PHYSICAL REVIEW A **73**, 041601(R) (2006)

Understanding the superfluid phase diagram in trapped Fermi gases

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(Received 22 December 2005; published 5 April 2006)

Trapped ultracold Fermi gases provide a system that can be tuned between the BCS and Bose-Einstein condensation regimes by means of a magnetic-field Feshbach resonance. Condensation of fermionic atom pairs in a ^{40}K gas was demonstrated experimentally by a sweep technique that pairwise projects fermionic atoms onto molecules. In this paper, we examine previous data obtained with this technique that probed the phase boundary in the temperature-magnetic field plane. Comparison of the ^{40}K data to a theoretically computed phase diagram demonstrates good agreement between the two.

DOI: 10.1103/PhysRevA.73.041601 PACS number(s): 03.75.Hh, 03.75.Ss, 74.20.-z

The field of ultracold Fermi gases has seen enormous progress in the years since the first observation of Fermi degeneracy [1]. Upon varying a magnetic field B, Feshbach resonances provide a means of controlling the strength of interactions between fermionic atoms, which is characterized by the s-wave scattering length a. The nature of the resultant superfluidity is expected to vary continuously [2] from a Bose-Einstein condensation (BEC) $(a>0)$ to that of BCS theory $(a<0)$ with magnetic field. In the BEC regime, evidence for condensation was obtained [3–5] by observing a bimodal distribution of the momentum profile, a standard technique originally developed for bosonic gases.

On the BCS side of the resonance, the situation is experimentally more complicated. To demonstrate condensation a momentum projection technique based on fast sweeps into the BEC regime was introduced [6]. Detailed time-dependent studies suggest that the sweeps used are sufficiently rapid that a condensate cannot be created during this sweep process. The presence of a condensate *after* a sweep then provides strong support for the existence of a condensate *before* the sweep on the BCS side of the resonance. In this way, the first Fermi gas normal-superfluid (NS) phase diagram was obtained experimentally for ^{40}K [6] and later for ^{6}Li [7]. Additional experiments in ^{6}Li have since added to the evidence for fermionic superfluidity, including collective mode observations [8,9], thermodynamic measurements near unitarity [10], and, most conclusively, the demonstration of quantized vortices [11]. These data, in conjunction with the sweep experiments, serve to further constrain the NS boundary.

The purpose of this paper is to present a comparison of this important phase boundary measured in fast sweep experiments involving ^{40}K to a theoretical computation of the NS boundary. Recent theoretical work examining the entropy of the trapped gas in the BCS-BEC crossover now makes it possible to make this comparison [12]. Thus, we present previous data [6] in an altogether different way and show that the experimentally obtained condensate fraction in ^{40}K provides a good measure of this phase boundary. While the emphasis of Ref. [6] was on providing evidence for the condensation of atom pairs in an ultracold Fermi gas, here we show that these data, moreover, provide a universal NS phase diagram (for broad resonances), as a function of temperature and interaction strength, that can be quantitatively compared with theory.

In making this quantitative comparison, several important issues need to be considered. First, the essential and unique feature of the fast sweep technique is that it can provide direct information about the condensate fraction in the fermionic regime by measuring the bimodal momentum distribution of the resultant weakly bound molecules in the BEC regime after a rapid sweep of the field B [6,7]. The projecting magnetic field sweep is completed on a time scale that allows molecule formation but is still too brief to allow additional pairs to condense.

In the ^{40}K experiment it was observed that the fast sweep resulted in significant number loss [3,6], presumably because of the relatively short lifetime of the molecules away from resonance [13]. The measured condensate fraction, which is defined as the number of condensed molecules divided by the total number of molecules observed after the fast sweep, could be affected by this loss. However, the loss process is almost certainly density dependent, and thus one expects only suppression (and never enhancement nor complete destruction) of the condensate fraction. Therefore, the NS boundary obtained in the experiment should be relatively unaffected. It corresponds to the threshold temperature below which a finite fraction of the molecules is observed to have zero momentum.

