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Illuminating Recommendation by Understanding the Explicit Item Relations

Qi Liu[1], *Member*, *ACM*, *IEEE*, Hong-Ke Zhao[1], Le Wu[2], Zhi Li[1,3]
and En-Hong Chen[1,*], *Fellow*, *CCF*, *Senior Member*, *IEEE*

[1] *Anhui Province Key Laboratory of Big Data Analysis and Application, University of Science and Technology of China Hefei 230027, China*

[2] *School of Computer and Information, Hefei University of Technology, Hefei 230009, China*

[3] *School of Software Engineering, University of Science and Technology of China, Hefei 230051, China*

E-mail: qiliuql@ustc.edu.cn; zhhk@mail.ustc.edu.cn; lewu@hfut.edu.cn; zhili03@mail.ustc.edu.cn
cheneh@ustc.edu.cn

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Abstract Recent years have witnessed the prevalence of recommender systems in various fields, which provide a personalized recommendation list for each user based on various kinds of information. For quite a long time, most researchers have been pursing recommendation performances with predefined metrics, e.g., accuracy. However, in real-world applications, users select items from a huge item list by considering their internal personalized demand and external constraints. Thus, we argue that explicitly modeling the complex relations among items under domain-specific applications is an indispensable part for enhancing the recommendations. Actually, in this area, researchers have done some work to understand the item relations gradually from "implicit" to "explicit" views when recommending. To this end, in this paper, we conduct a survey of these recent advances on recommender systems from the perspective of the explicit item relation understanding. We organize these relevant studies from three types of item relations, i.e., combination-effect relations, sequence-dependence relations, and external-constraint relations. Specifically, the combination-effect relation and the sequence-dependence relation based work models the intra-group intrinsic relations of items from the user demand perspective, and the external-constraint relation emphasizes the external requirements for items. After that, we also propose our opinions on the open issues along the line of understanding item relations and suggest some future research directions in recommendation area.

Keywords recommender system, item relation, recommendation interpretability

1 Introduction

Recommender systems are those techniques that automatically provide personalized item suggestions to interest target users based on various information, such as user-item interaction records, item contents and so on[1-2]. The suggestions provided by recommender systems aim at supporting users in various decision-making processes, such as what items to buy, what music to listen, or what news to read[2]. With the huge potential of recommender systems for improving user satisfaction and increasing company sales, recommender systems have become a hot topic in both academia and industry in recent years.

Generally speaking, the recommender systems can be grouped into three categories based on their utilized techniques, i.e., content-based recommendations[1,3], collaborative filtering[1,4], and hybrid approaches[1,5]. Specifically, the content-based recommendations utilize

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Survey

Special Section on Recommender Systems with Big Data

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*Corresponding Author

item descriptions for recommendation, while collaborative filtering relies on the collective user-item interactions (e.g., rating matrix, browsing history) for recommendation without any content information. To fully utilize the item contents and user-item interactive behaviors, the hybrid approaches exploit the complementary advantages of the above two kinds of methods for recommendation. Besides designing general recommender algorithms, a parallel line of research work focuses on designing domain-specific recommender systems by considering the unique domain knowledge, such as context-aware recommendations[6], mobile recommendations[7], and education exercise recommendations[8].

For a long time, all these studies have been largely evaluated by the predictive accuracy of the recommendation results, and they have achieved satisfactory performance on accuracy. However, in the real-world recommendation scenarios, the recommendation process is very complex. Users would choose from a huge item list by considering both their personalized demand with historical consumptions and external environment. For example, if a user has bought a camera recently, then a good recommender system would suggest camera accessories for this user. In the meantime, this target user may be limited by his/her budget constraint when further choosing accessories. Thus, it is necessary to directly utilize the item relationships in each specific domain to better understand user interest for recommendation. Nevertheless, most classical recommender algorithms have been making their main efforts on recommendation performances while neglecting to understand the item relations or only implicitly exploiting the item relations. For example, considering a shopping scenario of camera equipments, in the classical content-based recommender systems, the algorithms could only exploit similarities among different cameras while neglecting to understand the user demand or analyze the explicit item relations. Thus, the generated recommendations are all similar cameras though the active user has already bought a camera recently and will not buy one more in a short time. Hence, we argue that explicitly modeling the item relations, especially in various domain-oriented services, could largely enhance the customer understanding and recommendation performances.

Luckily, recently more and more researchers have been attempting to mine the explicitly item relations to better understand the specific recommendation domain and enhance recommendation performances. In this paper, we aim at summarizing a list of related studies that explore the explicit item relations to understand and illuminate the recommendation mechanism. Specifically, we review and organize the recent advances on recommendation area that involve explicit item relations from the following three categories, i.e., combination-effect relations, sequence-dependence relations, and external-constraint relations. Among these three categories, the combination-effect relation and the sequence-dependence relation based work models the intra-group intrinsic relations of items from the user demand perspective. For example, because of the user demand or item relations, one user may buy one item and then buy others, or buy several items together, or conversely, only select one from multiple competitive candidates. However, the external constraint relation emphasizes the external requirements of items. For example, limited by the budget, a user may only buy one or some of items. More particularly, we attempt to illustrate the combination-effect relations[9-10] from the broadly data-driven perspective in different domain-oriented services, which are similar to the complementary and substitutable products in economics. In this part, we group relevant studies from positive and negative directions of combination effects which are represented by the bundle recommendations[11-13] and comparison-choice ones[14-16] respectively. The sequence-dependence relations of items are often observed in various sequential application scenarios with temporal correlations, such as shopping[17], moving[18], singing songs[19], and learning courses[20]. In these scenarios, the prior items do have strong influences on users' decision-making on the following items. For this part, we organize these recent advances from the following representative areas, i.e., shopping trajectories[17,21], moving trajectories[18,22-23] and others. Especially, next-basket recommendations[17,24] and session-based recommendations[25-26] are two focuses in the shopping trajectories while the moving trajectories are widely seen and studied in some location-based services such as Point-of-Interest (POI) recommendations. Finally, we introduce the relevant studies on recommendation when items have the external requirements or constraints. These scenarios are often seen in some specific fields, such as finance and market[11,27-28], education and learning[29]. At the end of this paper, we also propose our opinions on the open issues along the line of understanding item relations and suggest some future research directions in recommendation area. Please note that extensive recommender system surveys appeared in the research community, such as general recommender algorithms[1], context-aware recommender systems[6,30], mobile recommender systems[7], and deep learning based recommendations[31]. To the best of our knowledge, this is one of the first few attempts that comprehensively review and summarize studies from the explicit item relation perspective to understand recommender systems.

