

Laplacian of Mesh

- Surface reconstruction

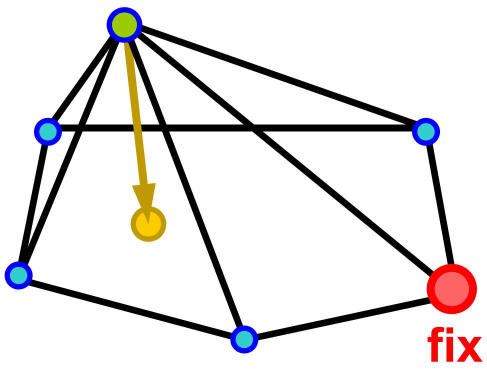


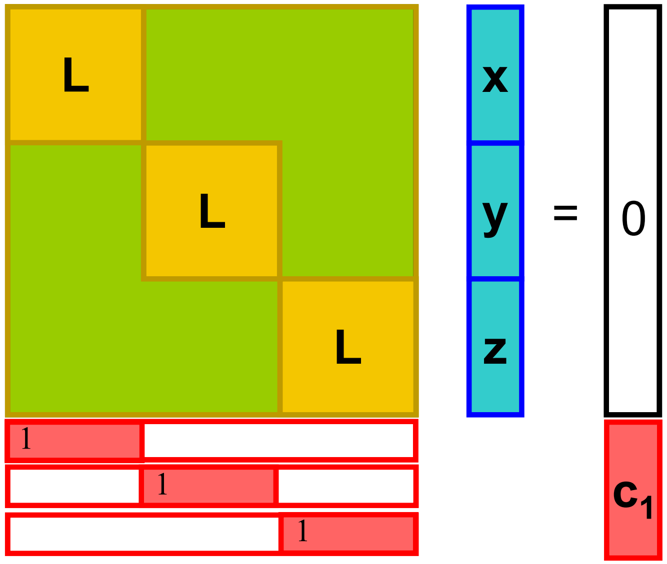


$$L(v_i) = v_i - \sum_{j \in N(i)} \omega_{ij} v_j = 0$$

Vertex Constraints

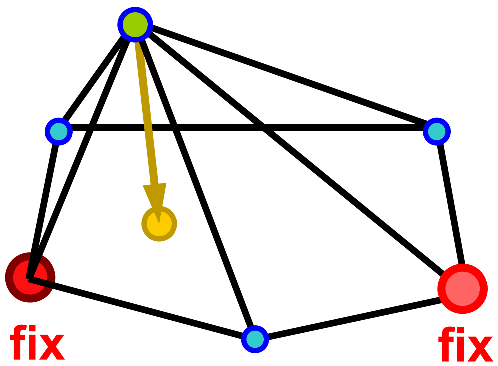
- Add position constraint for one vertex