Second, a general difficulty in experiments is the lack of model-independent thermometry in the strongly interacting regime. Therefore, experiments typically rely on the combination of temperature measurements made away from resonance and slow adiabatic sweeps to the strongly interacting regime. In this paper we use a superscript "0" to denote quantities measured away from the Feshbach resonance in the weakly interacting Fermi gas regime. The temperature relative to the Fermi temperature $(T/T_F)^0$ is determined from surface fits to absorption images of the gas taken after expansion from the trap. We have checked [14] that this yields accurate temperature measurements down to $(T/T_F)^0 \approx 0.1$, but becomes more difficult for lower $(T/T_F)^0$ because the momentum distribution of the Fermi gas approaches the $T=0$ limit. The NS phase diagram also depends on the adiabaticity of the slow sweep toward resonance. Studies of the condensate fraction as a function of sweep rate [3] suggest that the sweep toward resonance is sufficiently slow. More recent studies involving double ramps to the resonance and then back away suggest that extra heating during the ramp is not significant on the BCS side of the resonance [15].

Third, for comparison with theory, the magnetic-field values should be converted to the dimensionless parameter $1/k_F a$, which reflects the strength of the pairing interaction in BCS-BEC crossover theories. Here, k_F is the Fermi wave vector at the trap center. For a we use a previous measurement as a function of magnetic field [6], and for k_F we use k_F^0, measured in the weakly interacting regime. Here, $E_F^0 \equiv k_B T_F^0 = (\hbar k_F^0)^2/2m$ is the noninteracting Fermi energy, and $k_F^0 = (2m\bar{\omega}/\hbar)^{1/2}(3N_a)^{1/6}$, where N_a is the total number of atoms and $\bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$ is the geometric mean angular trap frequency. Results for the NS phase diagram are then plotted in terms of $(T/T_F)^0$ and the dimensionless parameter $1/k_F^0 a$. The phase diagram plotted in terms of the dimensionless parameter $1/k_F^0 a$ is universal, and should be applicable to ^{6}Li as well.

Our theoretical calculations are based on the finite temperature formalism described in Refs. [16,17], which presumes the usual BCS-Leggett wave function for the ground state. In this approach the magnitudes of the transition temperature T_c (in a trap) are similar to those found elsewhere [18] using a different ground state, where less is currently known about derived quantities such as the superfluid density and thermodynamics.

Indeed, comparisons with the experimental data require not only an understanding of the behavior of T_c in a trap but also an understanding [12] of the entropy S, which ultimately is related to thermometry. The behavior of the thermodynamics at unitarity, within this ground state framework, has been shown to compare favorably to experiment in Ref. [10]. More generally, the entropy $S(T)$ is dominated by fermionic excitations in the BCS regime and "bosonic" excitations (associated with finite momentum pairs) in the BEC regime. This theory for S is important because it provides the basis for determining the temperature after an adiabatic sweep, provided the initial temperature is known [12]. Earlier studies of adiabatic sweep thermometry can be found in Ref. [19], for a strictly noninteracting fermion-boson model.

The theoretical phase diagram we present here requires knowledge of the superfluid density N_s/N as well. Importantly, N_s/N reflects the same bosonic and fermionic contributions found in the thermodynamics, and can be readily calculated [20] within the present framework. As discussed above, the experimentally determined phase diagram relied on measurements of the condensate fraction N_0/N. In the BEC regime, these two quantities are equal. In the fermionic and unitary regimes, however, the condensate fraction is not unambiguously defined and could vary with the particular observation under consideration. The superfluid density should be regarded as a general upper bound to the condensate fraction. As an example, a recent Monte Carlo calculation defined a $T=0$ condensate fraction that is nearly a factor of 2 lower than N_s/N at unitarity [21]. However, for the phase diagram one expects that N_s/N and N_0/N should yield the same results for T_c. Our theoretical results are intended to represent the full equilibrium phase diagram for wide Feshbach resonances. Unlike in Refs. [22,23], we do not address specifics of the fast sweep projection process. However, it should be noted that in contrast to the present work, in Refs. [22,23], no attempt was made to distinguish between the actual physical temperature and that which was measured (in the weakly interacting regime).

