USTC, School of Mathematical Sciences — *Winter semester 2018/11/9*
Algebraic topology by Prof. Mao Sheng — *Exercise sheet 6*
MA04311 Tutor: Lihao Huang, Han Wu — *10 points*
Posted online by Dr. Muxi Li — *1 points for every problem*

Ex1. Show that the Δ-complex obtained from Δ^3 by performing the edge identifications $[v_0, v_1] \sim [v_1, v_3]$ and $[v_0, v_2] \sim [v_2, v_3]$ deformation retracts onto a Klein bottle. Find other pairs of identifications of edges that produce Δ-complexes deformation retracting onto a torus, a 2-sphere, and $\mathbb{R}P^2$.

Ex2. Construct a Δ-complex structure on $\mathbb{R}P^n$ as a quotient of a Δ-complex structure on S^n having vertices the two vectors of length 1 along each coordinate axis in $\mathbb{R}^{n+1}$.

Ex3. Compute the simplicial homology groups of the Klein bottle using the Δ-complex structure described at the beginning of this section.

Ex4. Compute the simplicial homology groups of the Δ-complex obtained from n + 1 2-simplices $\Delta_0^2, ..., \Delta_n^2$ by identifying all three edges of Δ_0^2 to a single edge, and for $i > 0$ identifying the edges $[v_0, v_1]$ and $[v_1, v_2]$ of Δ_i^2 to a single edge and the edge $[v_0, v_2]$ to the edge $[v_0, v_1]$ of Δ_{i-1}^2.

Ex5. Find a way of identifying pairs of faces of Δ^3 to produce a Δ-complex structure on S^3 having a single 3-simplex, and compute the simplicial homology groups of this Δ-complex.

Ex6. Compute the homology groups of the Δ-complex X obtained from Δ^n by identifying all faces of the same dimension. Thus X has a single k-simplex for each $k \leqslant n$.

Ex7. (a) Show the quotient space of a finite collection of disjoint 2 simplices obtained by identifying pairs of edges is always a surface, locally homeomorphic to $\mathbb{R}^2$.
(b) Show the edges can always be oriented so as to define a Δ-complex structure on the quotient surface.

Ex8. Show that if A is a retract of X then the map $H^n(A) \to H^n(X)$ induced by the inclusion $A \subset X$ is injective.

Ex9. Show that chain homotopy of chain maps is an equivalence relation.

Ex10. Verify that $f \simeq g$ implies $f_* = g_*$ for induced homomorphisms of reduced homology groups.

Please hand in this homework on 14th Nov. 2018.