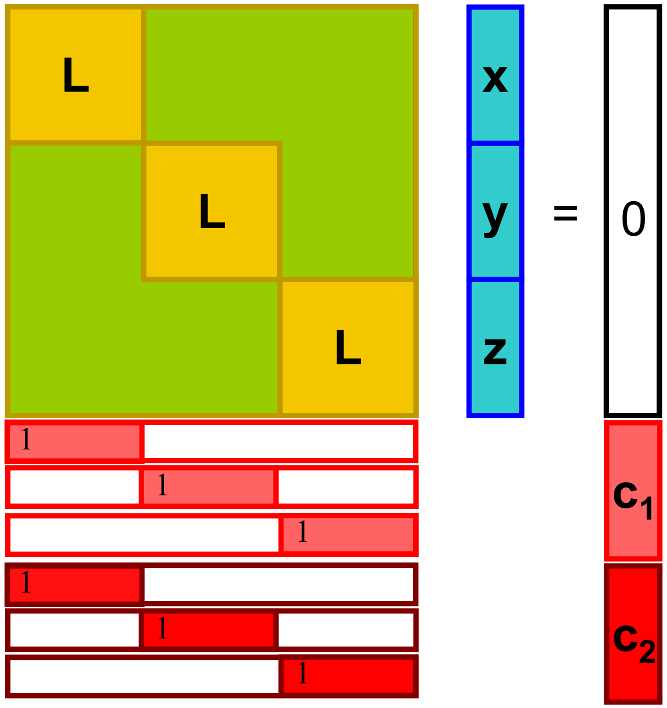




Vertex Constraints

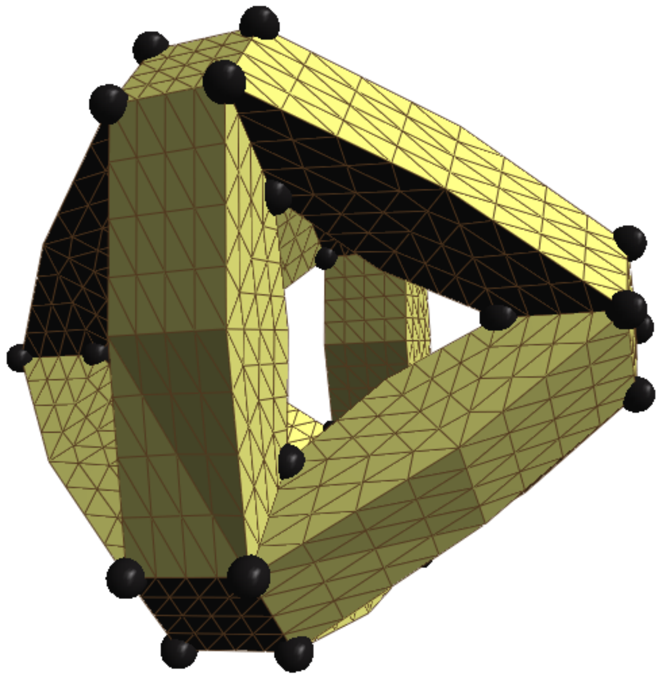
- Add position constraints for more vertices

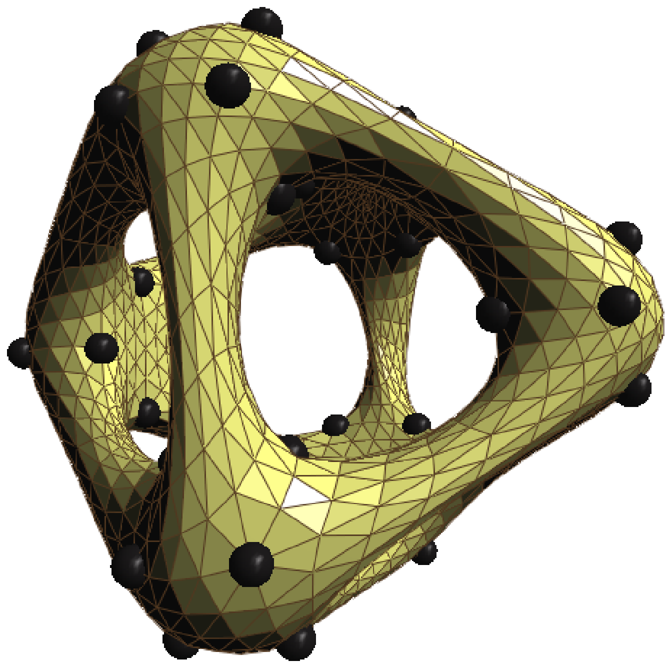




Adding Vertex Constraints

$$\min_{X'} \{ \|LX'\|^2 + \mu^2 \sum_{i \in C} \left| v_i' - v_i \right|^2 \}$$

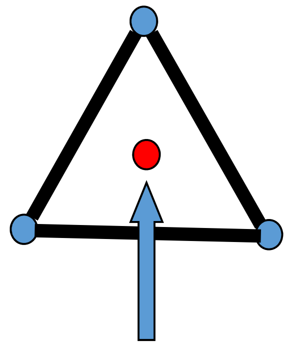




Face Constraints

$$
\begin{pmatrix} & L & & \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}
=
\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ c_1 \\ c_2 \\ t_1 \\ t_2 \end{pmatrix}
$$

