3D Printing Oriented Design: Geometry and Optimization

Siggraph Asia 2014 Course

Dec. 5, 2014 , Shenzhen

SIGGRAPH ASIA 2014 SHENZHEN

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Web: http://staff.ustc.edu.cn/~lgliu/Courses/SigAsia_2014_course_3Dprinting/index.html

Part 3: Structural Optimization

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- Dartmouth College, USA
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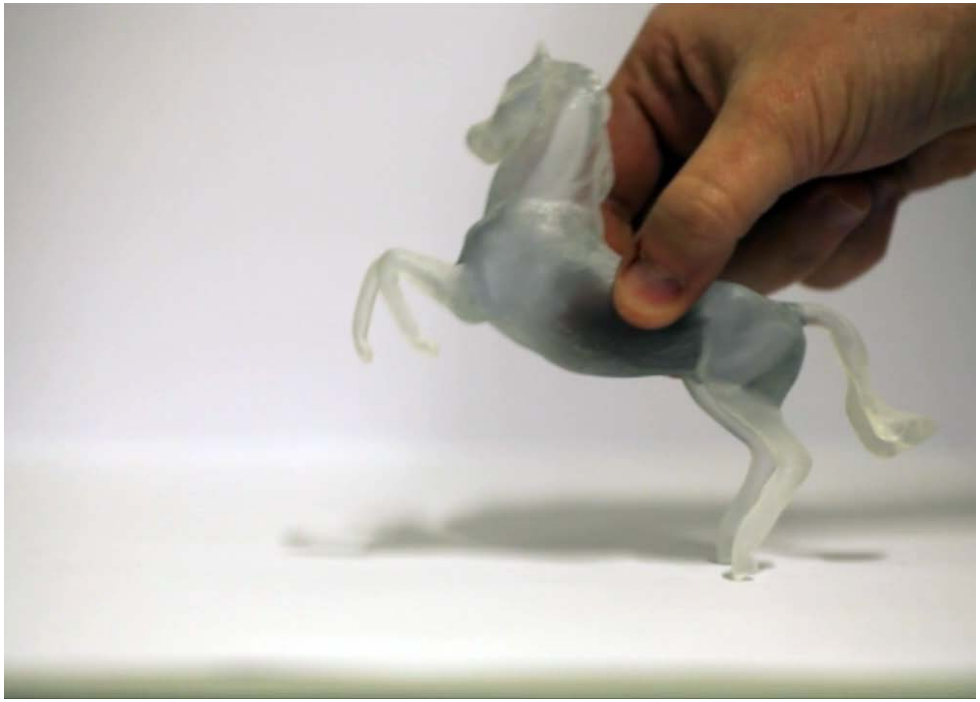
when we forget about structure…



[Prévost et al. 2013]

when we forget about structure…

[Stava et al. 2012

when we forget about structure…

[Zhou et al. 2013]

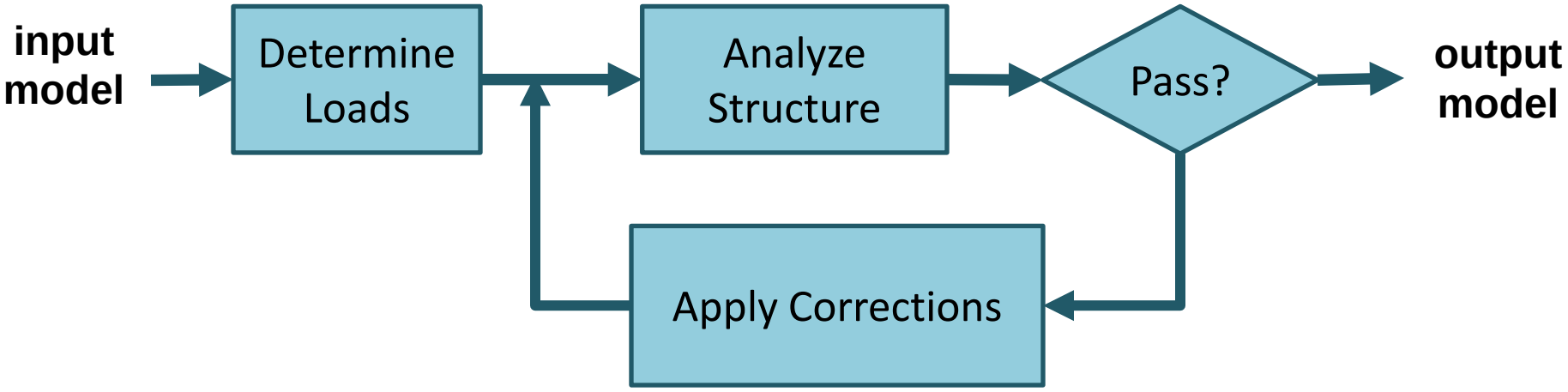
structural optimization

minimize: structural & design objectives

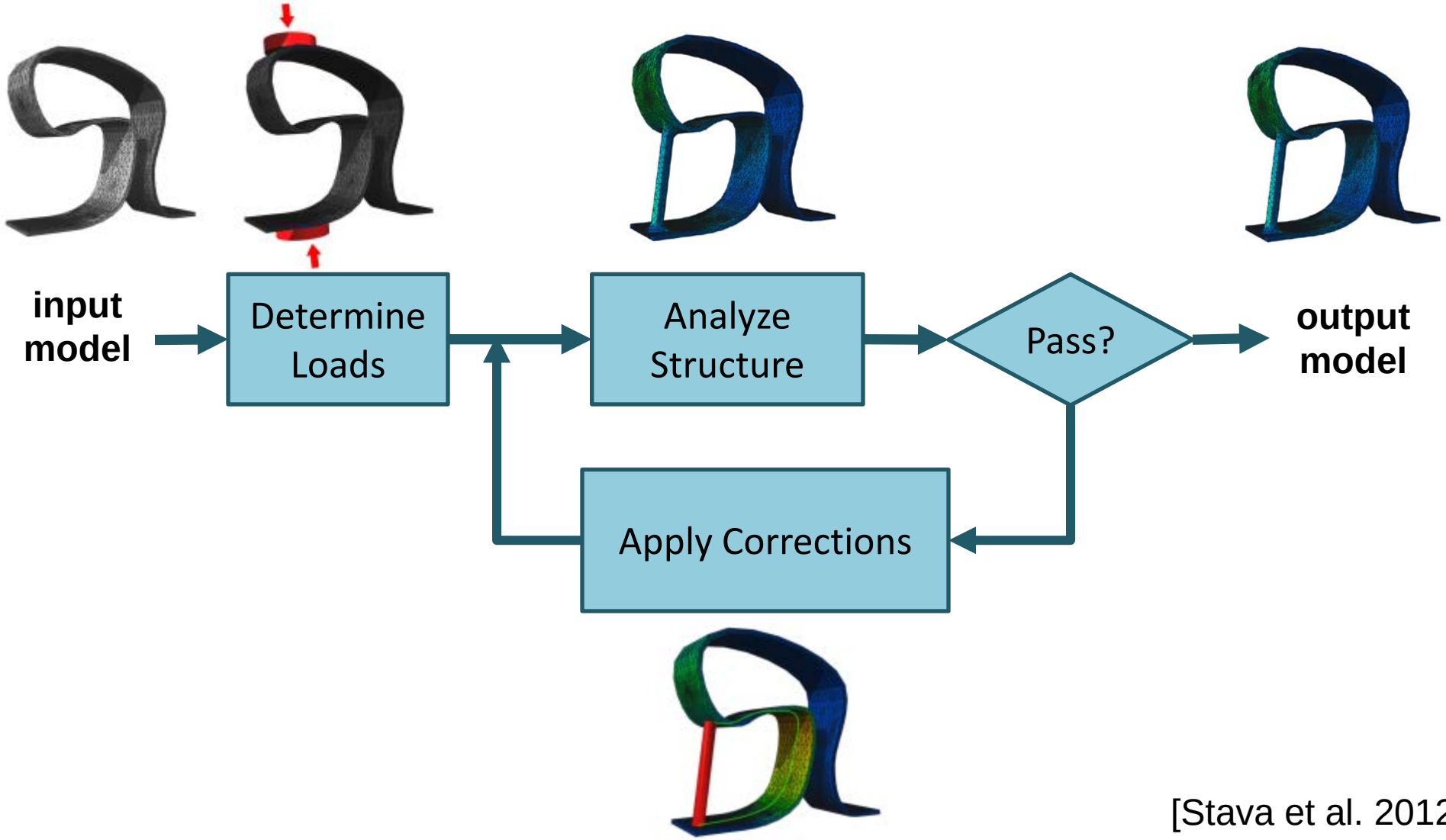
search domain: geometric variables

subject to: structural, material & manufacturing constraints

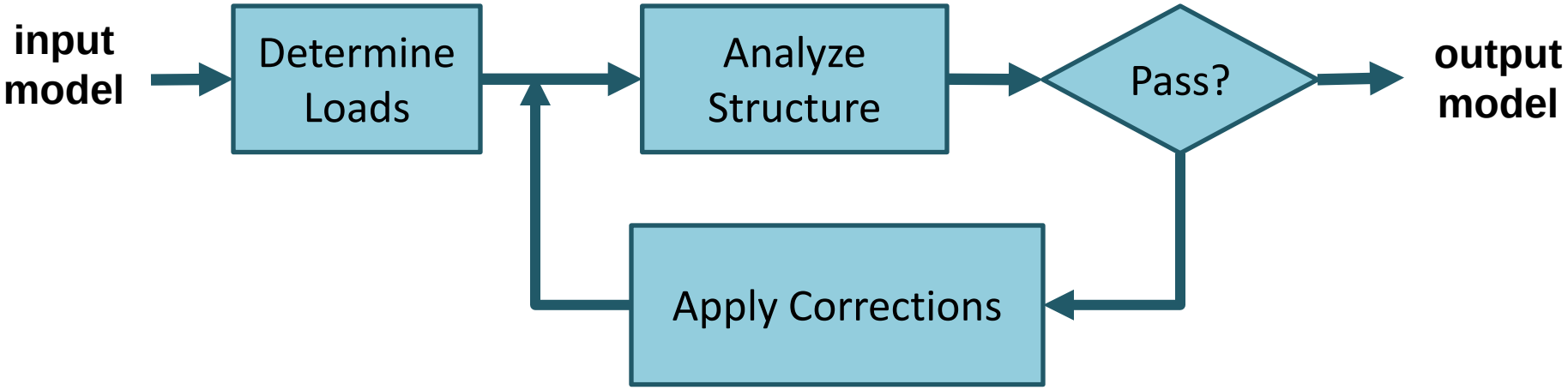
structural optimization

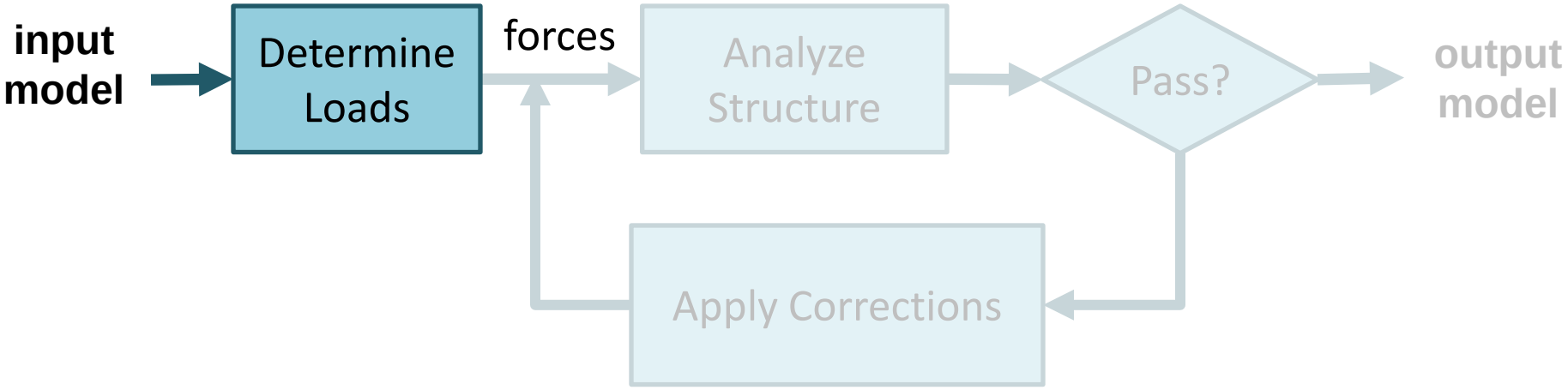


structural optimization



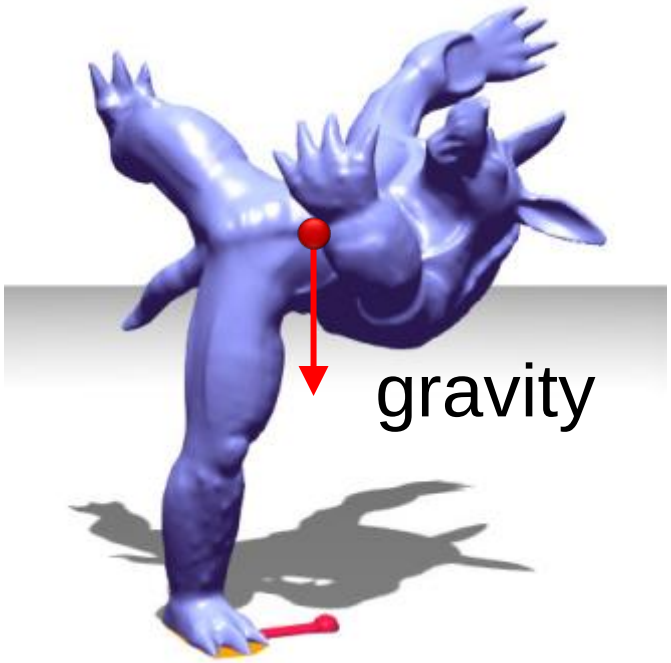
[Stava et al. 2012


input model
Determine Loads
Analyze Structure
Pass?
output model
Apply Corrections



- gravity
- gripping
- impact, wear & tear

determine loads **gravity**



center of mass $\mathbf{c} = \frac{1}{mass} \int_{vol} density \times \mathbf{r}\, d\mathbf{r}$

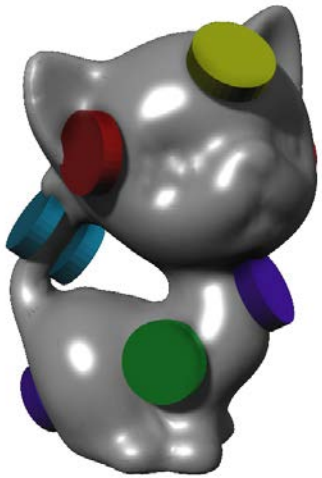
[Prévost et al. 2013]

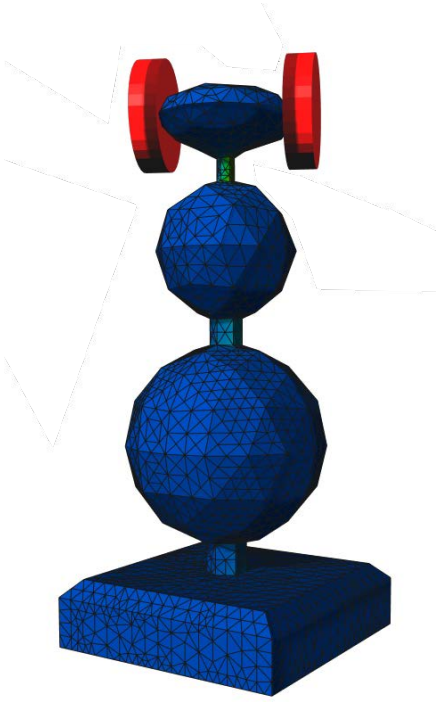
determine loads **gravity**

King’s College Chapel, UK

determine loads **pinch grip**

location of applied pressure

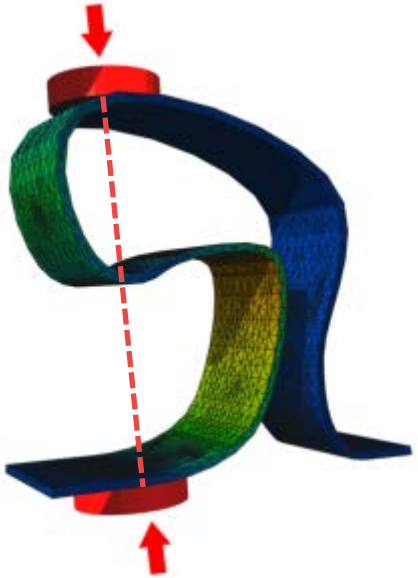


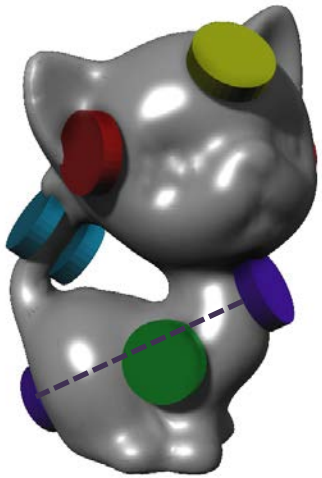


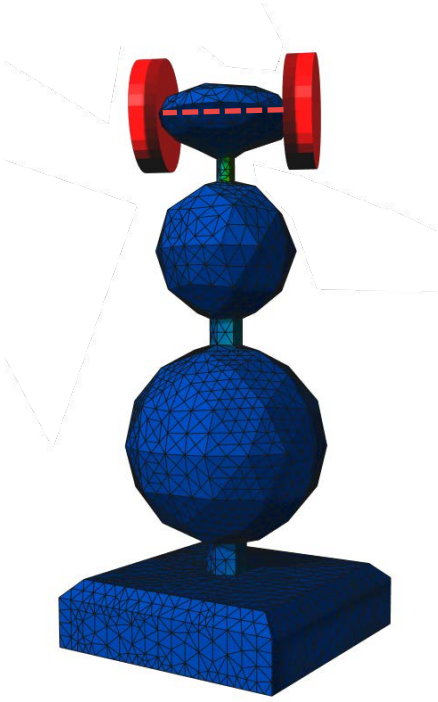
[Stava et al. 2012]

determine loads **pinch grip**

location of applied pressure







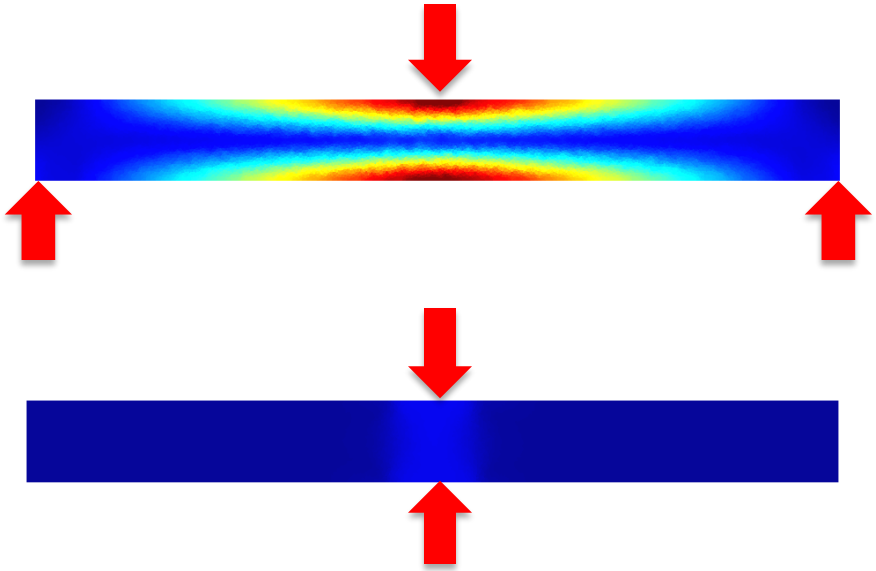
selection criteria

center of mass, medial axis, surface normal, occlusion, …

[Stava et al. 2012]

determine loads **worst case**

how will the shape fail?



[Zhou et al. 2013]

determine loads **worst case**

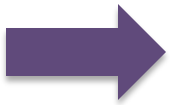
how will the shape fail?

need to check
all possible breakage points
all possible loading conditions

expensive question

[Zhou et al. 2013]

determine loads **worst case**

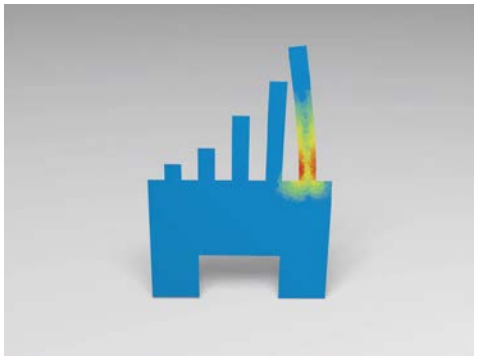


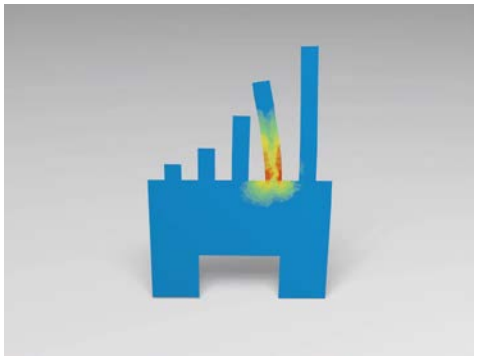
shape

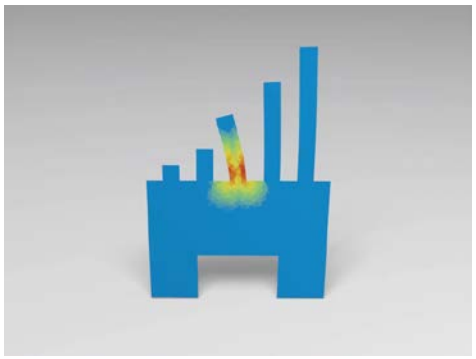
identify weak regions

[Zhou et al. 2013]

determine loads **worst case**





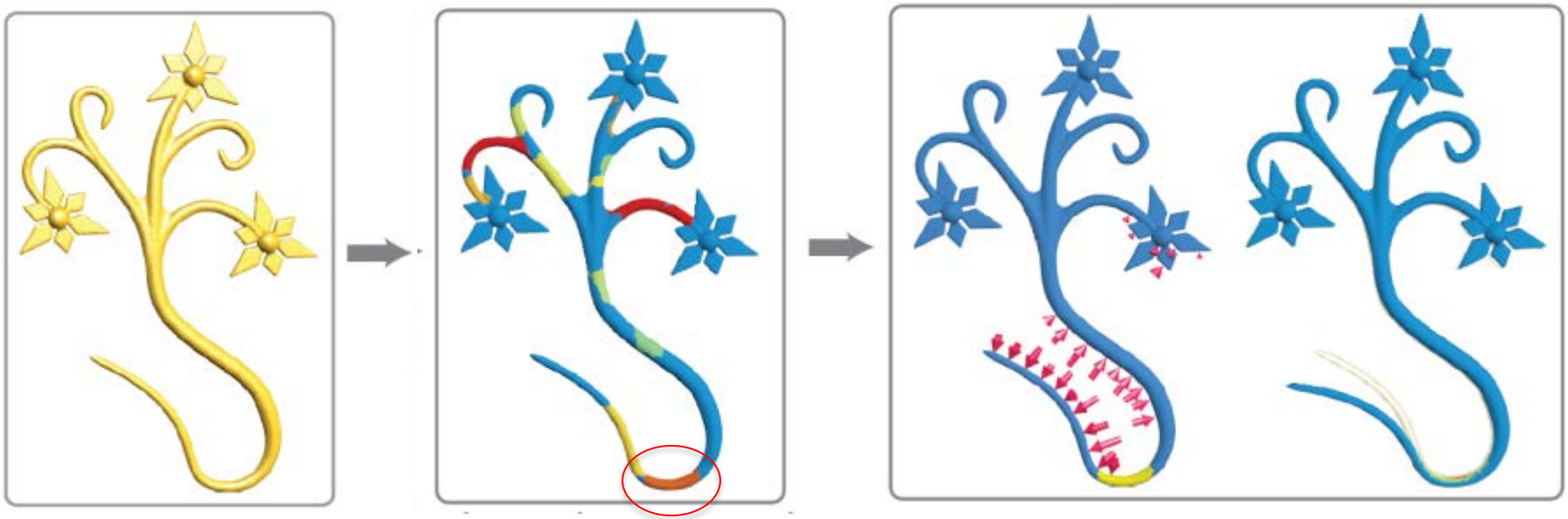


modal analysis

$$\nabla \cdot \underbrace{\sigma(u)}_{\text{stress}} = \underbrace{\rho}_{\text{density}} \underbrace{\ddot{u}}_{\text{acceleration}}$$