The remainder of this paper is organized as follows. In Section 2, we first briefly introduce the backgrounds and preliminaries of recommendation, including the conventional commendation techniques which implicitly modeled item relations. In Section 3, we make our main efforts to review and organize the recent advances on recommendation interpretability from the perspective of understanding explicit item relations. Finally, in Section 4, we conclude this paper and propose some opinions on the open issues in this area.

2 Backgrounds and Preliminaries

In this section, we briefly introduce the backgrounds of recommender systems and give some preliminaries. More details of recommender systems can be found in related surveys[1,7].

The primary purpose of recommender systems is to provide the items which are most likely to suit each user's personalized needs, thus leading to better user loyalty and future item consumptions[1]. Along this line, recommender systems benefit a broad range of application areas, such as e-commerce[13,21], media portals[2,19], and some other service providers[11,20,29]. In the meantime, with the rapid development of the related application scenarios, models and evaluations in different fields have been more and more diversified.

In this section, we first give some preliminaries about the conventional recommendations and then organize them from various aspects, i.e., model view (the recommendation models) and evaluation view (the evaluation metrics for recommender systems). Indeed, the conventional studies are used to be formalized as predictive or learning tasks in different forms while less efforts among them have been devoted to explicitly exploring and understanding the item relations.

2.1 Model View

From the model view, the typical approaches of recommendation algorithms are given in [1, 32]. Following them, recommender systems can be grouped into three categories, i.e., content-based recommendations[1,3], collaborative-filtering recommendations[1,4], and hybrid approaches[1,5].

Specifically, content-based models usually recommend items that are similar to those items that a user preferred or accepted in the past[3,33]. Thus, in content-based systems, the item similarity between items v_i and v_j is usually defined by a utility function $c(.)$,

$$S_{i,j} = c(v_i, v_j).$$

Indeed, the inputs of items are usually extracted from the item contents. In different scenarios, the utilized contents and functions are usually different. Many techniques from information retrieval, e.g., TF-IDF, item topics[34], and item content embeddings[35], are usually adopted in these studies. The content-based solutions work well with items which contain rich attributes and textual information, such as documents, news and websites. One strength of content-based recommender systems is the ability of dealing with cold-start problem[36] for new-coming items. With the employment of deep learning, the content-based methods can also easily handle the media items such as music and images[37]. Although conventional content-based methods compute the item similarities based on predefined features and metrics, they fail to understand the complex item relations but "seeming similarities". Thus, content-based methods often lead to the overspecialization problem[1], i.e., they fail to find novel items with only exploring the seeming contents or features of items.

In contrast, collaborative filtering methods will recommend items based on the user-item interaction records without any content information. The key idea of collaborative filtering is that users with similar consumption preferences in the past would possibly like similar items in the future. Thus, the estimated preference $C_{k,i}$ of user u_k to item v_i is usually computed as an aggregate of the preferences of some other similar users U_k for this item,

$$C_{k,i} = a(C_{k',i}), u_{k'} \in U_k,$$

where $a(.)$ is the aggregated function which may have different implements such as matrix factorization. From the above expression, we can see that collaborative filtering methods mainly exploit the implicit item relations, which means the intrinsic relations of items are not observable or understood yet. The item relations cannot be modeled directly but with the help of users indirectly. Indeed, since the emergence of collaborative filtering in the mid-1990s, researchers mainly focus on how to improve the personalized recommendations by mining the sparse interaction between users and items[38-39]. Collaborative filtering models, such as neighborhood-based[38], graph-based[40], and matrix factorization based models[41-43], have shown satisfactory performances with rather simple implementations in real scenarios. Among all collaborative-filtering methods, matrix factorization based models[41] are the most remarkable ones. However, as users typically showed limited actions compared with the huge items in the system, the user-item interaction matrix is very sparse. Thus, collaborative filtering methods often suffer from the cold-item, cold-user and data sparsity problems[41]. To avoid these limitations, hybrid approaches[5] appeared by combining the complementary advantages of content-based methods and collaborative filtering methods. For example, the performances of matrix factorization based collaborative filtering methods have been improved dramatically by incorporating additional content information of the users or items[42-43]. Based on different application scenarios, there are various types of hybrid strategies[5,44].

Besides the general classifications for recommender systems, there are also many relevant studies that are specific to the application scenarios. Indeed, with the prevalence of pervasive technologies and mobile services, it is much easier to collect various kinds of data. Thus, there are some emerging research topics, such as context-aware recommendations[6], mobile recommendations[7], and recommendation diversity[45-46]. These studies advance the general recommendation models by utilizing more kinds of data and applying them to specific scenarios.

2.2 Metric View

Besides the model view, recommender systems are evaluated by different metrics. In the most common formulation, the recommendation problem is reduced to the problem of rating prediction (e.g., a 1∼5 rating scale in Netflix and MovieLens, with larger values means higher preferences) for the items that have not been rated by an active user[1,41]. Usually, the goal of these recommendation algorithms is to estimate the missing entries of the user-item rating matrix. That is, for each user u and each item v, a system $\mathcal{R}$ is to estimate the rating $\widehat{r}_{u,v}$ from u to v,

$$\mathcal{R} : u \in U, v \in V \longrightarrow \widehat{r}_{u,v}.$$

Correspondingly, the effectiveness of the recommendation algorithms is evaluated by how close the predicted ratings ($\widehat{r}_{u,v}$) are to the real ratings of users ($r_{u,v}$). Thus, the mean absolute error (MAE) and the root mean squared error (RMSE) are the most widely-used metrics in these studies.

