算法基础

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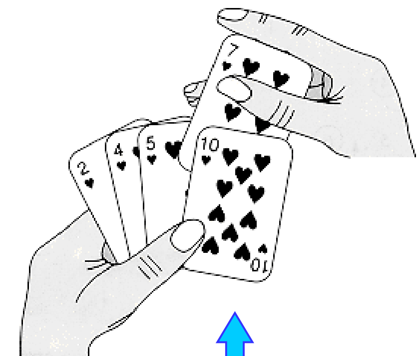
第五讲 概率分析与随机算法

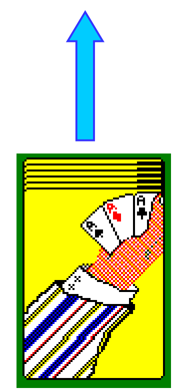
内容提要：

- 雇用问题
- 指示器随机变量
- 随机算法
- 在线雇用问题

算法运行时间与输入规模和输入分布有关，如插入排序：

INSERTION-SORT(A)	cost	times
for(j = 2; j <=$length[A]$; j++)	c_1	n
{ $key = A[j]$	c_2	n-1
// Insert $A[j]$ into the sorted sequence $A[1 .. j\text{-}1]$	0	n-1
$i = j$-1	c_4	n-1
while(i > 0 && $A[i] > key$)	c_5	
{ $A[i+1] = A[i]$	c_6	
$i = i$-1	c_7	
}		
$A[i+1] = key$	c_8	n-1
}		





$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1)$$

- 对于运算时间与输入数据分布有关的算法，时间复杂度分析一般有三种：最坏运行时间、最佳运行时间、平均运行时间。
- 平均运行时间是算法对所有可能情况的期望运行时间，与输入数据的概率分布有关。
- 分析算法的平均运行时间通常需要对输入分布做某种假定

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雇用问题

Scenario（情景）: hire the best office assistant in a month

- **You are using an employment agency to hire a new office assistant.（猎头公司帮你物色办公助理候选人）**
- **The agency sends you one candidate each day.**
- **You interview the candidate and must immediately decide whether or not to hire that person. But if you hire, you must also fire your current office assistant.**
- **Cost to interview is 1*K* per candidate (interview fee paid to agency).（面试一个候选人支付猎头公司 1*K*）**
- **Cost to hire is 1*W* per candidate (includes cost to fire current office assistant + hiring fee paid to agency).**

***Goal*: Determine what the price of this strategy will be?**

雇用问题

Pseudocode to model this scenario: Assumes that the candidates are numbered 1 to *n*. Uses a dummy candidate 0 that is worse than all others, so that the first candidate is always hired.

HIRE-ASSISTANT(n)	cost	times
best ← 0 // candidate 0 is a least-qualified dummy candidate		
for *i* ← 1 to *n*		
do interview candidate *i*	c_i	n
if candidate *i* is better than candidate *best*		
then *best* ← *i*		
hire candidate *i*	c_h	m

Cost: *n* candidates, we hire *m* of them, cost is $O(nc_i+mc_h)$

5.1.1 Worst-case analysis

HIRE-ASSISTANT(n)	cost	times
best ← 0 // candidate 0 is a least-qualified dummy candidate		
for *i* ← 1 to *n*		
do interview candidate *i*	c_i	n
if candidate *i* is better than candidate *best*		
then *best* ← *i*		
hire candidate *i*	c_h	m

Cost: n candidates, we hire m of them, cost is $O(nc_i+mc_h)$

- We focus on analyzing the hiring cost mc_h? ($c_h >> c_i$)
- mc_h varies with each run: it depends on the order in which we interview the candidates.
- Worst-case analysis
  - We hire all n candidates. $O(nc_i+mc_h)=O(nc_h)$? When?
  - The candidates appear in increasing order of qulity.

