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Radio-frequency spectroscopy of trapped Fermi gases with population imbalance

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Motivated by recent experiments, we address the behavior and evolution of radio-frequency (RF) spectra as temperature and polarization are varied in population-imbalanced Fermi gases from above to below T_c. We discuss a series of scenarios for the experimentally observed zero-temperature pseudogap phase and show how present and future RF experiments may help in its elucidation. We conclude that the experiments of Zwierlein and co-workers at the lowest T may well reflect ground-state properties, but take issue with their claim that the pairing gap survives up to temperatures of the order of the degeneracy temperature T_F at unitarity.

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The field of ultracold Fermi gases undergoing crossover from BCS superfluidity to Bose-Einstein condensate (BEC) is particularly exciting because these superfluids exhibit a rather novel form of fermionic superfluidity: pairing begins at temperature T^* while condensation take place at a significantly lower temperature T_c. Concomitantly, the normal-state fermionic spectrum exhibits an excitation gap or "pseudogap" [1–3]. Experiments by Zwierlein and co-workers [4] on population-imbalanced Fermi gases, based on combining vortex and radio-frequency (RF) spectroscopy [5], have thereby confirmed earlier indications [1,2] that a pairing gap is indeed visible *above* T_c. Importantly, it is claimed [5] that in a highly polarized gas one finds a ground state in which $T^* \neq 0$ while $T_c=0$. Throughout this paper we refer to this state as "a zero-temperature pseudogap phase."

It is the goal of the present paper to address these RF spectroscopy experiments on spin-imbalanced [4,6–8] unitary Fermi gases as temperature and polarization are varied [7,9,10]. With (roughly) decreasing T, one encounters [9] a Fermi gas, a pseudogap phase, a polarized superfluid (or "Sarma" state), and a phase-separated state, which is the ground state for all but possibly the highest polarizations at unitarity [6,8]. We discuss a series of (four) scenarios for the important zero-temperature pseudogap phase and show how present and future RF experiments may help clarify its origin. We conclude that, at the lowest T, the experiments of Zwierlein *et al.* reflect ground-state properties but disagree with the claim that the pairing gap survives up to the degeneracy temperature T_F.

We adopt a theoretical approach [2,11,12] to BCS-BEC crossover which appears to be uniquely positioned to address RF calculations [13,14] on polarized gases. We include trap and pseudogap effects at arbitrary T, and we systematically incorporate polarization effects, using our self-consistently determined temperature vs polarization phase diagram [9] within a BCS Leggett-like ground state. Bogoliubov–de Gennes (BdG) schemes presume a related ground state [15] but they apply strictly to $T=0$. Ours is a more consistent theory of pseudogap effects, because, as discussed in detail elsewhere [16], it introduces pairing fluctuations directly into both the gap and the number equations.

By contrast, essentially all other schemes, inspired by the Nozières–Schmitt-Rink approach [17], contain a problematic inconsistency in their incorporation of noncondensed pair effects [16] because they presume that pairing fluctuations enter only into the number equation but not in the gap equation. This is of concern for RF experiments which focus precisely on these pseudogap effects. Moreover, one needs to accommodate the effects of these noncondensed pairs in the spectral function, which we addressed a decade ago [18] above T_c, and, importantly, below T_c as well [11].

Here we use the standard one-channel grand canonical Hamiltonian $H-\mu_1 N_1-\mu_2 N_2$ which describes pairing between states $|1\rangle$ and $|2\rangle$ and for definiteness take state $|1\rangle$ as majority and state $|2\rangle$ as minority, unless indicated otherwise. We additionally ignore the interaction between state $|3\rangle$ and states $|1\rangle$ and $|2\rangle$, since mean-field energy shifts associated with the interaction between $|1\rangle$ and $|2\rangle$ and between $|1\rangle$ and $|3\rangle$ nearly cancel each other, as observed experimentally. Thus state $|3\rangle$ is associated with a noninteracting gas. In addition, there is a transfer matrix element $T_{\mathbf{k},\mathbf{p}}$ from $|2\rangle$to $|3\rangle$ given by $H_T=\Sigma_{\mathbf{k},\mathbf{p}}(T_{\mathbf{k},\mathbf{p}}c^\dagger_{3,\mathbf{p}}c_{2,\mathbf{k}}+\text{H.c.})$. For plane-wave states, $T_{\mathbf{k},\mathbf{p}}=\bar{T}\delta(\mathbf{q}_L+\mathbf{k}-\mathbf{p})\delta(\omega_{\mathbf{kp}}-\omega_L)$. Here $q_L\approx 0$ and ω_L are the momentum and energy of the RF laser field, and $\omega_{\mathbf{kp}}$ is the energy difference between the initial and final states. The RF current is defined as $I=\langle \dot{N}_3\rangle = i\langle [H,N_3]\rangle$. Using standard linear response theory, one finds