However, in some scenarios, instead of the explicit ratings, users often show some implicit feedbacks in the systems, e.g., clicking or visiting the recommended items. Thus, we could only observe the positive samples in the data. For these scenarios, researchers proposed a ranking based recommendation task[46]. These ranking-based methods aim to find out the items that each user would like to click, and then rank them properly to the given user. Indeed, in order to achieve better ranking-based performance, many studies focus on modifying rating-based methods to ranking-based optimization functions, such as the well known Bayesian Personalized Ranking (BPR) model[39]. This kind of recommender systems usually chooses ranking-based measures, such as precision, recall (or top-K, or hits), mean average precision (MAP). Clearly motivated, ranking-based methods $\mathcal{R}$ are often aimed at optimizing the recommending list $l(k)$ by minimizing its difference with the real list of interactive items $l(k')$,

$$\mathcal{R} : u \in U, v \in V \longrightarrow \min(l(k), l(k')).$$

In particular, MAP is one of the most commonly-used evaluation metrics in recommendation because of its sensitivity and reliability. More formally, if we are asked to recommend N items to users U, the number of relevant items in the full space is m, and then:

$$MAP@N = \frac{1}{|U|}\sum_{1}^{|U|}(\frac{1}{m}\sum_{k=1}^{N}p(k) \times rel(k)),$$

where $p(k)$ is the precision calculated by considering only the subset of recommendation from rank 1 through k and $rel(k)$ is an indicator that equals 1 if the k-th item is relevant and equals 0 otherwise.

Besides the above mentioned rating accuracy measures and the ranking accuracy measures, researchers also argued that the ultimate goal of recommender systems is to improve user satisfaction. Thus, some metrics, such as diversity and novelty[45-46], are also considered to evaluate recommendation performances.

However, for a quite long time, researchers have been pursuing recommendation performances on these specific metrics, especially accuracy, while ignoring the mission and essence of recommendation, i.e., understanding the user preferences and item relations. How to deep explore the explicit item relations to understand and illuminate the recommendation mechanism is still under-explored.

3 Explicit Relations of Items in Recommendation

In this section, we review and organize the recent advances that involve explicit modeling of item relations for recommendation. In some scenarios, items have the explicit relations[9,20] which are very important constraints or concerns when making recommendations. Then, according to different specific applications, we summarize the related work that explicitly models the item relations in recommender systems into three categories, i.e., the combination-effect relations[9,28], sequence-dependence relations[17,20], and external-constraint relations[11,27-28].

The first category of studies is based on the fact that some items have the combination-effects relations, which are usually defined as the "complementary products"[9,47], or "substitutable products"[9,47] in economics. The second category of studies is mainly focused on the sequential behaviors of users over items with significant partial ordering relations, for example, the travel spot recommendations[48-49], and song recommendations[19]. Different from the above two types of relations which are described from the intrinsic attributes of items themselves, the third category of item relations means that the items may have external constraints, e.g., budget[11], stock[27]. In the following, we will detail these studies in different categories respectively. Table 1 exhibits some representative researches in each category.

3.1 Combination-Effect Relations

The combination-effect relations of items are similar to the "complementary" or "substitutable" products in economics[47]. Specifically, "complementary" products are often the coefficient items in related categories and "substitutable" products are usually competitive items in the same category. In recommendation process, the combination-effect relations of items may be latent and very complex, which are often described from the data-driven perspectives (users' behaviors, e.g., buying together or comparing choices). Thus, the combination-effect relations in recommendation are much more broadly than the similar concepts in economics. For example, from the data-driven perspective, McAuley *et al.*[9] formulated the problem of inferring networks of substitutable and complementary products as a supervised link prediction task, where they learned the semantics of substitutes and complements from data associated with products. More specifically, based on the combination-effect relations of items, the relevant studies on recommendations around this issue can be mainly grouped into two subcategories, i.e., positive combination-effect relations (similar to "complementary"), and negative combination-effect relations

Table 1. Classification of Research with Explicit Relations of Items in Recommendation

Relation	Sub-Category	Representative Research	Technical View
Combination-effect	Positive combination-effect relations	Primary bundles[32,50], pre-generating bundles[51-52], dynamic bundles[11,13,28], post-generating bundles[53]	Factorization models[13,51-52], topic model[32,50], composite methods[11,28,53]
	Negative combination-effect relations	Comparison-choice-based recommendations[14-16]	Nantonac filtering[14], factorization models[16], composite methods[15]
Sequence-dependence	Shopping trajectories	Next-basket recommendations[17,24,54]	Markov chains[24], hierarchical representation model[17], RNN[54]
		Session-based recommendations[25-26,55]	RNN[25-26,55]
	Moving trajectories[56-62]	Moving trajectories[56-62]	Factorization models[58-61]
	Other-application trajectories	Music recommendations[19,63-64], course recommendations[20,65]	Graph methods[20], Markov models[19,63], composite methods[65], factorization models[64]
External-constraint	Budget-constraint recommendations[11,27-29]	Budget-constraint recommendations[11,27-29]	Factorization models[27,29], composite methods[11,28]

(similar to "substitutable"). Fig.1 shows a toy example of items' two types of combination-effect relations. We observe from Fig.1(a) that when a user is shopping for photographic equipments online, this user may buy multiple items as a camera package (these items that are bought together have positive relations). In the meantime, Fig.1(b) shows that he/she compared three similar SLR cameras before he/she finally bought one. The three similar SLR cameras comprise the negative relations in the recommendation process.

(a) (b)

Fig.1. Toy example of combination-effect relations of items. (a) Positive combinatorial relation of complementary items: buy together. (b) Negative combinatorial relation of substitutable items: compare and buy one.

Formally, we define the combination-effect relations from the view of probability, that is, we denote $e(v_i)$ as the event that item v_i is interacted in scenarios, such as selecting or clicking in shopping, visiting in tourism. Items with combination-effect relations mean,

$$P(e(v_i), e(v_j)) \geqslant P(e(v_i))P(e(v_j)) + \xi,$$

or,

$$P(e(v_i), e(v_j)) \geqslant P(e(v_i))P(e(v_j)) - \xi,$$

where $\xi \in (\delta, 1)$ is the parameter which reflects the degree of combination-effect relations and $P(e)$ devotes to the probability of event e occurring. In particular, the first case implies the positive combination-effect relations. In contrast, the second case implies the negative combination-effect relations. The larger $|\xi|$ is, the more significant the combination effect is.