5.1.2 Propbabilistic analysis

HIRE-ASSISTANT(n)	cost	times
best ← 0 // candidate 0 is a least-qualified dummy candidate		
for *i* ← 1 to *n*		
do interview candidate *i*	c_i	n
if candidate *i* is better than candidate *best*		
then *best* ← *i*		
hire candidate *i*	c_h	m

Assume that candidates come in a random order:

- Assign a rank to each candidate: *rank*(*i*) is a unique integer in the range 1 to *n*. （对每一个候选人分配一个秩）
- The list <*rank*(1), …, *rank*(*n*)> is a permutation of the candidate numbers <1, …, *n*>, such as <5, 2, 16, 28, 9, … , 11>
- The ranks form a uniform random permutation: each of the possible *n*! permutations appears with **equal probability.**

5.1.2 Propbabilistic analysis

HIRE-ASSISTANT(n)	cost	times
best ← 0 // candidate 0 is a least-qualified dummy candidate		
for *i* ← 1 to *n*		
do interview candidate *i*	c_i	n
if candidate *i* is better than candidate *best*		
then *best* ← *i*		
hire candidate *i*	c_h	m

Essential idea of probabilistic analysis: We must use knowledge of, or make assumptions about, the distribution of inputs.

（概率分析的本质：需已知或假定输入的概率分布）

- The expectation is taken over this distribution.
  （依据概率分布来求期望，期望值即是平均 hire 的人数）
- Section 5.2 contains a probabilistic analysis of the hiring problem.

5.1.3 Randomized algorithms

- **Idea**
  - **We might not know the distribution of inputs, or we might not be able to model it computationally.**

    (我们不知道输入的分布，也不可能为输入的分布进行可计算的建模)
  - **Instead, we use randomization within the algorithm in order to impose a distribution on the inputs.**

    (在算法中通过对输入进行随机化处理，从而为输入强加一种分布)

5.1.3 Randomized algorithms

- *For the hiring problem,* change the scenario:
  - The employment agency sends us a list of all *n* candidates in advance.

    （猎头公司预先提供 *n* 个候选人列表）
  - On each day, we randomly choose a candidate from the list to interview.

    （每天，我们随机选取一人进行面试）
  - Instead of relying on the candidates being presented to us in a random order, we take control of the process and enforce a random order. (Chap 5.3)

    （无须担心候选人是否被随机提供，我们通过随机运算预处理可以控制候选人的随机顺序）

5.1.3 Randomized algorithms

- **What makes an algorithms randomized: An algorithm is randomized if its behavior is determined in part by values produced by a *random-number generator*.**

  （算法随机化：由随机数产生器生成数值……）

  - **RANDOM(*a*, *b*) returns an integer *r*, where *a*≤*r*≤*b* and each of the *b*-*a*+1 possible values of *r* is equally likely.**
  - **In practice, RANDOM is implemented by a pseudorandom-number generator, which is a deterministic method returning numbers that “look” random and pass statistical tests.（RANDOM实际上由一个确定的算法〔伪随机产生器〕产生，其结果表面上看上去像是随机数）**

5.1.3 Randomized algorithms

- Random number generator（随机数产生器）
  - Most random number generators generate a sequence of integers by the following recurrence
  - X_0= a given integer (seed), $X_{i+1} = aX_i \bmod M$
  - For example, for X_0=1, a=5, M=13, we have X_1=5%13=5, X_2=25%13=12, X_3= 60%13=8. Each integer in the sequence lies in the range [0, M - 1].

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Indicator random variables（指示随机变量）

Given a sample space and an event *A*, we define the indicator random variable（给定采样空间S和事件 *A*，指示随机变量…）

$$\mathrm{I}\{A\} = \begin{cases} 1 & \text{if } A \text{ occur,} \\ 0 & \text{if } A \text{ does not occur.} \end{cases}$$

- ***Lemma(引理)***

  For an event, let $X_A = \mathrm{I}\{A\}$. Then $\mathrm{E}[X_A] = \Pr\{A\}$.

