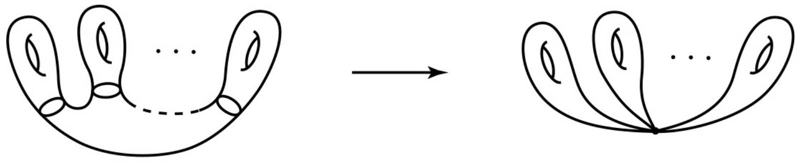
Ex1. Show that the functors $h^n(X) = \text{Hom}(H_n(X), \mathbb{Z})$ do not define a cohomology theory on the category of CW complexes.

Ex2. Many basic homology arguments work just as well for cohomology even though maps go in the opposite direction. Verify this in the following cases:
(a) Compute $H^i(S^n; G)$ by induction on n in two ways: using the long exact sequence of a pair, and using the MayerVietoris sequence.
(b) Show that if A is a closed subspace of X that is a deformation retract of some neighborhood, then the quotient map $X \to X/A$ induces isomorphisms $H^n(X, A; G) \approx \widetilde{H}^n(X/A; G)$ for all n.
(c) Show that if A is a retract of X then $H^n(X; G) \approx H^n(A; G) \oplus H^n(X, A; G)$.

Ex3. Show that if $f : S^n \to S^n$ has degree d then $f_* : H^n(S^n; G) \to H^n(S^n; G)$ is multiplication by d.

Ex4. Let $\langle X, Y \rangle$ denote the set of basepoint-preserving homotopy classes of basepoint-preserving maps $X \to Y$. Using Proposition 1B.9, show that if X is a connected CW complex and G is an abelian group, then the map $\langle X, K(G, 1) \rangle \to H^1(X; G)$ sending a map $f : X \to K(G, 1)$ to the induced homomorphism $f_* : H_1(X) \to H_1(K(G, 1)) \approx G$ is a bijection, where we identify $H^1(X; G)$ with $\text{Hom}(H_1(X), G)$ via the universal coefficient theorem.

Ex5. Assuming as known the cup product structure on the torus $S^1 \times S^1$, compute the cup product structure in $H^*(M_g)$ for M_g the closed orientable surface of genus g by using the quotient map from M_g to a wedge sum of g tori, shown below.



Please hand in this homework on 19th Dec. 2018.