$$I = 2\bar{T}^2\,\mathrm{Im}[X_{\mathrm{ret}}(-\omega_L+\mu_3-\mu_2)],$$

$$X(i\omega_n) = T\sum_{m,\mathbf{k}} G_3(\mathbf{k},i\nu_m)G_2(\mathbf{k}+\mathbf{q}_L,i\nu_m+i\omega_n), \tag{1}$$

where μ_3 is the chemical potential of $|3\rangle$ and ω_{23} is the energy splitting between $|3\rangle$ and $|2\rangle$. After Matsubara summation and using $A_3(\mathbf{k},\nu)=2\pi\delta(\nu-(\epsilon_{\mathbf{k}}+\omega_{23}-\mu_3))$ as well as $A_2(\mathbf{k},\nu)\equiv -2\,\mathrm{Im}\,G_2(\mathbf{k},\nu+i0^+)$ to rewrite the spectral functions for states $|3\rangle$ and $|2\rangle$, respectively, we have

$$I(\omega) = \frac{\bar{T}^2}{2\pi}\sum_{\mathbf{k}} A_2(\mathbf{k}+\mathbf{q}_L,\epsilon_{\mathbf{k}}-\omega-\mu_2)$$
$$\times[f(\epsilon_{\mathbf{k}}-\omega-\mu)-f(\epsilon_{\mathbf{k}}+\omega_{23}-\mu_3)], \tag{2}$$

where $\omega\equiv\omega_L-\omega_{23}$ is defined to be the RF detuning and $f(x)$ is the Fermi distribution function. In the above equations the retarded response function $X_{\mathrm{ret}}(\omega)=X(i\omega_n\to\omega+i0^+)$, and we have expressed the linear response kernel X in terms of single-particle Green's functions. We define ω_n and ν_m as even and odd Matsubara frequencies, respectively, and G_2 is the fully dressed Greens function for the state $|2\rangle$ spins. (We use the convention $\hbar = k_B = 1$.)

In our T-matrix formalism [11,12], $G_2(\mathbf{k},\nu)$ contains two self-energy contributions deriving from condensed Cooper pairs (Σ_{sc}) as well as from finite-momentum pairs (Σ_{pg}). Both have an important role in the spectral function [11,18]. We have $\Sigma = \Sigma_{pg} + \Sigma_{sc}$, where $\Sigma_{pg}(\mathbf{k},\nu) = \Delta_{pg}^2/(\nu + \xi_{k,1} + i\gamma)$ and $\Sigma_{sc}(\mathbf{k},\nu) = \Delta_{sc}^2/(\nu + \xi_{k,1})$. Here Δ_{sc} is the superfluid order parameter, and $\gamma \neq 0$ is associated with the lifetime effects of noncondensed pairs. The resulting spectral function [11] is

$$A_2(\mathbf{k},\nu) = \frac{2\Delta_{pg}^2 \gamma (\bar{\nu} + \xi_{\mathbf{k}})^2}{(\bar{\nu} + \xi_{\mathbf{k}})^2 (\bar{\nu}^2 - E_{\mathbf{k}}^2)^2 + \gamma^2 (\bar{\nu}^2 - \xi_{\mathbf{k}}^2 - \Delta_{sc}^2)^2}. \quad (3)$$