Proof **Letting ~*A* be the complement of *A*, we have**

$$\begin{aligned} \mathrm{E}[X_A] &= \mathrm{E}[\mathrm{I}\{A\}] \\ &= 1.\Pr\{A\} + 0.\Pr\{\sim A\} \qquad \{\text{definition of expected value}\} \\ &= \Pr\{A\}. \end{aligned}$$

Indicator random variables（指示随机变量）

- *Lemma*

  For an event A, let X_A = I{A}. Then E[X_A]=Pr{A}.

- Simple example: Determine the expected number of heads when we flip a fair coin one time.

  （投一次硬币，正面向上的期望〔平均数〕）

  - Sample space is {H, T }
  - Pr{H} = Pr{T} = 1/2
  - Define indicator random variable X_H = I{H}.

    X_H counts the number of heads in one flip.
  - Since Pr{H} =1/2, lemma says that E[X_H] = 1/2 .

Indicator random variables（指示随机变量）

- *Lemma*

  For an event A, let X_A = I{A}. Then E[X_A]=Pr{A}.

- **Slightly more complicated example**: Determine the expected number of heads when in n coin flips.
  （投 n 次硬币，正面向上的期望〔平均数〕）
  - Let X be a random variable for the number of heads in n flips.（令随机变量 X 表示投 n 次硬币正面向上的数）
  - Then, $\mathrm{E}[X] = \sum_{k=0}^{n} k \cdot \Pr\{X = k\}$

    a slightly more complicated? Yes!
  - Instead, we’ll use indicator random variables……

Indicator random variables（指示随机变量）

- ***Lemma***

  For an event A, let X_A = I{A}. Then E[X_A]=Pr{A}.

- **Slightly more complicated example: Determine the expected number of heads when in n coin flips.**

  （投 n 次硬币，正面向上的期望〔平均数〕）

  - **For i=1, …, n, define X_i=I{the ith flip results in event H}**
  - **Then,** $X = \sum_{i=1}^{n} X_i$
  - **Lemma says that E[X_i] = Pr{H} = 1/2 for i=1, 2, …, n**
  - **Expected number of heads is**

$$E[X] = E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} 1/2 = n/2$$

Analysis of the hiring problem

Assume that the candidates arrive in a random order.

Let X be a random variable that equals the number of times we hire new office assistant.

（令随机变量 X 表示我们雇用新助手的人数）

- **Use a probabilistic analysis**

Then $$E[X]=\sum_{i=1}^{n} x_i \Pr\{X = x_i\}$$

The calculation would be cumbersome（计算烦琐）

Analysis of the hiring problem

Assume that the candidates arrive in a random order.

random variable X = the number of times we hire new office assistant （随机变量 X 表示我们雇用新助手的人数）

- **Use indicator random variables**
  - Define indicator random variables $X_1, X_2, \ldots, X_n$, where X_i = I{ candidate i is hired}
  - Useful properties:

$$X = X_1 + X_2 + \ldots + X_n$$

Lemma => $E[X_i]$ = Pr{ candidate i is hired}.

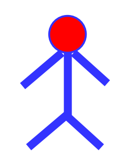
We need to compute Pr{ candidate i is hired}?