[Zhou et al. 2013]

determine loads **worst case**

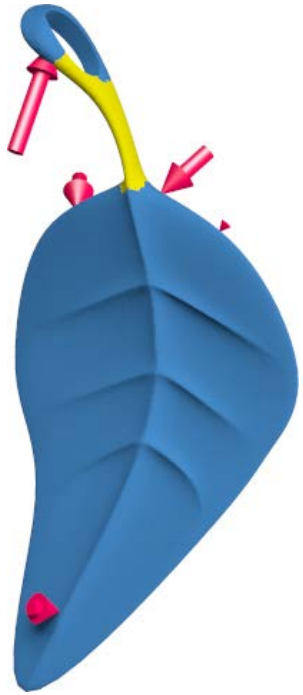


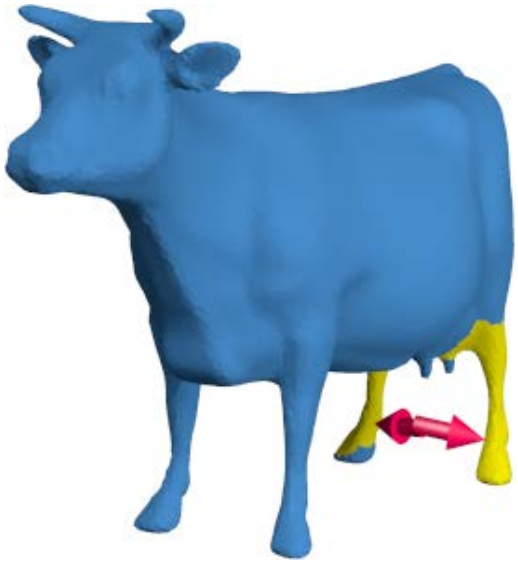
weak regions

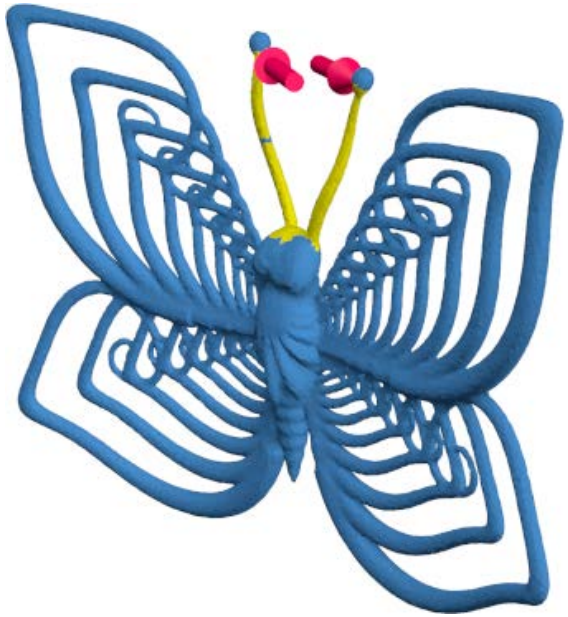
worst case pressure distribution

[Zhou et al. 2013]

determine loads **worst case**

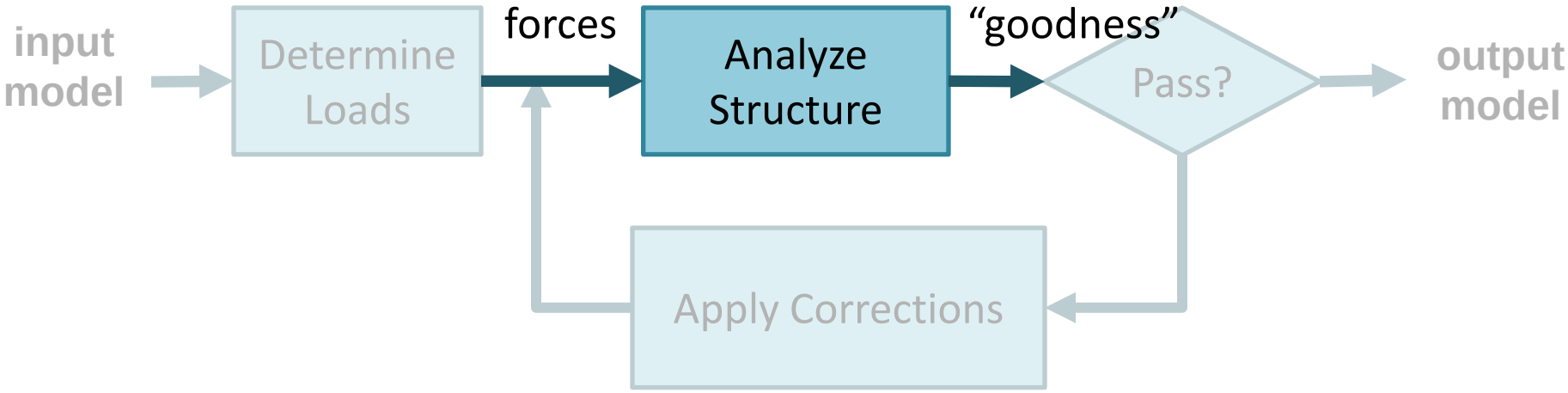






optimal pressure distributions

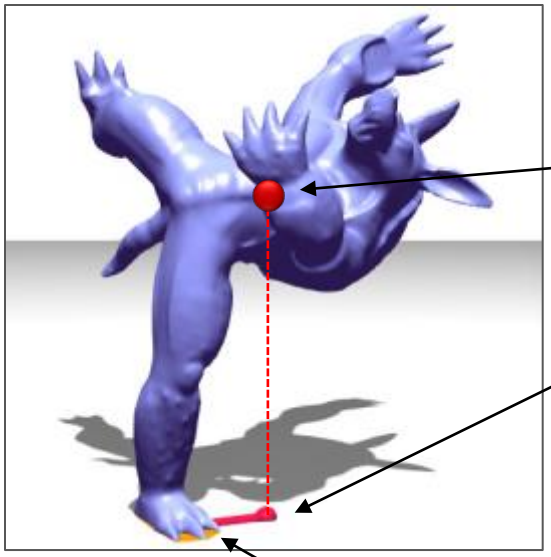
[Zhou et al. 2013]



- static equilibrium
- finite element method
- cross-sectional analysis
- frame structure

analyze structure static equilibrium

one rigid body



center of mass

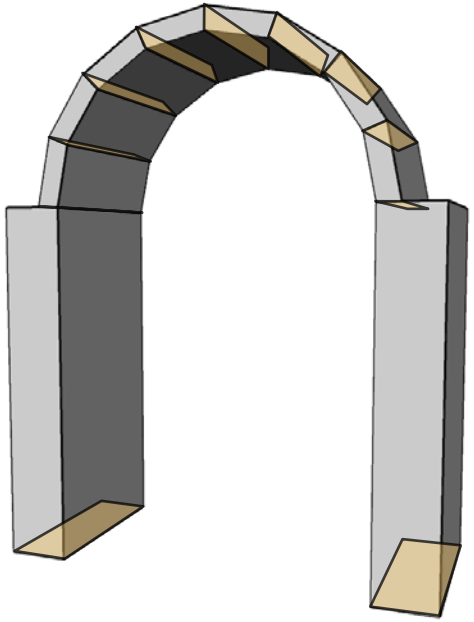
“goodness”
= distance to support region

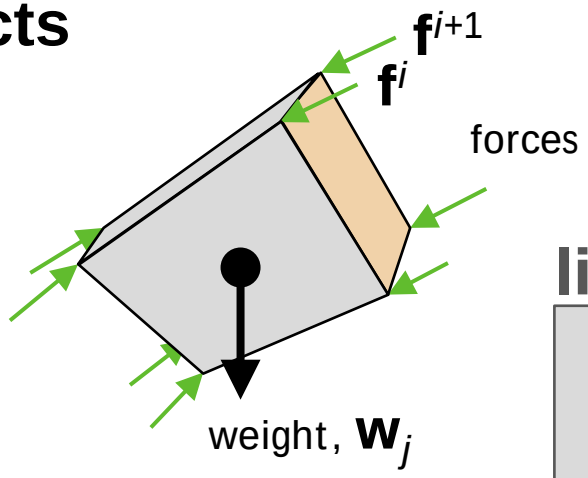
contact area

[Prévost et al. 2013]

analyze structure static equilibrium

system of objects





linear constraints

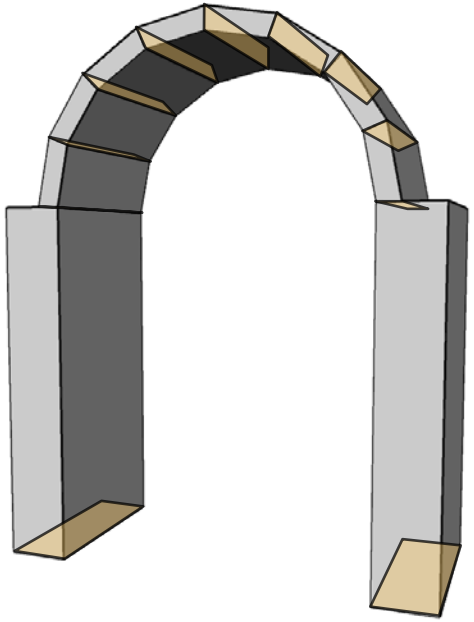
static equilibrium

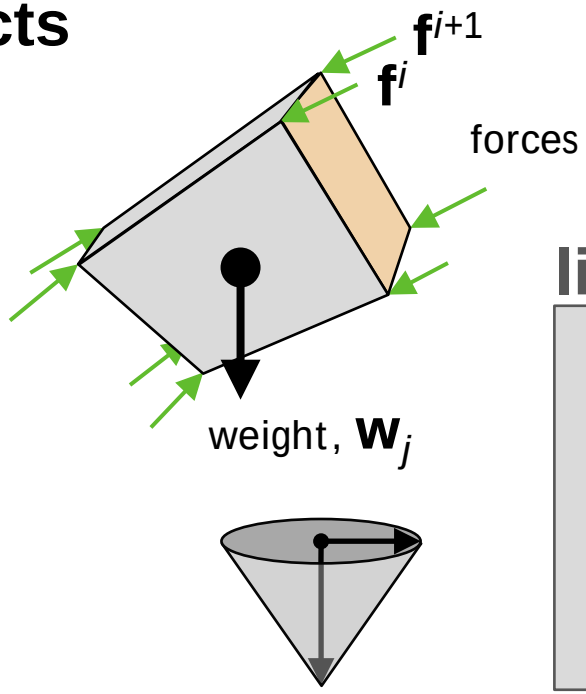
Σ forces = 0, Σ torques = 0

[Whiting et al. 2009]

analyze structure static equilibrium

system of objects





linear constraints

static equilibrium

Σ forces = 0, Σ torques = 0

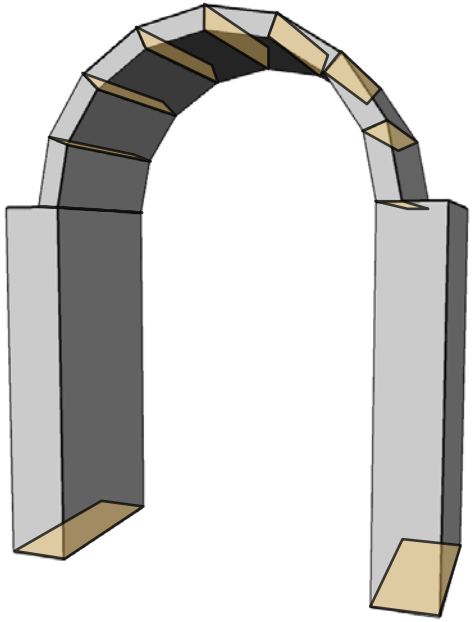
friction

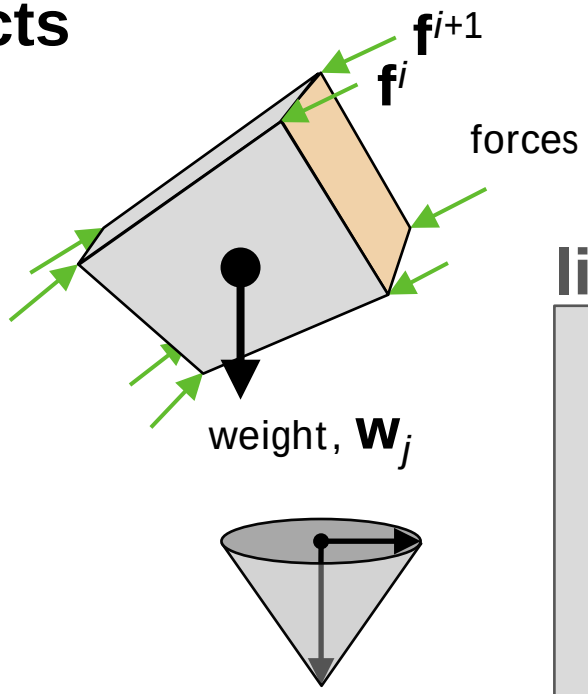
in-plane force $\leq \alpha$(normal force)

[Whiting et al. 2009]

analyze structure static equilibrium

system of objects





linear constraints

static equilibrium

Σ forces = 0, Σ torques = 0

friction

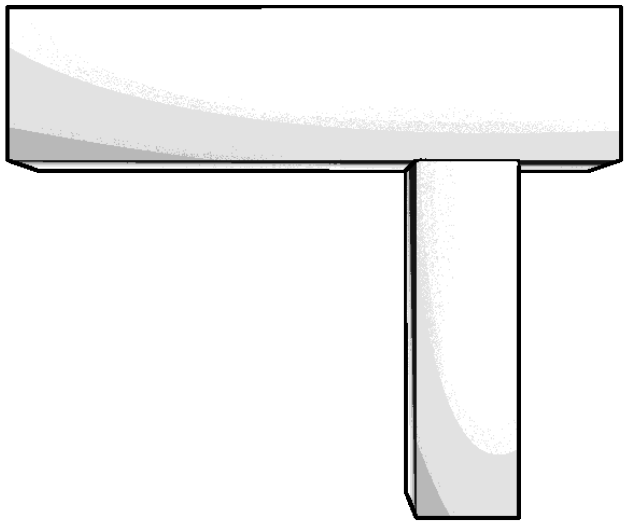
in-plane force ≤ α(normal force)

compression-only

normal forces ≥ 0

[Whiting et al. 2009]

analyze structure static equilibrium

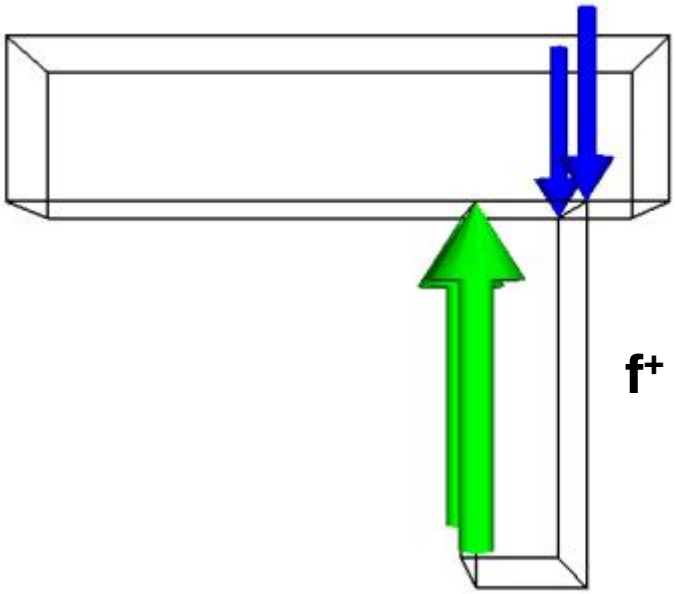


unstable

[Whiting et al. 2009]

analyze structure static equilibrium

forces on top block for equilibrium



$\mathbf{f}^-$ tension

$\mathbf{f}^+$ compression

unstable

"goodness"

quadratic program

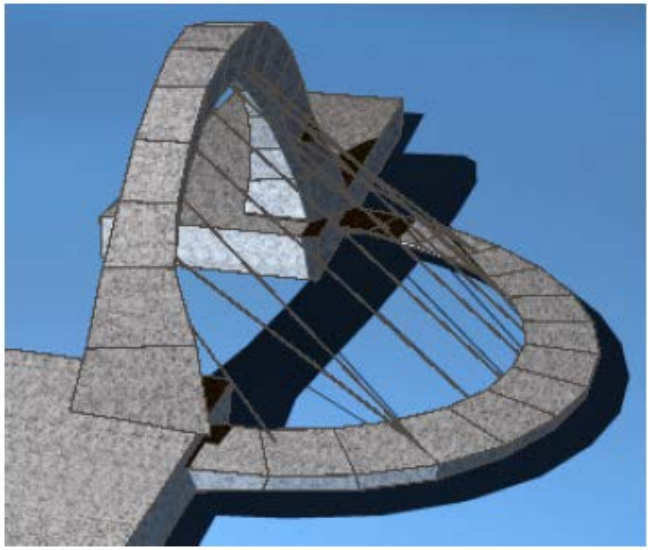
min $|\text{tension}|^2$

s.t. static equilibrium

friction

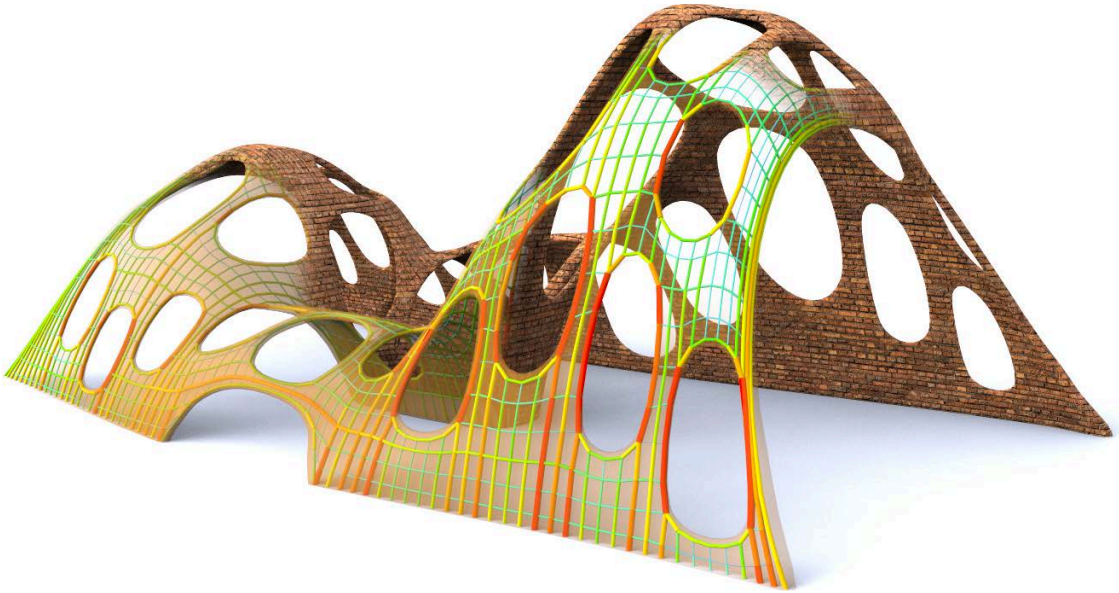
[Whiting et al. 2009]

analyze structure static equilibrium



[Whiting et al. 2009, 2012

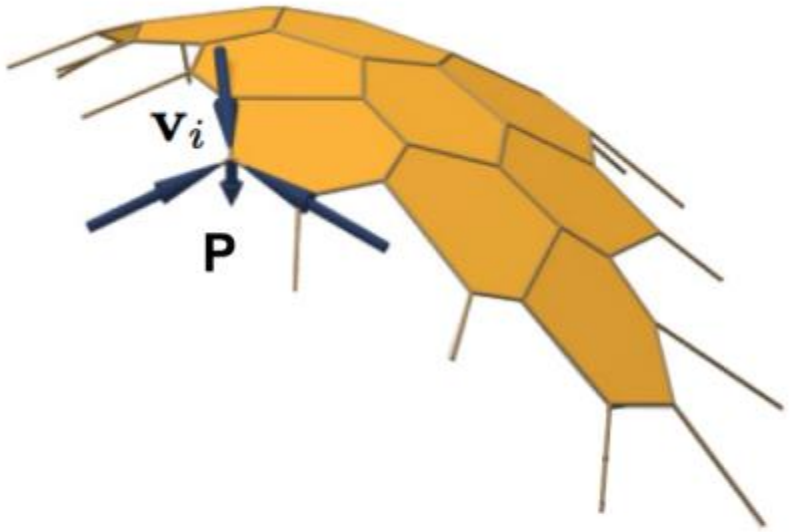
static equilibrium thrust network analysis



[Vouga et al. 2012]

static equilibrium thrust network analysis

3D “thrust” network



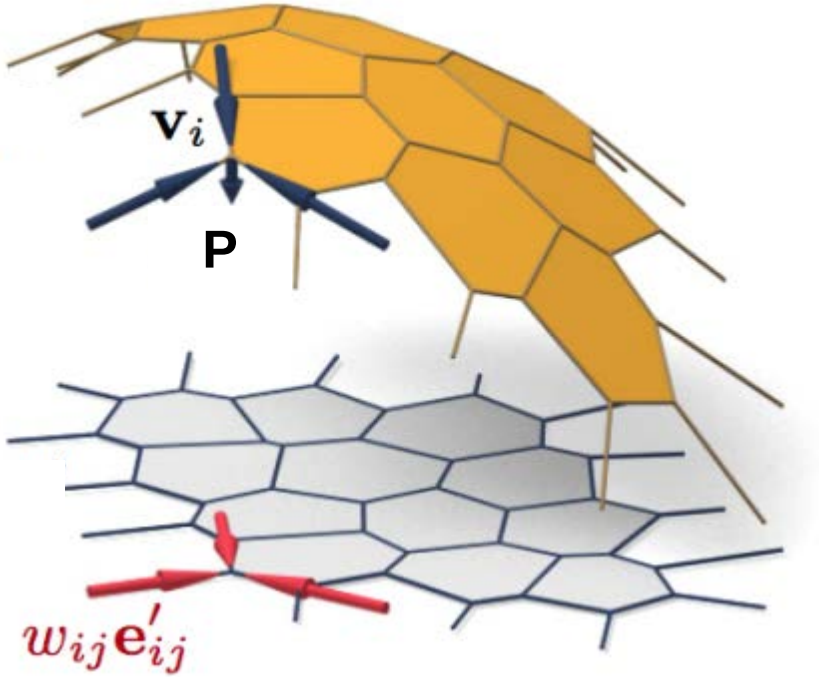
node = block
edge = force

[Block 2009, Vouga et al. 2012]

static equilibrium thrust network analysis

3D “thrust” network

2D projection



node = block
edge = force

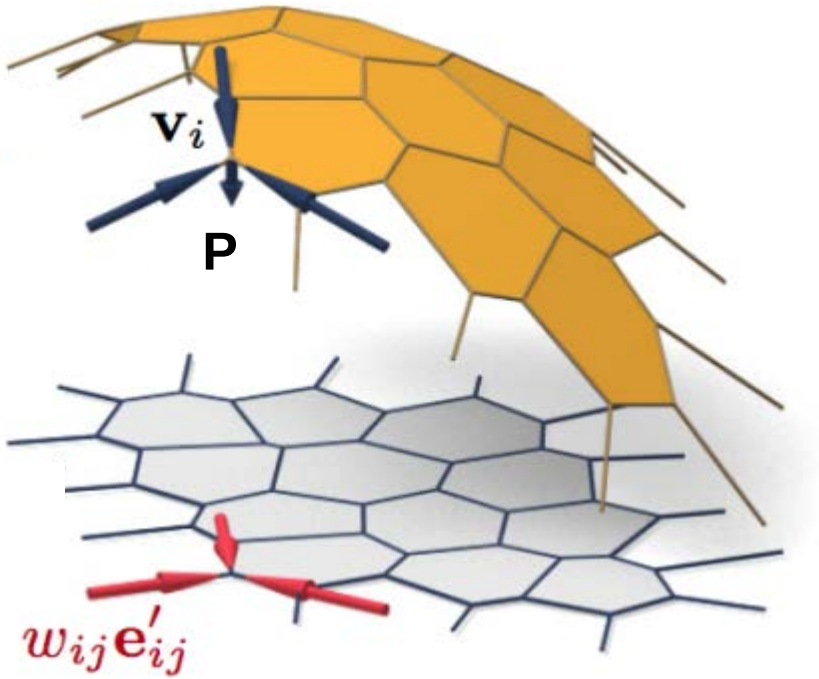
[Block 2009, Vouga et al. 2012]

static equilibrium thrust network analysis

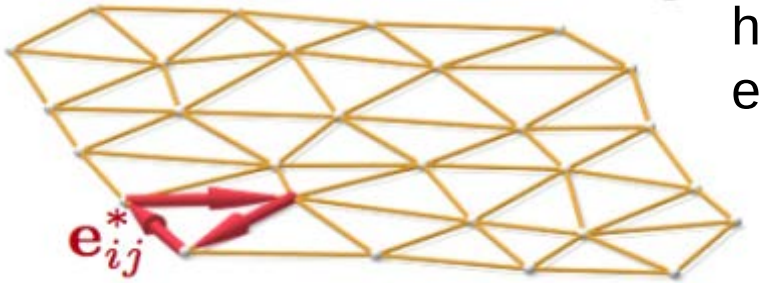
3D “thrust” network

2D projectior

reciprocal diagram



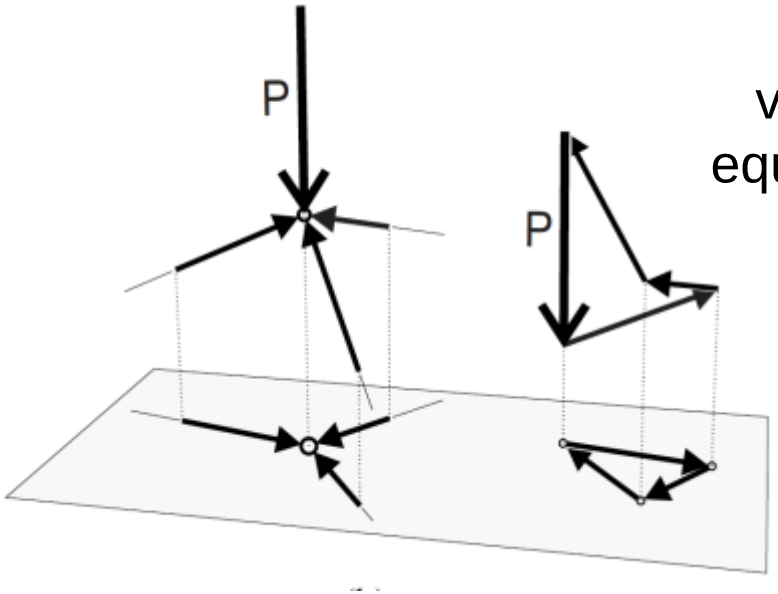
node = block
edge = force



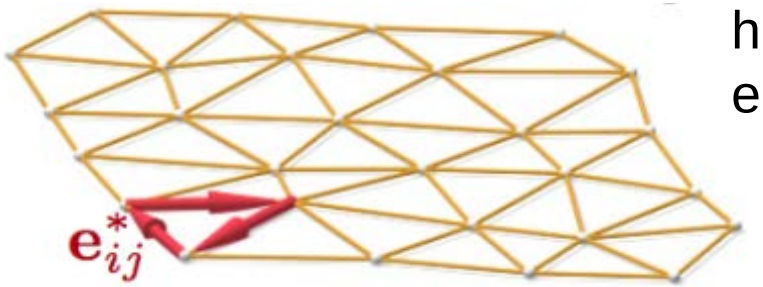
horizontal equilibrium

[Block 2009, Vouga et al. 2012]

static equilibrium thrust network analysis



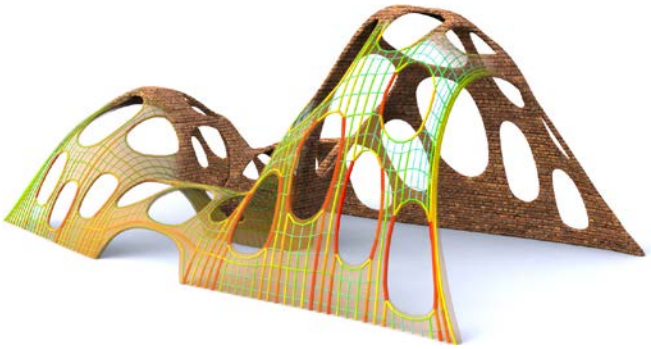
vertical
equilibrium



horizontal
equilibrium

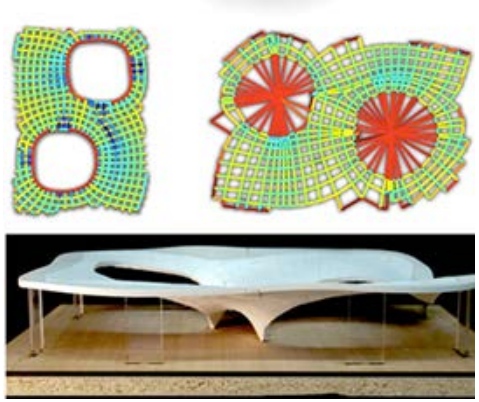
[Block 2009, Vouga et al. 2012]

static equilibrium thrust network analysis

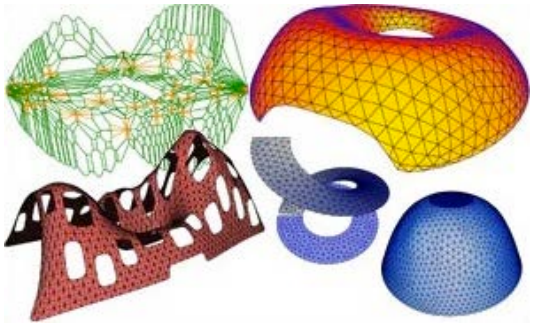


[Vouga et al. 2012]

