Ex1. Construct a map $S^n \to S^n$ of degree k, for each $n \geqslant 1$ and $k \in \mathbb{Z}$.

Ex2. Given a map $f : S^{2n} \to S^{2n}$, show that there is some point $x \in S^{2n}$ with either $f(x) = x$ or $f(x) = -x$. Deduce that every map $\mathbb{R}P^{2n} \to \mathbb{R}P^{2n}$ has a fixed point. Construct maps $\mathbb{R}P^{2n} \to \mathbb{R}P^{2n}$ without fixed points from linear transformations $\mathbb{R}^{2n} \to \mathbb{R}^{2n}$ without eigenvectors.

Ex3. For an invertible linear transformation $f : \mathbb{R}^n \to \mathbb{R}^n$ show that the induced map on $H_n(\mathbb{R}^n, \mathbb{R}^n\{0\}) \approx \widetilde{H}_{n-1}(\mathbb{R}^n - \{0\}) \approx \mathbb{Z}$ is $\mathbf{1}$ or $-\mathbf{1}$ according to whether the determinant of f is positive or negative. [Use Gaussian elimination to show that the matrix of f can be joined by a path of invertible matrices to a diagonal matrix with 1s on the diagonal.]

Ex4. A polynomial $f(z)$ with complex coefficients, viewed as a map $\mathbb{C} \to \mathbb{C}$, can always be extended to a continuous map of one-point compactifications $f : S^2 \to S^2$. Show that the degree of f equals the degree of f as a polynomial. Show also that the local degree of f at a root of f is the multiplicity of the root.

Ex5. Compute the homology groups of the following 2-complexes:
(a) The quotient of S^2 obtained by identifying north and south poles to a point.
(b) $S^1 \times (S^1|S^1)$.
(c) The space obtained from D^2 by first deleting the interiors of two disjoint subdisks in the interior of D^2 and then identifying all three resulting boundary circles together via homeomorphisms preserving clockwise orientations of these circles.
(d) The quotient space of $S^1 \times S^1$ obtained by identifying points in the circle $S^1 \times \{x_0\}$ that differ by $2\pi/m$ rotation and identifying points in the circle $\{x_0\} \times S^1$ that differ by $2\pi/n$ rotation.

Please hand in this homework on 5th Dec. 2018.