International Journal of Modern Physics B, Vol. 16, Nos. 20, 21 & 22 (2002) 3176–3179


# MAGNETIC FIELD EFFECTS ON $T_C$ AND THE PSEUDOGAP ONSET TEMPERATURE IN CUPRATE SUPERCONDUCTORS

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We study the sensitivity of $T_c$ and the pseudogap onset temperature, $T^*$, to low fields, $H$, for cuprate superconductors, using a BCS-based approach extended to arbitrary coupling. We find that $T^*$ and $T_c$, which are of the same superconducting origin, have very different $H$ dependences. The small coherence length makes $T^*$ rather insensitive to the field. However, the presence of the pseudogap at $T_c$ makes $T_c$ more sensitive to $H$. Our results for the coherence length $\xi$ fit well with existing experiments. We predict that very near the insulator $\xi$ will rapidly increase.

The pseudogap phenomena have been a great challenge to condensed matter physicists since last century. Yet there has been no consensus on the origin of the pseudogap and its relation to the superconducting order parameter. Theories about the pseudogap physics fall into two categories: (1) precursor versus (2) non-precursor superconductivity. For the former, pseudogap forms as a consequence of precursor superconducting pairing, and therefore, shares the same origin as the order parameter. In contrast, for the latter category, pseudogap has a different origin, e.g., a hidden DDW (d-density wave) order.[1]

On the other hand, experiment has revealed different behaviors of $T_c$ and the pseudogap onset temperature $T^*$ in magnetic fields.[2] However, there is still no proper theoretical explanation. This difference has been used as evidence against precursor superconductivity scenarios. Here we show that it can be well explained within the present precursor superconductivity theory.[3,4,5]

Our calculation is based on an extension of BCS theory which incorporates incoherent pair excitations. These pair excitations become increasingly important at large coupling $g$, and lead to a pseudogap in the single-particle excitation spectrum, as seen in the cuprates. As the temperature increases from $T = 0$, these pairs can survive a higher temperature ($> T_c$) than the condensate, until they are completely destroyed by the thermal effect at $T^*$. In agreement with experiment, we find that $T^*$ and $T_c$ have very different field ($H$) dependences. The small coherence length ($\xi$) makes $T^*$ rather insensitive to the field. However, the presence of the pseudogap at $T_c$ (at optimal and under- doping) makes $T_c$ relatively more sensitive to $H$. Our results for the coherence length $\xi$ fit well with existing experiments. Furthermore, we predict that very near the insulator $\xi$ will rapidly increase.

We first consider the zero magnetic field case. We include, in addition to time-reversal state pairing, finite center-of-mass momentum pair excitations in the problem and then treat the interrelated single- and two-particle propagators self-consistently. We truncate the infinite series of equation of motion at the three-particle level, and then factorize the three-particle Green's function $G_3$ into single-particle ($G$) and two-particle ($G_2$) Green's functions.[6]

Here we consider an electron system near half filling on a quasi-two dimensional (2D) square lattice, with tight-binding dispersion $\epsilon_{\mathbf{k}}$. The electrons interact via a separable potential $V_{\mathbf{k},\mathbf{k}'} = g\varphi_{\mathbf{k}}\varphi_{\mathbf{k}'}$, where $\varphi_{\mathbf{k}} = \cos k_x - \cos k_y$ (for $d$-wave). We use a T-matrix approximation for the self-energy, and have

$$\Sigma(K) = G_0^{-1}(K) - G^{-1}(K) = \sum_Q t(Q)\, G_0(Q-K)\, \varphi^2_{\mathbf{k}-\mathbf{q}/2}\,, \tag{1}$$

$$t(Q) = -\frac{|\Delta_{sc}|^2}{T}\delta(Q)\theta(T_c - T) + \frac{g}{1+g\chi(Q)}\,, \tag{2}$$

where $\Delta_{sc}$ is the order parameter, and $\chi(Q) = \sum_K G(K)G_0(Q-K)\,\varphi^2_{\mathbf{k}-\mathbf{q}/2}$. For small $Q \neq 0$, $t(Q)$ can be written in a standard propagator form. The pseudogap is given by $\Delta^2_{pg} \equiv -\sum_Q t(Q)$, the total gap[4] by $\Delta = \sqrt{\Delta^2_{sc} + \Delta^2_{pg}}$, and the quasiparticle dispersion by $E_{\mathbf{k}} = \sqrt{\epsilon^2_{\mathbf{k}} + \Delta^2\varphi^2_{\mathbf{k}}}$.