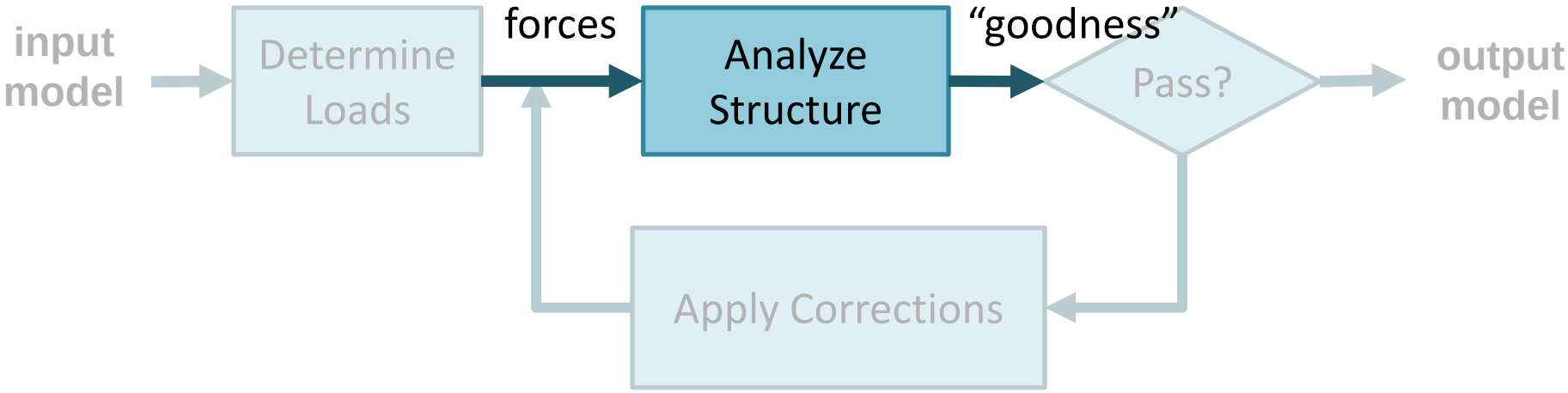
[Liu et al. 2013]



[Panozzo et al. 2013]

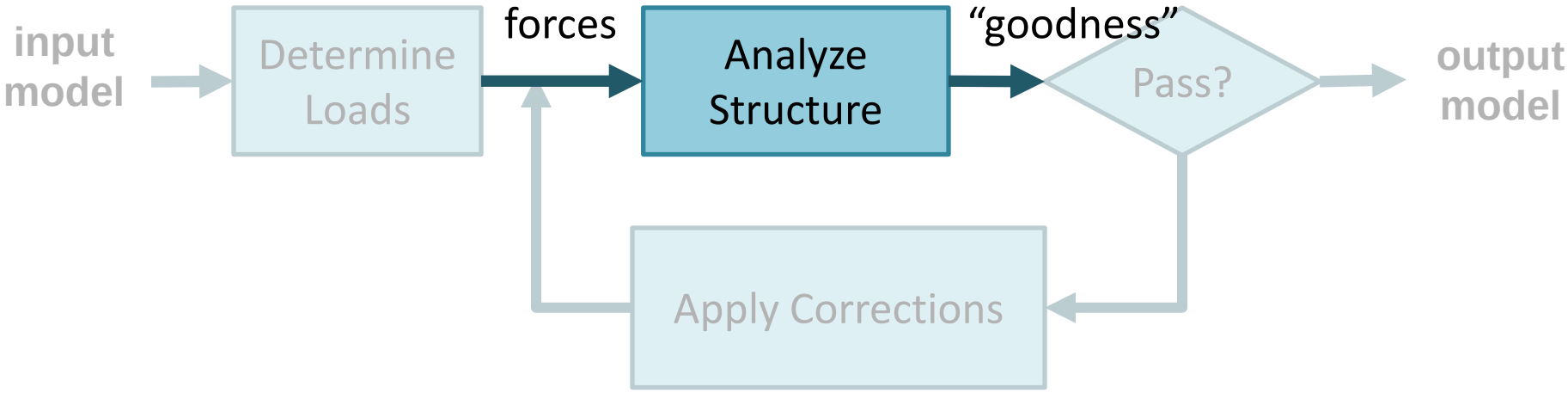


[De Goes et al. 2013]



- static equilibrium
- finite element method
- cross-sectional analysis
- frame structure

assumed rigid bodies
- elements infinitely strong
- ignore internal stress
- no fracture



- static equilibrium
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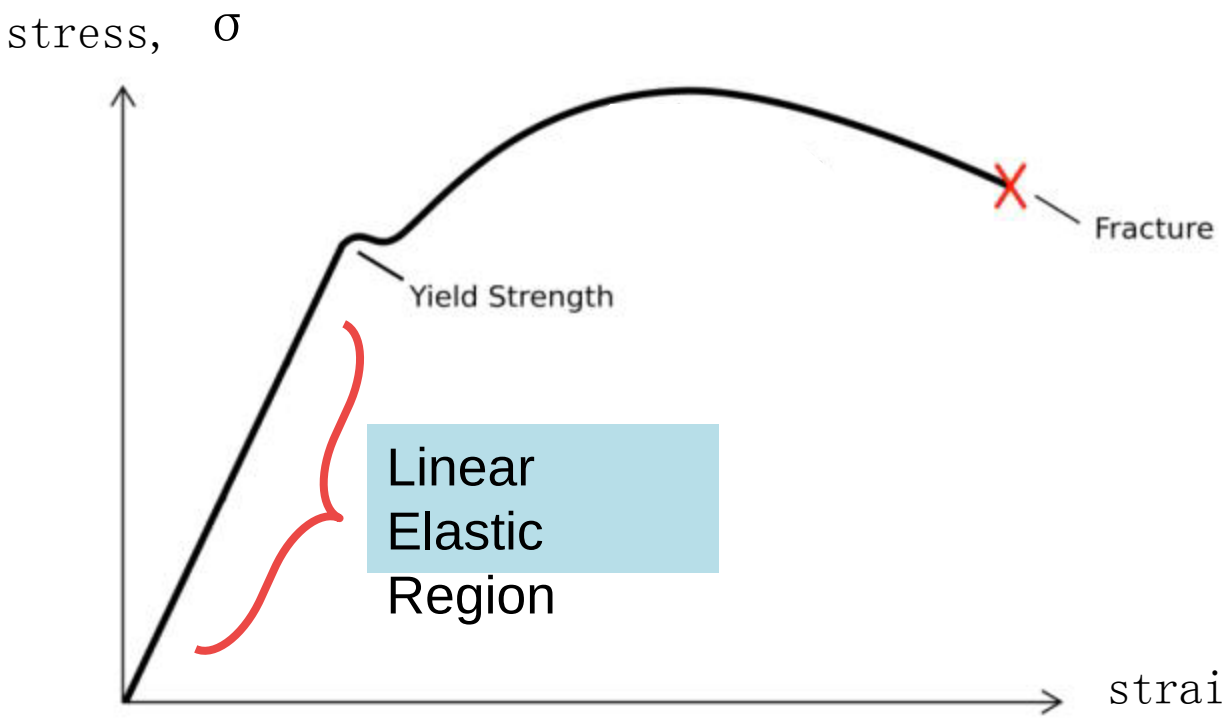
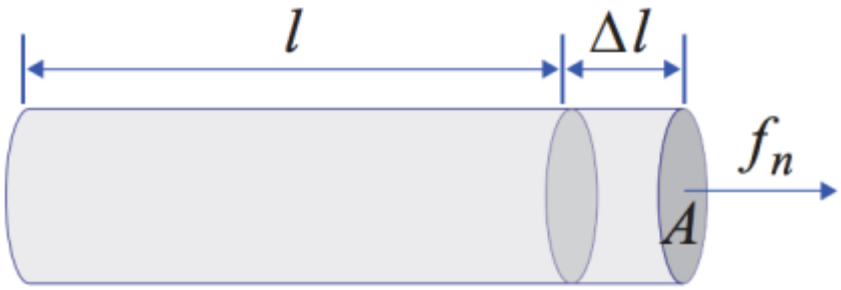
analyze structure **finite element method**



$$\sigma = E\,\varepsilon$$

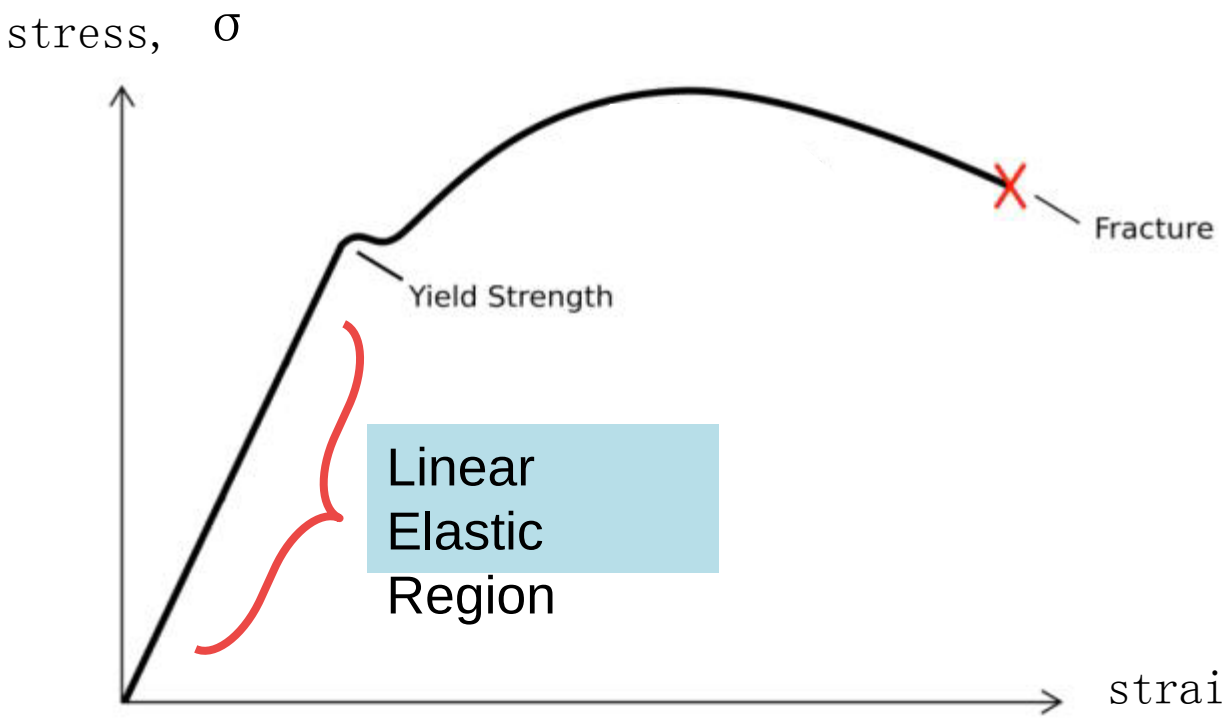
analyze structure **finite element method**

[Mueller et al. 2008



strain: $\varepsilon = \Delta l / l$

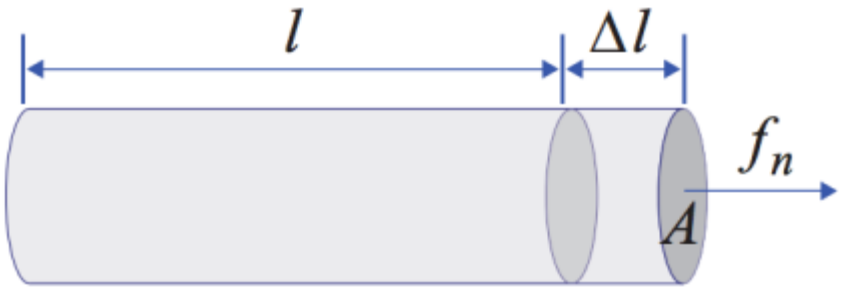
stress: $\sigma = f_n / A$



$\sigma = E\,\varepsilon$

analyze structure **finite element method**

[Mueller et al. 2008



strain: $\varepsilon = \Delta l / l$

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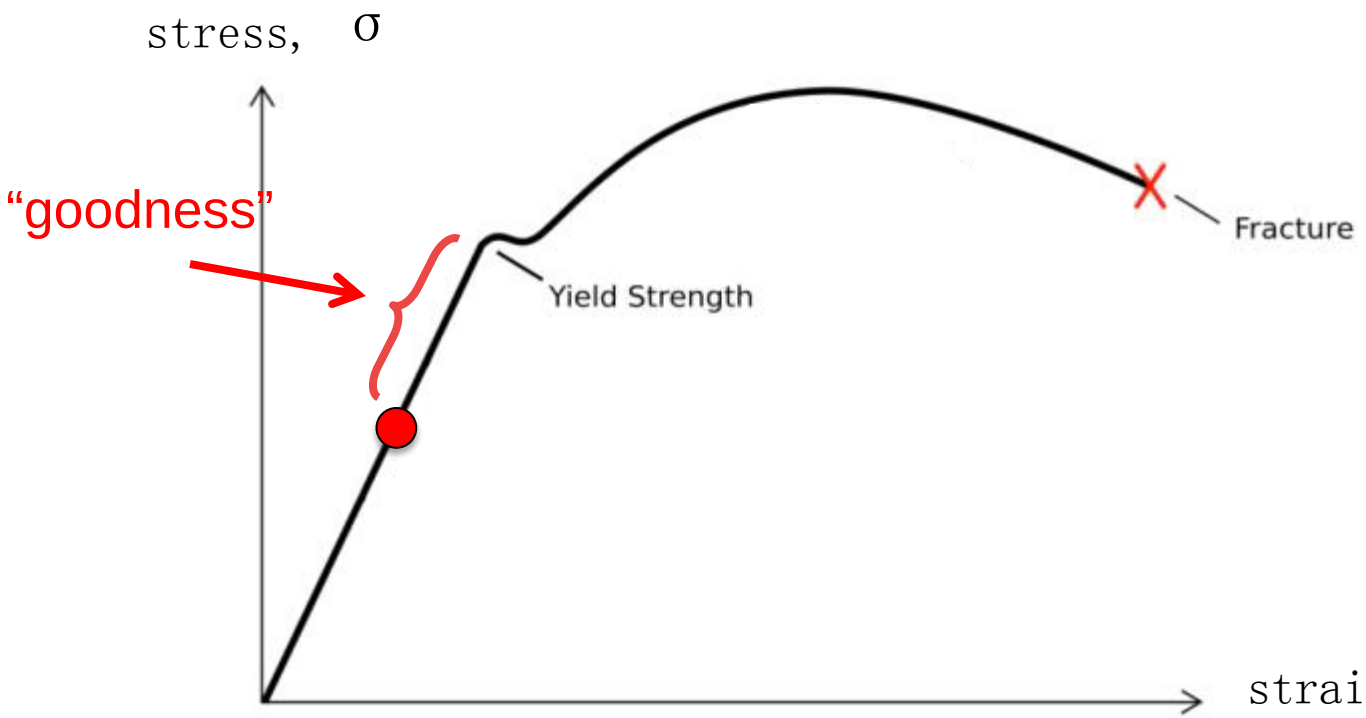
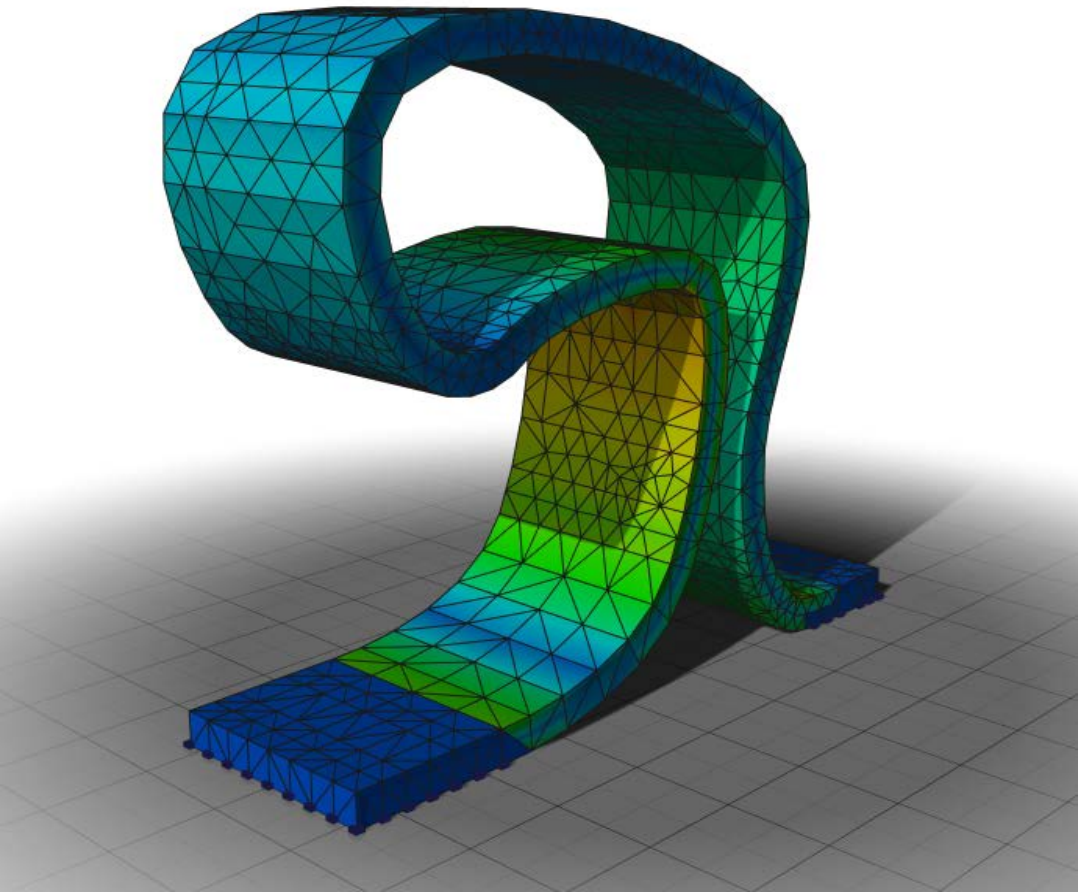


$\sigma = E\,\varepsilon$

analyze structure **finite element method**



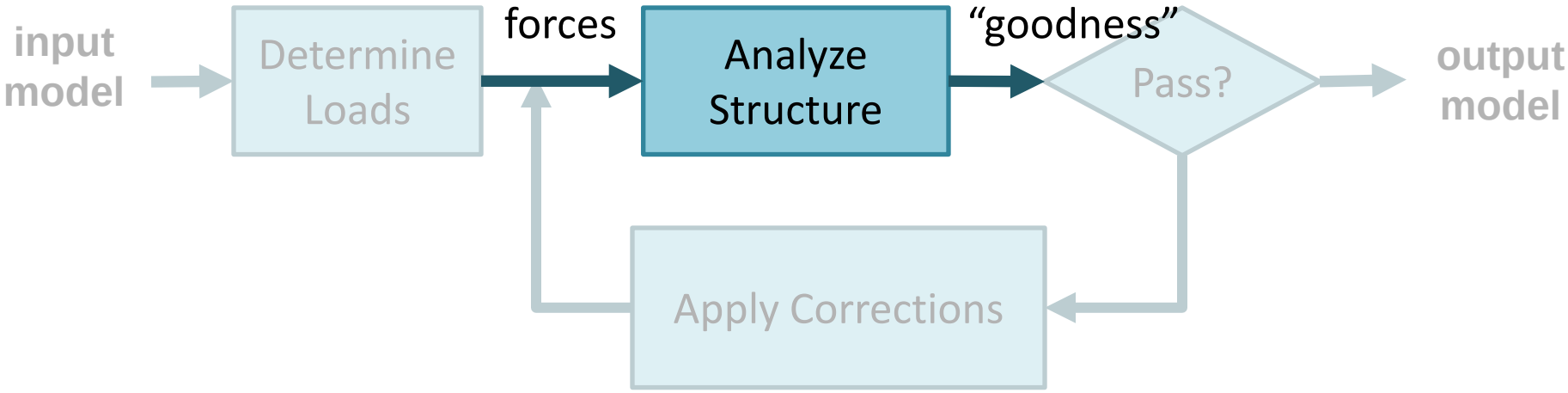
tetrahedra

stiffnes
s

$$\mathbf{K}\,\mathbf{D} = \mathbf{F}$$

deformation forces

“goodness” = $\sigma_{yield} - \sigma_{max}$



- static equilibrium
- finite element method
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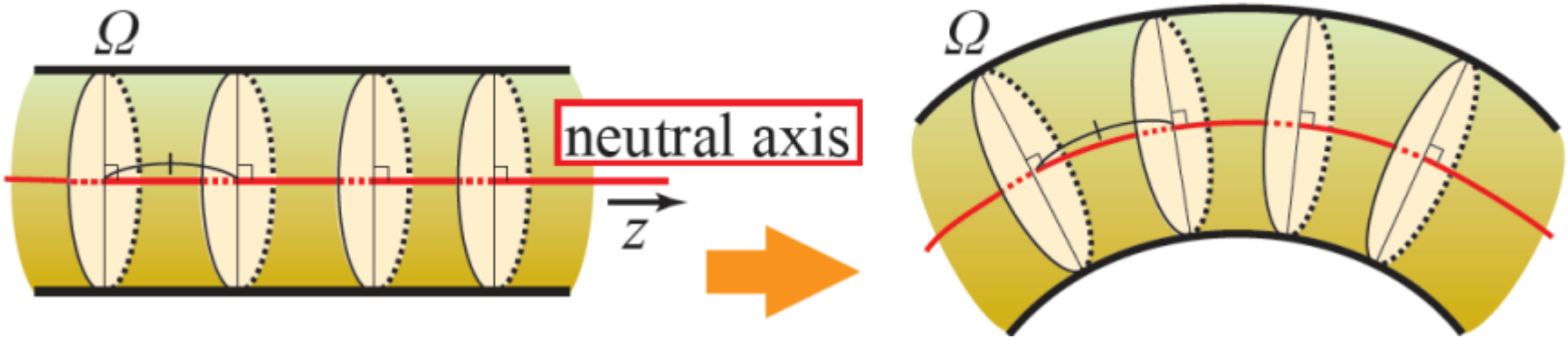
analyze structure **cross-sections**

Euler-Bernoulli beam assumption

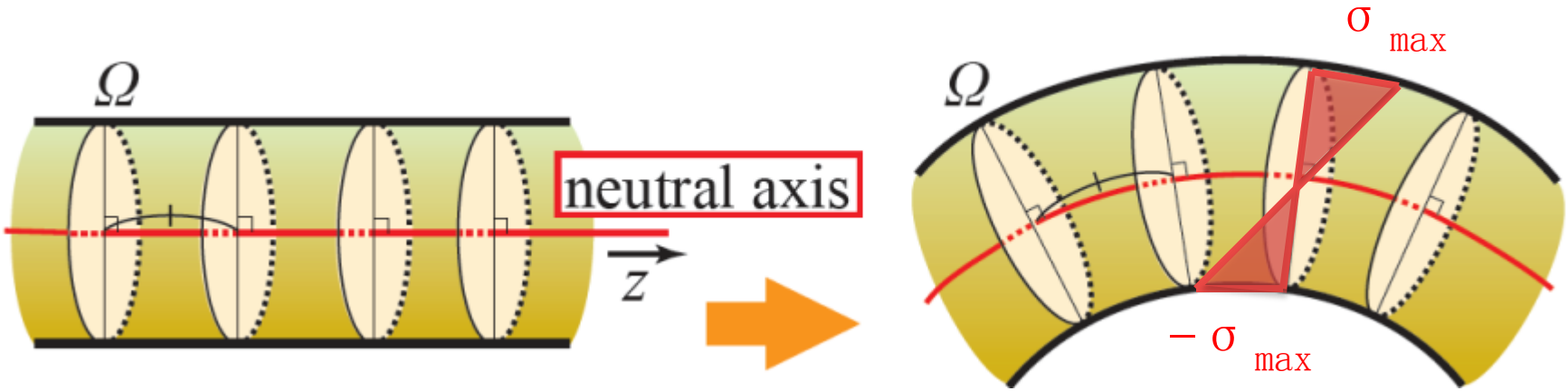


- neutral axis does not stretch – zero stress
- max stress at boundary

[Umetani et al. 2013]

analyze structure **cross-sections**

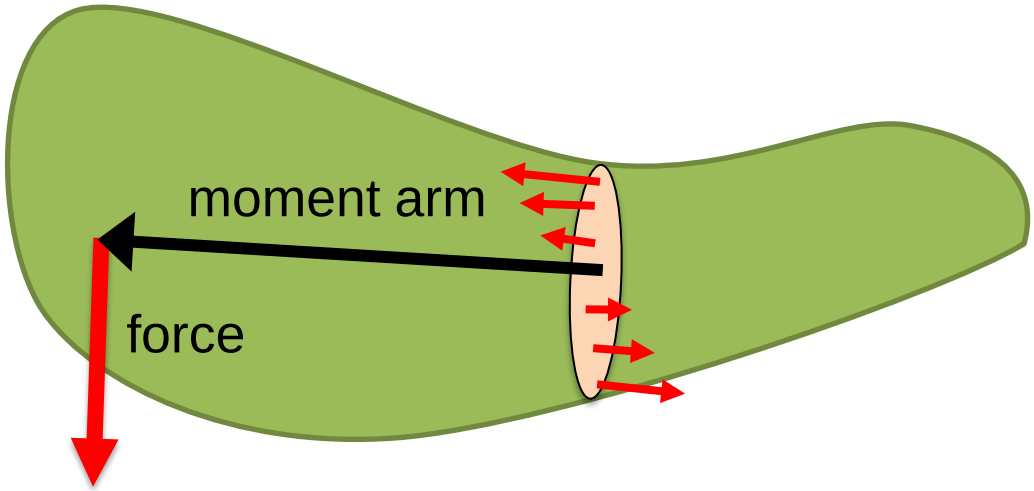
Euler-Bernoulli beam assumption

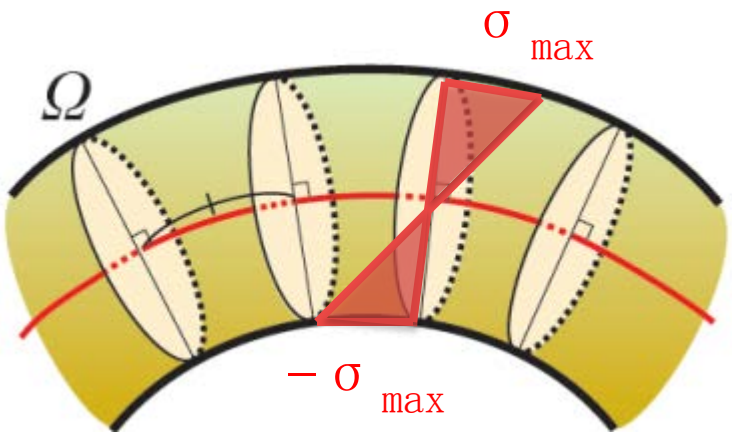


- neutral axis does not stretch – zero stress
- max stress at boundary

[Umetani et al. 2013]