$$A \quad \mathbf{x} \quad = \quad \mathbf{b}$$

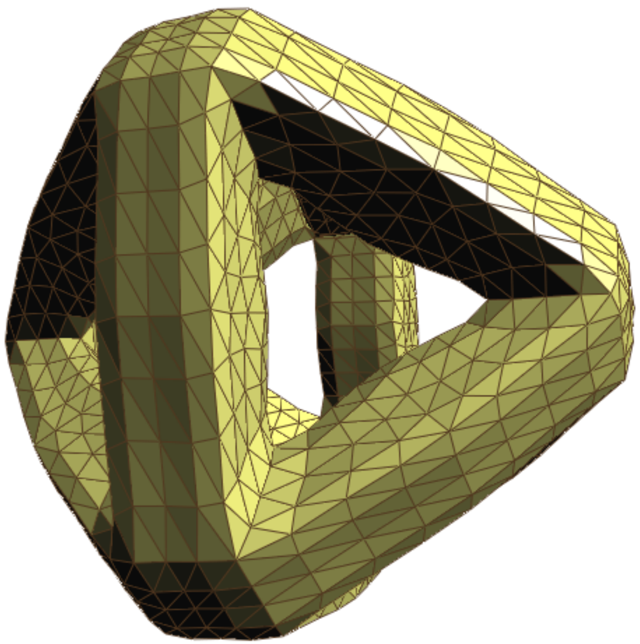


barycenter

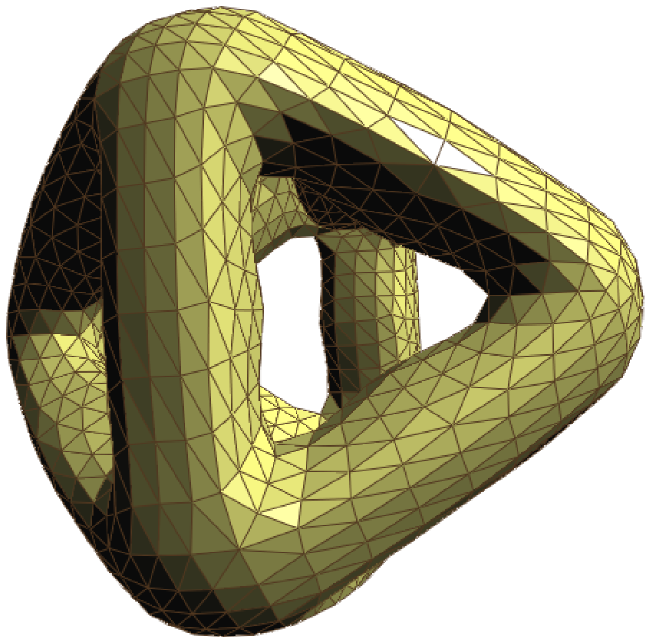
$$v_{center} = \frac{1}{3}(v_i + v_j + v_k)$$

Adding Face Constraints

$$\min_{X'}\left\{\|LX'\|^2 + \sum_{<i,j,k>\in T} \lambda^2 \left|(v_i' + v_j' + v_k') - (v_i + v_j + v_k)\right|^2\right\}$$



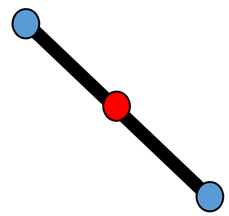
$\lambda = 0.5$



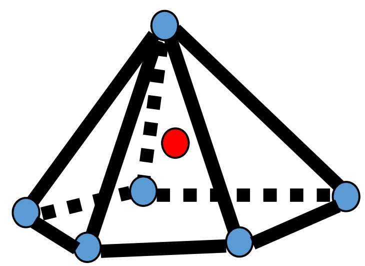
$\lambda = 0.3$

Other Constraints

- Edge constraints



- 1-ring barycenter constraints



- Other linear constraints

Minimizing Energy

$$\min_{X'} \left\{ \|LX'\|^2 + \sum_{i \in C} \mu^2 \left| v_i' - v_i \right|^2 + \sum_{<i,j,k> \in T} \lambda^2 \left| (v_i' + v_j' + v_k') - (v_i + v_j + v_k) \right|^2 \right\}$$

$$A\mathbf{x} = \mathbf{b}$$

Least Square Solution

- An over-determined system:

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}$$

- Normal equation:

$$\mathbf{A}^T\mathbf{A}\,\mathbf{x} = \mathbf{A}^T\mathbf{b}$$

$$\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\,\mathbf{A}^T\mathbf{b}$$

One Channel Solution

- Very efficient solution by Cholesky factorization of $A^T A$:

$$\boldsymbol{A}^T \boldsymbol{A} = \boldsymbol{R}^T \boldsymbol{R}$$

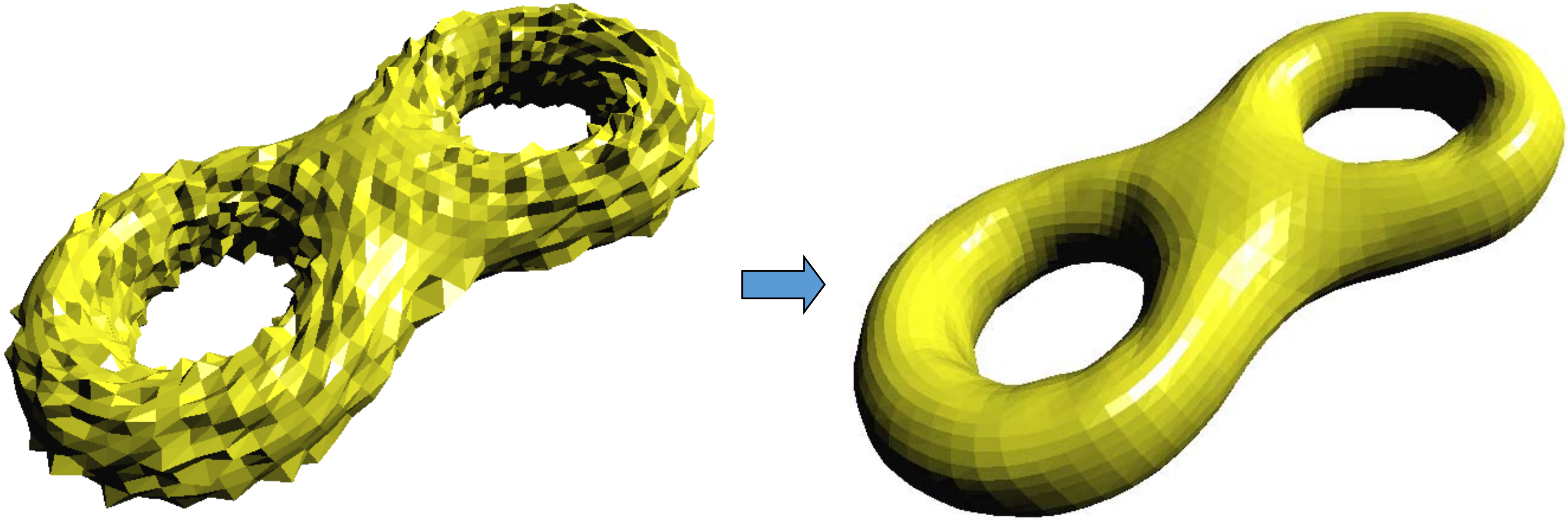
R is upper-triangular and sparse

Once R is computed, solving for **x**, **y**, **z** by back-substitution:

$$\boldsymbol{R}^T \xi = \boldsymbol{A}^T \mathbf{b}$$

$$\boldsymbol{R}\mathbf{x} = \xi$$

Results

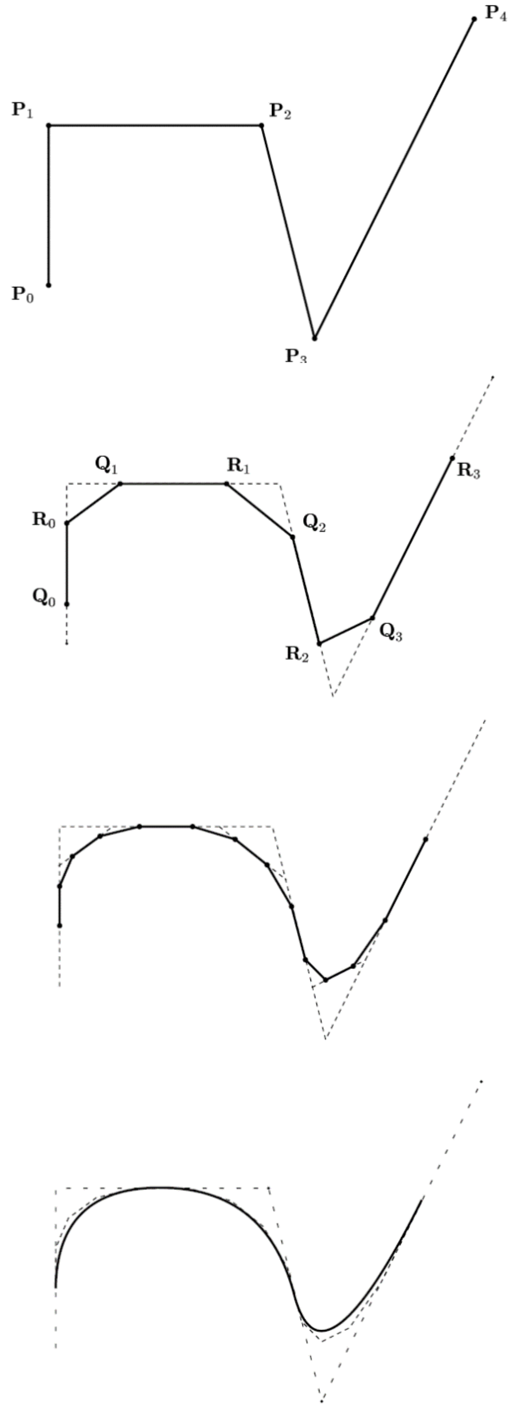


‘8’-like mesh model
3070 vertices, 6144 triangles

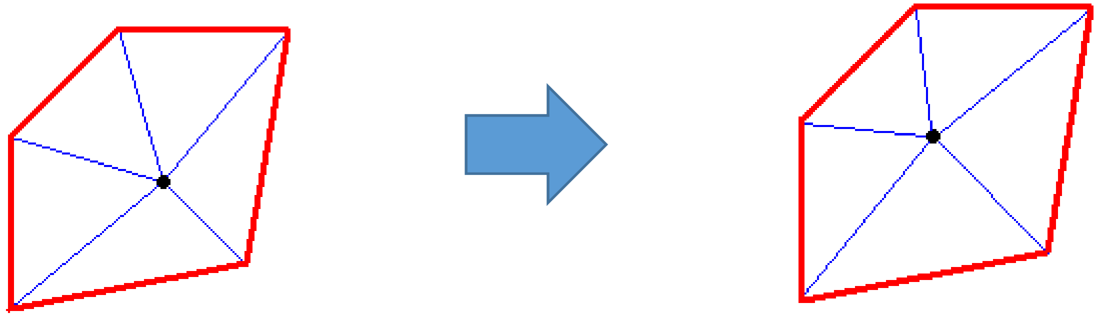
4. Mesh Improvement

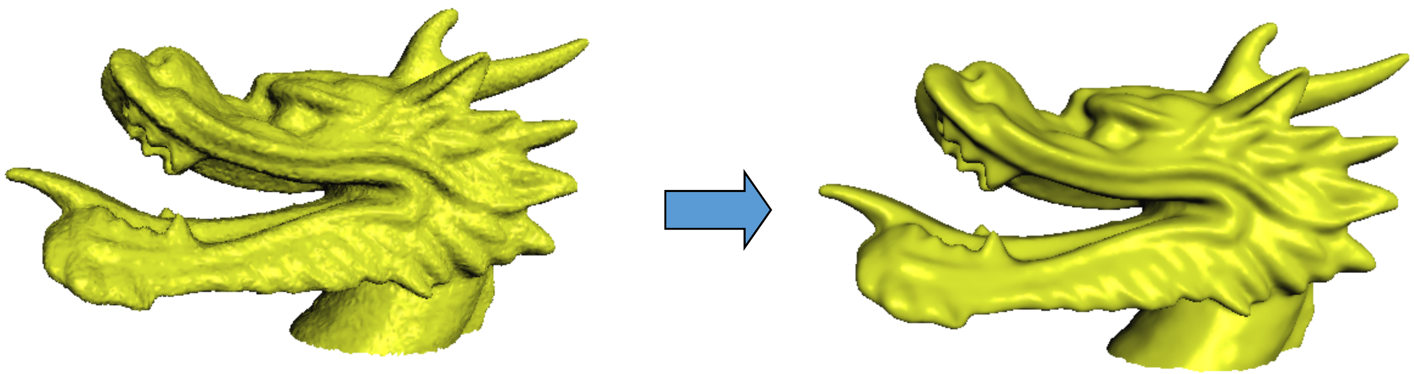
Smoothing Everywhere

- Real life applications
  - Sculpture
  - Decoration
- Methods
  - Corner cutting
- Geometric modeling
  - Chaikin's scheme
  - Bézier: de Castljau algorithm
  - B-spline: knot insertion
  - Subdivision surface