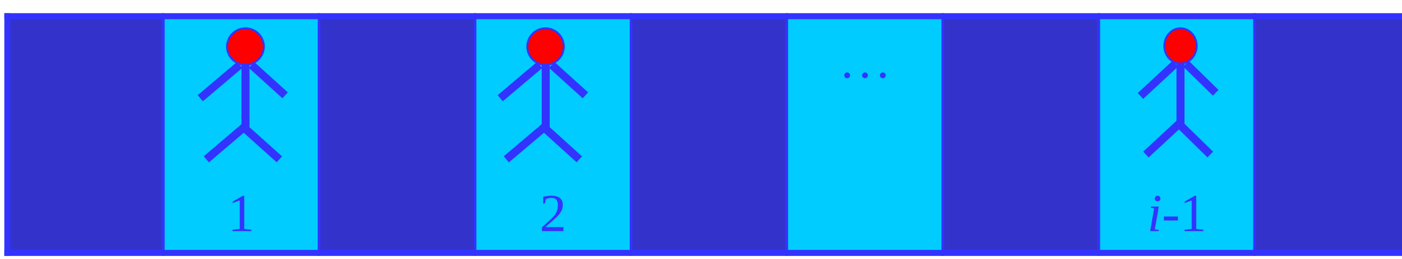
Analysis of the hiring problem

HIRE-ASSISTANT(n)	cost	times
best ← 0 // candidate 0 is a least-qualified dummy candidate		
for *i* ← 1 to *n*		
do interview candidate *i*	c_i	n
if candidate *i* is better than candidate *best*		
then *best* ← *i*		
hire candidate *i*	c_h	m

We need to compute Pr{ candidate *i* is hired}?

- *i* is hired ⇔ *i* is better than each of candidates 1, 2, … , *i*-1.
- Assumption that the candidates arrive in random order => any one of these candidates is equally likely to be the best one.
  （若候选人随机到来，则每一个候选人为最佳人选的概率相等）
- Thus, E{X_i} = Pr{ candidate *i* is the best so far} = 1/*i* ?





Analysis of the hiring problem

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- *i* is hired ⇔ *i* is better than each of candidates 1, 2, … , *i*-1.
- Assumption that the candidates arrive in random order => any one of these candidates is equally likely to be the best one.
  （若候选人随机到来，则每一个候选人为最佳人选的概率相等）
- Thus, $E\{X_i\}$ = Pr{ candidate *i* is the best so far} = $1/i$?

Then

$$E[X] = E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} 1/i = \ln n + O(1)$$

Thus, the expected hiring cost is $O(c_h \ln n)$, which is much better than the worst-case cost of $O(nc_h)$.

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5.3 Randomized algorithms

5.3.1 The hiring problem

For the hiring problem, the algorithm is deterministic.

- For any given input, the number of times we hire a new office assistant will always be the same.（给定输入，则雇用的人数确定）
- The number of times we hire a new office assistant depends only on the input.（雇用的人数〔资源消费〕依赖于输入）
  - Some rank orderings will always produce a high hiring cost. Example: <1,2,3,4,5,6>, where each is hired.
  - Some will always produce a low hiring cost. Example: <6,*,*,*,*,*>, where only the first is hired.
  - Some may be in between.

5.3.1 The hiring problem

Instead of always interviewing the candidates in the order presented, what if we first randomly permuted this order.

- The randomization is now in the algorithm, not in the input distribution. （随机化过程在算法中，而不是在输入中体现）
- Given a particular input, we can no longer say what its hiring cost will be. Each time we run the algorithm, we can get a different hiring cost. （算法的运行时间与输入无关）
- No particular input always elicits worst-case behavior. （算法的最坏运算时间不取决于特定的输入）
- Bad behavior occurs only if we get “unlucky” numbers from the random-number generator.（只有当随机数产生器产生很不幸运的数时，算法的运算时间最坏）

5.3.1 The hiring problem

Algorithm for randimized hiring problem

RANDOMIZED-HIRE-ASSISTANT(*n*)

  Randomly permute the list of candidates

  HIRE-ASSISTANT(*n*)

- *Lemma*

  The expected hiring cost of RANDOMIZED-HIRE-ASSISTANT is $O(c_h \ln n)$.

Proof

After permuting the input array, we have a situation identical to the probabilistic analysis of deterministic HIRE-ASSISTANT.

（对输入矩阵进行随机置换后，情况同HIRE-ASSISTANT相同）

5.3.2 Randomly permuting an array

Goal:

Produce a uniform random permutation (each of the *n*! permutations is equally likely to be produced),

that is

$A = \langle 1, 2, 3, \ldots, n \rangle$

The numbers of permutation of A is $P_n^n = n!$, each of that is equally likely to be produced.