Euler-Bernoulli beam assumption

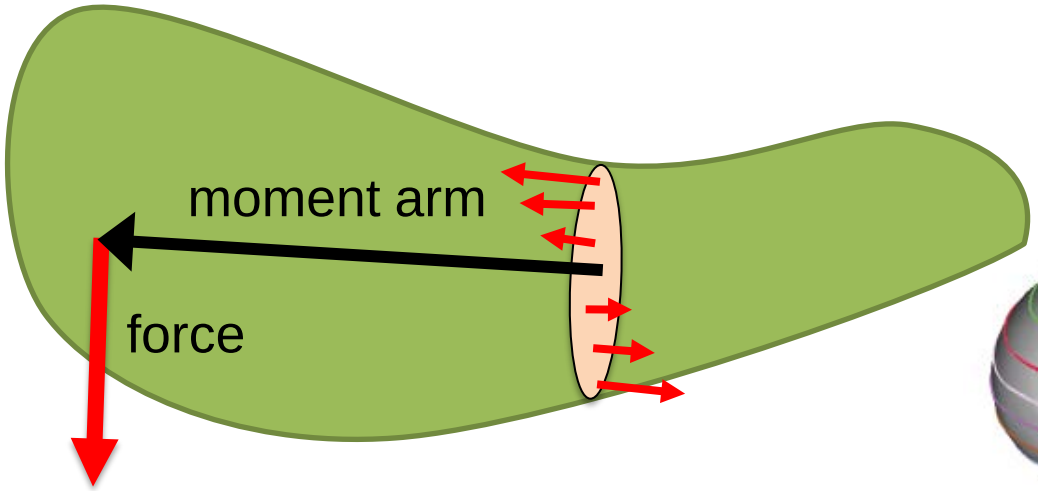


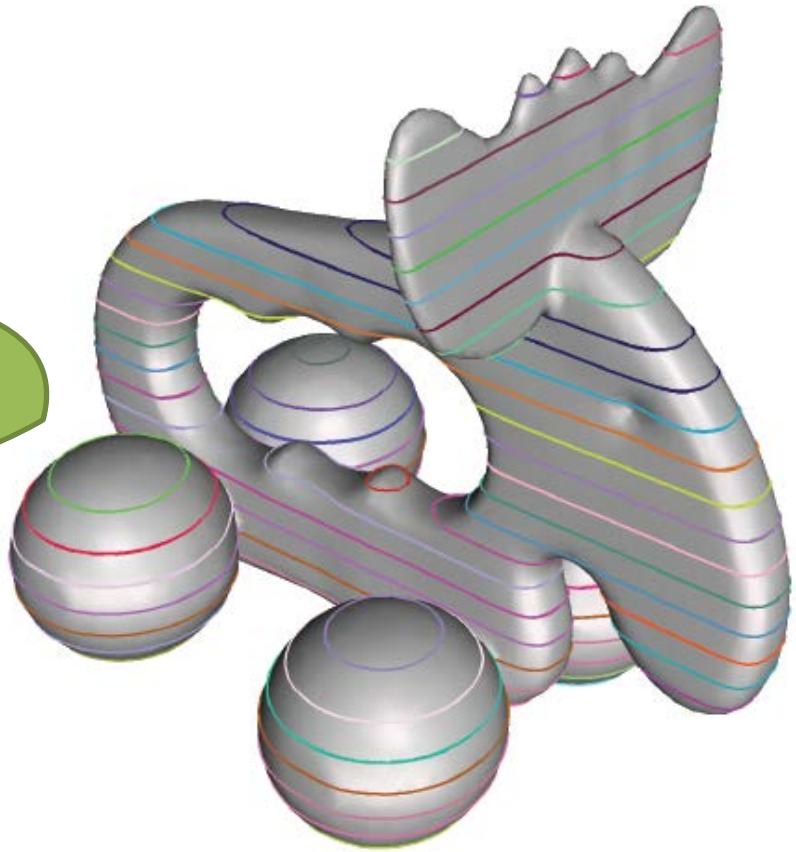


- neutral axis does not stretch – zero stress
- max stress at boundary

[Umetani et al. 2013]

analyze structure **cross-sections**

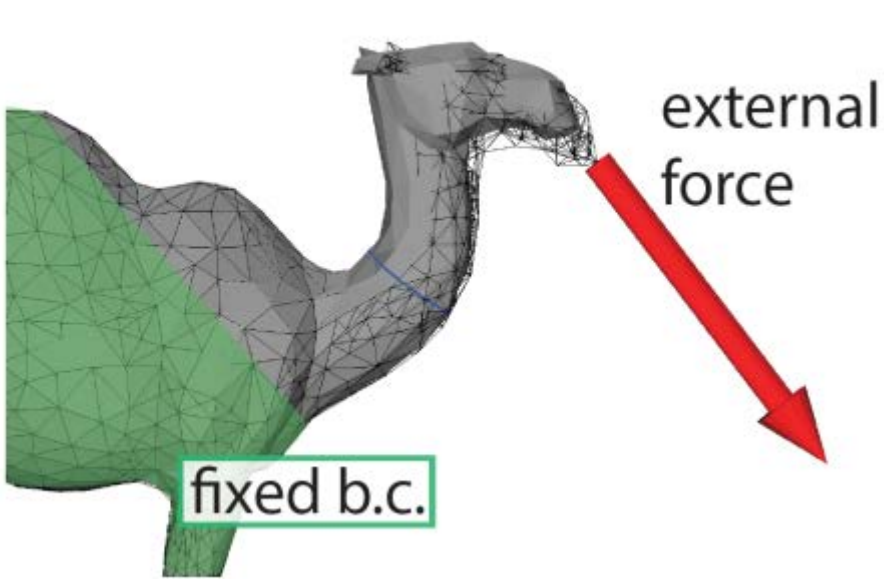




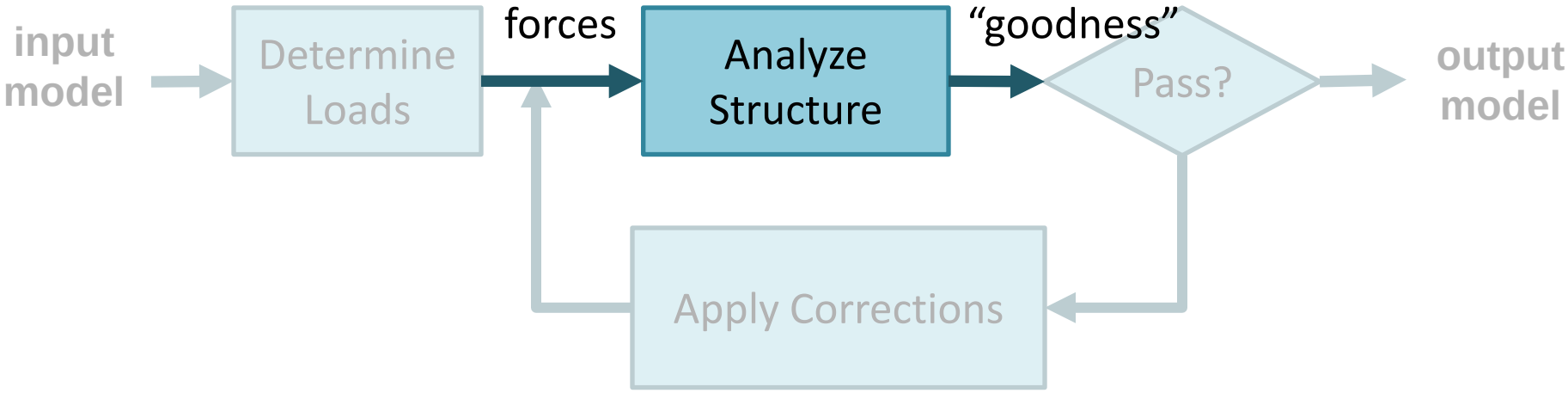
“goodness” = $\sigma_{yield} - \max(\sigma_{cross})$

[Umetani et al. 2013]

analyze structure **cross-sections**



[Umetani et al. 2013]



- static equilibrium
- finite element method
- cross-sectional analysis
- frame structure

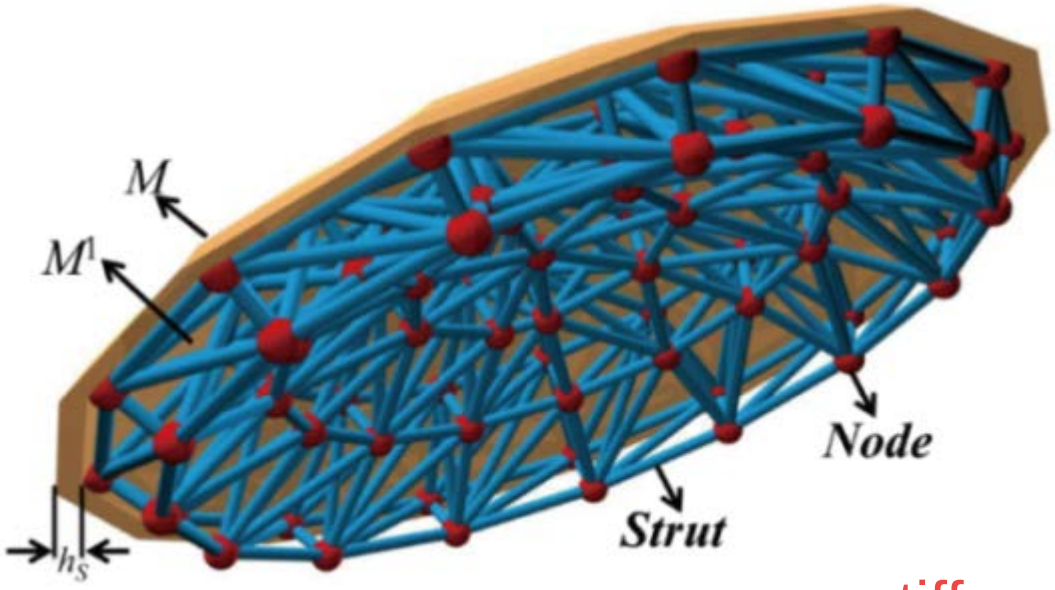
assumed rigid or solid

- consider alternative structures

analyze structure **skin-frame**

[Wang et al. 2013]

analyze structure **skin-frame**



finite element method

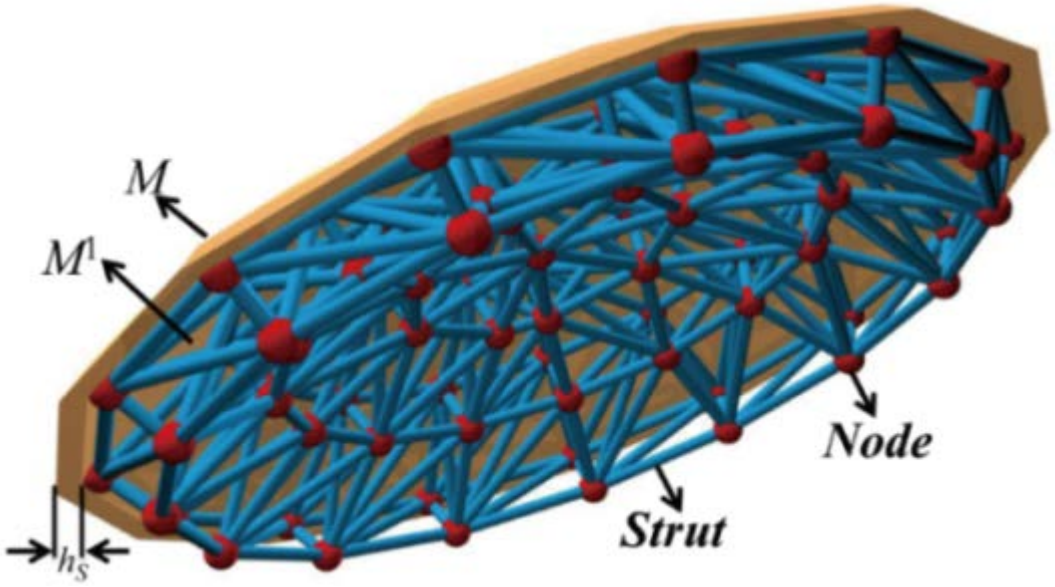
stiffness: node positions, strut radii

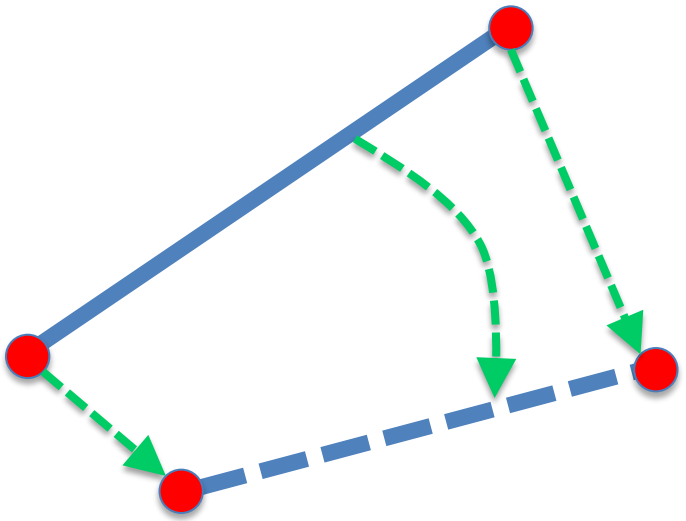
$$\mathbf{K}\,\mathbf{D} = \mathbf{F}$$

deformation forces

[Wang et al. 2013]

analyze structure **skin-frame**

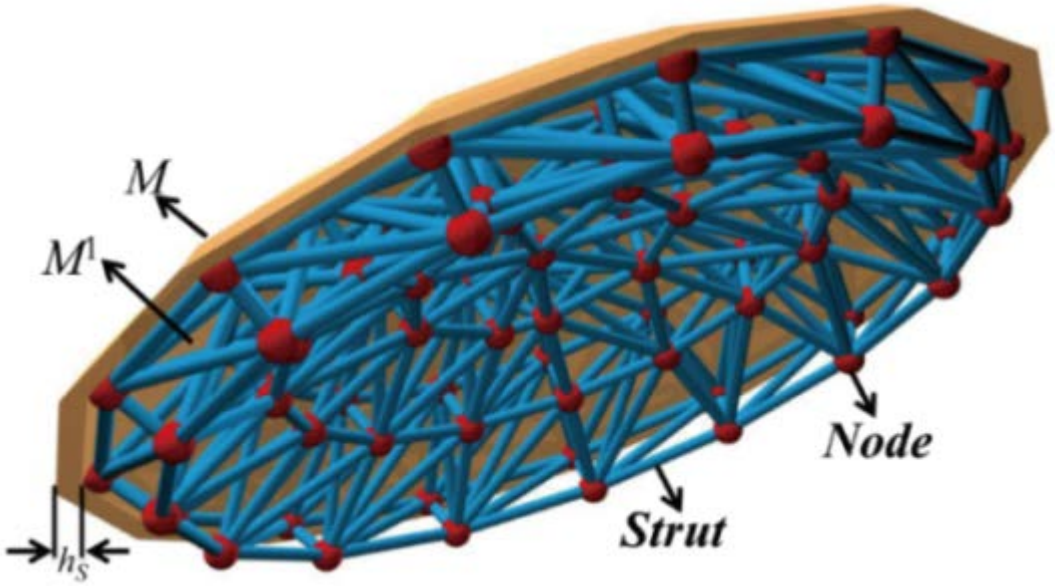


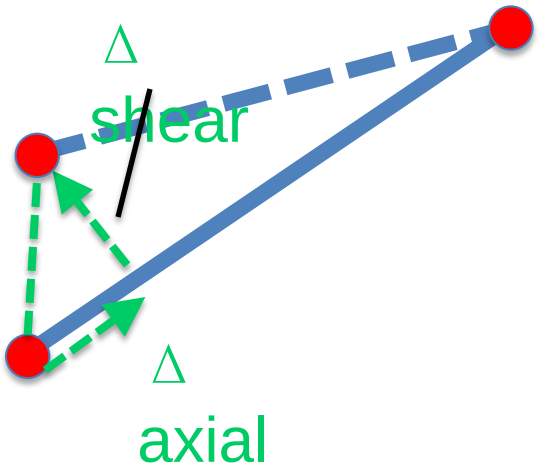


elasticity

[Wang et al. 2013]

analyze structure **skin-frame**

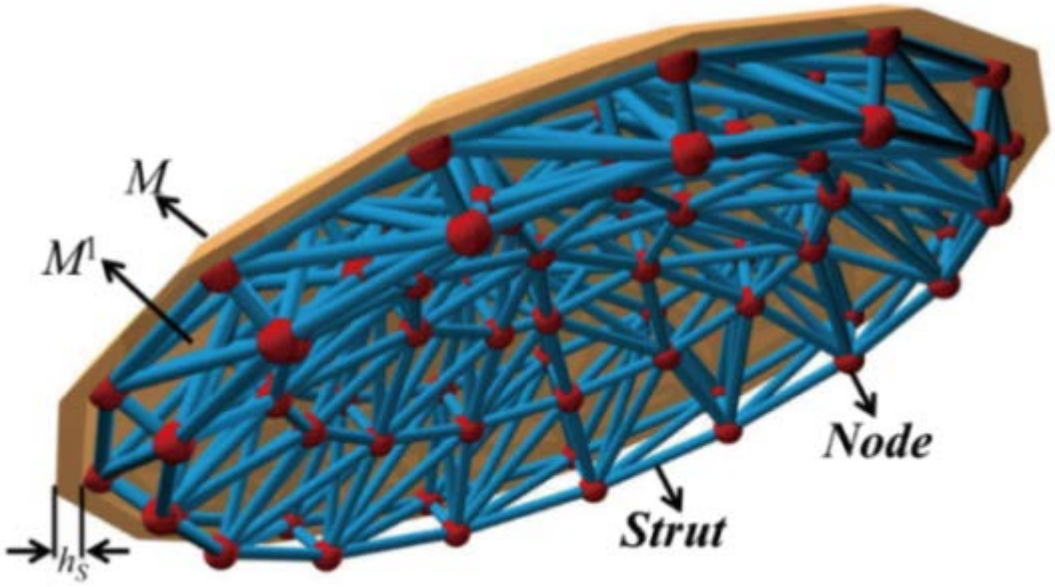


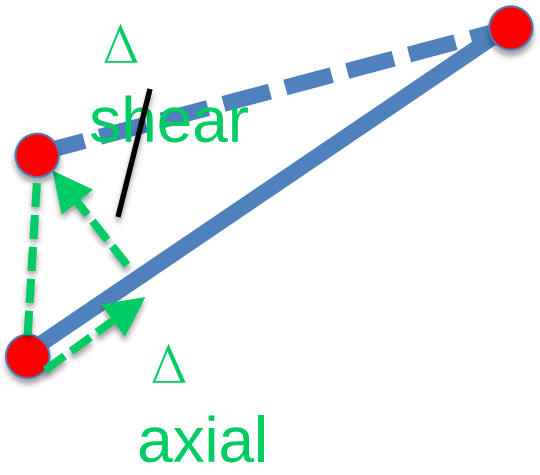


elasticity

[Wang et al. 2013]

analyze structure **skin-frame**





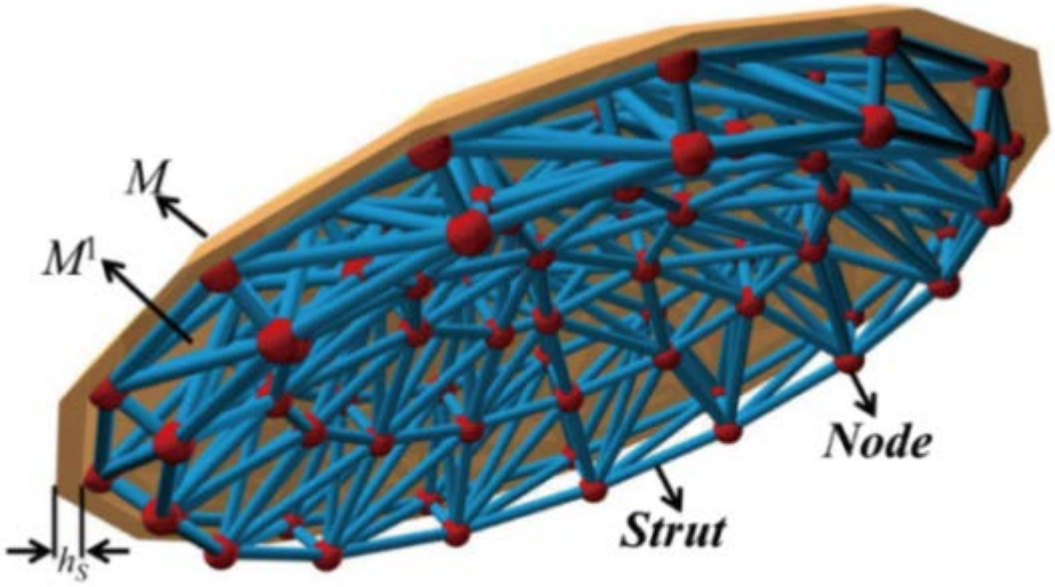
elasticity

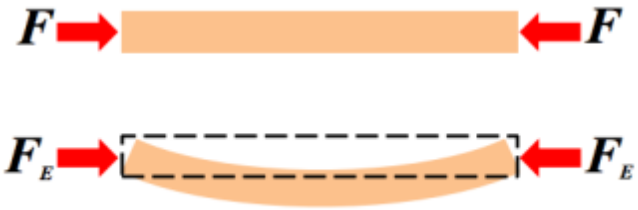
$$\frac{\Delta axial}{length(strut)}\gamma \leq \text{max axial stress}$$

$$\frac{\Delta shear}{length(strut)}\mu \leq \text{max shear stress}$$

[Wang et al. 2013]

analyze structure **skin-frame**

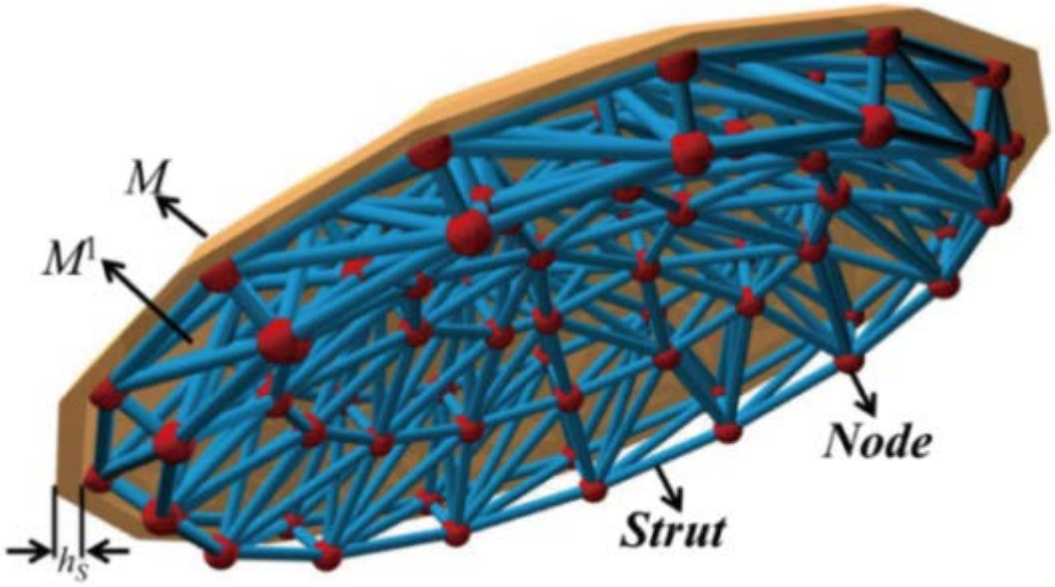




Euler buckling strut radius limited by slenderness (length/radius)

[Wang et al. 2013]

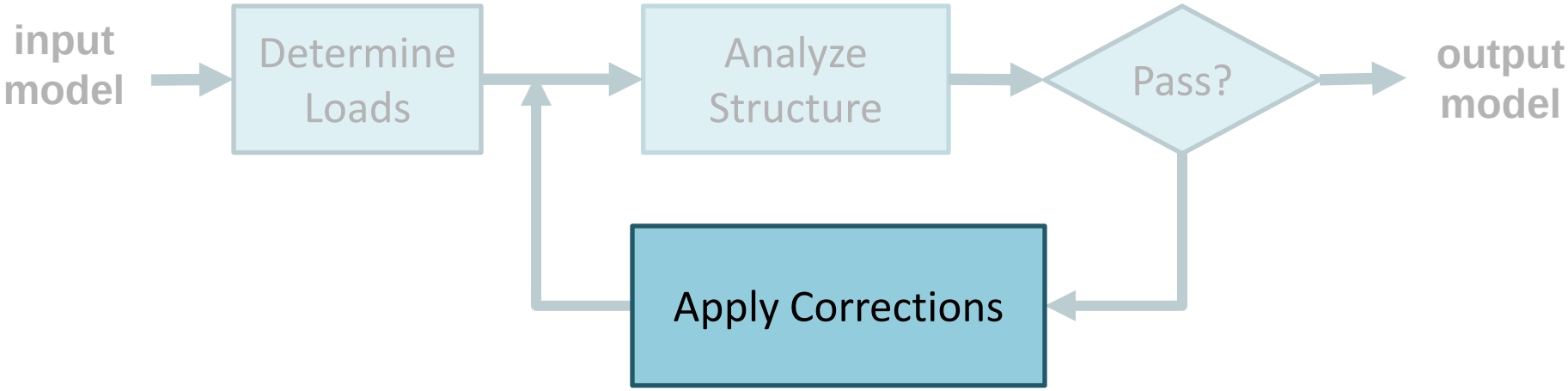
analyze structure **skin-frame**



goal: make structure light under structural constraints

“goodness” = material reduction

[Wang et al. 2013]