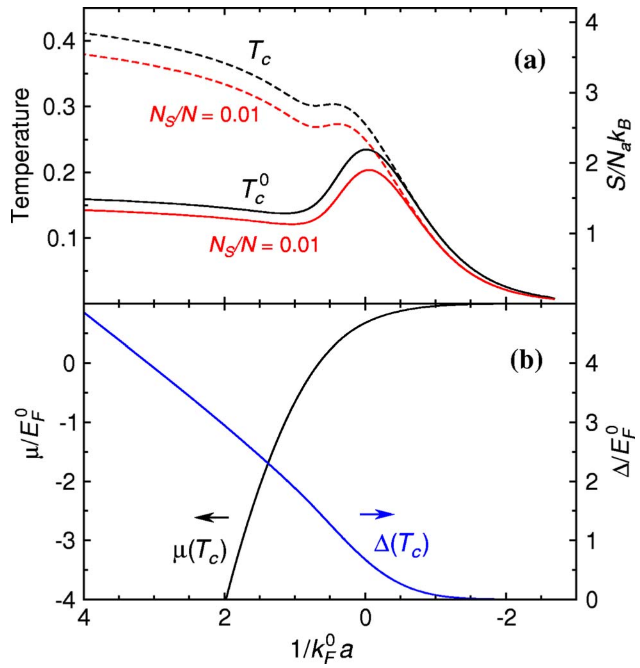

FIG. 1. (Color online) (a) Physical temperature T/T_F^0 (dashed curves), effective temperature $(T/T_F)^0$ (solid curves) at the superfluid transition (black curves) and $N_s/N=0.01$ (red curves). (b) $\mu(T_c)$ (black curve) and $\Delta(T_c)$ (blue curve) at the trap center as functions of $1/k_F^0 a$. In (a), the solid lines also represent *approximately* $S/N_a k_B$ ($\propto T^0$), where N_a is the total number of atoms of both spins. $(T/T_F)^0$ is the temperature measured in the noninteracting Fermi gas limit.

We now address the superfluid phase diagram. In the experiment the temperature of the gas is determined in the noninteracting Fermi gas limit at high field; we denote this temperature by T^0. The destination field is accessed adiabatically by a slow magnetic-field sweep. Using the theory in Ref. [12], we calculate the entropy at different magnetic fields and temperatures. In this way, we can associate the physical temperature T with the effective temperature T^0. We then calculate the condensate fraction N_0/N, here approximated by the superfluid density N_s/N, as a function of temperature T or T^0 and of magnetic field B. The latter parameter is appropriately characterized by the dimensionless variable $1/k_F^0 a$, which provides a measure of the strength of pairing interaction. At a given field this parameter varies with the Fermi temperature.

In Fig. 1(a), we show the results for the superfluid transition temperature T_c (black dashed line) and its corresponding value T_c^0 (black solid line) for an isentropic sweep into the

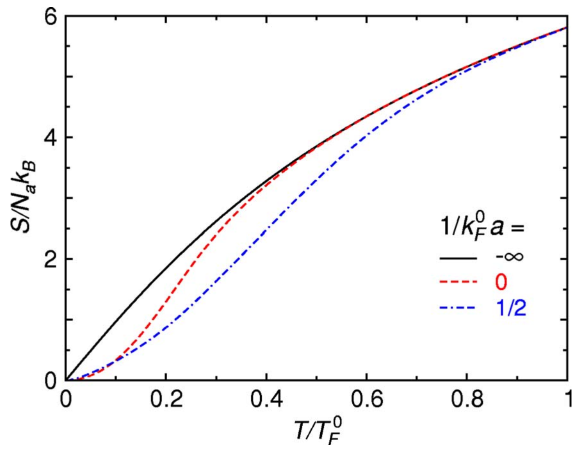

FIG. 2. (Color online) Entropy per particle $S/N_a k_B$ as a function of physical temperature T/T_F^0 for $1/k_F^0 a = -\infty$ (black), 0 (red, dashed), and 1/2 (blue, dot-dashed line), representing the ideal Fermi gas, unitary, and strongly interacting near-BEC cases, respectively.

