Valley current splitter in minimally twisted bilayer graphene

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We study the electronic transport properties at the intersection of three topological zero lines as the elementary current partition node that arises in minimally twisted bilayer graphene. Unlike the partition laws of two intersecting zero lines, we find that (i) the incoming current can be partitioned into both left-right adjacent topological channels and (ii) the forward-propagating current is nonzero. By tuning the Fermi energy from the charge-neutrality point to a band edge, the currents partitioned into the three outgoing channels become nearly equal. Moreover, we find that the current partition node can be designed as a perfect valley filter and energy splitter controlled by electric gating. By changing the relative electric-field magnitude, the intersection of three topological zero lines can transform smoothly into a single zero line, and the current partition can be controlled precisely. We explore the available methods for modulating this device systematically by changing the Fermi energy, the energy gap size, and the size of the central gapless region. The current partition is also influenced by magnetic fields and the system size. Our results provide a microscopic depiction of the electronic transport properties around a unit cell of minimally twisted bilayer graphene and have far-reaching implications in the design of electron-beam splitters and interferometer devices.

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I. INTRODUCTION

Twisted van der Waals layers provide intriguing platforms to investigate novel physics. On one hand, twisted bilayer graphene (r-BG) systems provide ideal platforms to explore the electronic correlation effect on the flat bands at magic angles [1–6]. On the other hand, when the twist angle decreases from the magic angles, the reconstruction of atomic lattices in r-BG becomes more and more important [7]. As a result, the incommensurate moiré structure at the magic angles will gradually become an array of commensurate domains with soliton boundaries. The presence of a minimal twist changes the local stacking order, arranging the AB/BA stacking regions periodically in space. The perpendicular electric field in r-BG or the sublattice potential difference in a graphene/h-BN bilayer leads to an energy gap that results in opposite valley Chern numbers at the AB/BA stacking regions. At the interfaces between different topological regions, topologically confined states (i.e., zero-line modes; ZLMs) appear and form networks [24–35]. Recently, a network of topologically protected helical states was discovered in minimally twisted bilayer graphene [7–21] and graphene/hexagonal boron nitride (h-BN) superlattices [22,23]. Because of the threefold rotational symmetry of the lattices, the elementary component of this network is the intersection of three ZLMs. In contrast to pristine atomic crystals where the electrons can be regarded as a particle hopping from one site to another, the electrons in a topological network are more like a wave propagating along the domain wall without distribution inside the domains. Thus, novel transport properties are expected. However, the microscopic electronic transport properties of this intersecting point remains poorly understood.

In this paper, we study the transport properties of three intersecting ZLMs connected to six terminals. When current is injected from one terminal, we find notable partitioning towards the forward channel; this situation is qualitatively different from the intersection of two ZLMs, where no forward propagation of current is observed. In the case of three intersecting ZLMs, the incoming current is partitioned towards the forward and the two adjacent zero lines. This current partitioning depends strongly on the size of the central region when the region is small but saturates when it is large. By tuning the Fermi level, we find that the current partition can be controlled over a wide range. A perpendicular magnetic field can tune the currents propagating to the adjacent lines, whereas the forward-propagating current remains quite robust. By changing the size of the AA stacking zone, we also study the effect of a twist angle on the transport properties. We find that (i) our device can support stable current partitioning even without a perpendicular electric field and (ii) the electric field allows us to tune the partition properties and turn the device into a valley current splitter.

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FIG. 1. (a) Moiré pattern of twisted bilayer graphene and identification of regions with AB, BA, and AA stacking. (b) Schematic of six-terminal device with three intersecting zero lines. The plus and minus signs indicate the alternating sublattice potentials. Throughout this paper and unless specified otherwise, we use a side length of 25 nm for our regular hexagonal device. Blue and red arrows correspond to modes that carry valley indices $K$ and $K'$, respectively. Transmission is forbidden along the zero lines with reversed chirality indicated by the red arrows. (c) Dependence of current partitioning as a function of the Fermi level to the different output terminals. The plus and minus signs indicate the alternating sublattice potentials. Throughout this paper and unless specified otherwise, we use a side length of 25 nm for our regular hexagonal device. Blue and red arrows correspond to modes that carry valley indices $K$ and $K'$, respectively. (d, e) Local density of states at Fermi level energies 10 and 0.1 eV, respectively. Green arrows show the forward propagation directions of valley $K$ currents, while white arrows represent the zero lines with valley index $K'$ where the currents do not propagate. The color bar shows the linear gradient of values.