Objectives

- balanced
- self-supporting
- robust
- material cost

corrections

- interior fill
- surface geometry

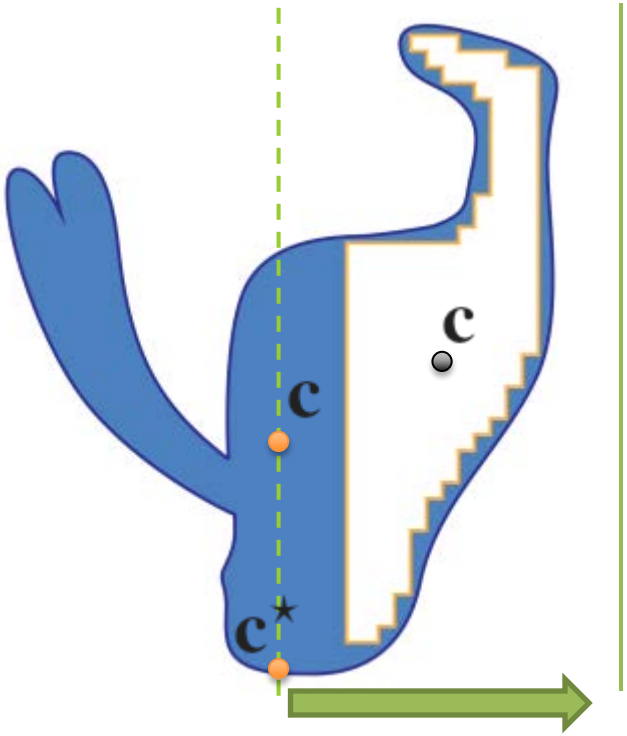
apply corrections **balance**

voxel grid represents volume

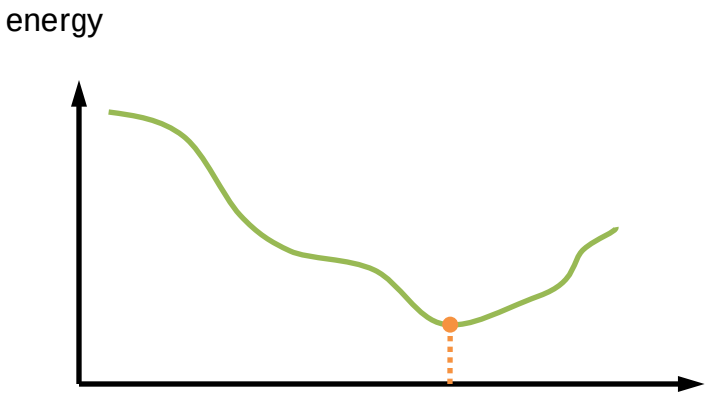
- binary fill values
- printability constraint

[Prévost et al. 2013]

apply corrections **balance**



optimal plane



[Prévost et al. 2013]

apply corrections **balance**

shape = $\sum$ deformation handles

reduced number of variables

smooth and intuitive deformations

[Lander 1998; Jacobson 2011]

[Prévost et al. 2013]

apply corrections **balance**

gradient descent on reduced model:

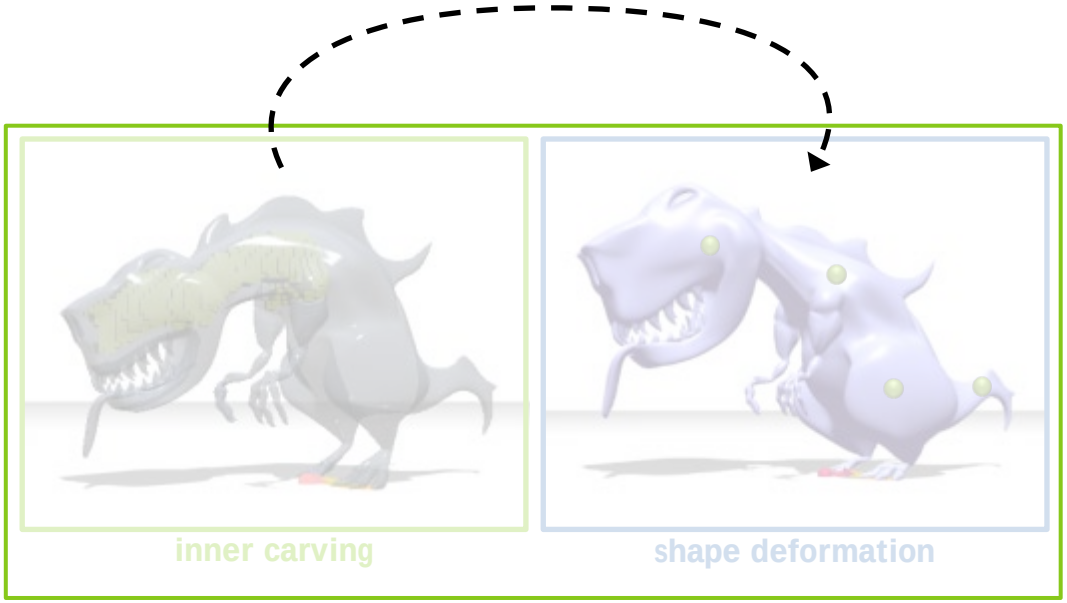
translation and scale at handles

[Prévost et al. 2013]

apply corrections **balance**

Fix: handles

Optimize: voxel fill



Fix: voxels

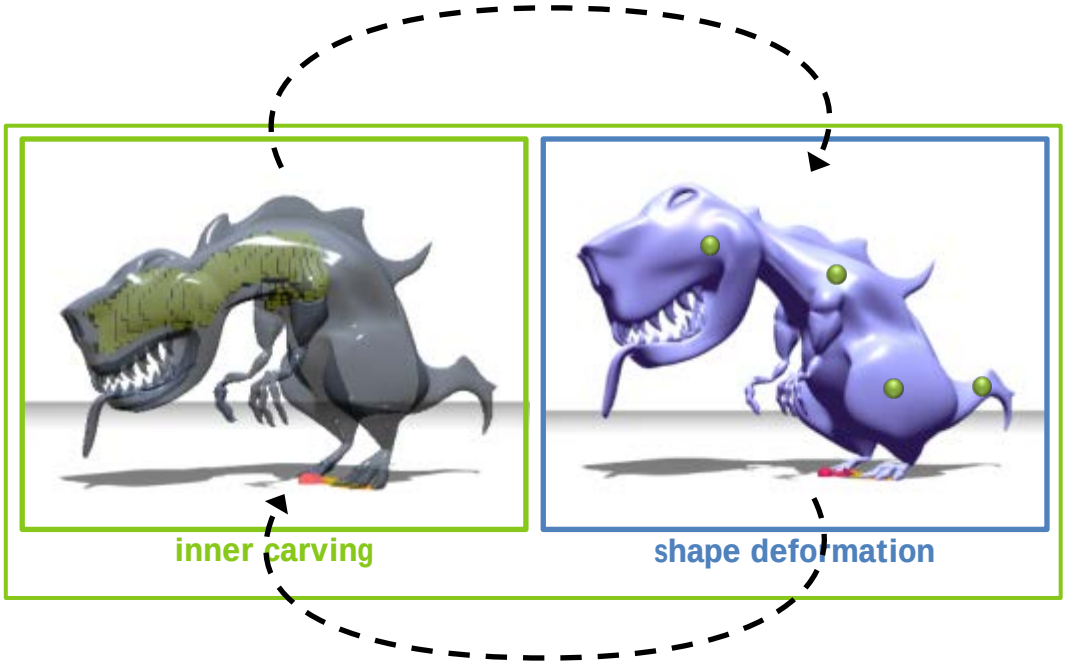
Optimize: handle transformations

[Prévost et al. 2013]

apply corrections **balance**

Fix: handles

Optimize: voxel fill

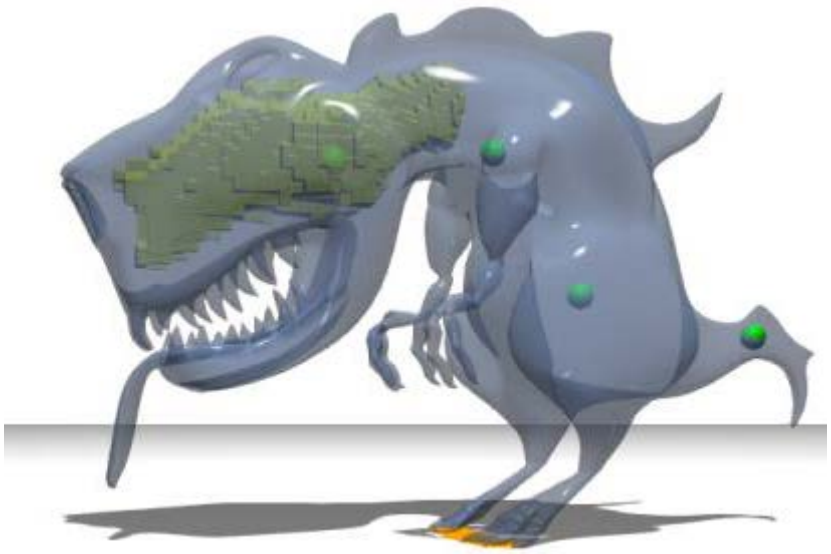


Fix: voxels

Optimize: handle transformations

[Prévost et al. 2013]

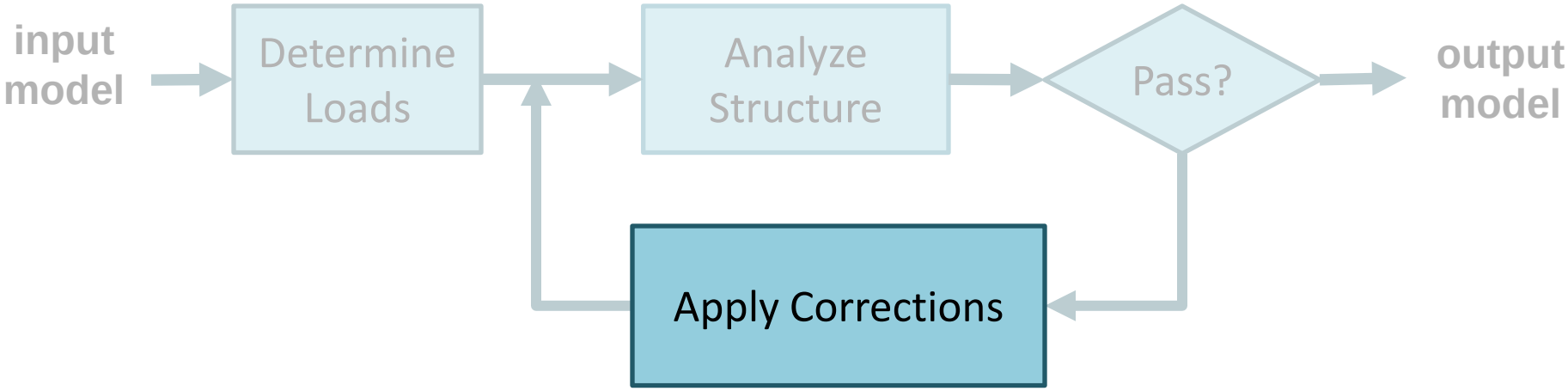
apply corrections **balance**



[Prévost et al. 2013]

apply corrections **balance**

[Prévost et al. 2013]



Objectives

- balanced
- self-supporting
- robust
- material cost

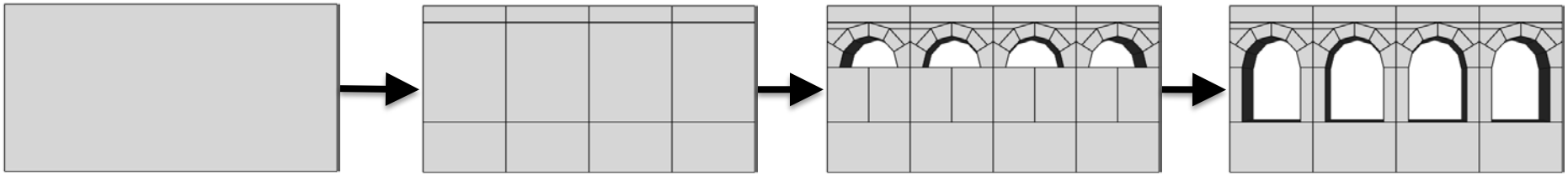
masonry structures

- procedural modeling
- thrust networks

apply corrections **procedural modeling**

[Muller et al. 2006, Whiting et al. 2009]

apply corrections **procedural modeling**

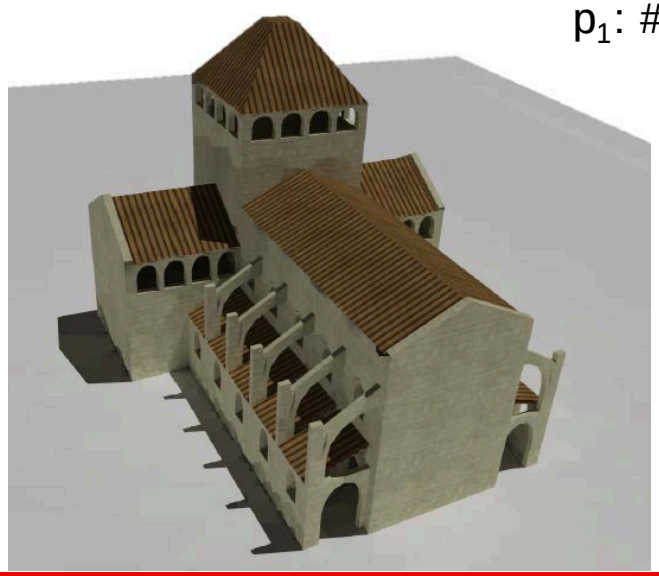


starting shape | subdivide | replace with arch, columns | scale columns

discrete parameter
p_1: #segments

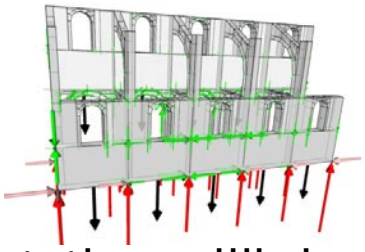
continuous parameters
p_2: arch thickness
p_3: column width

design variables



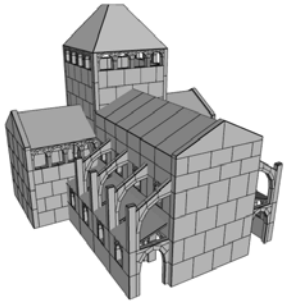
[Muller et al. 2006, Whiting et al. 2009]

apply corrections **procedural modeling**



self-weight

static equilibrium



input model

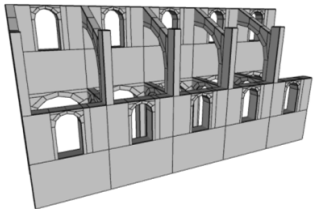
Determine Loads

Analyze Structure

Pass?

output model

Apply Corrections

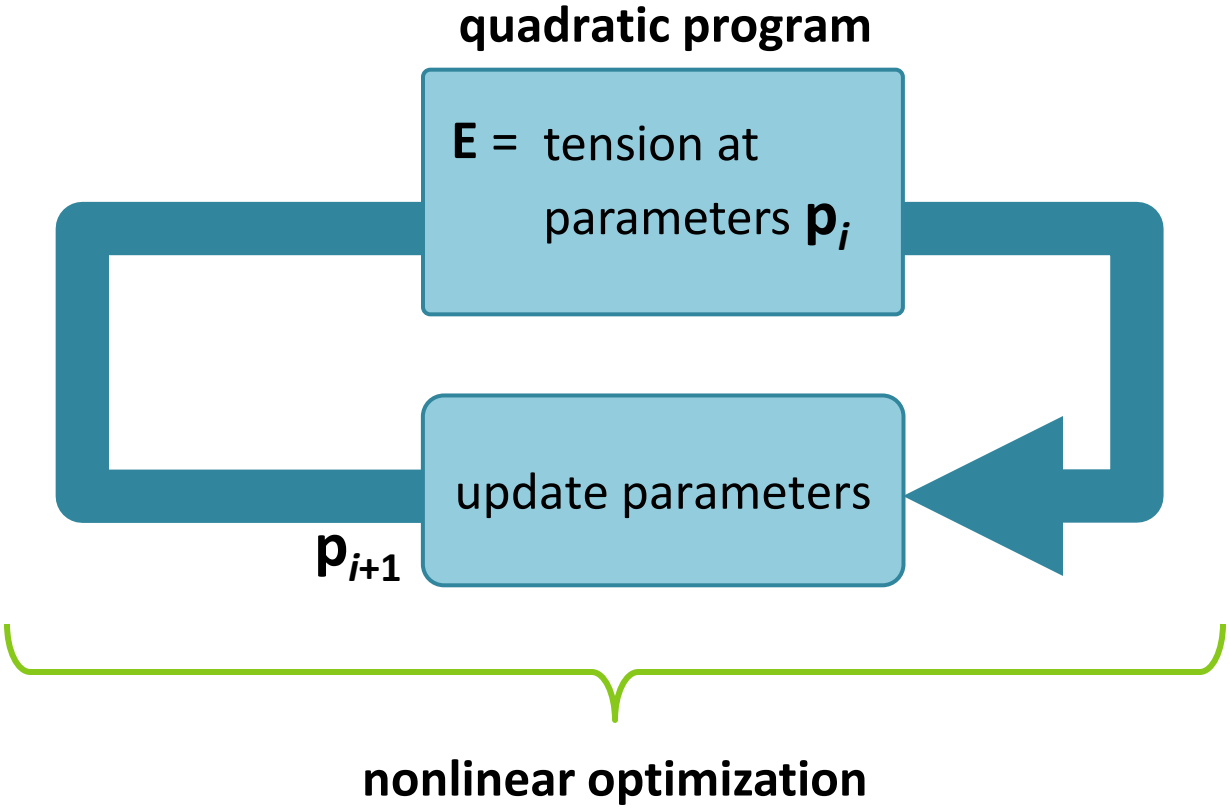


update procedural model

[Whiting et al. 2009]

nested optimizations

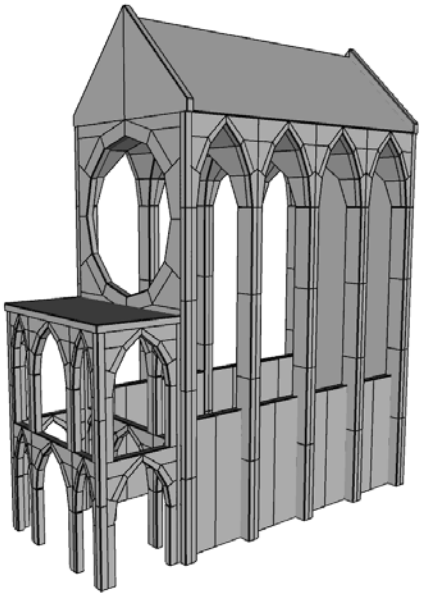
minimize: feasibility energy



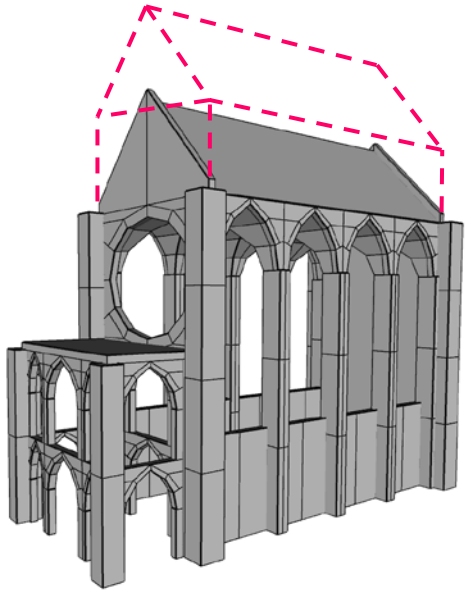
[Whiting et al. 2009]

apply corrections **procedural modeling**

sainte chapelle



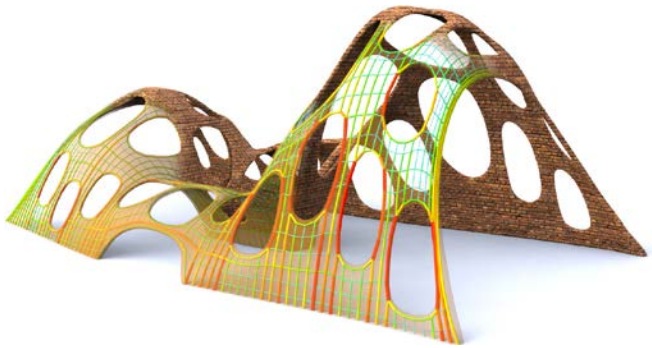
input: unstable model



4 parameter optimization

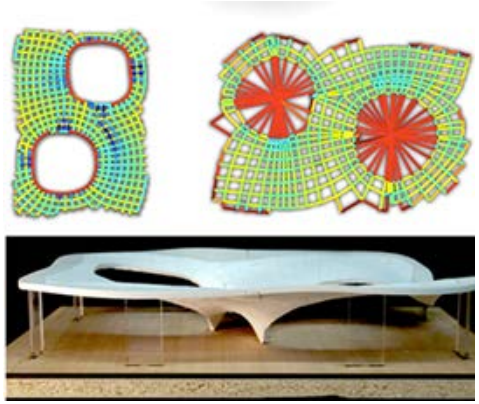
[Whiting et al. 2009]

static equilibrium thrust network analysis

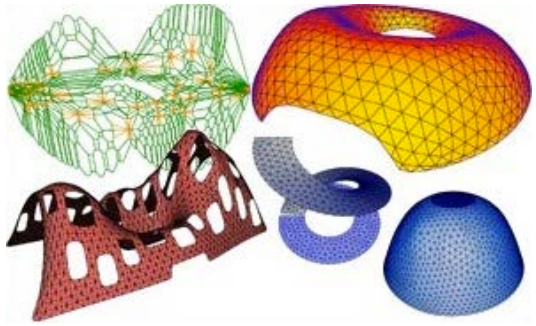


[Vouga et al. 2012]

[Liu et al. 2013]



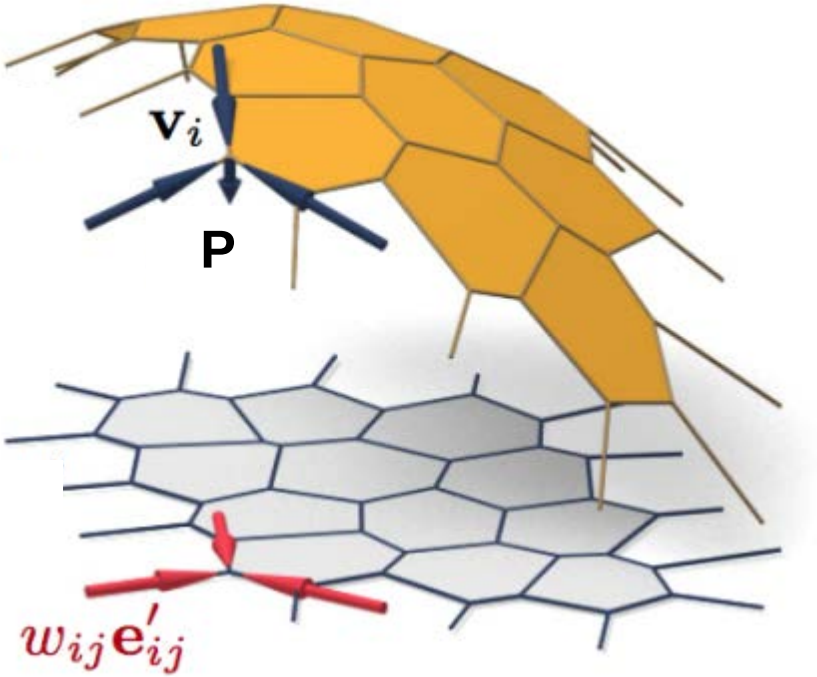
[Panozzo et al. 2013]



[De Goes et al. 2013]

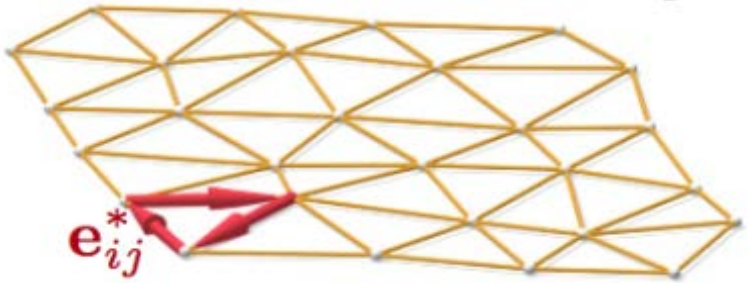
static equilibrium thrust network analysis

3D “thrust” network



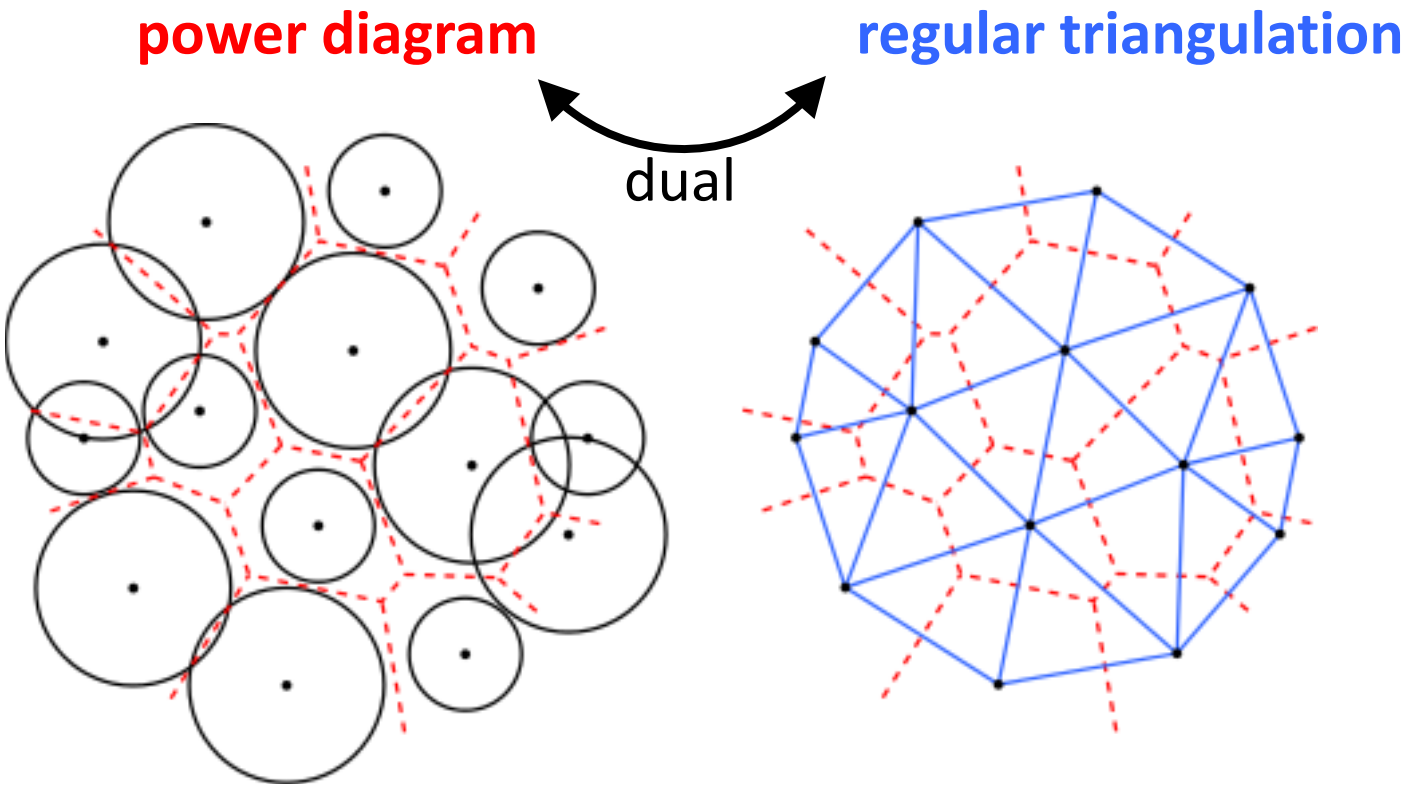
2D projectior

reciprocal diagram



[Block 2009, Vouga et al. 2012]