5.3.2 Randomly permuting an array

(1) Permute-by-sorting — The method is not very good

- Assume the given array *A* contains the element 1 through *n*.
  （设数组 $A = <1, 2, \dots, n>$）
- Assign each element *A*[*i*] a **random priority** *P*[*i*] , then sort the elements of *A* according to these priorities. Example
  （为每个元素 *A*[*i*] 分配一个随机数 *P*[*i*] 作为其优先权，然后依据这些优先权对数组 *A* 进行排序。）
  - If initial array is *A*=<1,2,3,4>, choose random priorities *P*=<36,3,97,19>, then produce an array *B*=<2,4,1,3>

```
PERMUTE-BY-SORTING(A)
n = length[A]
for(i=1; i<=n; i++)
        P[i] = RANDOM(1, n^3)
sort A, using P as sort keys
return A
```

We use a range of 1 to n^3 in RANDOM to make it likely that all the priorities in *P* are unique.(Exercise 5.3-5)

5.3.2 Randomly permuting an array

The method is better

(2) RANDOMIZE-IN-PLACE

RANDOMIZE-IN-PLACE(A, n)
for(i=1; i<=n; i++)
swap($A[i]$, A[RANDOM(i, n)])

1	2	3	…	n
$A(1)$	$A(2)$	$A(3)$	…	$A(n)$

1	2	3	…	i_1	…	n
$A(i_1)$	$A(2)$	$A(3)$	…	$A(1)$	…	$A(n)$

1	2	3	…	i_2	…	n
$A(i_1)$	$A(i_2)$	$A(3)$	…	$A(2)$	…	$A(n)$

Idea:

- **In iteration i, choose $A[i]$ randomly from $A[i .. n]$.**
- **Will never alter $A[i]$ after iteration i.** （第 i 次迭代后不再改变 $A[i]$）

Merit:

- **It runs in linear time without requiring sorting ($O(n)$).**
- **It needs fewer random bits (n random numbers in the range 1 to n rather than the range 1 to n^3)**（仅需更小范围的随机数产生器）
- **No auxiliary array is required.**（不需要辅助空间）

5.3.2 Randomly permuting an array

(2) RANDOMIZE-IN-PLACE

RANDOMIZE-IN-PLACE(A, n)
for(i=1; i<=n; i++)
    swap(A[i], A[RANDOM(i, n)]

1	2	3	…	n
$A(1)$	$A(2)$	$A(3)$	…	$A(n)$

1	2	3	…	i_1	…	n
$A(i_1)$	$A(2)$	$A(3)$	…	$A(1)$	…	$A(n)$

1	2	3	…	i_2	…	n
$A(i_1)$	$A(i_2)$	$A(3)$	…	$A(2)$	…	$A(n)$

Correctness:

- Given a set of n elements, a k-permutation is a sequence containing k of the n elements. There are $n!/(n-k)!$ possible k-permutations ? （给定 n 个元素，从其中任取 k 个元素进行排列，则有 $n!/(n-k)!$ 种不同的 k-排列，或 k-置换 ?）

$$P_n^k = C_n^k \cdot P_k^k = \frac{n!}{k!(n-k)!} \cdot k! = \frac{n!}{(n-k)!}$$

- *Lemma*

  RANDOMIZE-IN-PLACE computes a uniform random permutation.