Here $\xi_{\mathbf{k},1} = \epsilon_{\mathbf{k}} - \mu_1$, $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$, $\mu = (\mu_1 + \mu_2)/2$, $h = (\mu_1 - \mu_2)/2$, and $\bar{\nu} = \nu - h$. The quasiparticle dispersion $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2(T)}$ where $\Delta^2(T) = \Delta_{sc}^2(T) + \Delta_{pg}^2(T)$. The precise value of γ and even its T dependence are not particularly important, as long as it is nonzero at finite T. As is consistent with the standard ground-state constraints, Δ_{pg} vanishes at $T \equiv 0$, where all pairs are condensed. Above T_c, we have Eq. (3) with $\Delta_{sc} = 0$. Because the energy level difference ω_{23} (≈ 80 MHz) is large compared to other energy scales, the state $|3\rangle$ is initially empty and thus $f(\epsilon_{\mathbf{k}} + \omega_{23} - \mu_3) = 0$ in Eq. (2). Once the trap is incorporated, Eqs. (2) and (3) can then be used to compute the local current density $I(r,\omega)$ and then to obtain the total net current $I_\sigma(\omega) = \int d^3r\, I(r,\omega) n_\sigma$ with $\sigma = 1, 2$. Unless stated otherwise, the energy unit T_F represents the Fermi temperature for the noninteracting unpolarized Fermi gas with the same total particle number.

To treat the trap, we assume a spherically symmetrical harmonic oscillator potential $V(r) = m\bar{\omega}^2 r^2/2$. The density, excitation gap, and chemical potential, which vary along the radius, can be determined using the local density approximation (LDA). The phase diagram, representing the stable regimes for phase separation, the Sarma phase as well as the normal Fermi gas phases as a function of temperature and polarization, has been mapped out [7,10]. Since it is at the heart of the current experiments, one must also determine [9] where *pairing occurs without superfluidity*. These noncondensed pair effects (which are generally ignored in the literature) are also essential for arriving at physical values for T_c. *Important for the present purposes, the phase-separated state is not [16] associated with pseudogap effects, unlike the Sarma state*. The same behavior is mirrored in the density profiles [16]. The Sarma phase consists of a superfluid core followed by a correlated "mixed normal" or pseudogap regime, followed by a Fermi gas in the outer regions of the trap. The phase-separated state, by contrast, has an essentially unpolarized superfluid core separated from a noncorrelated normal Fermi gas by a sharp interface.

To begin, it is useful to present the prototypical behavior for the RF spectra. Quite generally we find that in the phase-separated state (low T) there is a single pairing peak, whereas in the pseudogap phase (higher T) there are two peaks, and the Sarma phase (intermediate T) may have either one or two, depending on T and δ. Finally, at high T, we have only an atomic peak, located precisely at the atomic level separation $\omega_{23} = 0$. For a range of lower T, the atomic peak persists, deriving from the effectively noninteracting Fermi gas contribution at the trap edge; the pairing peak arises from the superfluid or pseudogap region in the trap center.

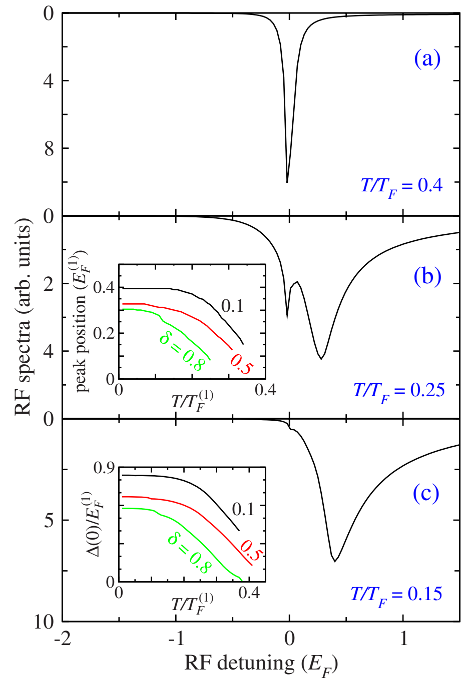

FIG. 1. (Color online) RF spectra for polarized gases in a harmonic trap at unitarity and polarization $\delta = 0.5$, for $T/T_F =$ (a) 0.4, (b) 0.25, and (c) 0.15. The insets in (b) and (c) are, respectively, the pairing peak position and the energy gap $\Delta(T)$ at the trap center as a function of $T/T_F^{(1)}$ (in units of the majority Fermi energy $E_F^{(1)}$), for $\delta = 0.1$ (black), 0.5 (red), and 0.8 (green lines), as labeled. The corresponding $T_c/T_F = 0.28$ (black), 0.25 (red), and 0.19 (green lines), respectively, and the estimated T^* can also be read off from the insets where the gap vanishes. For illustrative purposes, we choose $\gamma = 0.05 E_F$.