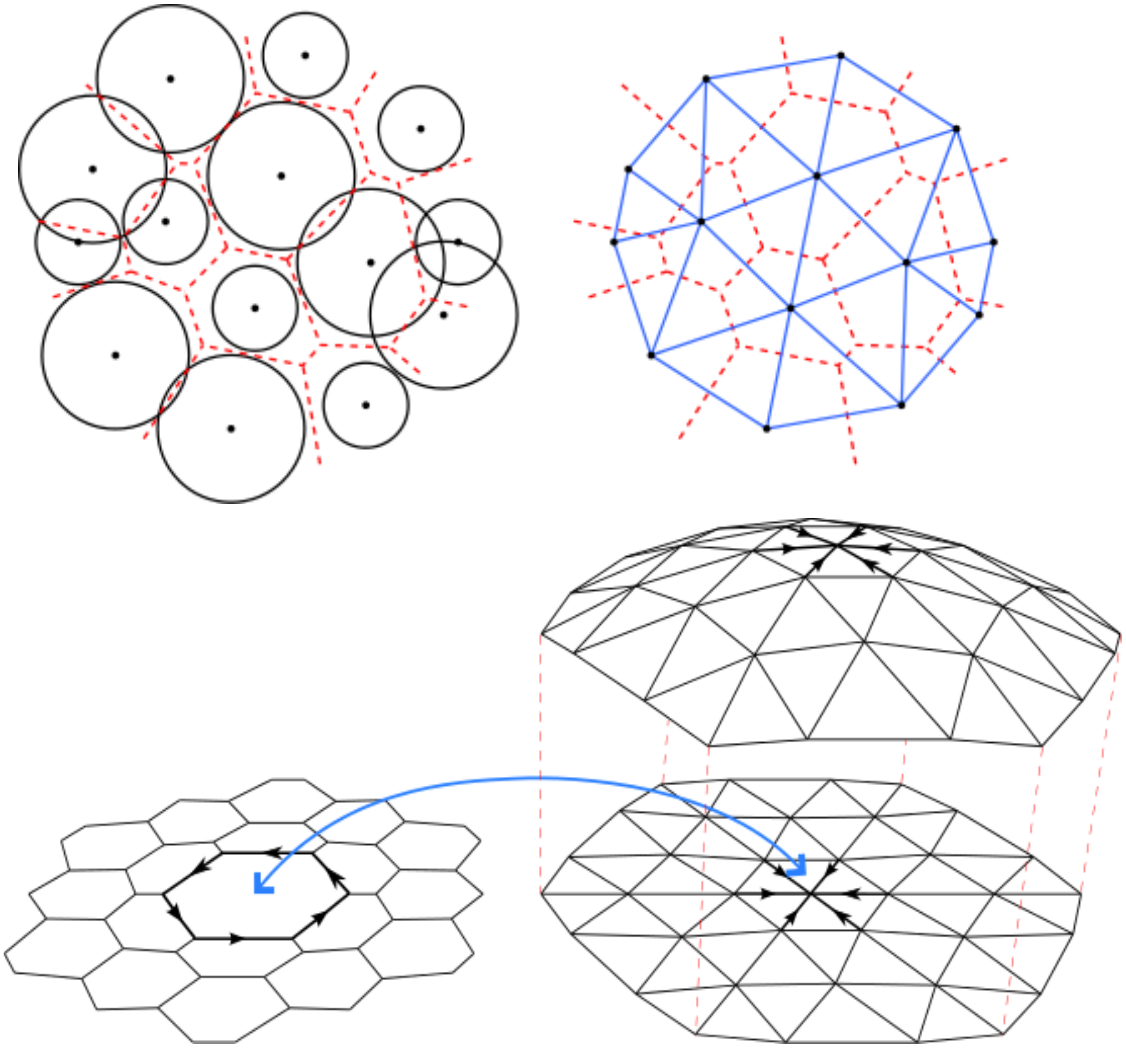
apply corrections **self-supporting**



[Aurenhammer 1987, Liu et al. 2013]

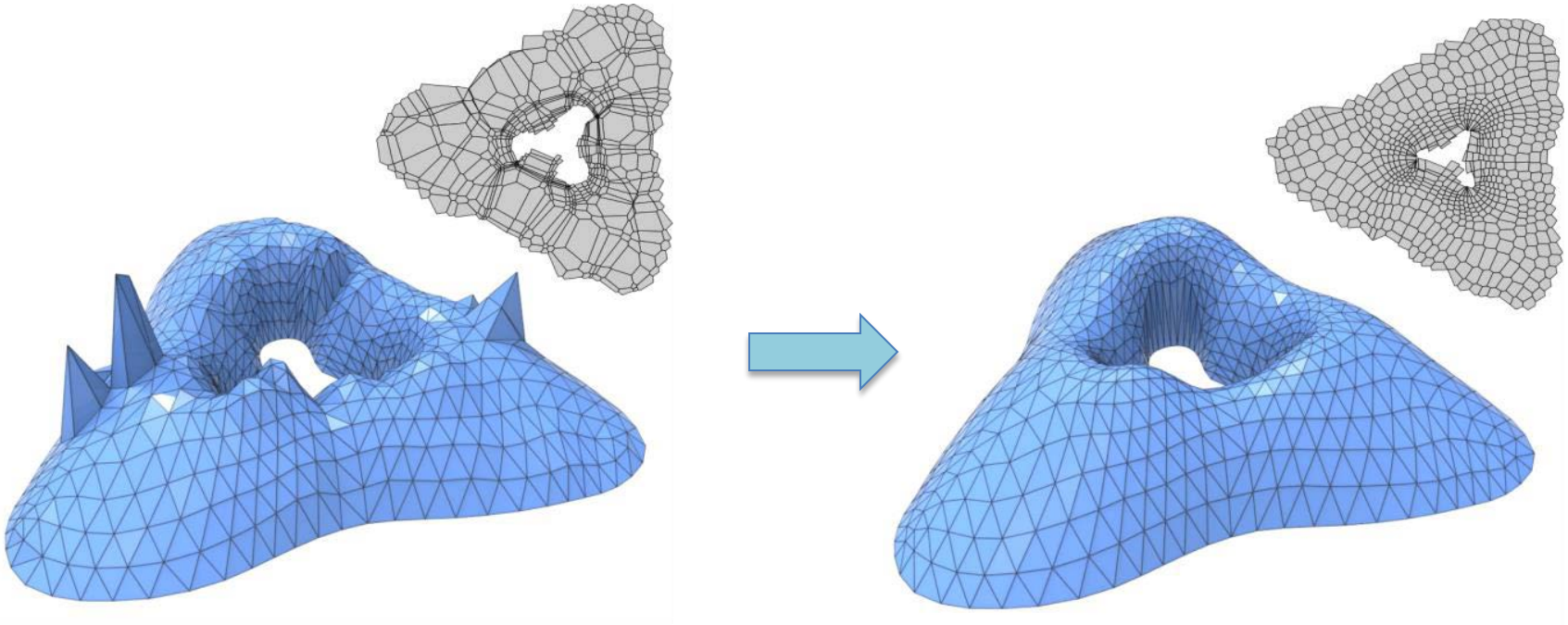
apply corrections **self-supporting**

power diagram **regular triangulation**



[Liu et al. 2013]

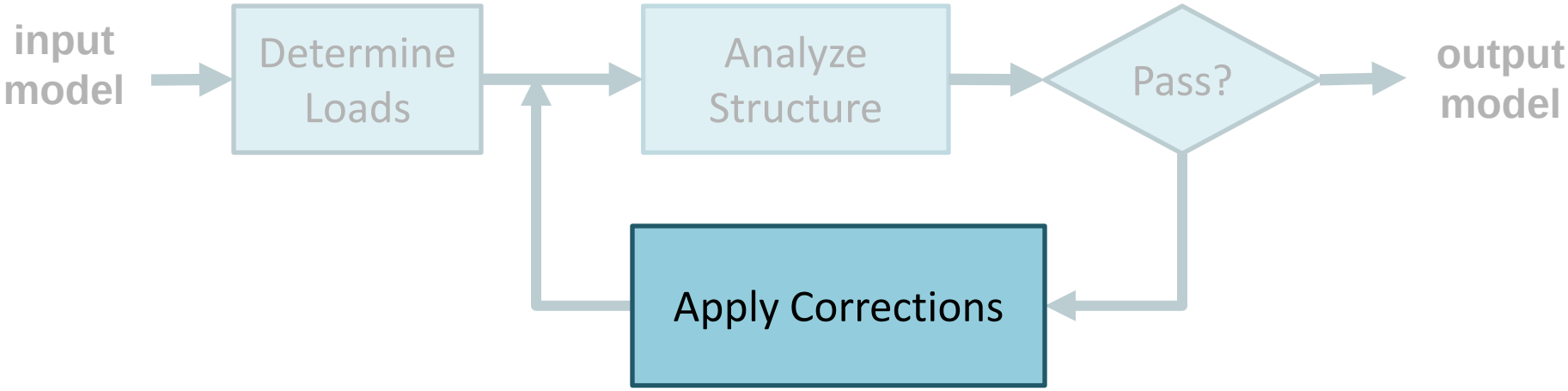
apply corrections **self-supporting**



smoothing scheme

update weights of power diagram: uniformly sized cells

[Liu et al. 2013]



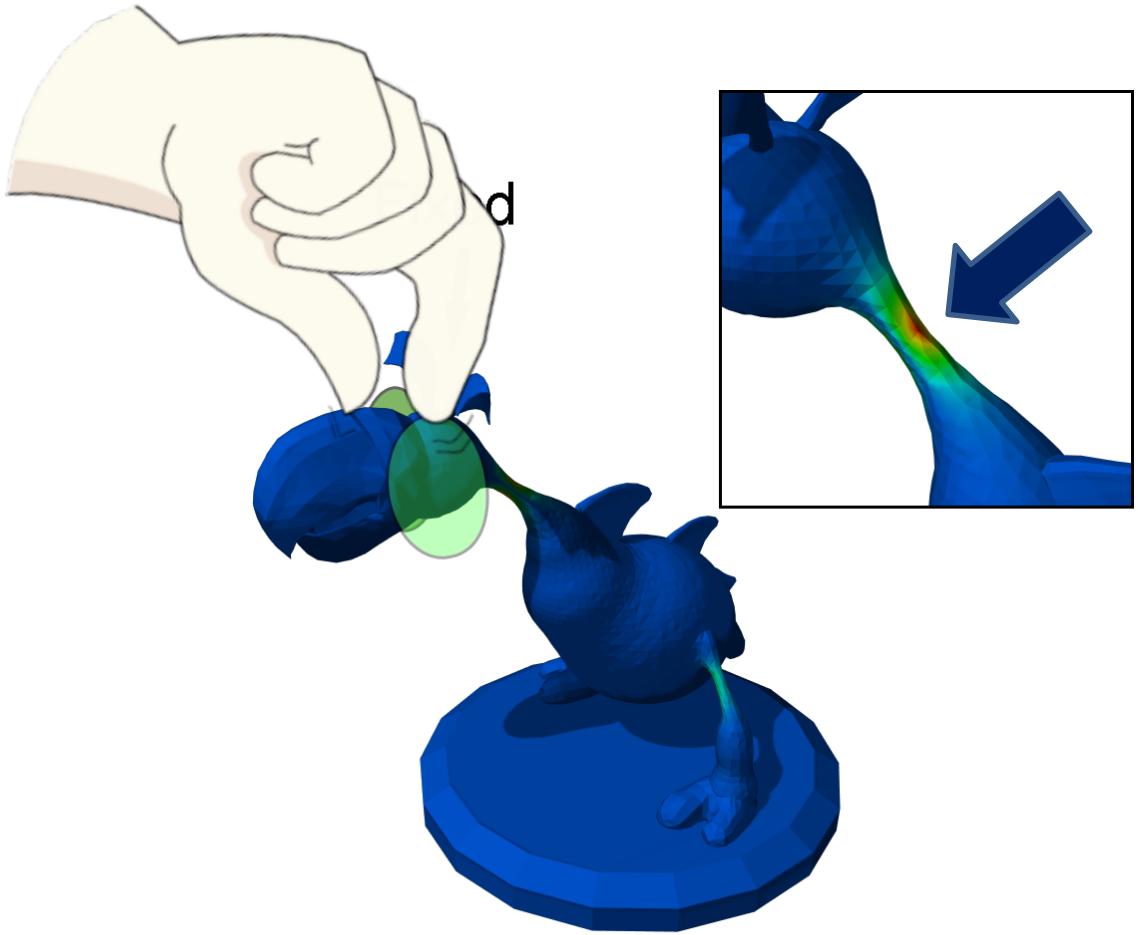
Objectives

- balanced
- self-supporting
- robust
- material cost
- deformation

corrections

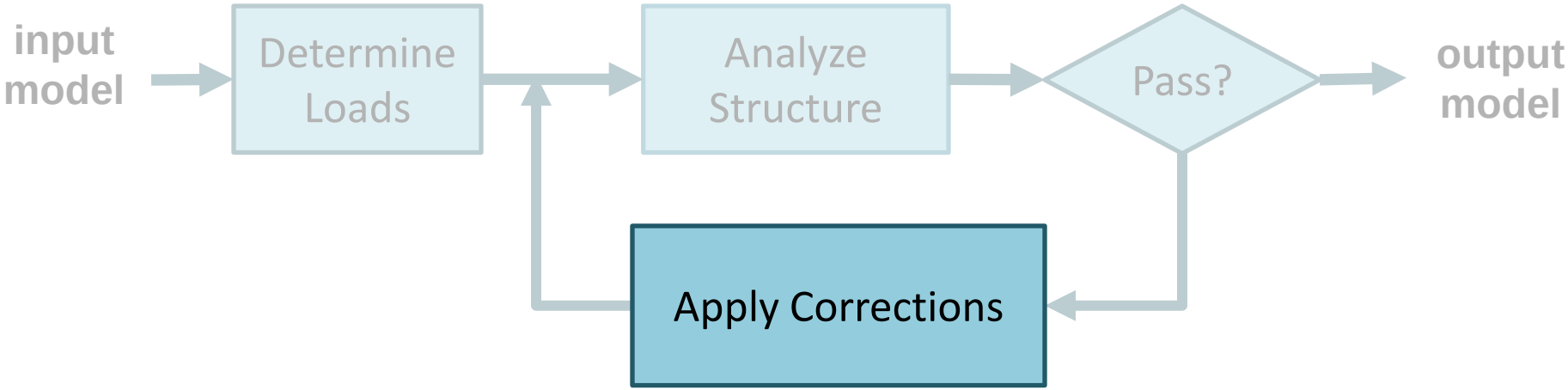
- struts, thickening, hollowing

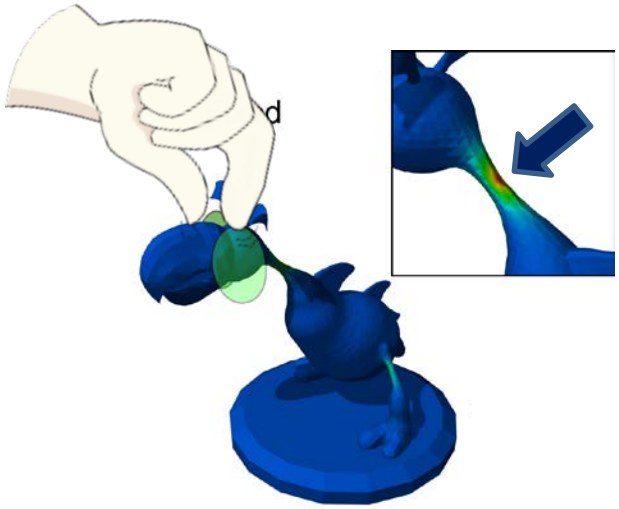
apply corrections **robustness**



[Stava et al. 2012]

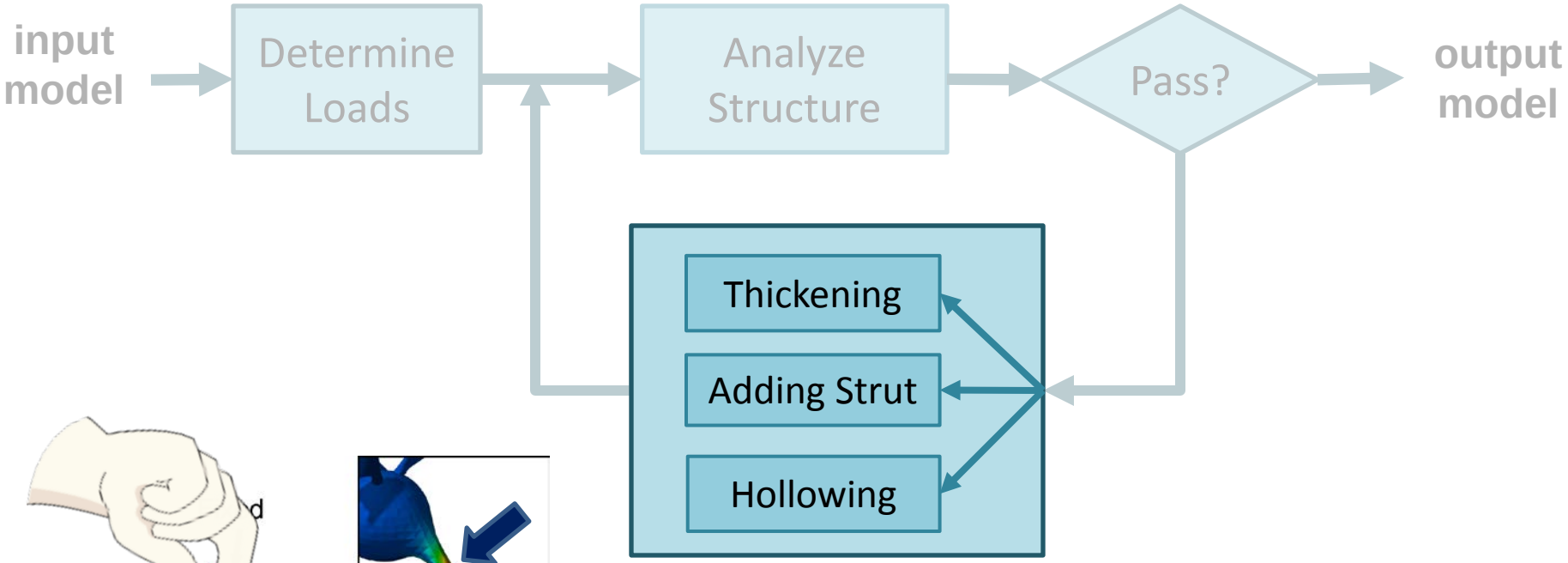
apply corrections **robustness**

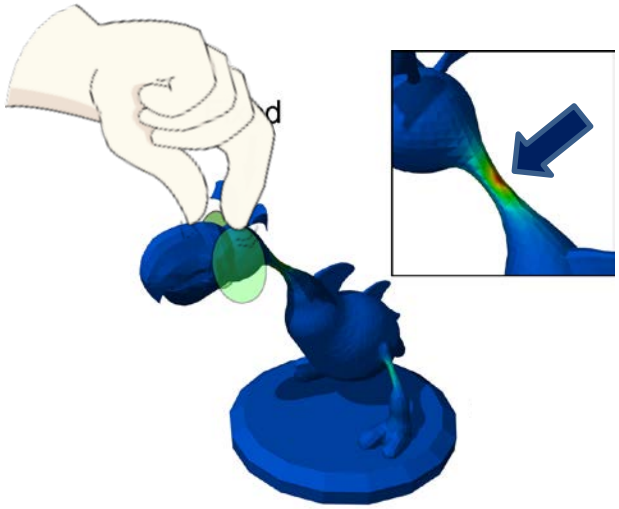




[Stava et al. 2012]

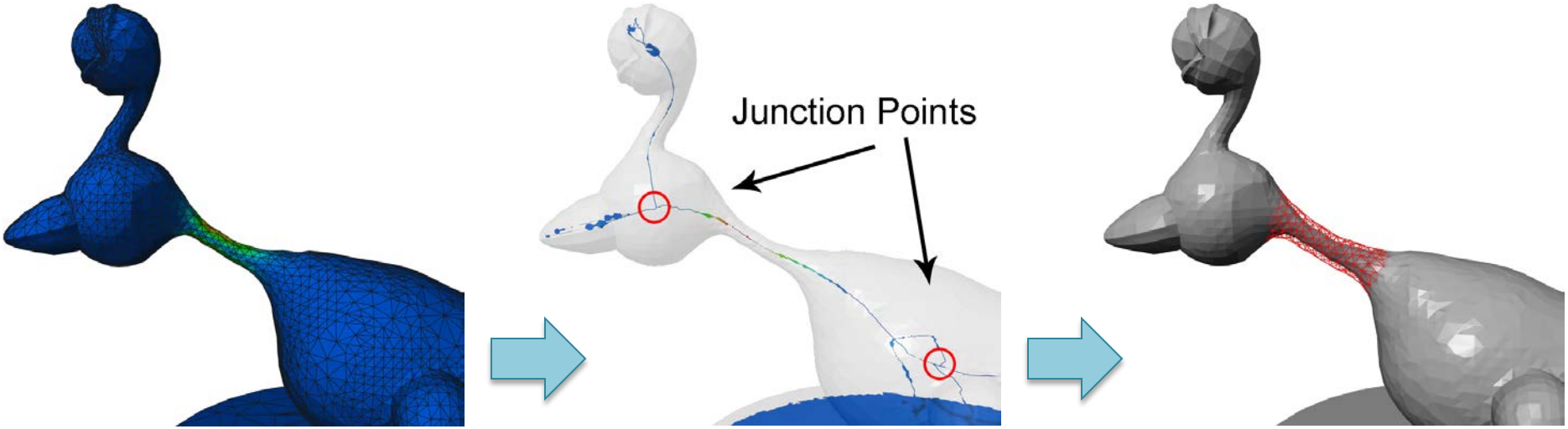
apply corrections **robustness**





[Stava et al. 2012]

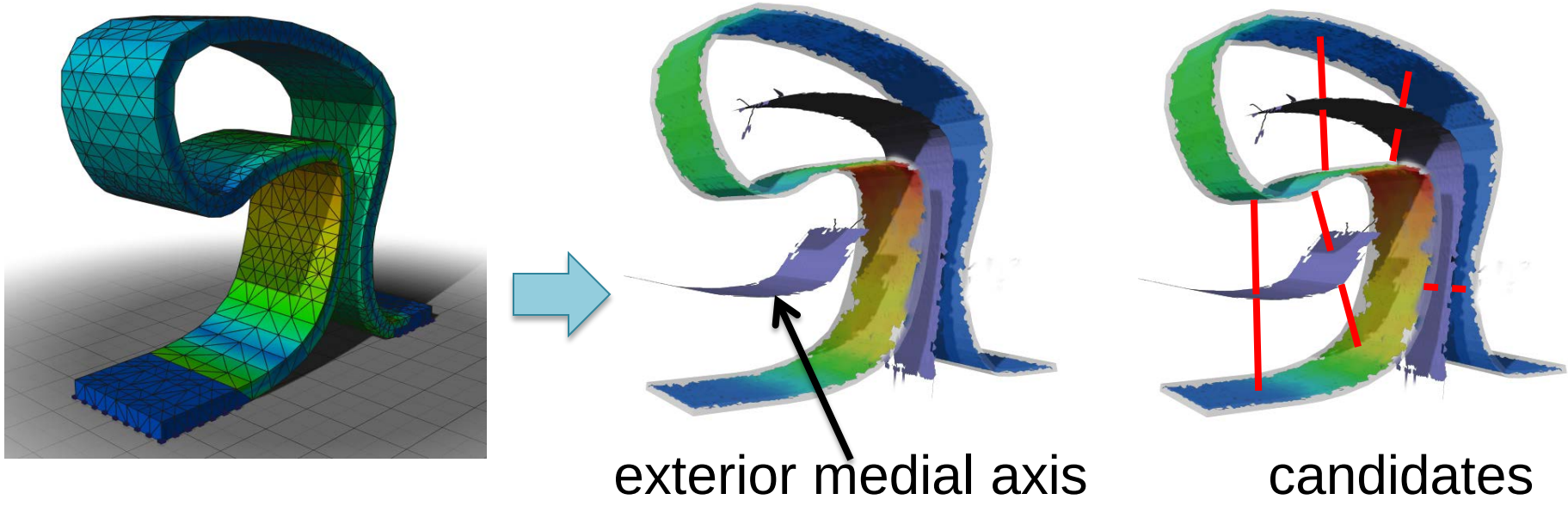
thickening



medial axis

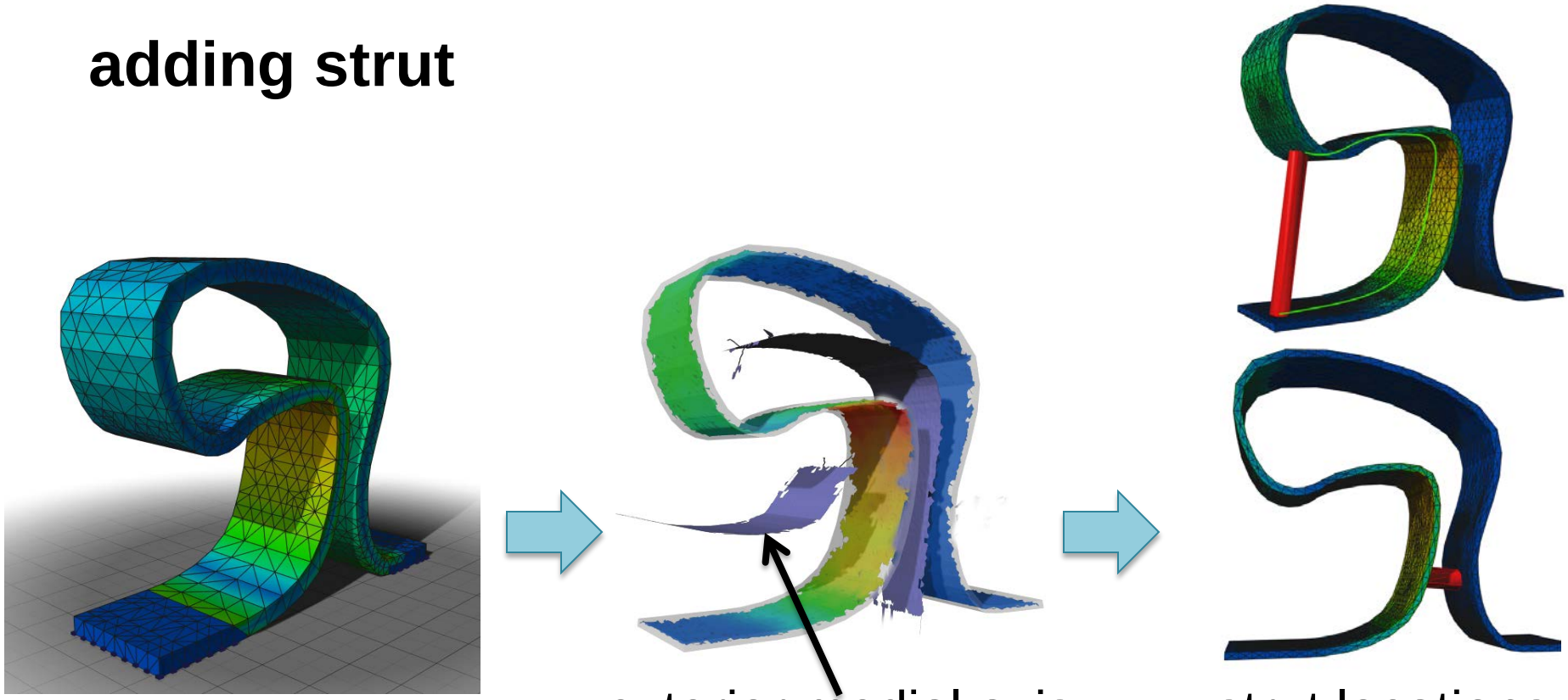
[Stava et al. 2012]

adding strut



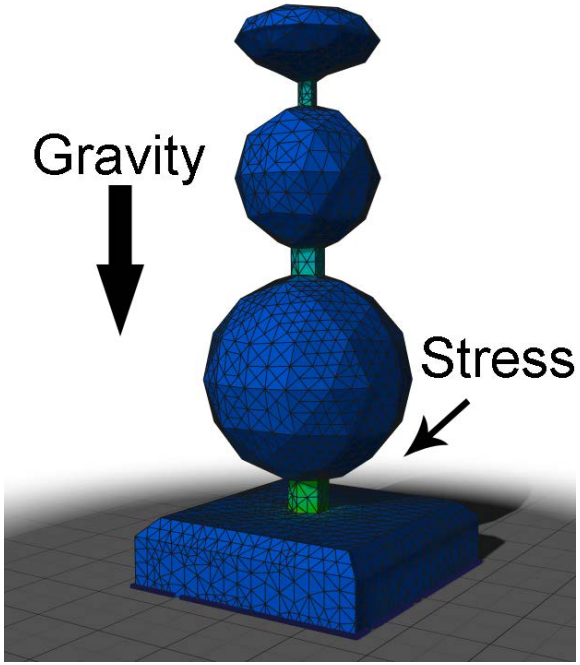
[Stava et al. 2012]