3.1.1 *Positive Combination-Effect Relations*

In e-commerce, the items with positive combination-effect relations are those which are often bought together, i.e., item bundle or item set. Usually, they have positive correlation in consumption. In addition, these effects in recommendation are more general than those in economics. For example, diapers and beer may have positive combination-effect relations from the data-driven views[9]. Specifically, He *et al.*[10] studied the complicated and heterogeneous relationships between items in product recommendation settings. In the past, the positive combination-effect relations of items have been widely studied with the association rules[66]. In recent years, in order to more clearly exploit the positive combination-effect relations of items, researchers conducted extensive focused studies which were known as set or bundle recommendations[11,13].

Bundle (Also Known as Set, Package, etc.). Recommendation is a system that is capable of simultaneously recommending multiple items in the form of sets or packages to users[11]. In these scenarios, users are usually exposed to a set of items and may buy them together or more than one item in one order[13]. This mechanism considers the whole set of recommended items as a bundle by modeling the relevance of items. It is reported that 1/3 of orders from Walmart.com① contain at least two items[13]. In domain-specific areas, bundles have some different forms, such as travel packages in tourism[11-12,32], courses with prerequisites[67-68], baskets in retail[69-70] and massive shopping packages in e-commerce[13]. In fact, researches on bundle marketing (pricing, generating, etc.) have a long history in economics, which have mainly been discussed as a promotional strategy[71-72]. Bundle recommendations provide massive win-win values for both merchants (sales increase) and customers (cost saving)[13]. In recent years, bundle recommendations have been widely studied in academia.

Specifically, based on the types and stages of generating bundles when making recommendations, the studies of bundle recommendations can be classified into four categories, i.e., primary bundles, pre-generating bundles, dynamic bundles, and post-generating bundles based studies. Specifically, Fig.2 shows the flows of different types of bundles and their typical application cases in recommendation.

Primary Bundles. In this type of scenarios, bundles are the minimum consumption units instead of items, that is to say, bundles are primary or predefined for marketing and there is no need to consider the item relations within bundles when making recommendations. Under this scenario, the techniques of bundle recommendations are similar to the techniques aimed at individual items. In some specific applications, such as travel packages (spots, flights, hotels, etc.) in tourism or monthly plans (talk, messages, data, etc.) in mobile

①https://www.walmart.com, Dec. 2017.

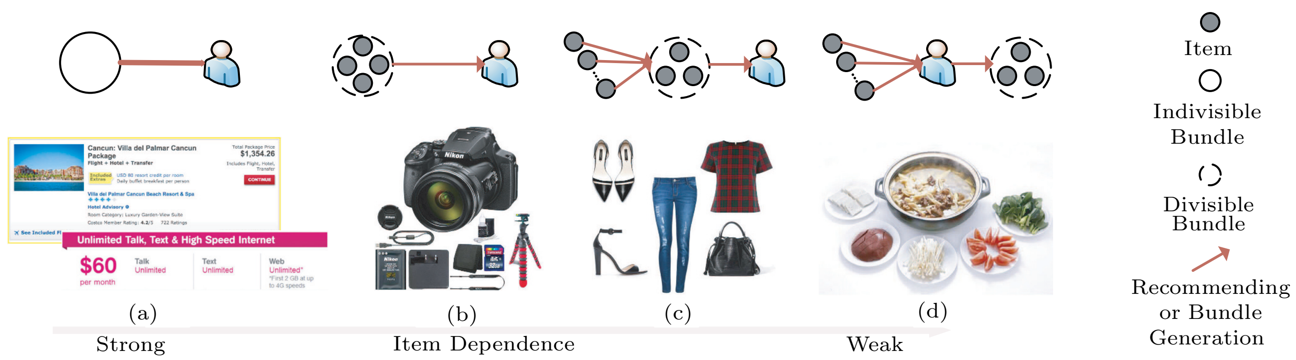


Fig. 2. Four types of bundles in recommendation. (a) Primary bundles. (b) Pre-generating bundles. (c) Dynamic bundles. (d) Post-generating bundles.

service, this type of bundles and relevant studies are widely researched[32,50].

Pre-Generating Bundle. In this type of scenarios, bundles are a pre-existing prior to making recommendations, such as bundles from history transactions or firstly generating bundles[52]. However, different from the primary bundles, items are interactive so that influences from both items and bundles should be considered when making recommendations in this scenario.

For example, Liu *et al.*[73] proposed a probabilistic model to capture the relationships of items in each bundle. Based on the preferences inferred from the model, an approach for recommending items to form product bundles is developed by estimating the probability that a consumer would buy an associative item together with the item already bought in the shopping cart. Liu *et al.*[74] studied properties of these user-generated item lists and proposed a Bayesian ranking model which considers users' previous interactions with both item lists and individual items. Cao *et al.*[51] devised embedding factorization models by incorporating item-item (item-item-list) co-occurrence with embedding-based algorithms. Specifically, in their study, a factorization model was used to capture users' preferences over items and item lists, and embedding-based models were utilized to discover the co-occurrence information among items and item lists. Besides, there are some other considerations when making bundle recommendations in this type of scenarios, such as personalized pricing[75].

Further, in these pre-generated bundle based scenarios, some researchers studied more complex problems, i.e., recommending bundles or packages to user groups. For example, Qi *et al.*[76-77] presented two probabilistic models for recommending packages to groups: one model could compute the probabilities that the user group likes individual items before deriving the probability that the group would select a package of items, and the other model formed item packages that were favored by the individual group members before identifying those that had a high likelihood to be selected by the group.