II. MODEL HAMILTONIAN

The general stacking order of the moiré pattern is shown in Fig. 1(a), where the bright zone in the center corresponds to AA stacking. The size of the AA stacking region decreases with a decrease in the twist angle. Around this central zone, alternating periodic chiral AB or BA stacking regions are formed. At the interfaces between the AB and the BA stacking regions, domain walls appear, indicated by the dashed white lines. By applying a perpendicular electric field, the centers of the chiral stacking regions become gapped, while gapless ZLMs form at the domain walls [8,15,16].

ZLMs form at the domain walls [8,15,16]. The AA stacking zone is located at the intersection of three concurrent ZLMs, which are successively rotated by 60°. Without loss of generality, in our calculations we consider a monolayer graphene flake with position-dependent staggered sublattice potentials to form six adjacent regions with different valley Hall topologies labeled by plus/minus signs in Fig. 1(b), which correspond to AB/BA stacking regions under an electric field in a twisted bilayer graphene. The valley Hall domain walls form three intersecting zero lines, while the intersection region corresponding to the AA stacking zone in the moiré pattern of a twisted bilayer is not gapped locally. The monolayer graphene flake with staggered site potentials can be described by the $\pi$-orbital tight-binding Hamiltonian

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j + \sum_{i \in A} U_{Ai} c_i^\dagger c_j + \sum_{i \in B} U_{Bi} c_i^\dagger c_j,$$

where $c_i^\dagger$ ($c_j$) is a creation (annihilation) operator for an electron at site $i$, and $t = 2.6$ eV is the nearest-neighbor hopping amplitude. The sublattice potentials are spatially varying, with $U_{Ai} = -U_{Bi} = \lambda \Delta$ in regions labeled by $\lambda = \pm$ as shown in Fig. 1(b), where $2\Delta$ measures the magnitude of the staggered sublattice potential difference.

The zero lines are connected to six reservoirs labeled $T_i$ ($i = 1-6$) as shown in Fig. 1(b). Herein, we take $T_1$ to be the injection terminal. The electronic transport calculations are based on the Landauer-Büttiker formula [36] and recursively constructed Green’s functions [37]. The conductance from the $q$ terminal to the $p$ terminal is evaluated from

$$G_{qp} = \frac{2e^2}{h} \text{Tr}[\Gamma_p G^\pi \Gamma_q G^\pi],$$

where $G^{\pi}$ is the retarded/advanced Green’s function of the central scattering region, and $\Gamma_p$ is the line-width function describing the coupling between the $p$ terminal and the central scattering region. The propagation of a ZLM coming in from the $p$ terminal is illustrated by the local density of states at energy $\epsilon$, which can be calculated by

$$\rho_p(\epsilon, r) = 1/2\pi |G^{\pi}(\epsilon, r)|^2,$$

where $r$ is the actual spatial coordinate.

III. RESULTS AND DISCUSSION

A. Current partition laws

In our calculations, the central region is a hexagon with circumcircle diameter $D = 50$ nm, and the diameter of the AA stacking zone is 0.28 nm unless stated otherwise. With $\Delta = 0.1t$, we calculate how the current partition depends on the Fermi energy $E_F$ as shown in Fig. 1(c). The current partition laws can be summarized as

$$G_{31} = G_{51} = 0,$$

$$G_{21} = G_{61},$$

$$G_{\text{tot}} = G_{21} + G_{41} + G_{61} = \epsilon^2/h.$$  

Here, Eq. (1) restricts current partitioning into zero lines of opposite chirality, while the condition in Eq. (2) requires the

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mirror reflection symmetry of the partition, which is broken in the presence of a magnetic field as we show later. Equation (3) implies that there is no backscattering due to intervalley scattering, and indeed any reflected current remains very weak even in the presence of disorders [26].