Mesh Improvement

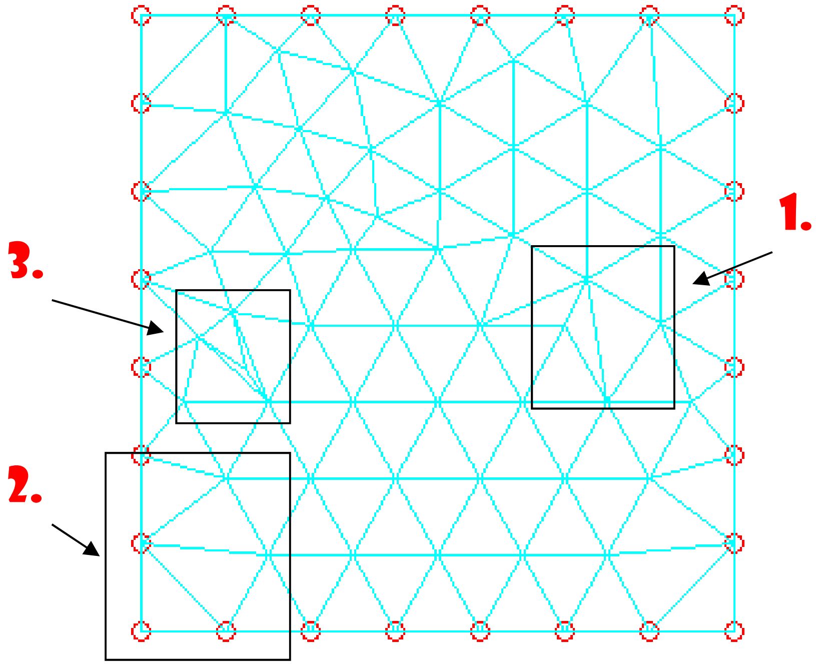




Mesh Improvement

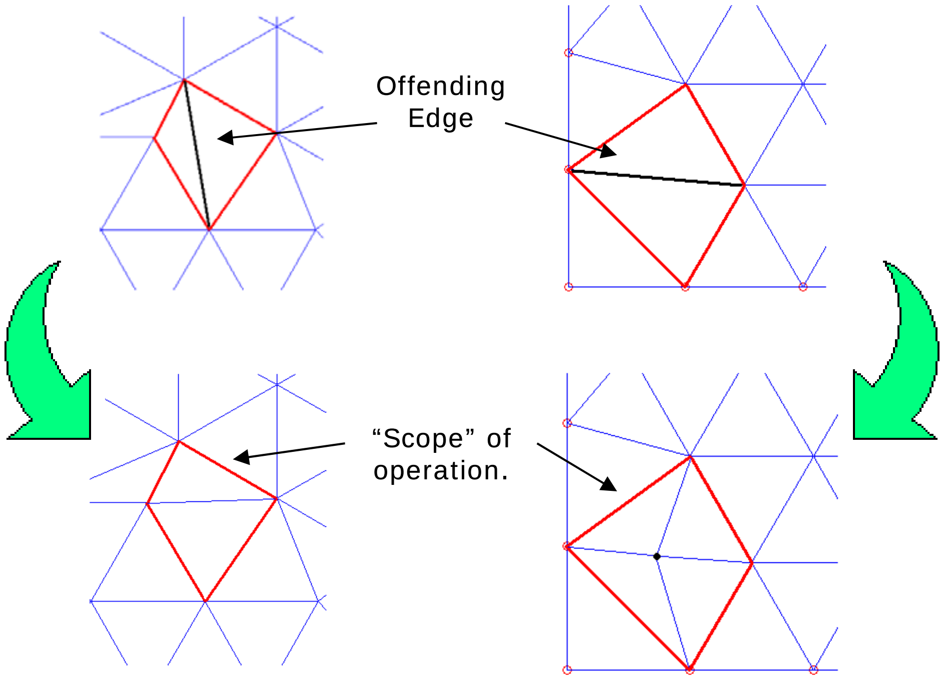
<u>Topology changes</u>

<u>Local</u> correction strategies

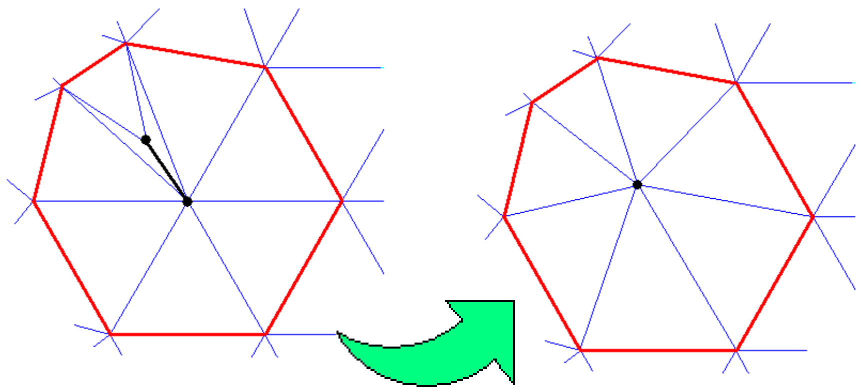


1. **Flip** an edge.

2. **Split** an edge.