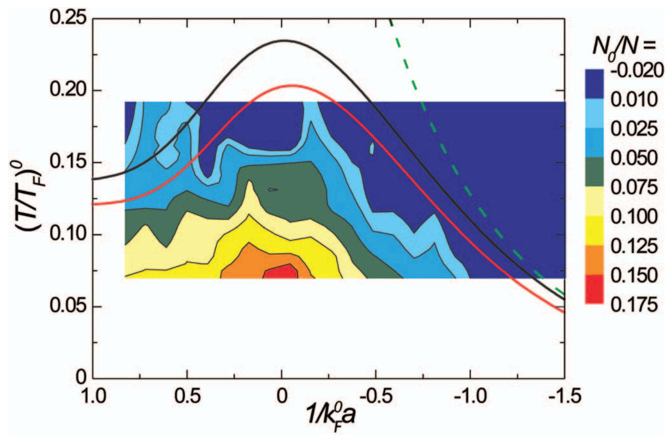

FIG. 3. (Color) Phase diagram of ^{40}K. A contour plot of the measured condensate fraction N_0/N as a function of $1/k_F^0 a$ and effective temperature $(T/T_F)^0$ is compared with theoretically calculated contour lines at $N_s/N=0$ (at the superfluid transition, black curve) and 0.01 (red curve). The experimental data have an overall systematic uncertainty of approximately 0.1 in $1/k_F^0 a$. The overall trend of the experimental contour of $N_0/N=0.01$ and the theoretical line for $N_s/N=0.01$ are in good agreement. The dashed line represents the naive BCS result $T_c/T_F^0 \approx 0.615 e^{\pi/2k_F^0 a}$. Here all temperatures are measured in the Fermi gas regime.

Fermi gas regime as functions of $1/k_F^0 a$. In a similar fashion, we plot the physical (red dashed line) and effective temperatures (red solid line) corresponding to $N_s/N=0.01$. In Fig. 1(b), we plot the fermionic chemical potential $\mu(T_c)$ and the excitation gap $\Delta(T_c)$ at the trap center as a function of $1/k_F^0 a$. When the chemical potential is negative the system can be viewed as "bosonic," whereas when μ is positive it is "fermionic."

Because the entropy for a noninteracting gas at low T/T_F^0 is *nearly* linearly dependent on the temperature, one can conclude that T_c^0 is approximately proportional to the entropy at the transition $S(T_c)$. The latter is labeled on the right hand axis of Fig. 1(a). It follows that as a natural consequence of an isentropic sweep, T_c^0 is reduced substantially from the physical T_c except in the BCS regime. As can be seen from the figure, this reduction is dramatic in the BEC regime ($1/k_F^0 a > 0.7$) and persists essentially to unitarity ($1/k_F^0 a = 0$). One can understand this reduction as reflecting the presence of bosonic degrees of freedom at T_c. Once noncondensed bosons or preformed pairs are present at the temperature of their condensation, the entropy curve for $S(T)$ for $T \leq T_c$ drops substantially below its counterpart for a noninteracting Fermi gas at the same T. One can alternatively say that when T_c and T_c^0 are significantly different, a normal state excitation gap or "pseudogap" [16,17,24] is present at T_c. In the fermionic regime ($\mu > 0$) and at the transition temperature, this pseudogap is parametrized by $\Delta(T_c)$, which is also shown in Fig. 1(b) and should be viewed as an alternative measure of bosonic degrees of freedom. One can see that the difference between T_c and T_c^0 reflects rather nicely the behavior of $\Delta(T_c)$ as a function of $1/k_F^0 a$. Beginning at unitarity and moving towards the BEC regime, $\Delta(T_c)$ increases rapidly, reflecting the rapid increase in the bosonic degrees of freedom. This leads then to a strong reduction from T_c to T_c^0 and explains the existence of the maximum seen in T_c^0.

To illustrate these effects, in Fig. 2 the T dependence of the entropy $S(T)$ is shown for selected values of $1/k_F^0 a$ representing the Fermi gas, unitary, and near-BEC cases. Here we see that the noninteracting gas result at low T/T_F^0 is close to a straight line, and that as the system becomes more strongly interacting, $S(T)$ acquires a higher power law in T and is suppressed relative to this noninteracting line once the temperature goes below the pair formation temperature T^*.