Note that the forward-propagating conductance $G_{41}$ is nonzero because the zero line from $T_1$ to $T_4$ has the same chirality, in contrast to the case of two intersecting zero lines, where the forward propagation is forbidden by the chirality conservation rule. Moreover, the forward-current transmission strongly depends on $E_F$ as shown in Fig. 1(c). When $E_F$ is close to the charge-neutrality point (CNP), $G_{41} \approx 0.11e^2/h$ is less than the conductance of $G_{21} = G_{61} \approx 0.44e^2/h$ towards the sides. When $E_F$ is shifted away from the CNP and moves toward the bulk band edges, $G_{41}$ increases gradually and exceeds $G_{21}$ when $E_F > 0.08t$. The current partition at different $E_F$ values is shown more clearly in Figs. 1(d) and 1(e), wherein the local density of states for current injected from $T_1$ is plotted at $E_F = 0.001t$ and 0.100$t$, respectively. This Fermi-energy dependent current partition is because the AA stacking region is not gapped locally, which can be seen as a finite potential barrier located at the zero-lines intersection. Hence, with respect to quantum mechanics, when we increase the Fermi energy, the energy of injecting electrons will also increase, which improves the probability of electrons transmitting the finite potential barrier (AA stacking region). In other words, more injecting current will transmit the AA stacking region and propagate to $T_4$ when we increase the Fermi energy; meanwhile, the current portioned to $T_2$ and $T_6$ will decrease, which suggests that longitudinal transport may be greatly modified through a perpendicular electric gate by altering the current percolation properties through multiple partition nodes.

B. Influence of a magnetic field

In addition to control through electrical means, the current partition is also strongly affected by applying a magnetic field. The effect of a magnetic field $B = \nabla \times A$ can be included by attaching a Peierls phase factor to the hopping term

$$t_{ij} \rightarrow t_{ij} \exp \left(-i\frac{e}{\hbar} \int A \cdot dl \right),$$

where $\int A \cdot dl$ is the integral of the vector potential along the path from site $i$ to site $j$. The presence of a magnetic field changes the current partition as shown in Fig. 2(a) for $\Delta = 0.1t$ and $E_F = 0.001t$. We find that the presence of the magnetic field removes the equality between $G_{21}$ and $G_{61}$ because the magnetic field breaks the mirror reflection symmetry. However, the forward-propagating current remains unaffected.

By changing $E_F$ under a given magnetic field as shown in Fig. 2(b), we find that the current partitions to both the forward and the side directions vary simultaneously. As $E_F$ approaches the bulk band edge, the current partitioned to $T_4$ increases gradually but does not reach the same magnitude as that for vanishing magnetic flux. Figures 2(c) and 2(d) show more clearly the current partition under a magnetic flux of $\Phi_B = 0.05 \mu$Wb for $E_F = 0.001t$ and 0.1$t$, respectively, wherein more current is partitioned into $T_2$ than into $T_6$. This difference is more obvious at the higher Fermi energy, in agreement with previous work [18,19].

To show the influences of the magnetic field and Fermi energy more systematically, in Fig. 2(e) we show the phase diagram for conductance $G_{41}$ with different values of $E_F$ and $\Phi_B$ for $\Delta = 0.1t$. The color bar shows the linear gradient of values in units of $e^2/h$.