Proof Use a loop invariant:

5.3.2 Randomly permuting an array

(2) RANDOMIZE-IN-PLACE

```
RANDOMIZE-IN-PLACE(A, n)
for(i=1; i<=n; i++)
        swap(A[i], A[RANDOM(i, n)]
```

1	2	3	…	n
$A(1)$	$A(2)$	$A(3)$	…	$A(n)$

1	2	3	…	i_1	…	n
$A(i_1)$	$A(2)$	$A(3)$	…	$A(1)$	…	$A(n)$

1	2	3	…	i_2	…	n
$A(i_1)$	$A(i_2)$	$A(3)$	…	$A(2)$	…	$A(n)$

- *Lemma*

  RANDOMIZE-IN-PLACE computes a uniform random permutation.

$$1/P_n^k = 1/\frac{n!}{(n-k)!} = \frac{(n-k)!}{n!}$$

Proof **Use a loop invariant:**

Loop invariant: Just prior to the *i*th iteration of the for loop, for each possible (*i*-1)-permutation, subarray *A*[1 .. *i*-1] contains this (*i*-1)-permutation with probability (*n*-*i*+1)!/*n*! ?

（第 *i* 次迭代之前，对 (*i*-1)-置换，任意一个(*i*-1)-置换*A*[1 .. *i*-1]的概率为(*n*-*i*+1)!/*n*! ?）

5.3.2 Randomly permuting an array

(2) RANDOMIZE-IN-PLACE

1	2	3	…	n
$A(1)$	$A(2)$	$A(3)$	…	$A(n)$

1	2	3	…	i_1	…	n
$A(i_1)$	$A(2)$	$A(3)$	…	$A(1)$	…	$A(n)$

1	2	3	…	i_2	…	n
$A(i_1)$	$A(i_2)$	$A(3)$	…	$A(2)$	…	$A(n)$

```
RANDOMIZE-IN-PLACE(A, n)
for(i=1; i<=n; i++)
        swap(A[i], A[RANDOM(i, n)]
```

- ***Lemma*** RANDOMIZE-IN-PLACE computes a uniform random permutation.

Proof Use a loop invariant:

Loop invariant: $A[1 .. i-1]$ contains each $(i-1)$-permutation with probability $(n-i+1)!/n!$.

- **Initialization:** Just before first iteration, $i=1$. Loop invariant says for each possible 0-permutation, subarray $A[1 .. 0]$ contains this 0-permutation with probability $n!/n!=1$. $A[1 .. 0]$ is an empty subarray, and a 0-permutation has no elements. So, A[1 .. 0] contains any 0-permutation with probability 1.

（空集包含空置换的概率为1）

5.3.2 Randomly permuting an array

(2) RANDOMIZE-IN-PLACE

```
RANDOMIZE-IN-PLACE(A, n)
for(i=1; i<=n; i++)
        swap(A[i], A[RANDOM(i, n)]
```

- ***Lemma*** RANDOMIZE-IN-PLACE computes a uniform random permutation.

Proof **Loop invariant:** Pr{A[1 .. i-1] contains each (i-1)-permutation} = (n-i+1)!/n! .

- **Maintenance:** Assume that prior to the ith iteration, Pr{A[1 .. i-1] contains each (i-1)-permutation} = (n-i+1)!/n! , we will show that after the ith iteration, Pr(A[1 .. i] contains each i-permutation) = (n-i)!/n! .

  （第 i 次迭代前，设(i-1)-置换 A[1 .. i-1] 中，任一置换发生的概率为 (n-i+1)!/n!，则需证明在第i 次迭代后，任一 i-置换 的概率为(n-i)!/n! ）

  Consider a particular i-permutation R=<x_1, x_2, … , x_i>. It consists of an (i-1)-permutation R'=<x_1, x_2, … , x_{i-1}>, followed by x_i .（考虑一个特别的 i-置换 R，其前 i-1 个元素组成 (i-1)-置换 R'，最后一个元素为 x_i ）. ……

5.3.2 Randomly permuting an array

(2) RANDOMIZE-IN-PLACE

```
RANDOMIZE-IN-PLACE(A, n)
for(i=1; i<=n; i++)
        swap(A[i], A[RANDOM(i, n)]
```

- ***Lemma*** RANDOMIZE-IN-PLACE computes a uniform random permutation.

Proof **Loop invariant:** Pr{A[1 .. i-1] contains each (i-1)-permutation} = $(n-i+1)!/n!$.