Dynamic Bundles. In this type of scenarios, bundles are dynamically generated along with the recommending process. Compared with the above mentioned two types of bundles, recommending dynamic bundles is more challenging. In these scenarios, the crucial step for bundle generation is often formalized as a dynamic programming problem, e.g., Knapsack problem. For example, Xie *et al.*[11] defined composite recommendation where each item had both a rating value and a cost associated with it, and the user specified a maximum total cost (budget) for any recommended set of items. They developed greedy algorithms to generate top-K recommendations to maximize the total value of items in the package for users. Zhu *et al.*[13] proposed a bundle recommendation problem which models cross-item dependencies with expected reward vector and cross-dependency matrix explicitly and rigorously. Further, the bundle recommendation problem was formalized as a Quadratic Knapsack problem. Serbos *et al.*[78] focused on an aspect of fairness of package-to-group recommendations. In their study, the problem of finding the most fair package is solved by introducing two definitions of fairness. In P2P lending market, Zhao *et al.*[28] presented a focused study on personalized portfolio (loan bundle) selection. They proposed a dynamic programming algorithm and an evolutionary algorithm respectively for solving the loan portfolio selection problem under different assumptions.

Post-Generating Bundles. Different from the above three types, in the type of post-generating-bundles scenarios, individual items are selected as candidates firstly and then generate the final bundles to target users with some constraints or objectives. For example, Zhao *et al.*[53] proposed to recommend loan item bundles with monetary amount on each loan in P2P lending market. In their study, they first recommended single loan items as candidates for lenders using collaborative filtering, and then optimized the final loan bundles based on the lenders' context, i.e., currently holding loan portfolios.

Besides, there are also some other relevant studies of recommendation that involve the positive combination-effect relations of items. For example, Le *et al.*[70] addressed the problem scenario where the user was currently holding a basket of items (bundle), and the studied task was to recommend an item to be added to the basket.

3.1.2 *Negative Combination-Effect Relations*

Involving the combination-effect relations of items, users may face another scenario when making decision or selection in shopping, that is, users may select one or some items from the accessing item set, i.e., these items are substitutable or competitive[9,47]. For example, Liu *et al.*[16] proposed that users suffered from the problem of indecisiveness when they chose among competing product options. Fig.3 shows a typical example of comparison-choice shopping from several competing cameras in Amazon②.

In fact, when shopping online, users' behaviors such as clicks or some other implicit feedbacks often reflect their comparison and decision process. In recent years, some studies have focused on explicitly understanding the users' preferences and decision processes from these behavior logs[14,79]. For example, many researchers exploit pairwise strategies to learn the users' preferences[80-81]. Indeed, previous research shows that users are more accurate by making relative indirect judgments using pairwise comparison than by directly ranking or rating items[15]. Thus, the pairwise strategies are suitable and also widely used for modeling the relative preferences on competing items[80-81]. In [79], an adaptive scheme in which users are explicitly asked for their relative preference between a pair of items was proposed. Thus, users give pairwise feedback to the underlying algorithm, which updates user parameters as it receives more responses. Furthermore, in [14], users were asked to order a set of items instead of pairs of items. On the other hand, aimed at the case of helping users make comparison and better decision, many shopbots (comparison shopping agents) are developed in e-commerce[82-83].

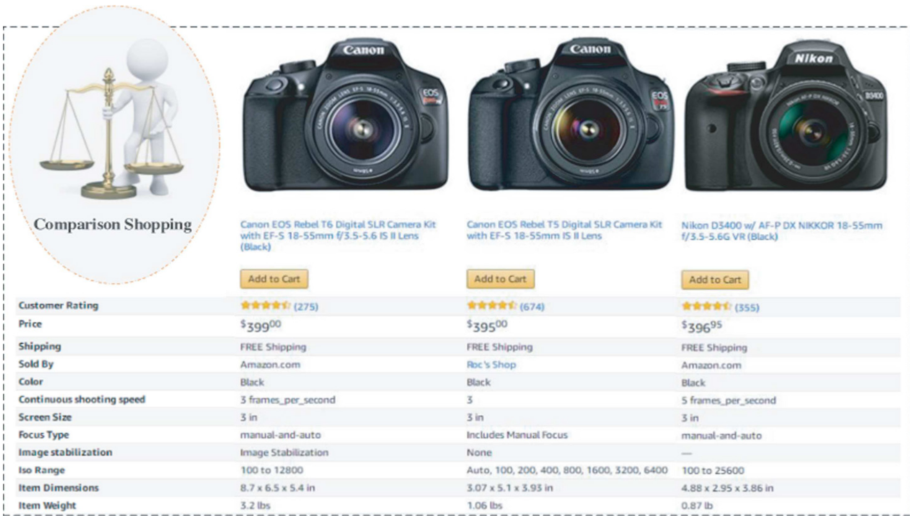


Fig.3. Comparison-choice shopping for items with negative combination-effect relations.

3.2 Sequence-Dependence Relations

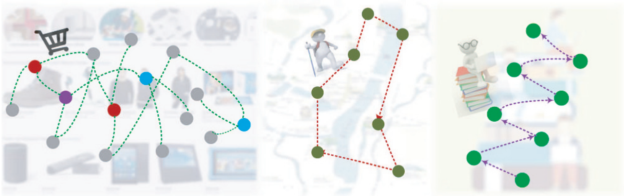
Another most widely available type of item relations in recommendation is the sequence-dependence relations. Involving this sequence-dependence relations, researchers also conduct massive relevant studies in various types of sequential scenarios, such as shopping[17], traveling[18], singing[19], and learning[20]. Actually, we can also define the sequence-dependence relations from the probabilistic view, that is, we denote $e(v_i)$ as the

②https://www.amazon.com/, Dec. 2017.

event that item v_i is interacted in scenarios, such as selecting or clicking in shopping, visiting in tourism. Items with sequence-dependence relations mean,

$$P(e(v_i)|e(v_j)) \geqslant P(e(v_i)) + \xi,$$

where $\xi \in (0,1)$ is the parameter which reflects the degree of sequence-dependence relations. The larger ξ is, the more significant the sequence-dependence effect is, i.e., if $P(e(v_i)|e(v_j)) > P(e(v_i)|e(v_k))$, v_i has stronger sequential relations dependent on v_j than v_k. In this part, we organize these recent advances from the following representative categories, i.e., shopping trajectories, moving trajectories and others. Fig.4 shows three representative application scenes that involve sequence-dependence relations of items, which are shopping, moving/traveling, and learning-skill trajectories respectively.



(a) (b) (c)

Fig.4. Cases of items with sequence-dependence relations. (a) Shopping trajectories. (b) Moving trajectories. (c) Studying trajectories.

