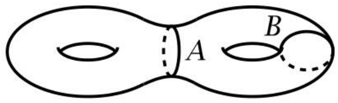
USTC, School of Mathematical Sciences — *Winter semester 2018/11/16*
Algebraic topology by Prof. Mao Sheng — *Exercise sheet 6*
MA04311 Tutor: Lihao Huang, Han Wu — *10 points*
Posted online by Dr. Muxi Li

Ex1.(1pt) For an exact sequence $A \to B \to C \to D \to E$ show that $C = 0$ iff the map $A \to B$ is surjective and $D \to E$ is injective. Hence for a pair of spaces (X, A), the inclusion $A \hookrightarrow X$ induces isomorphisms on all homology groups iff $H_n(X, A) = 0$ for all n.

Ex2.(1pt) (a) Show that $H_0(X, A) = 0$ iff A meets each path-component of X.
(b) Show that $H_1(X, A) = 0$ iff $H_1(A) \to H_1(X)$ is surjective and each path-component of X contains at most one path-component of A.

Ex3.(2pts) (a) Compute the homology groups $Hn(X, A)$ when X is S^2 or $S^1 \times S^1$ and A is a finite set of points in X.
(b) Compute the groups $H_n(X, A)$ and $H_n(X, B)$ for X a closed orientable surface of genus two with A and B the circles shown. [What are X/A and X/B?]



Ex4.(1pt) Show that for the subspace $\mathbb{Q} \subset \mathbb{R}$, the relative homology group $H_1(\mathbb{Q}, \mathbb{R})$ is free abelian and find a basis.

Ex5.(1pt) Compute the homology groups of the subspace of $I \times I$ consisting of the four boundary edges plus all points in the interior whose first coordinate is rational.

Ex6.(2pts) Show that $\widetilde{H}_n(X) \approx \widetilde{H}_{n+1}(SX)$ for all n, where SX is the suspension of X. More generally, thinking of SX as the union of two cones CX with their bases identified, compute the reduced homology groups of the union of any finite number of cones CX with their bases identified.

Ex7.(2pts) Making the preceding problem more concrete, construct explicit chain maps $s : C_n(X) \to C_{n+1}(SX)$ inducing isomorphisms $\widetilde{H}_n(X) \approx \widetilde{H}_{n+1}(SX)$.

Please hand in this homework on 21st Nov. 2018.