C. Effect of system size

For an electronic nanodevice, its size plays a crucial role. Herein, we investigate the current partition of a device whose
Phase diagrams for conductance

The linear gradient of values in units of \( D \) circle diameter

different Fermi energies

3(d), we plot the current partition as a function of circumcircle diameter \( L \) and \( G \) when the system size has only a weak influence on the current partition increases, the backscattering vanishes. However, increasing the conductance is not quantized. When the system size which is mainly due to the backscattering because the total fluctuation appears for small sizes and exhibits a weak dependence on \( \Delta \). The current partition fluctuates obviously for \( D > 6 \) nm, which is mainly due to the backscattering because the total conductance is not quantized. When the system size \( L \) increases, the backscattering vanishes. However, increasing the system size has only a weak influence on the current partition when \( E_F \) is close to the CNP but has a definite influence of bringing \( G_{41} \) and \( G_{21} \) closer together when \( E_F \) lies at the band edge. This behavior suggests that the equal-current partition approximation is valid at a small twist angle but invalid at a large twist angle when the Fermi energy \( E_F \) is far from charge neutrality.

The length dependence of the current partition at different \( E_F \) values with \( \Delta = 0.1r \) is shown in Fig. 3(e), wherein the fluctuation appears for small sizes and exhibits a weak dependence on \( E_F \). As the system size increases, \( G_{41} \) gradually increases and then saturates at a magnitude that increases with \( E_F \). By setting \( E_F = 0.001r \), we plot the phase diagram of \( G_{41} \) as a function of the energy gap \( \Delta \) and circumcircle diameter \( D \) in Fig. 3(f), wherein similar behaviors are observed. Moreover, we find that the saturated \( G_{41} \) at larger sizes also increases with the energy gap.

D. Effect of AA stacking size

We now study how the AA stacking size at the intersection affects the current partition, as shown schematically in Fig. 4(a). This can simulate the effect of the twisting angle \( \theta \) in \( r \)-BG, where the size of the AA stacking zone
quantitatively, we add two sets of electric fields \( EF \) first decreases and then increases to the quantized value near \( \theta \). In particular, we only consider the twist angle range from 0.1° to 0.2°, in which cases the domains are observable and the domain walls are sharp enough to be described by our model Hamiltonian. We vary the circumsphere diameter of the AA stacking region, namely, \( D_{AA} \), from the smallest size to 25.2 nm while fixing the device size at \( D = 50 \) nm. At \( EF = 0.001\tau \) and \( \Delta = 0.1\tau \), we find that \( G_{41} \) increases with \( D_{AA} \) and becomes quantized at \( D_{AA} \approx 14 \) nm, where the current partition of \( G_{21,61} \) vanishes. This behavior suggests that the central AA stacking region acts as a scattering zone whose scattering weakens as its size increases, thereby increasing the forward-propagation current. As a result, most of the current flows along one zero line, with weak partition to the other allowed zero lines.

For the largest AA stacking region, i.e., when there is no longer an energy gap in the central scattering region, the current is fully partitioned into the forward-propagating zero line. Upon changing \( EF \) from the CNP, we find that the current partition becomes tunable. As we increase \( EF \), \( G_{41} \) first decreases and then increases to the quantized value near \( EF \approx 0.05\tau \). When we further increase \( EF \), \( G_{41} \) decreases to a small value. For \( D_{AA} = 14 \) nm and \( \Delta = 0.1\tau \), the current partitioned into \( L_4 \) increases with \( |\Delta| \) and those into \( L_2 \) and \( L_6 \) decrease. Note that all of the above results were calculated under the condition that \( U_{AA} = \Delta \). At \( EF = 0.001\tau \) and \( \Delta = 0.1\tau \), we find that \( G_{41} \) increases with \( |U_{AA}| \) and becomes quantized at \( |U_{AA}| = \Delta \), where the current partitioning of \( G_{21,61} \) vanishes.