3.2.1 Shopping Trajectories

From some point of view, items with sequence-dependence relations also imply their combination-effect relations, i.e., positive or negative. For example, users may also buy multiple items with positive combination-effect relations, or select and compare items with negative combination-effect relations one by one, which are the most common sequence scenes in commerce. However, differently, these studies put more emphasis on the sequential dependencies of items.

In commerce, especially the online shopping, recent years have witnessed the researches on recommendation of exploring not only the user preferences but also the sequential behaviors to highlight relevant next-items, i.e., the next few items that the users probably would like. For utilizing the sequential behaviors of users to predict the users' next action based on the last actions, many sequential approaches were explored, for example, the Markov chain model[24], and also some other sequential pattern mining techniques[84]. Along this line, one step further, Chen *et al.*[21] focused on making recommendations in right orders, i.e., items should be better recommended in consideration of the relations of each other, by a graph that combines both relevance and order effects in recommendation. In the past few years, thanks to the tremendous success of deep neural networks, approaches to sequence data modeling have made significant strides and Recurrent Neural Network (RNN) models have been applied to modeling the sequential item relations in recommendation[25-26,54-55].

Besides the scenarios involve common sequence-dependence relations of items in shopping, there are two special cases, i.e., next-basket recommendations, and session-based recommendations.

Next-Basket Recommendations. Specifically, next-basket recommendations attempt to predict the next few items that the user most probably would like[17]. Although next-basket recommendations are also to recommend multiple items as a basket, they are quite different with the bundle recommendations. Next-basket recommendation focuses on modeling the recent behaviors of users or learning the sequence-dependence relations between prior items and those in the recommended baskets (i.e., buying one item leads to buying another next)[17] while ignoring the relations of items within the recommended baskets (which is just the concern of bundle recommendations). In recent years, researchers have made many efforts towards this task.

For example, Rendle *et al.*[24] presented a method bringing both matrix factorization and Markov chains for next basket recommendations. In their study, matrix factorization is used to learn the general taste of a user while Markov chain is used to predict the next action based on the recent actions of a user by learning a transition graph over items. Besides, Wang *et al.*[17] introduced a novel hierarchical representation model for next basket recommendation. The proposed model can well capture both sequential behavior and users' general taste by involving transaction and user representations in prediction. In recent years, with the prevalence of deep learning models, many scholars have proposed to exploit RNN to the next-basket recommendation task. For example, a dynamic recurrent basket model is proposed in [54] to learn a dynamic representation of a user. The dynamic representation of a specific user can reveal users' dynamic interests at different time, and the global sequential features reflect interactions of all baskets of the user over time.

Session-Based Recommendations. As another special case in this category, session-based recommendation also received massive attention in academia[25-26].

For many recommending scenarios (particularly news portals and small retails), we do not have the identity of user-ids and personal visit-logs over a long period of time. Therefore, such recommender systems are based on short session data without the user profile. However, the traditional matrix factorization methods rely on decomposing the user-item interactions matrix for each item and user. That makes matrix factorization methods hardly perform well. In this situation, most session-based recommender systems are simply deployed by item-to-item similarity, such as neighborhood methods[38,85]. However, these item-to-item methods just take the items similarity into consideration and ignore the sequential ordering within the session.

In fact, considering the scenario and the collected data in sessions, it is very suitable to use the sequential methods to discover the relations of items and improve the recommendation performance. Thanks to the tremendous success of deep neural networks in the past few years, approaches to sequence data modeling have made significant progress. For example, Hidasi *et al.*[55] firstly applied recurrent neural network by modeling the whole session for the session-based recommendations, which outperformed item-based methods significantly. Tan *et al.*[86] took one step further to improve the RNN-based model performance by incorporating data augmentation and a new method for shifts in the input data distribution. Since above RNN-based models show promising improvements over traditional methods[25], RNN in session-based recommendations has attracted great attention and also leads to many novel models to improve the performance on this task.

On the one hand, some session-based recommender systems combine the sequence data with the item features[55,87-88]. For example, Hidasi *et al.*[55] introduced a parallel RNN (p-RNN) architecture to model sessions based on the clicks and the features (images and texts) of the clicked items, which has significant performance improvements over feature-less session models. Besides, some studies aim to understand the session context and user purpose[26]. For example, Li *et al.*[89] proposed a neural framework to model the user's sequential behavior and capture the user's main purpose in the current session.

3.2.2 Moving Trajectories

Another typical scenario involving sequence-dependence relations of items in recommendation is the moving-trajectory-based applications[18]. In these applications, items (e.g., locations) have strong dependencies or constraints (e.g., spatial distance). In this specific area, there are massive studies along with rich research topic, for example, point of interest (POI) recommendations[56,90-92], location or trajectory prediction and recommendations[22-23], travel planing[49,93]. It is worth noting that in the location-based services, like POI recommendations, the sparsity (users may visit only a limited number of spatial items/places)[94-96] and cold-start (users travel out of town or to unfamiliar regions)[95-97] problems are more serious than those in other common scenarios. Also, in these applications, when making sequential recommendations, items may be limited by spatial constraints (e.g., distances between neighbor items should not be very large, geographical correlation, interest drift)[98-99]. Since there have been already massive relevant studies and surveys efforts around this topic, we do not detail them in this part. Readers could refer to some references for more details in this area, for example, [56-57,62].

3.2.3 Other-Application Trajectories

Besides the location-based services, users' preferences in some other scenarios may also imply various latent trajectories[18]. For example, users listen to music or practice singing from basics to proficiency[63], and students learn courses from preliminaries to advanced ones[20]. Indeed, these trajectories from the sequentially-ordered user-item interaction logs can be leveraged in the recommendations. At the end of this subsection, we mainly focus on two other common scenarios, i.e., music recommendations, and course recommendations.

