USTC, School of Mathematical Sciences — Winter semester 2018/19
Algebraic topology by Prof. Mao Sheng — Exercise sheet 2
MA04311 Tutor: Lihao Huang, Han Wu — 8 points
Posted online by Dr. Muxi Li

Ex 1. (1 pt) *Let $X \subset \mathbb{R}^m$ be the union of convex open sets $X_1, \cdots, X_n$ such that $X_i \bigcap X_j \neq \varnothing$ for all i, j. Give an example to show that X is not necessarily simply-connected.*

Ex 2. (1 pt) *Let $X \subset \mathbb{R}^m$ be the union of simply-connected open sets $X_1, \cdots, X_n$ such that $X_i \bigcap X_j \bigcap X_k$ are path-connected for all i, j, k. Show that X is simply-connected.*

Ex 3. (1 pt) *Let $X \subset \mathbb{R}^3$ be the union of n lines through the origin. Compute $\pi_1(\mathbb{R}^3 - X)$.*

Ex 4. (1 pt) *Let $X \subset \mathbb{R}^2$ be a connected graph that is the union of a finite number of straight line segments. Show that $\pi_1(X)$ is free with a basis consisting of loops formed by the boundaries of the bounded complementary regions of X, joined to a basepoint by suitably chosen paths in X. [Assume the Jordan curve theorem for polygonal simple closed curves, which is equivalent to the case that X is homemorphic to S^1.]*

Ex 5. (1 pts) *Suppose a space Y is obtained from a path-connected subspace X by attaching n-cells for a fixed $n \geq 3$. Show that the inclusion $X \hookrightarrow Y$ induces an isomorphism on π_1. [See the proof of Proposition 1.26.] Apply this to show that the complement of a discrete subspace of $\mathbb{R}^n$ is simply-connected if $n \geq 3$.*

Ex 6. (1 pts) *Let X be the quotient space of S^2 obtained by identifying the north and south poles to a single point. Put a cell complex structure on X and use this to compute $\pi_1(X)$.*

Ex 7. (1 pts) *Compute the fundamental group of the space obtained from two tori $S^1 \times S^1$ by identifying a circle $S^1 \times \{x_0\}$ in one torus with the corresponding circle $S^1 \times \{x_0\}$ in the other torus.*

Ex 8. (1 pts) *Show that $\pi_1(\mathbb{R}^2 - \mathbb{Q}^2)$ is uncountable.*

Note: Please hand in this homework on 10^{th} Oct. 2018.