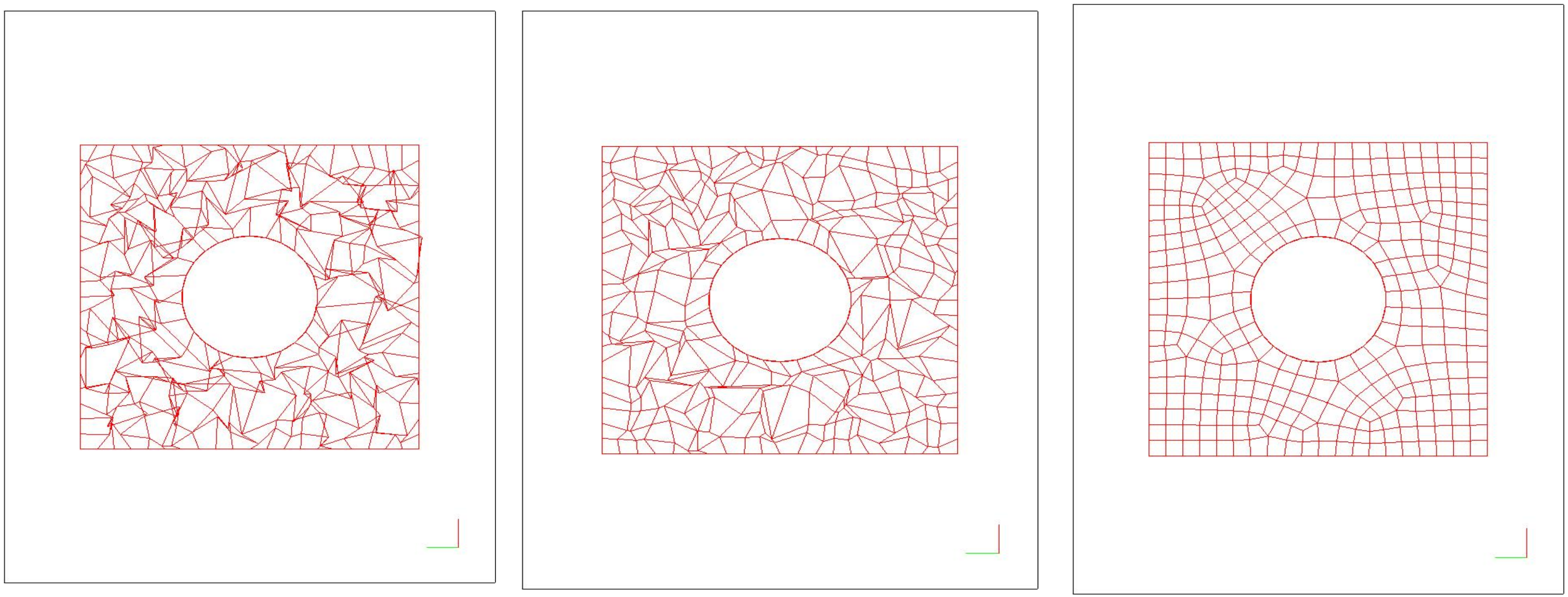


3. **Collapse** an edge.



Mesh Improvement

- Example



其他去噪方法

- 基于稀疏优化的方法
  - He and Schaefer. Mesh denoising via L0 minimization. Siggraph 2013.
- 基于压缩感知的方法
  - Wang et al. Decoupling Noises and Features via Weighted L1-analysis Compressed Sensing. ACM TOG, 2014.
- 基于机器学习的方法