Music Recommendations. Firstly, taking music-oriented applications as examples, compared with general behaviors, user listening-music or practicing-singing behaviors have a very strong sequential dependence. For exploring the preferences of users over sequences, researchers conduct many sequential modeling for music recommendations, such as Markov models. For example, Wu *et al.*[19] proposed personalized Markov embedding, a next-song recommendation strategy for online karaoke users by modeling the sequential singing behavior. Chen *et al.*[100] presented latent Markov embedding, a machine learning algorithm for generating music playlists. Ji *et al.*[101] proposed time-based Markov embedding model which boosts the recommendation performance by leveraging temporal information. Cheng *et al.*[102] proposed to use word embedding techniques in music play sequences to estimate the similarity between songs. In their study, the learned similarity is embedded into matrix factorization to make recommendations.

Besides, some researchers utilize context-ware recommendations for the next-song prediction task. Specifically, in music-oriented applications, context is usually reflected in the sequence of songs liked or played by the user in his/her current interaction with the system, such as a playlist. For example, Hariri *et al.*[64] presented a context-aware music recommender system. Wang *et al.*[103] inferred music pieces' latent low dimensional representations (embeddings) from users' music listening sequences using neural network models. In their study, users' general preferences are inferred from their complete listening records and context is inferred from her/his current interaction session (music pieces recently listened by a user) using the learned embeddings.

Course Recommendations. In education scenarios, students' knowledge is changing over learning courses. With the development of online education and big data collection in this area, some researchers proposed to model the evolution of student knowledge over time[104]. Also, for providing better services to students, researchers study the course effects on student knowledge trace for sequential course recommendations. For example, Lan *et al.*[104] proposed to model time-varying student knowledge and the effects of lesson modules. Chen *et al.*[105] devised an explanatory probabilistic approach to track the knowledge proficiency of students over time by leveraging educational priors.

When making course recommendations, sequence dependence of items (e.g., courses) is one important consideration. For example, Parameswaran *et al.*[20] argued that the recommended courses must satisfy sequential relationships (e.g., constraints or requirements), i.e., the courses taken by a student must satisfy requirements (e.g., taking two out of a set of five math courses) in order for the student to graduate. Along this line, they proposed to recommend to these students courses that not only are desirable (e.g., popular or taken by similar students), but also help satisfy constraints. Chen *et al.*[106] proposed a hybrid recommender system which can discover content-related item sets using item-based collaborative filtering and then apply the item sets to sequential pattern mining algorithm to filter items according to common learning sequences. Besides, Reddy *et al.*[107] presented a probabilistic model of students and educational content that can be used to recommend personalized sequences of lessons with the goal of helping students prepare for specific assessments. Xu *et al.*[65] presented a systematic methodology for offering personalized course sequence recommendations to students. Specifically, a forward-search backward-induction algorithm was developed and could optimally select course sequences to decrease the time required for a student to graduate.

3.3 External-Constraint Relations

In this subsection, we introduce the research on recommendation which involves the external-constraint relations of items. In [20], the authors organized some relevant studies on recommendation with requirements or constraints. Different from that, we focus on the external constraints rather than the internal relations of items (such as the referred combination-effect, sequence-dependence relations) when items are put together.

Specially, the external-constraint item relations are enforced rules and recommender systems must take these mandatory constraints into account, which makes the recommended items satisfy the external-constraint relations. Formally, this type of relations can be stated as follows. Suppose $e(v_i)$ is the event that item v_i is interacted in scenarios, such as selecting or clicking in shopping, visiting in tourism; $b(e_i)$ is the external utility accompanying the event, such as the expense in shopping. The external-constraint recations between two items can be defined as follows,

$$\begin{aligned}&P(e(v_i), e(v_j)) \leqslant P(e(v_i))P(e(v_j)) - \xi,\\ &\text{if} \qquad I(b(e_i) + b(e_j)) = 0,\end{aligned}$$

where $I(x)$ is an index function that equals 1 if x satisfies the external constraint and equals 0 otherwise, and $\xi \in (0, 1)$ is the parameter which reflects the influence of violating the external constraint. Obviously, the larger ξ is, the more significant the influence is.

Indeed, the external-constraint recommendation scenarios are often seen in some specific areas, such as finance, market, and education. Fig.5 shows two cases in e-commerce that items have external-constraint relations (i.e., budget or capacity constraint, and stock constraint).

Budget-Constraint Recommendations. In some consumption activities, such as shopping[11], traveling[108], budget is a very important constraint when making decisions. Budget constraint is often involved in recommending scenarios when several items are recommended synchronously, e.g., package or bundle recommendations. Especially for the recommendations with dynamic bundles, budget is one of the most important constraints[11,28]. For example, in [11], the total cost of the recommended item set is within a pre-defined budget. In [109], by using a context-aware approach, the authors proposed a recommendation model which could identify users' state of mind and budget based on click stream data. In [110], authors studied the personalized recommendation as a budget-constrained user selection problem. Specific to tourism, Benouaret and Lenne[108] proposed to recommend the most interesting travel packages for the user, where each package satisfies the budget constraint. In their study, they formally presented a novel composite recommender system for budget-constraint travel package recommendation.

(a) (b)

Fig.5. Exampled cases of items with external-constraint relations. (a) Budget constraint. (b) Stock constraint.

Other External-Constraint Recommendations. Besides the budget constraint in consumption activities, researchers exploited some other types of external constraints in recommendation, such as profit constraint[28] in investment, stock constraint[27] in retail, and load constraint[29] in education. Specifically, for example, Zhao *et al.*[28] proposed to recommend loan item set to customers which satisfies the profit constraint, i.e., the profit return (average lend rate) must be greater than the lenders' expectation. Zhong *et al.*[27] studied the stock constrained recommendation in e-commerce that jointly optimized the recommended items for all users based on both user preference and inventory sizes of different items. For some popular products in the market, such as the newly released iPhone in Fig.5, considering stock constraints when making recommendation is necessary. In education, Yang *et al.*[29] formulated a novel constrained question recommendation problem (recommending questions to students for solving) with load balance constraints. In their study, they considered the following constraints when making recommendation: students should not be over-burdened with too many requests, and students should not be requested to address problems beyond their capabilities.