In Fig. 4(f), we plot the phase diagram of \( G_{41} \) with different \( U_{AA} \) and \( D_{AA} \) at fixed \( EF = 0.001\tau \) and \( \Delta = 0.1\tau \); for \( D_{AA} < 4.3 \) nm, \( U_{AA} \) no longer influences the current partitioning. In Fig. 4(g), we also plot the phase diagram of \( G_{41} \) with different \( \Delta \) and \( D_{AA} \) at fixed \( EF = 0.001\tau \). Now the resonant transmission appears for a different band gap \( \Delta \). As we decrease \( |\Delta| \) from 0.1\( \tau \), the value of \( D_{AA} \) for resonant transmission increases until it equals 25.2 nm, and the resonant transmission disappears when \( |\Delta| < 0.6\tau \). Figure 4(h) plots the phase diagram of \( G_{41} \) with different \( EF \) values and \( D_{AA} \) at fixed \( \Delta = 0.1\tau \). One finds that the resonant transmission appears at \( EF < 0.05\tau \) and the resonant diameter increases with \( EF \). Note that at the resonant transmission point of \( \Delta = 0.1\tau \) and \( D_{AA} \approx 14 \) nm, increasing \( EF \) drives \( G_{41} \) to decrease gradually as in Fig. 4(c), suggesting that the current partitioning goes from 0 to a finite value. Such tunability by means of electric gating could be used to construct a field-effect transistor based on the dissipationless topological ZLMs for switching on and off the forward propagation.

**E. Tunable valley current splitter**

In order to control the current partition effectively and quantitatively, we add two sets of electric fields \( \Delta_1 \) and \( \Delta_2 \) to the six-terminal device, as shown in the inset in Fig. 5(a). We vary the magnitude of \( \Delta_2/\Delta_1 \) while fixing \( EF = 0.001\tau \), the device size \( D = 50 \) nm, and \( \Delta_1 = 0.1\tau \). We find that \( G_{41} \) becomes quantized when \( \Delta_2/\Delta_1 < 0 \), where the current partition of \( G_{21,61} \) vanishes, which means that there exists only a single zero line. With \( \Delta_2/\Delta_1 > 0 \), we find that \( G_{41} \) decreases and \( G_{21,61} \) increase with \( \Delta_2/\Delta_1 \). When \( \Delta_2/\Delta_1 \approx 0.6 \), \( G_{41,21,61} \) become the same value. When we further increase \( \Delta_2/\Delta_1 \) to 2, \( G_{41} \) decreases to 0 and \( G_{21,61} \) become 0.5\( e^2/h \), as shown in Fig. 5(c), which shows the current partition just like the intersection of two zero lines. By fixing one of the electric fields and varying the other, we can modulate the partition \( G_{41} \) from 0 to quantized, precisely, making this device more qualified as a valley current splitter.

**IV. SUMMARY**

We have presented a systematic study of the electronic transport properties of three intersecting zero lines in a twisted bilayer graphene, which form six topological zones stacked either in AB or in BA patterns and are centered in a gapless region with AA stacking. We have investigated the influence of the system size, AA-region size, \( EF \), energy gap, and magnetic field on the current partitioning. When current flows through the device, the forward-propagating current partitioning \( G_{41} \) is nonzero and can be tuned by mediating \( EF \). This is in contrast to the topological intersection of two zero lines, where forward propagation is forbidden.

When the central AA region is small, increasing \( EF \) to the band edge enhances \( G_{41} \) to partition the current into three nearly equal zero lines, and the potential of the AA region does not affect the current partitioning. By making the central AA region larger, which corresponds to decreasing the twist angle, resonant forward transmission can be realized at a proper \( EF \), in which case the partitioning to the side zero lines vanishes. Starting from the case of resonant transmission, further increasing \( EF \) (e.g., by means of electric
gating) increases the current partitioning to the side zero lines from 0 to a definite level, thereby suggesting that the system could be used as a dissipationless field-effect transistor. Moreover, decreasing the system size gives rise to strong backscattering and conductance fluctuations. In the absence of a magnetic field, the symmetric geometry of our central region leads to $G_{21} = G_{61}$ current partitioning, but the presence of a magnetic field breaks this symmetry.

Our theoretically proposed device can precisely find experimental realization in graphene moiré structures and can also find realizations in phononic crystals. Specifically, our findings are the first prediction of the current partition at the zero-line intersection node of a triangular network of topological channels in small-twist-angle bilayer graphene and pave the way towards the realization of low-power topological quantum devices.

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