- **Maintenance:** … …

i-permutation R=$<x_1, x_2, \dots, x_i> = <x_1, x_2, \dots, x_i> \cup x_i$ = R’ $\cup x_i$.

Let E_1 be the event that the algorithm actually puts R’ into A[1 .. i-1] . By the loop invariant, Pr{E_1}=$(n-i+1)!/n!$.

Let E_2 be the event that the ith iteration puts x_i into A[i].

We get the i-Permutation R in A[1 .. i] if and only if both E_1 and E_2 occur => the probability that the algorithm produces R in A[1 .. i] is Pr{$E_2 \cap E_1$} = ?. … （令事件E_1表示算法实际输出 (i-1)-置换R’为A[1 .. i-1]，根据循环不变量， Pr{E_1}=$(n-i+1)!/n!$，令事件E_2表示第 i 次迭代后输出 A[i] 为x_i，则当且仅当E_1 和 E_2同时发生时我们得到 i-置换 R 为A[1 .. i]，其概率为Pr{$E_2 \cap E_1$} = ? ）…

5.3.2 Randomly permuting an array

(2) RANDOMIZE-IN-PLACE

```
RANDOMIZE-IN-PLACE(A, n)
for(i=1; i<=n; i++)
        swap(A[i], A[RANDOM(i, n)]
```

- ***Lemma*** RANDOMIZE-IN-PLACE computes a uniform random permutation.

Proof **Loop invariant:** Pr{A[1 .. i-1] contains each (i-1)-permutation} = (n-i+1)!/n! .

- **Maintenance:**

i	i+1	…	n
$A(j_i)$	$A(j_{i+1})$	…	$A(j_n)$

… …

$\Pr\{E_2 \cap E_1\} = \Pr\{E_2 | E_1\} \Pr\{E_1\}$.

The algorithm choose x_i randomly from the n-i+1 possibilities in $A[i .. n]$ => $\Pr\{E_2|E_1\} = 1/(n-i+1)$? Thus,

$$\Pr\{E_2 \cap E_1\} = \Pr\{E_2 | E_1\}\Pr\{E_1\} = \frac{1}{n-i+1} \cdot \frac{(n-i+1)!}{n!} = \frac{(n-i)!}{n!}$$

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- ***Lemma*** RANDOMIZE-IN-PLACE computes a uniform random permutation.

Proof **Loop invariant:** Pr{A[1 .. i-1] contains each (i-1)-permutation} = (n-i+1)!/n! .

- **Termination:**

  At termination, i=n+1, it is true prior to the ith iteration, so we conclude that A[i .. n] is a given n-permutation with probability (n-n)!/n! = 1/n! . ■ (lemma)

第五讲 概率分析与随机算法

内容提要：

- 雇用问题
- 指示器随机变量
- 随机算法
- 在线雇用问题

在线雇用问题

- **情景描述：**

  假设现在我们不希望面试所有的应聘者来找到最好的一个，也不希望因为不断有更好的申请者出现而不停地雇用信任解雇旧人。我们愿意雇用接近最好的应聘者，只雇用一次。但是，我们必须遵守猎头公司的一个规定：在每次面试之后，必须给出面试结果，要么雇用候选人，要么拒绝。

- **Goal：** 最小化面试次数和最大化雇用应聘者的质量取得平衡。

在线雇用问题

- 解决思路：
- 1）面试一个应聘者之后，能他一个分数，令*score(i)*表示给第i个应聘者的分数，并且假设所有应聘者得分都不相同；
- 2）面试前面k个（k<n）应聘者然后拒绝他们，再雇用其后比前面的应聘者更高分数的第一个应聘者。

```
ON-LINE-MAXIMUM (k, n)
  bestscore ← -∞
  for i ← 1 to k
      do if score(i) > bestscore
           then bestscore ← score(i)
  for i ← k+1 to n
      do if score(i) > bestscore
           then return i
  return n
```

谢谢！

Q & A