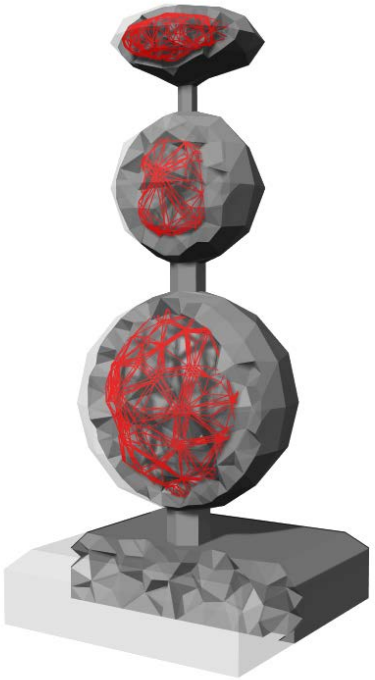
adding strut



[Stava et al. 2012]

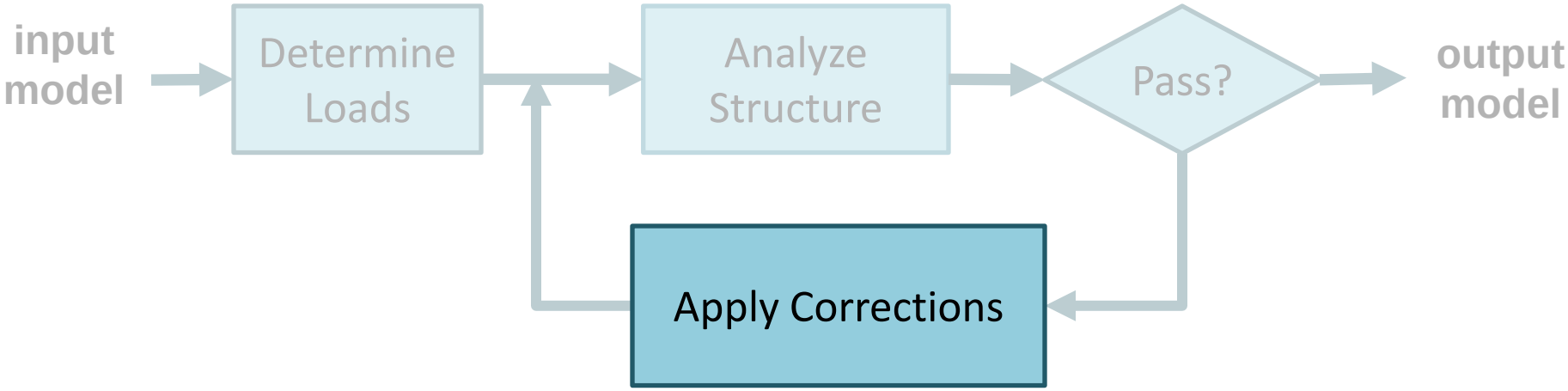
hollowing





reduced mass

[Stava et al. 2012]



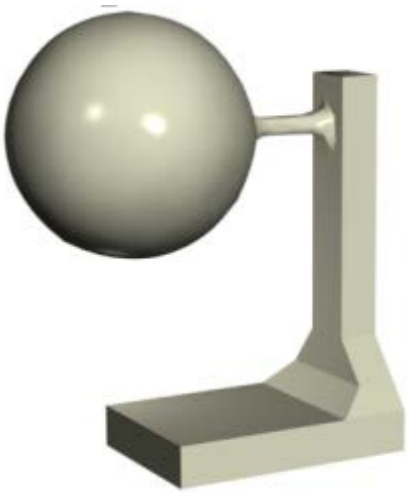
Objectives

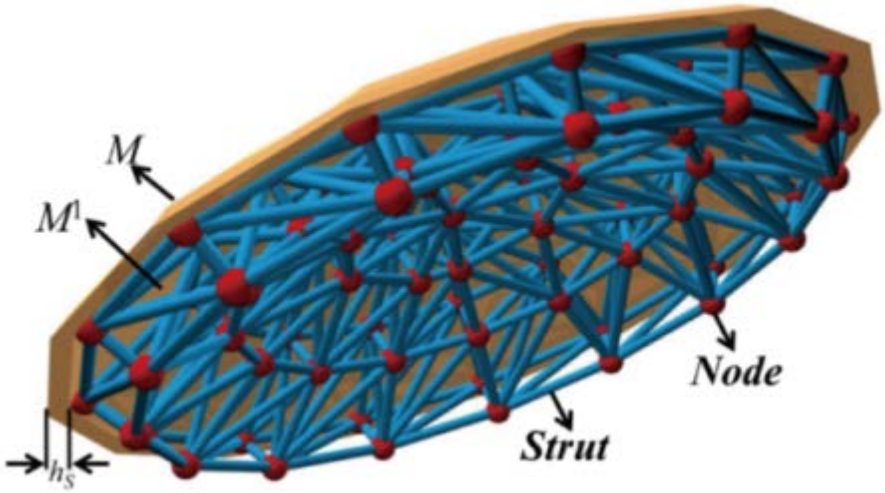
- balanced
- self-supporting
- robust
- material cost
- deformation

corrections

- interior frame

apply corrections **skin frame**



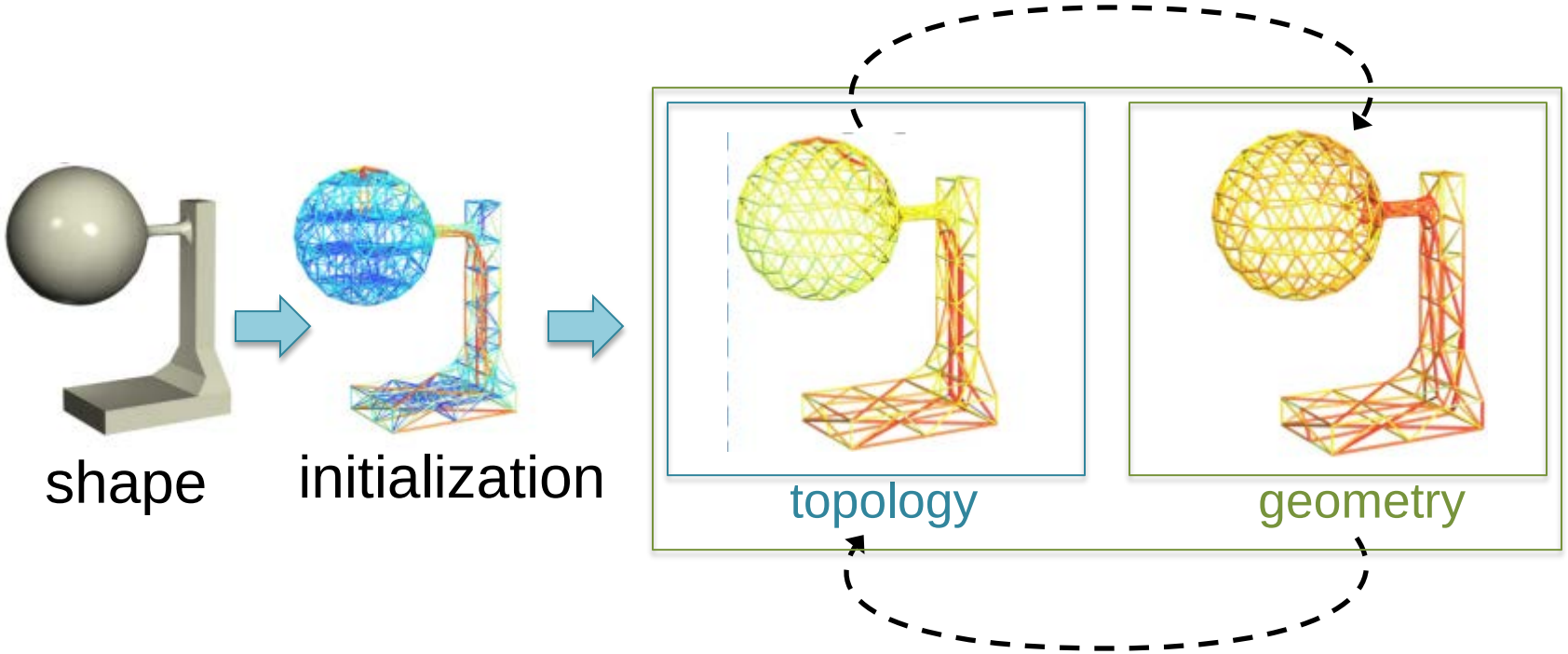


multi-objective optimization

major goal min: volume of all struts
minor goal min: #of struts

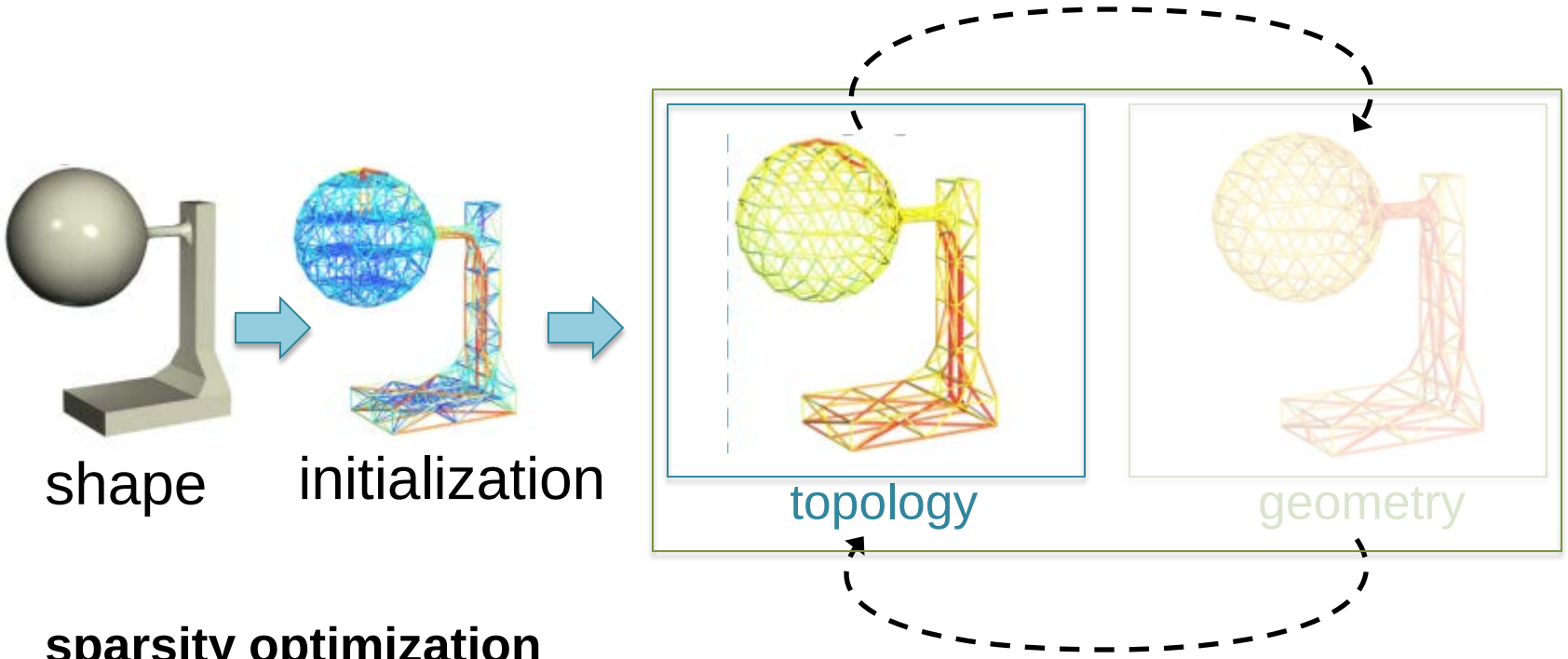
[Wang et al. 2013]

apply corrections **skin frame**



[Wang et al. 2013]

apply corrections **skin frame**



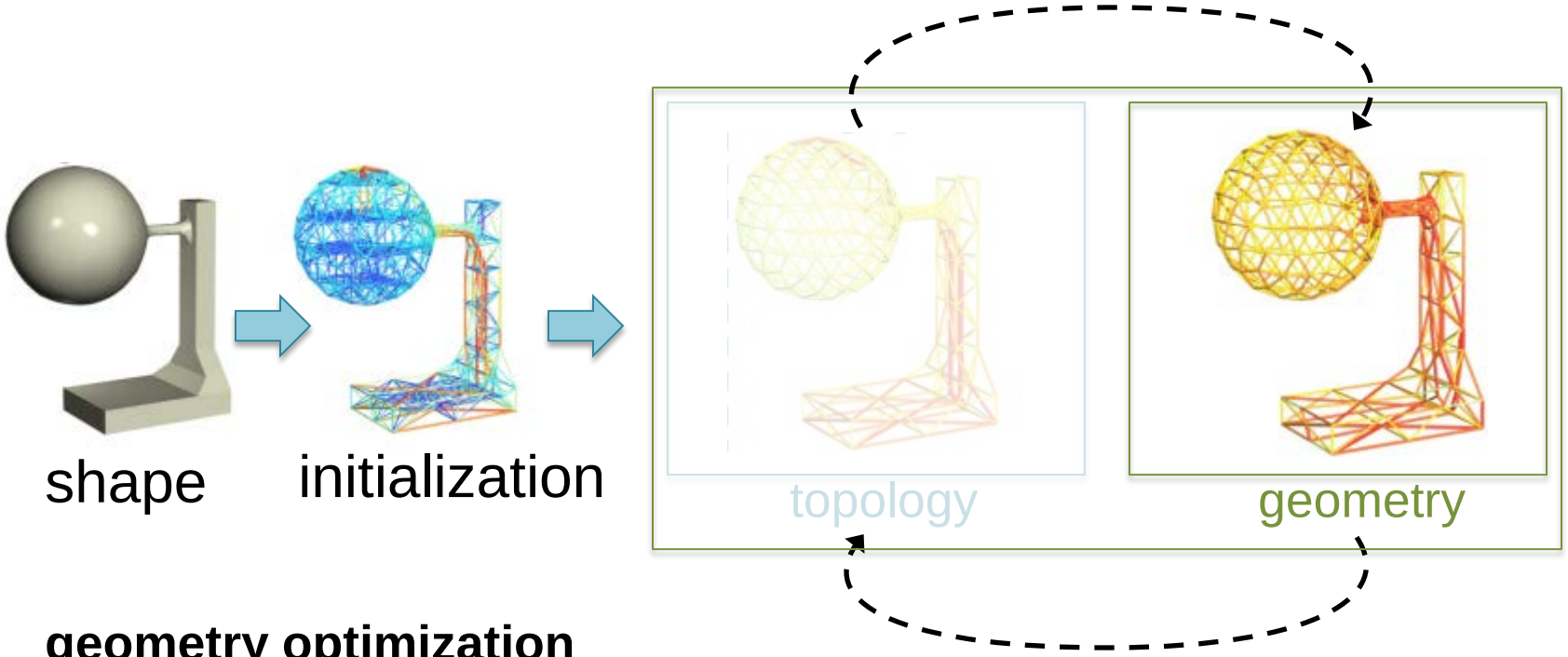
sparsity optimization

min: L0 norm → approximated reweighted L1 norm

s.t.: stress bounded
volume does not increase

[Wang et al. 2013]

apply corrections **skin frame**



geometry optimization

min: material volume
vars: strut radii, internal nodes

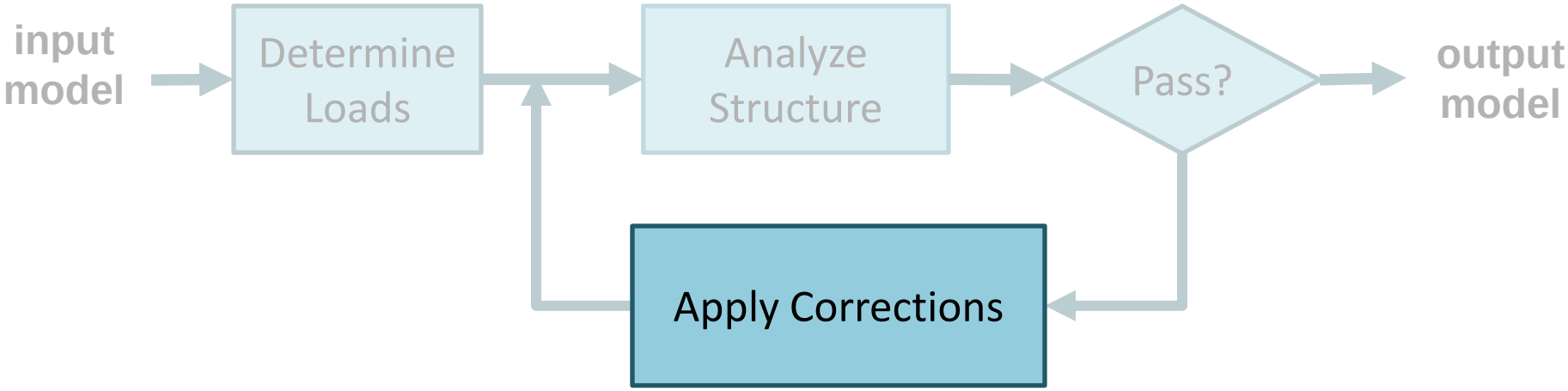
s.t.: stress bounded, buckling, printability

[Wang et al. 2013]

apply corrections **skin frame**

Stress Relief
[Stava et al. 2012]

Skin Frame
[Wang et al. 2013]



Objectives

- balanced
- self-supporting
- robust
- material cost
- deformation

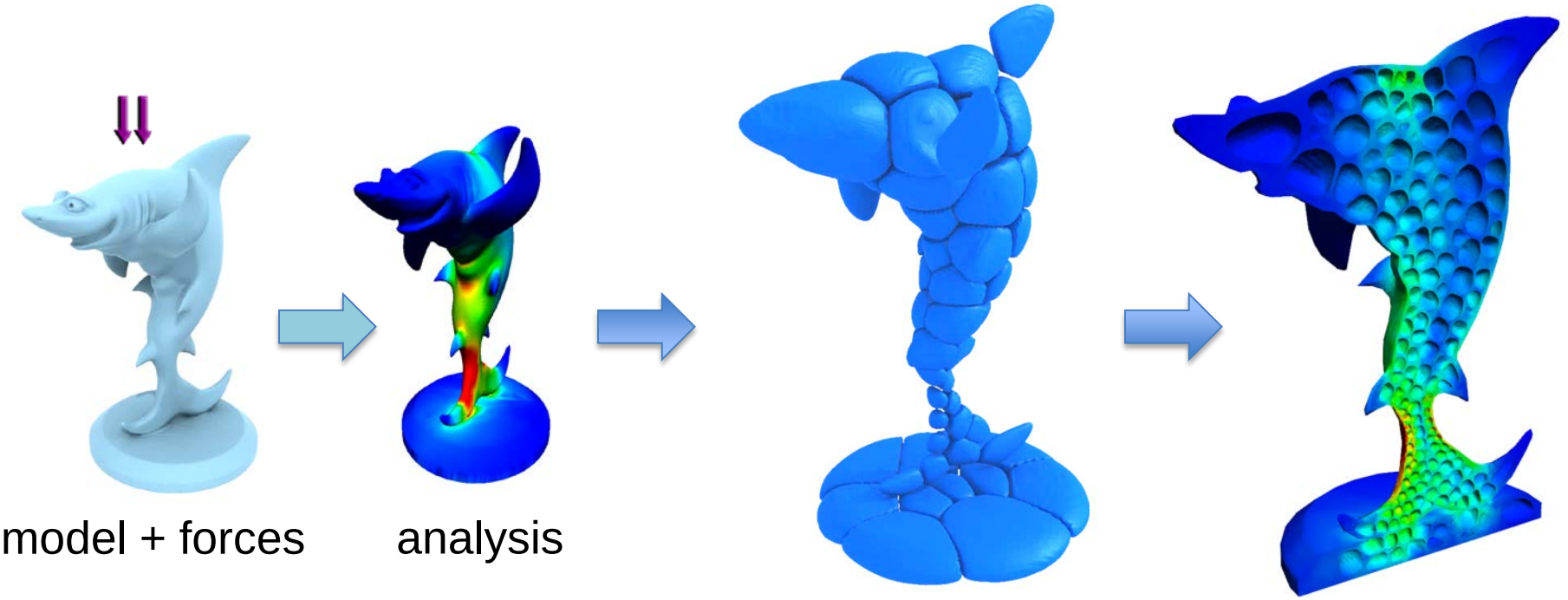
corrections

- interior voids

apply corrections **honeycomb cell structure**

[Lu et al. 2014]

apply corrections **honeycomb cell structure**



model + forces

analysis

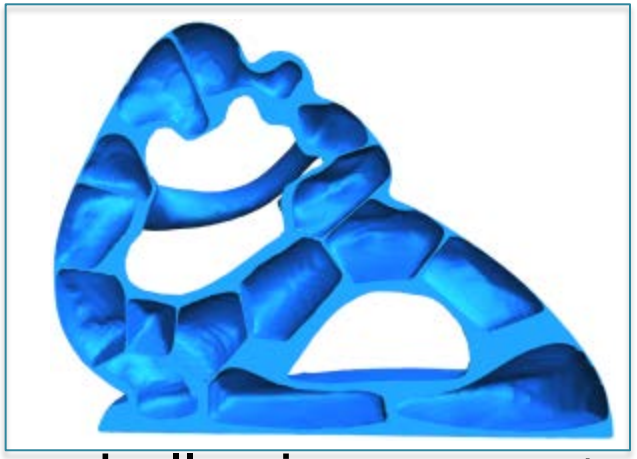
Voronoi tessellation
+ pore extraction

strength-to-weight
optimization

[Lu et al. 2014]

apply corrections **honeycomb cell structure**

#cells



hollowing amount

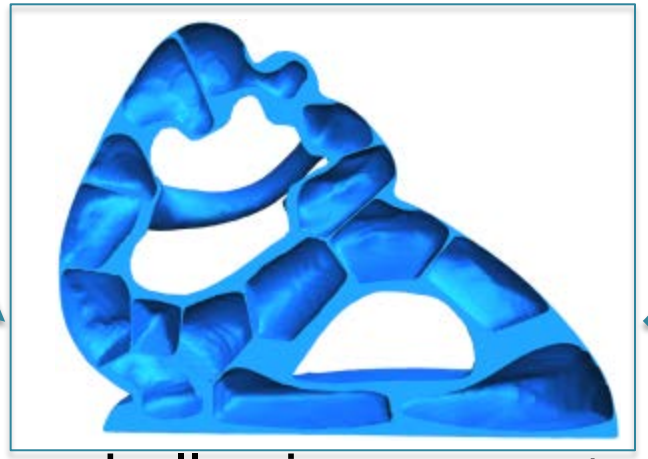
[Lu et al. 2014]

apply corrections **honeycomb cell structure**

outer loop

#cells

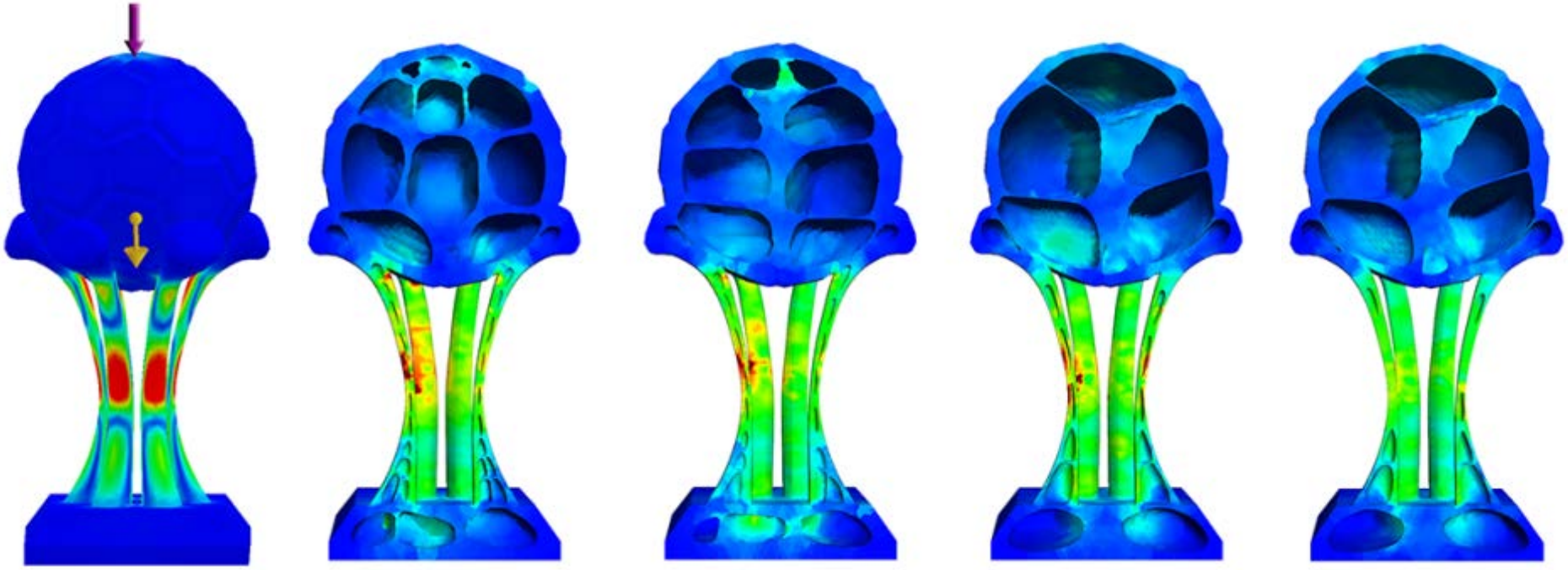
inner loop



hollowing amount

[Lu et al. 2014]