  - Wang et al. Mesh Denoising via Cascaded Normal Regression. Siggraph 2016.
- 很多很多工作…

其他数据的去噪

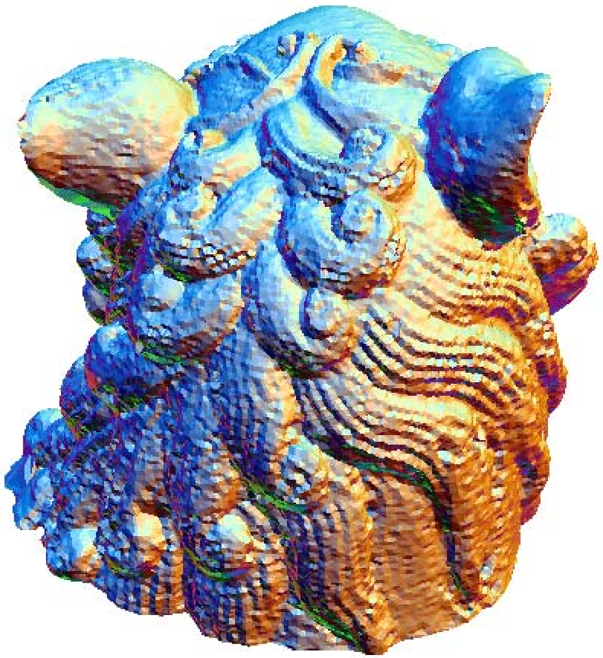
- Point cloud
- Volumetric data
- Depth images

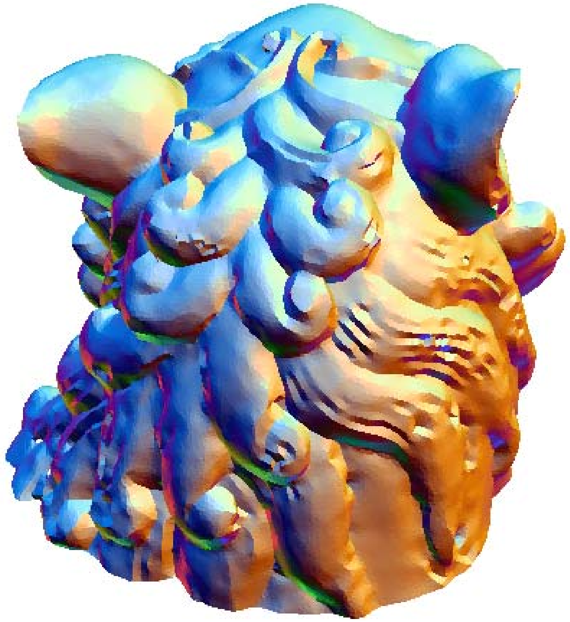
Many Problems Remain

Mesh smoothing remains to be an active research area

Photo

Scanned mesh





Smoothed mesh

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谢　谢　！