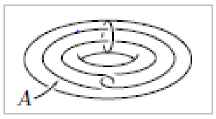
Ex 1. (1 pt) *Define $f : S^1 \times I \to S^1 \times I$ by $f(\theta, s) = (\theta + 2\pi s, s)$, so f restricts to identity on the two bundary circles of $S^1 \times I$. Show that f is homotopic to the identity by a homotopy f_t that is stationary on one of the boundary circles, but not by any homotopy f_t that is stationary on both boundary circles. [Consider what f does to the path $s \mapsto (\theta_0, s)$ for fixed $\theta_0 \in S^1$.]*

Ex 2. (1 pt) *Does the Borsuk-Ulam theorem hold for the torus? In other words, for every map $f : S^1 \times S^1 \to \mathbb{R}^2$ must there exist $(x, y) \in S^1 \times S^1$ such that $f(x, y) = f(-x, -y)$?*

Ex 3. (1 pt) *Let A_1, A_2, A_3 be compact sets in $\mathbb{R}^3$. Use the Borsuk-Ulam theorem to show that there is one plane $P \subset \mathbb{R}^3$ that simultaneously divides each A_i into two pieces of equal measure.*

Ex 4. (1 pt) *From the isomorphism $\pi_1(X \times Y, (x_0, y_0)) \approx \pi_1(X, x_0) \times \pi_1(Y, y_0)$ it follows that loops in $X \times \{y_0\}$ and $\{x_0\} \times Y$ represent commuting elements of $\pi_1(X \times Y, (x_0, y_0))$. Construct an explicit homotopy demonstrating this.*

Ex 5. (3 pts) *Show that there are no retraction $r : X \to A$ in the following cases:*



(a) $X = \mathbb{R}^3$ with A any subspace homeomorphic to S^1.
(b) $X = S^1 \times D^2$ with A its boundary torus $S^1 \times S^1$.
(c) $X = S^1 \times D^2$ and A the circle shown in the figure.
(d) $X = D^2 \vee D^2$ with A its boundary $S^1 \vee S^1$.
(e) X a disk with two points on its boundary identified and A its boundary $S^1 \vee S^1$.
(f) X the Möbius band and A its boundary circle.

Ex 6. (2 pts) *Using the technique in the proof of Proposition 1.14, show that if a space X is obtained from a path-connected subspace A by attaching a cell e^n with $n \geq 2$, then the inclusion $A \hookrightarrow X$ induces a surjection on π_1. Apply this to show:*
(a) The wedge sum $S^1 \vee S^2$ has fundamental group $\mathbb{Z}$.
(b) For a path-connected CW complex X the inclusion map $X^1 \hookrightarrow X$ of its 1-skeleton induces a surjection $\pi_1(X^1) \to \pi_1(X)$. [For the case that X has infinitely many cells, see Proposition A.1 in Appendix.]

Note: Please hand in this homework on next Wednesday (26$^{\text{th}}$ Sep. 2018).