4 Conclusions and Open Issues

In this paper, we attempted to illuminate the recommendation by reviewing and organizing the recent advances from the perspective of understanding the explicit relations of items. Specifically, we summarized three types of item relations in recommendation, i.e., combination-effect relations, sequence-dependence relations, and external-constraint relations. Specific to each type of relations, there are many specific hot research topics which are also involved. Relevant studies of combination-effect relations of items contain the bundle recommendations, and comparison-choice-based recommendations. Those of sequence-dependence relations contain next-basket recommendations, session-based recommendations, and some other sequence-dependence studies. In the studies with external-constraint relations of items, budget constraint is widely studied in the market or commerce. In the meantime, with the summary of these relevant studies, many representative applications are also involved in each category.

In this paper, we tried to provide an explainable view for recommendation. However, around this line, there is still a long way and our review is just an exploratory attempt. By attracting more researchers' attention around this topic, more and more studies on understanding and explaining recommendation will be done. Here, we provide our opinions on several open issues for future research.

Collecting Large-Scale Data with Explicit Item Relations. Data collection is the fundamental work for conducting data-driven studies. Indeed, the large-scale data can open an opportunity not only to mine user preferences but to understand explicit item relations. However, the available data which is feasible for studying explicit item relations in academia is limited. Also, collecting large-scale data with explicit user feedbacks or labels is much more difficult. Thus, in this area, collecting large-scale available data is one of the most crucial tasks in the future. In recent years, many researchers have paid more attention to this crucial task, and more and more large-scale datasets have been collected and disclosed[111-112].

Exploring Rich Metadata for Understanding Explicit Item Relations. As described in this paper, massive studies have illustrated the power of rich metadata (e.g., images, videos) on recommendation performances, especially with the prevalence of deep learning and reinforcement learning. For example, many studies used deep learning methods to extract features from images[113] or texts[114]. These studies incorporated the metadata to obtain features of single item, but how to exploit item relations from the metadata is still largely unexplored. Thus, how to explore rich metadata for better understanding the explicit item relations is another promising research direction in the future.

Understanding the Complex Item Relations. In this paper, we organized the recent advances on recommendations with explicit item relations and also the prior studies mainly involved one type of item relations. However, in many real-world applications, item relations are complex and multi-fold. For example, when making tourism recommendations, systems may consider both the internal combination effects and the external budget constraints of items at the some time, such as load recommendations in P2P lending[28], stock recommendations[27]. Many tasks in these scenarios must consider cross-domain relations of items, which make it more difficult to model item relations. Thus, how to comprehensively understand the complex item relations is another challenge and open issue in this area.

Improving the Interpretability of Recommendation. The interpretability is important for recommender systems to improve the user experience and satisfaction. Also, some existing studies made efforts on several areas to improve the interpretability of recommendation, such as context-aware[7], sequence-aware[111], and psychology[16]. And also, exploring the explicit item relations for recommendation interpretability is a possible direction.

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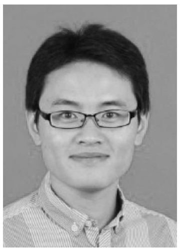
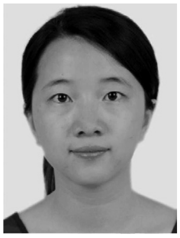
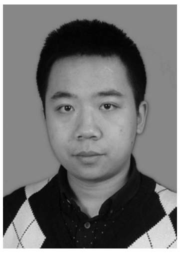


Qi Liu received his Ph.D. degree in computer science from University of Science and Technology of China (USTC), Hefei, in 2013. He is an associate professor with USTC, Hefei. His general area of research is data mining and knowledge discovery. He has published prolifically in refereed journals and conference proceedings, e.g., IEEE Transactions on Knowledge and Data Engineering, ACM Transactions on Information Systems, ACM Transactions on Knowledge Discovery from Data, ACM Transactions on Intelligent Systems and Technology, ACM SIGKDD, IJCAI, AAAI, IEEE ICDM, SDM, and ACM CIKM. He has served regularly on the program committees of a number of conferences, and is a reviewer for the leading academic journals in his fields. He received the ICDM 2011 Best Research Paper Award. He is a member of ACM and IEEE.

Hong-Ke Zhao received his B.E. degree in software engineering from South China University of Technology, Guangzhou, in 2013. He is currently pursuing his Ph.D. degree with the School of Computer Science and Technology, University of Science and Technology of China, Hefei. His current research interest includes data mining, deep learning and Fintech, such as Crowdfunding and P2P lending. He has published papers in refereed journals and conference proceedings, such as ACM Transactions on Intelligent Systems and Technology, IEEE Transactions on Systems, Man, and Cybernetics: Systems, Information Sciences, ACM SIGKDD, IEEE ICDM, AAAI, and IJCAI.



Le Wu received her Ph.D. degree in computer science from University of Science and Technology of China, Hefei, in 2015. She is currently a faculty member with Hefei University of Technology, Hefei. Her general areas of research are data mining, recommender system, and social network analysis. She has published several papers in referred conference proceedings and journals, such as AAAI, SIAM SDM, IEEE ICDM, ACM CIKM, and ACM Transactions on Intelligent Systems and Technology.



Zhi Li received his B.E. degree in software engineering from Xi'an Jiaotong University, Xi'an, in 2015. He is currently pursuing his M.E. degree with the School of Software Engineering, University of Science and Technology of China, Hefei. His current research interest includes data mining, sequential learning and recommender systems. He has published papers in Journal of Shandong University (Engineering Science) and ACM SIGKDD.

En-Hong Chen received his Ph.D. degree from University of Science and Technology of China (USTC), Hefei, in 1996. He is a professor and the vice dean of the School of Computer Science and Technology, USTC, Hefei. His general areas of research include data mining and machine learning, social network analysis, and recommender systems. He has published more than 100 papers in refereed conferences and journals, including IEEE Transactions on Knowledge and Data Engineering, IEEE Transactions on Mobile Computing, IEEE Transactions on Industrial Electronics, ACM Transactions on Knowledge Discovery from Data, ACM SIGKDD, IEEE ICDM, and NIPS. He was on program committees of numerous conferences including SIGKDD, ICDM, and SDM. He received the Best Application Paper Award on KDD-2008, and the Best Research Paper Award on ICDM 2011. His research is supported by the National Science Fund for Distinguished Young Scholars of the National Natural Science Foundation of China. He is a fellow of CCF, and a senior member of IEEE.