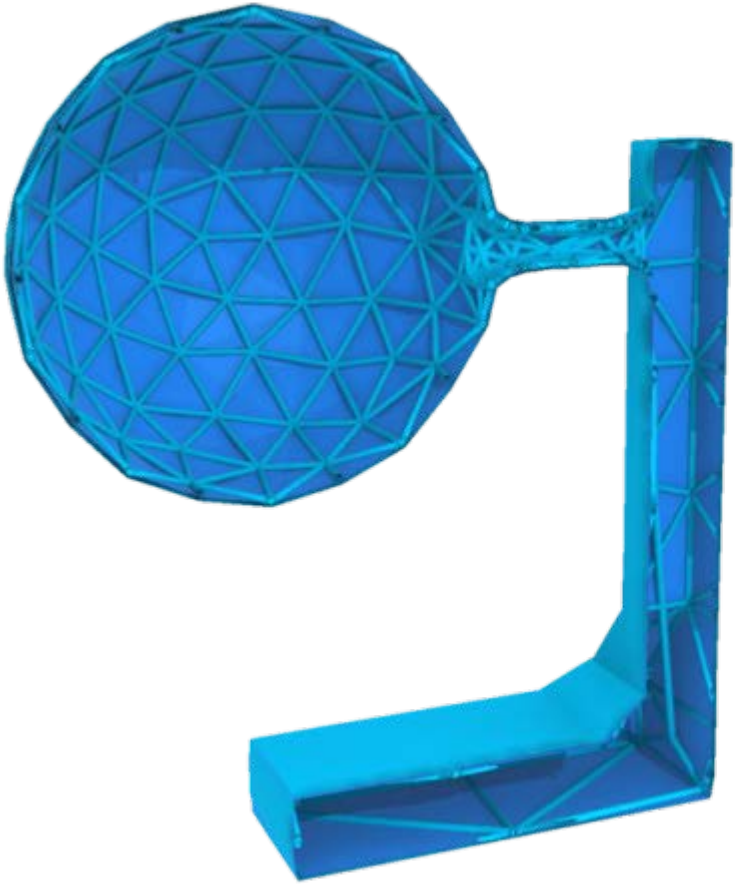
apply corrections **honeycomb cell structure**



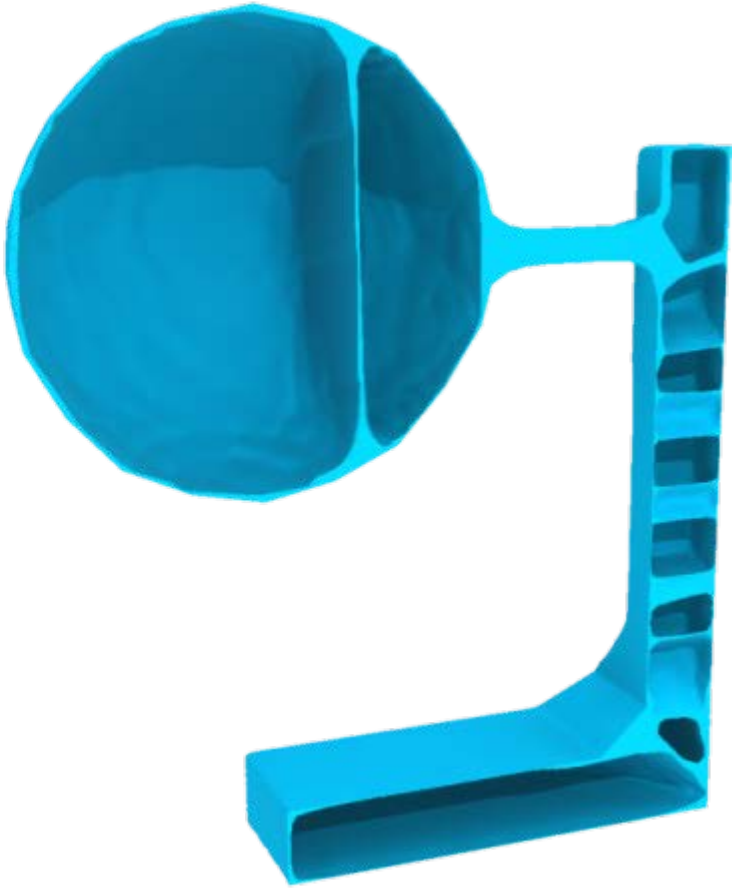
stress map

[Lu et al. 2014]

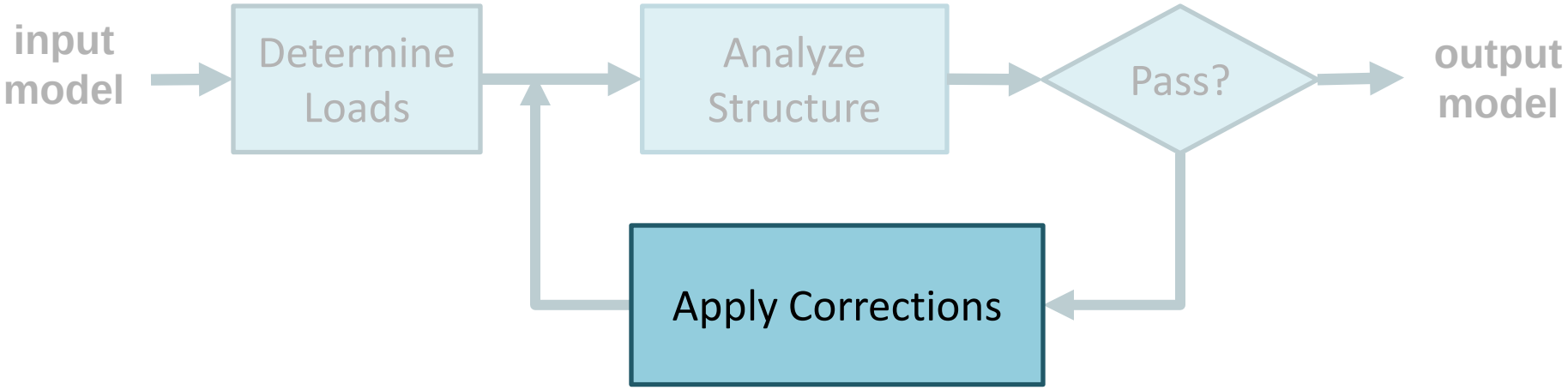
apply corrections honeycomb cell structure



[Wang et al. 2013]
(resists 5N with the weight 109.3g)



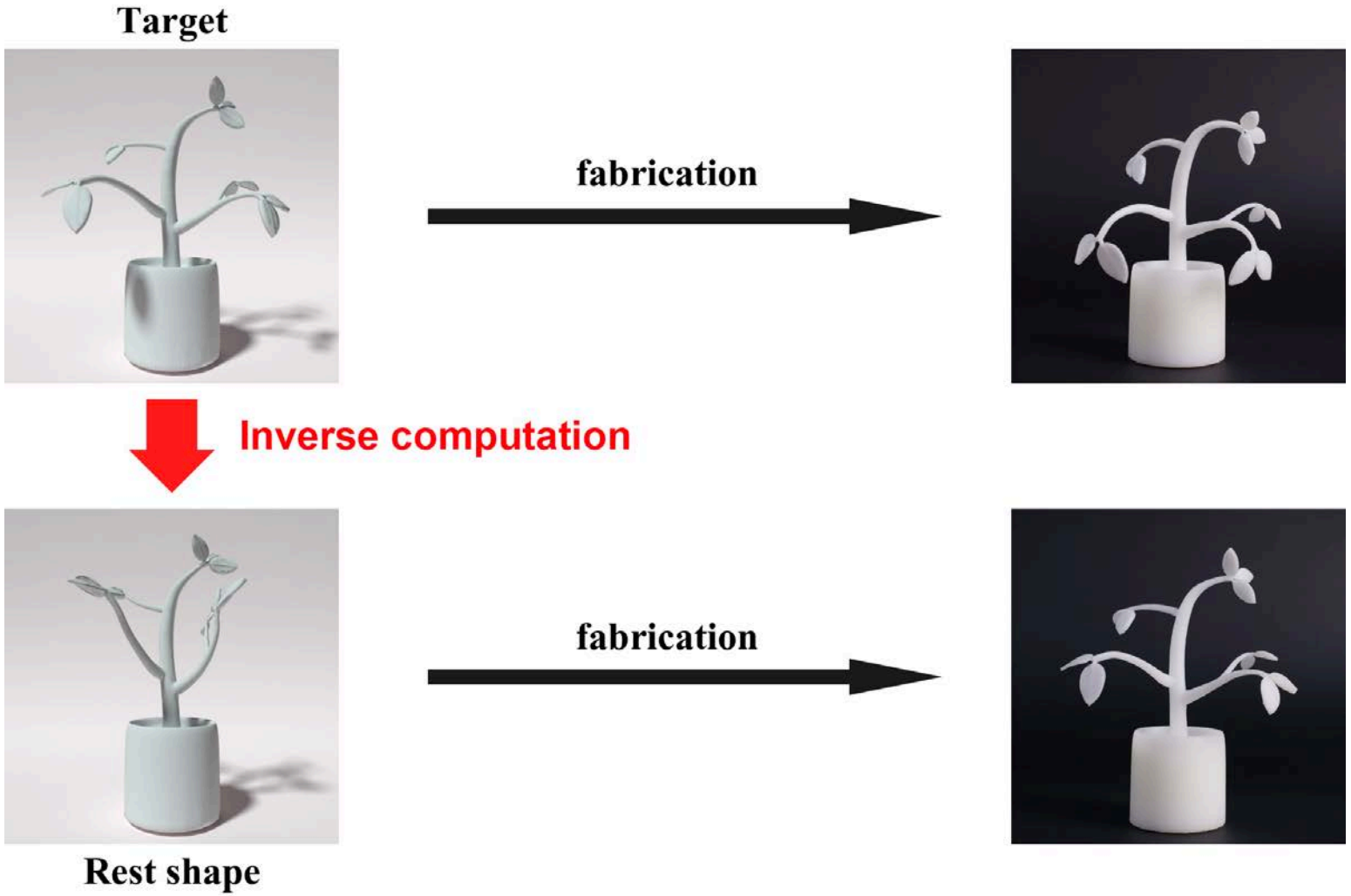
[Lu et al. 2014]
(resists 20N with the weight 92.5g)



Objectives

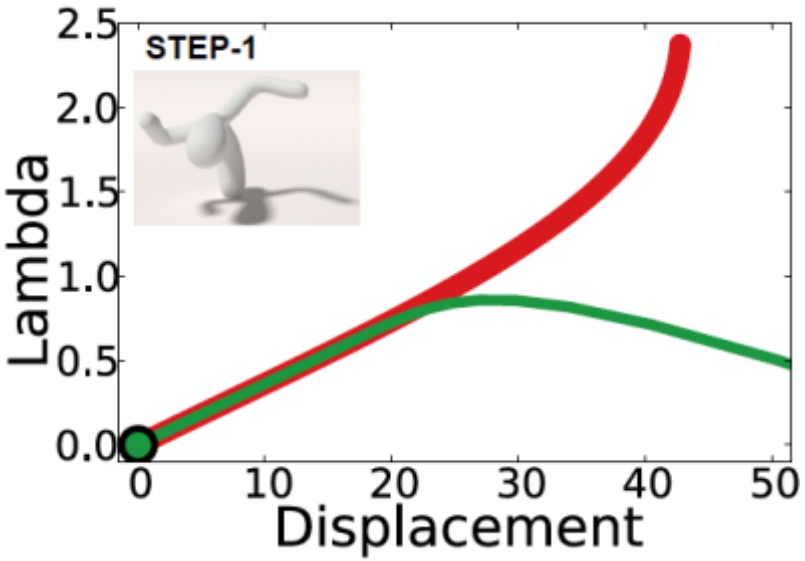
- balanced
- self-supporting
- robust
- material cost
- deformation

apply corrections **inverse elastic deformation**

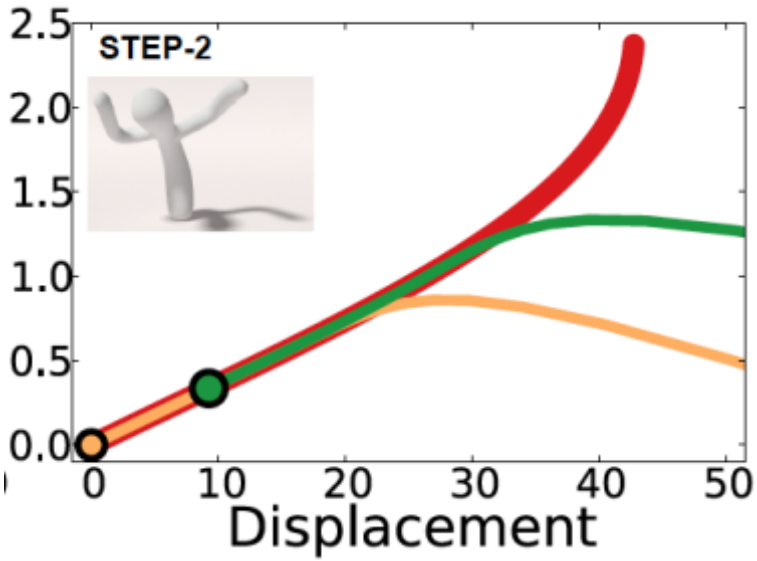


[Chen et al. 2014]

apply corrections **inverse elastic deformation**



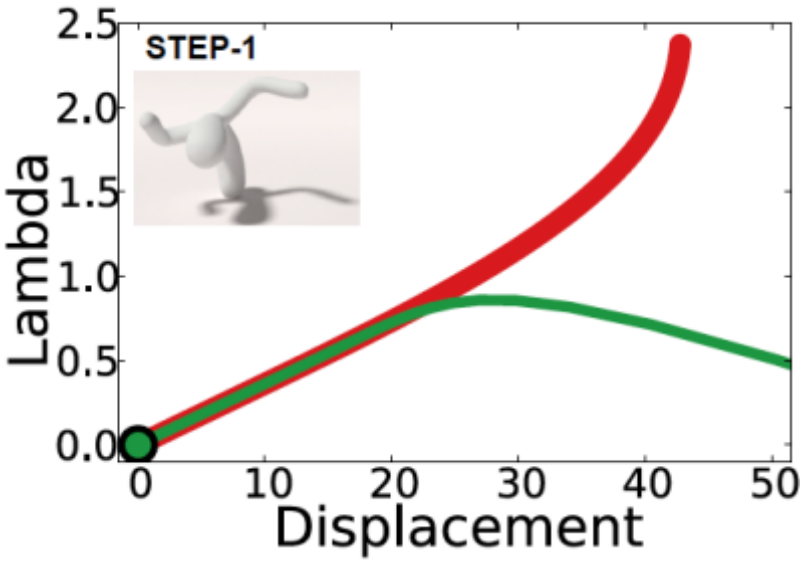
gravity = 0



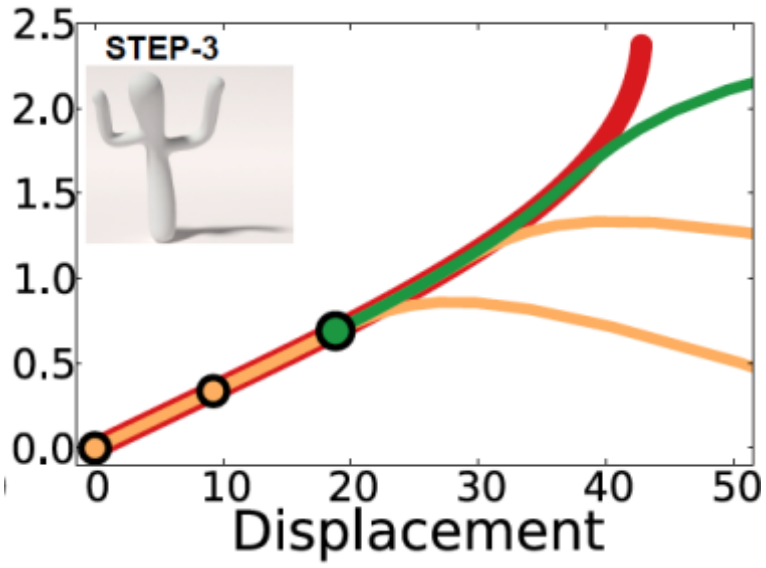
gravity = λ g

[Chen et al. 2014]

apply corrections **inverse elastic deformation**



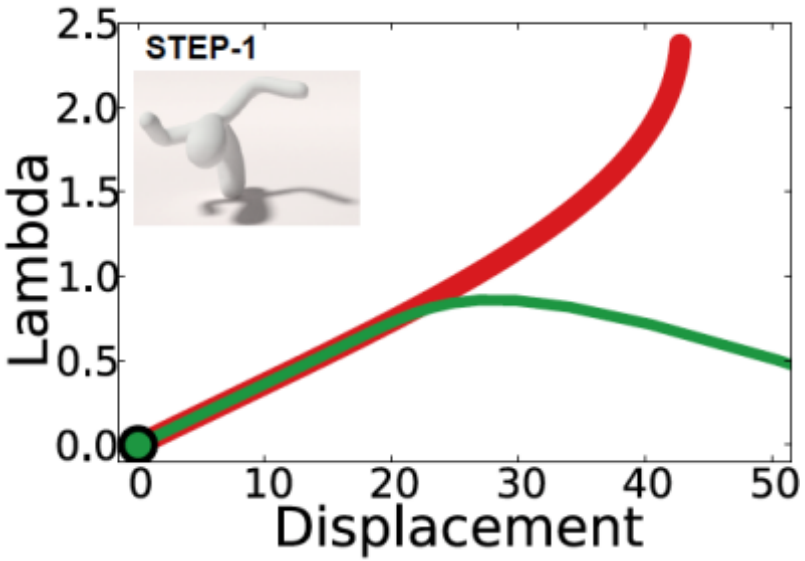
gravity = 0



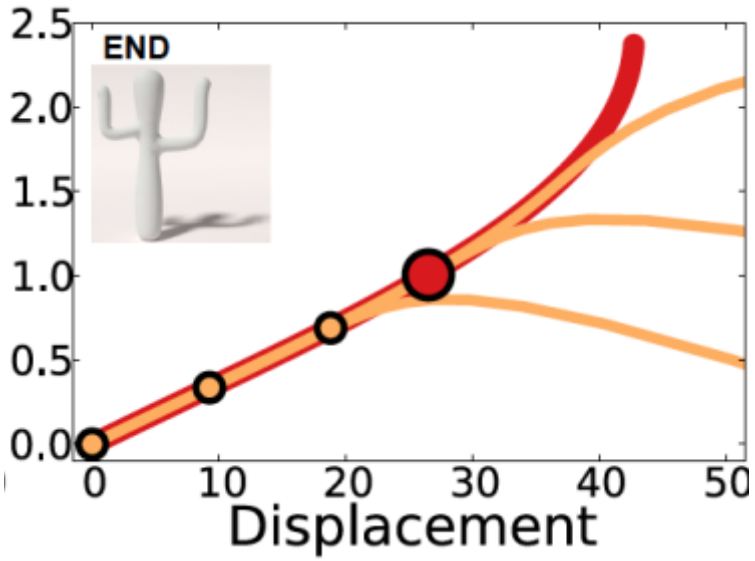
gravity = λ g

[Chen et al. 2014]

apply corrections **inverse elastic deformation**



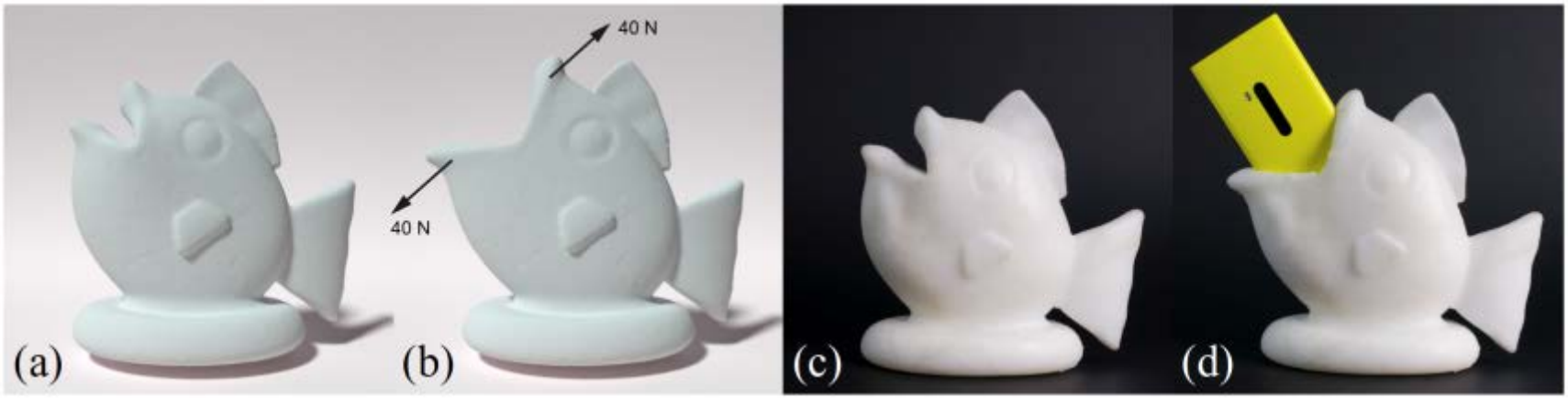
gravity = 0



gravity = g

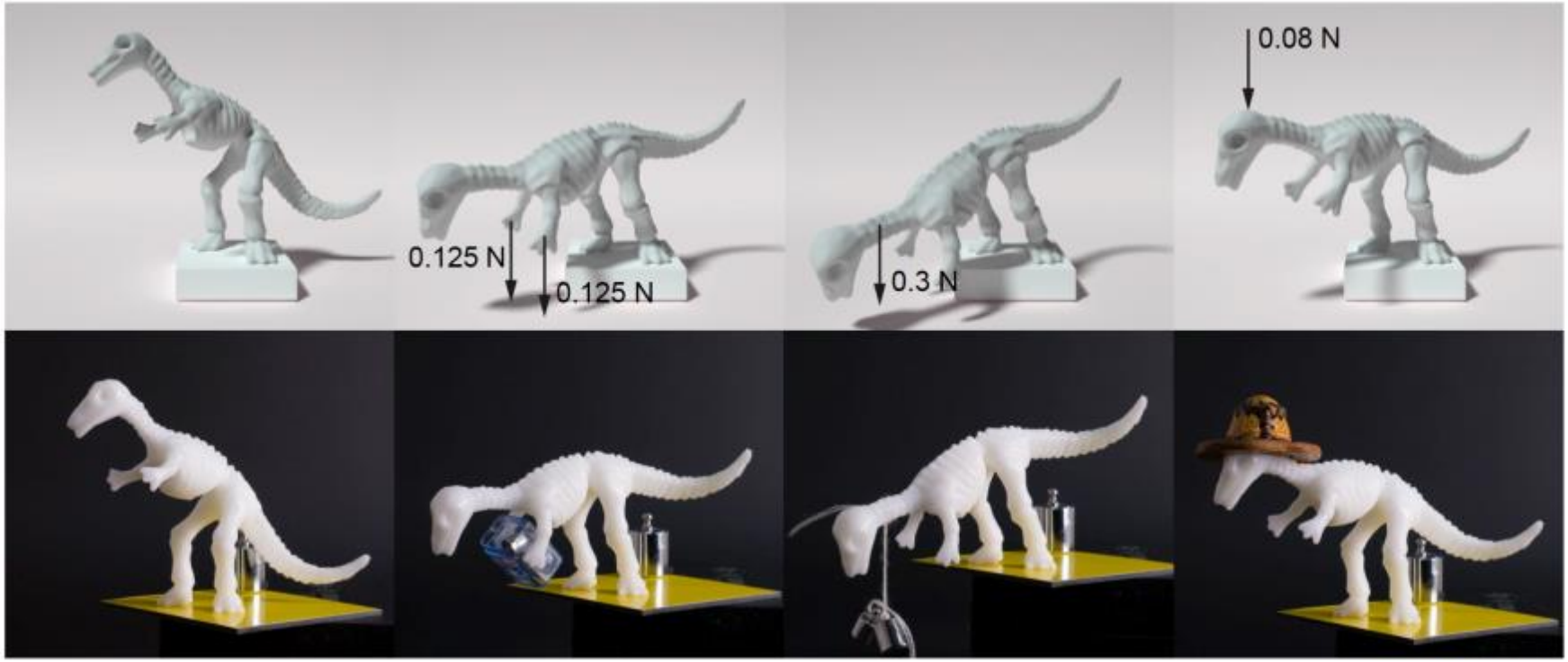
[Chen et al. 2014]

apply corrections **inverse elastic deformation**



[Chen et al. 2014]

apply corrections **inverse elastic deformation**



[Chen et al. 2014]

structural optimization

Analysis Methods

- static equilibrium
- cross-sectional analysis
- finite element method
- modal analysis

Objectives

- balanced / self-supporting
- robust
- material cost
- visual impact

Material Model

- rigid
- fractures
- deformable

Corrections

- interior fill
- surface geometry
- supporting structure: struts, frames

Thank you!

SIGGRAPH ASIA 2014 SHENZHEN

Comment and feedback via course webpage