$T_c$ is determined by the superconducting instability condition $1 + g\chi(0) = 0$, in conjunction with the number constraint $n = 2\sum_K G(K)$. We obtain a set of three equations.[4] Taking into account that the cuprates is close to the Mott insulator and thus in-plane hopping $t_{\parallel}(x) = t_0(1-n) = t_0 x$, where $x$ is the doping concentration, we solve for $T_c$, $\Delta$, chemical potential $\mu$, and $\Delta_{pg}$. The results for $T_c$, $\Delta_{pg}(T_c)$, and $\Delta(T=0)$ as a function of $x$ are summarized in Fig. 1(a). Our predictions fit experiment well with $-g/4t_0 = 0.045$ and $t_0 \approx 0.6$ eV. For more details, see Refs. 3–5.

In a finite field, the Ginzburg-Landau free energy functional near $T_c$ can be expanded to quadratic order in the order parameter $\Delta_{sc}$:

$$F \sim \left(\tau_0(T) + \eta^2\left(\frac{\nabla}{i} - \frac{2e\mathbf{A}}{c}\right)^2\right)|\Delta_{sc}|^2\,, \tag{3}$$

where $\tau_0(T) = \bar{\tau}_0\left(1 - \frac{T}{T_c}\right)$, and $-\frac{1}{T_c}\frac{dT_c}{dH}\Big|_{H=0} = \frac{2\pi}{\Phi_0}\xi^2 = \frac{2\pi}{\Phi_0}\frac{\eta^2}{\bar{\tau}_0}$. As an estimate, one has $H_{c2}(0) \approx \Phi_0/(2\pi\xi^2)$. ($\Phi_0 = hc/2e$ is the flux quantum).

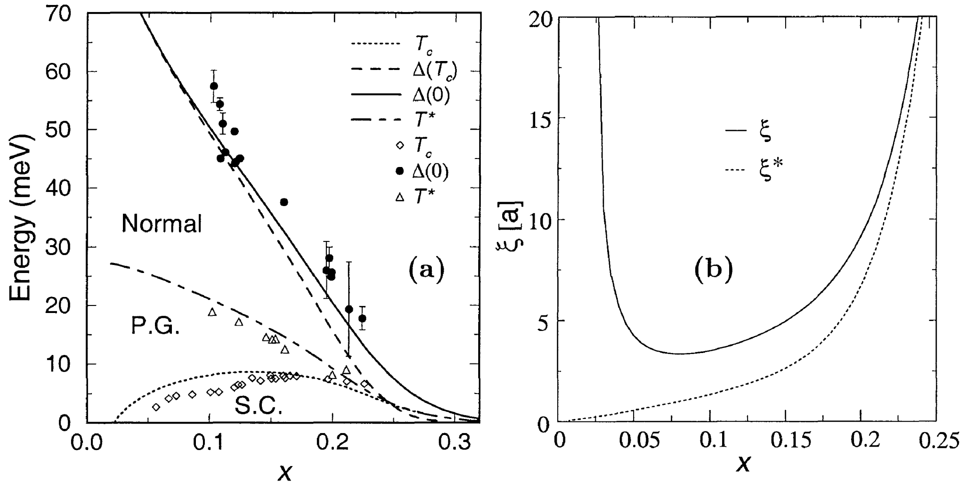


Figure 1. (a) Calculated cuprate phase diagram. $T^*$ was estimated using the BCS mean-field solution. Experimental data are taken from: (•) Ref. 7; (◇) Ref. 8; (△) Ref. 9. For more details see Ref. 4. Note here $\Delta(0)$ has been multiplied by 2 to compare with experiment, since $\varphi_{\mathbf{k}} = 2$ at $\mathbf{k} = (\pi, 0)$. (b) Magnetic length scales associated with $T_c$ and $T^*$ as a function of doping concentration in the cuprates.

For 3D weak coupling (BCS), $\bar{\tau}_0 = N(0)$ and the phase stiffness $\eta^2 = N(0)7\zeta(3)/48\pi^2(v_F/T_c)^2$. Therefore $N(0)$ is canceled in $\xi_{BCS}^2 = 7\zeta(3)/48\pi^2(v_F/T_c)^2$. In general, $\tau_0$ and $\eta^2$ can be determined from the expansion of $t^{-1}(Q)$:

$$\tau_0 = \frac{1}{g} + \chi(\mathbf{0}, 0), \qquad \eta^2 = \frac{1}{2}\sqrt{\det\left[\partial_{q_i}\partial_{q_j}\chi(Q)\right]}\Bigg|_{Q=0}. \tag{4}$$

In weak field, $T \gg eH/mc$, we use semiclassical phase approximation to treat the single-particle and pair propagators. Both the single-particle and pair momenta can be modified by the interaction with the field. But formally, the Dyson's equation remains the same, and the superconducting transition is still determined by the pairing instability (Thouless) condition: $g^{-1} + \hat{\chi}(0) = 0 \approx \tau_0 + \eta^2 \cdot \left(\frac{2e}{c}H\right)$, where $\hat{\chi}(Q)$ is the pair susceptibility in the field. To linear order in $H$, we can evaluate $\tau_0$ and $\eta^2$ at $H = 0$.