We are now in a position to compare our calculated phase diagram with experimental measurements. In Fig. 3, we replot the measured phase diagram from Ref. [6] as a function of $1/k_F^0 a$ and overlay our theoretical curves. The top (black) curve corresponds to the theoretical calculation for $(T_c/T_F)^0$, whereas the remaining (red) line is the effective temperature $(T/T_F)^0$ corresponding to the superfluid fraction $N_s/N = 0.01$. We present both theoretical curves because (as can be seen from the disproportionate breadth of the contour swath for $0 < N_s/N < 0.01$ in Fig. 4, see also Ref. [12]), the super-

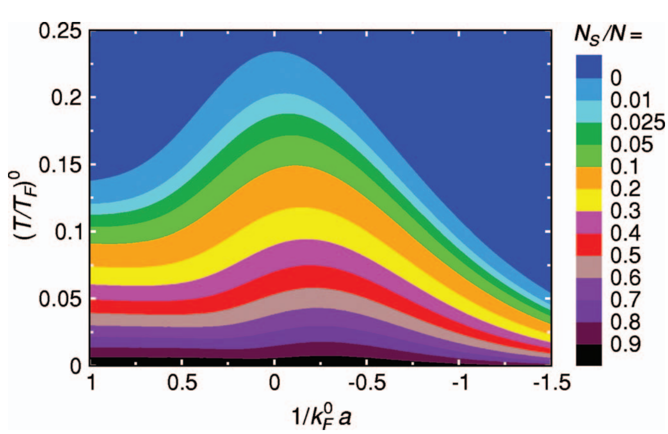

FIG. 4. (Color) The theoretically computed equilibrium phase diagram and contour plot of the superfluid density N_s/N as a function of $(T/T_F)^0$ and $1/k_F^0 a$.

fluid density has a flat tail close to the transition temperature due to trap inhomogeneity effects. Consequently, experimental noise may add a large uncertainty to the temperature for which $N_s/N=0$; the experimentally measured 1% contour should be a more robust boundary.

In general, the phase boundary for $N_s/N=0.01$ is in good agreement with the experimentally measured phase boundary for $N_0/N=0.01$. However, in the near-BEC regime there are a few experimental data points that show a finite condensate fraction above the theoretical transition line. The two well-known weaknesses of the mean-field approach may be partly responsible for this discrepancy: Both the overestimate of the interboson scattering length in the BEC regime and underestimate of the interaction energy at unitarity lead to a slightly underestimated peak density of the trap profile, which in turn leads to an underestimate of T_c. In addition, in the experiment, the slow sweeps that extend to the BEC side of the resonance are not perfectly adiabatic. Moreover, the experiment finds less than 100% conversion of atoms to molecules or pairs in this regime [25].

In Fig. 4 we plot the theoretical phase diagram and contour plot for N_s/N, which should be appropriate to the vortex experiments as well [11]. We can compare this calculation with its experimental counterpart in Fig. 3. When comparing the theoretical values of N_s/N at the lowest temperature $[(T/T_F)^0 \approx 0.07]$ accessed experimentally, we find that, at unitarity, the theoretical value is about two times as large as in the ^{40}K experiment. This difference may be attributed to a number of factors. First, in the theoretical calculations, we do not address the details of the projection sweep process. Second, sweeps may not be 100% adiabatic; minor heating is known to be present for sweeps from the noninteracting Fermi gas to the BEC regime. Given these factors, the agreement between theory and experiment is reasonably good.

In summary, we have shown that previous measurements of Ref. [6] of the normal state superfluid phase boundary in ^{40}K are in reasonable agreement with theoretical calculations. The theory presented here is consistent with previous claims that fast sweep experiments do, indeed, provide a reliable indication of the NS phase boundary. A feature of the predicted phase boundary is that, when it is plotted in terms of the temperature T_c^0 in the noninteracting regime, there is a maximum near unitarity as a function of $1/k_F^0 a$. Indications for this maximum have also been observed [7] in ^{6}Li, where one finds that the biggest condensate fraction occurs near unitarity, as in Fig. 4.

We thank Eric Cornell for providing the impetus for this paper and Cheng Chin for very helpful comments. This work was supported by NSF, NASA, and NSF-MRSEC Grant No. DMR-0213745. C.A.R. acknowledges support from the Hertz foundation.

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