At $T^*$, the pseudogap is zero, only the bare Green's function is involved, $\chi_0(Q) = \sum_K G_0(K)G_0(Q-K)\,\varphi^2_{\mathbf{k}-\mathbf{q}/2}$. We have

$$-\frac{1}{T^*}\frac{dT^*}{dH}\bigg|_{H=0} = \frac{\eta^{*2}}{\bar{\tau}_0^*}\frac{2\pi}{\Phi_0}, \qquad \xi^{*2} = \frac{\eta^{*2}}{\bar{\tau}_0^*}, \tag{5}$$

where $\bar{\tau}_0^* = \sum_{\mathbf{k}} \varphi_{\mathbf{k}}^2 \left[ -f'(\epsilon_{\mathbf{k}}) + \frac{d\mu}{dT}\frac{T}{\epsilon_{\mathbf{k}}}\left(\frac{1-2f(\epsilon_{\mathbf{k}})}{2\epsilon_{\mathbf{k}}} + f'(\epsilon_{\mathbf{k}})\right)\right]_{T=T^*}$

At weak coupling (for $s$-wave $\varphi_{k_F} = 1$), we recover the BCS limit: $\mu \approx E_F$, $\bar{\tau}_0^* = -\sum_{\mathbf{k}} f'(\epsilon_{\mathbf{k}})\varphi_{\mathbf{k}}^2 \approx N(0)\varphi_{k_F}^2 \approx N(0)$. And $\eta^{*2}$ is determined by expanding $\chi_0(\mathbf{q},0) = \sum_{\mathbf{k}} \frac{1-f(\epsilon_{\mathbf{k}})-f(\epsilon_{\mathbf{k-q}})}{\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{k-q}}}\varphi_{\mathbf{k-q}/2}^2$ to the $q^2$ order.

At large $g$ (for the underdoped cuprates), $\Delta_{pg}(T_c)$ is large. Noticing that $T^*$ is very weakly $H$ dependent in the strong pseudogap regime [see Fig. 1(b)], and that $T^* \propto \Delta_{pg}$ in zero field, we assume $\Delta_{pg}$ is only weakly $H$ dependent. In other words, only the superconducting order parameter is strongly coupled to the field. Then we obtain $\bar{\tau}_0 \approx -\sum_{\mathbf{k}} \varphi_{\mathbf{k}}^2 f'(E_{\mathbf{k}})$, which decreases rapidly as $\Delta_{pg}$ increases. $\eta^2$ is obtained by expanding to the $q^2$ order

$$\chi(\mathbf{q},0) = \sum_{\mathbf{k}} \left[\frac{1-f(E_{\mathbf{k}})-f(\epsilon_{\mathbf{k-q}})}{E_{\mathbf{k}}+\epsilon_{\mathbf{k-q}}}u_{\mathbf{k}}^2 - \frac{f(E_{\mathbf{k}})-f(\epsilon_{\mathbf{k-q}})}{E_{\mathbf{k}}-\epsilon_{\mathbf{k-q}}}v_{\mathbf{k}}^2\right]\varphi_{\mathbf{k-q}/2}^2 . \quad (6)$$

At weak coupling, $\xi$ and $\xi_{BCS}$ coincide. But they split apart as $g$ increases and the pseudogap opens (see Ref. 10 for details). In Fig. 1(b), we plot the doping dependence of the calculated $\xi$ and $\xi^*$. At large $x$ (overdoping, weak coupling), the two are nearly equal. But for underdoping, while $\xi^*$ continues to decrease with decreasing $x$, $\xi$ remains nearly flat for a broad range of $x$ until its final rapid increase toward the insulator limit. Since $dT/dH \propto \xi^2$, $T^*$ is rather insensitive and $T_c$ is relatively more sensitive to $H$. These results are in agreement with experimental observations.[2,11]

This work was supported by NSF-MRSEC, grant No. DMR-9808595, and by the State of Florida (Q.C.).

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