Figure 1 presents numerical results for the minority RF spectra at unitarity and at moderate polarizations $\delta = (N_\uparrow - N_\downarrow)/N = 0.5$. The temperature gradually increases from bottom to top. We consider lower temperatures (by a factor of about 2) to arrive at results which are comparable to those in [5]. The two insets show the pairing peak position and trap center gap as a function of $T/T_F^{(1)}$. In these two insets, we follow the experiments of Zwierlein and coworkers and use the majority-component Fermi energy $E_F^{(1)}$ as a unit of energy for both temperature and gap. The black, red, and green curves correspond to three polarizations $\delta = 0.1$, 0.5, and 0.8, respectively. One can see that the higher polarization is associated with a smaller peak position and energy gap. We see that the magnitudes of the pairing gap are rather comparable to their experimental counterparts. As in the experiment, the pairing gap increases with decreasing temperature. The energy scale at which it smoothly vanishes can be read off in the insets, which yield T^*. There is no sharp feature at T^*, so experimentally it cannot be precisely defined. Nevertheless, we see that there is a clear separation between the peak location curves for the three polarizations. By contrast the experimental data for all measured polariza-

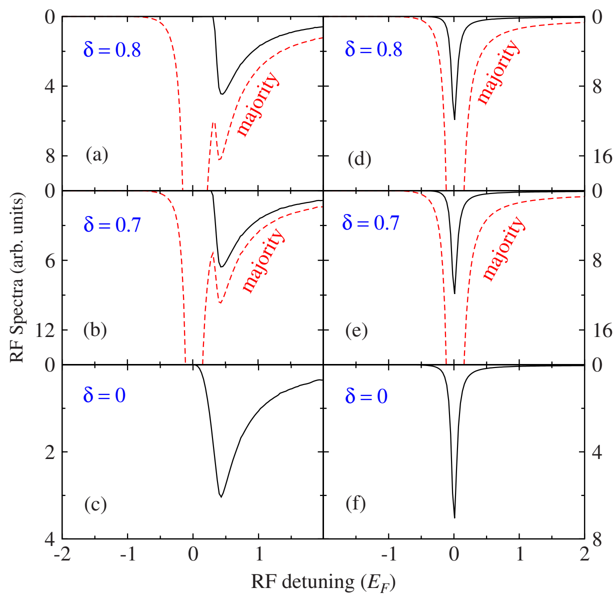

FIG. 2. (Color online) Low-temperature (T=0.01) RF spectra at different polarizations (as labeled) for a unitary (left) and noninteracting Fermi gas. When $\delta \neq 0$, the solid (black) and dashed (red) curves show the result when state $|2\rangle$ is the minority and majority, respectively.

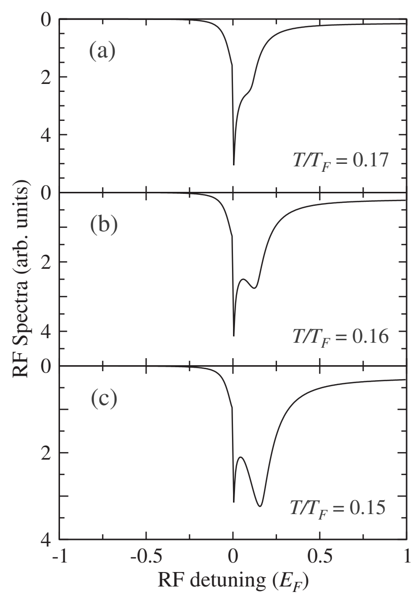

FIG. 3. RF spectra at unitarity when $|2\rangle$ is the minority with polarization δ=0.95, for different temperatures T/T_F=(a) 0.17, (b) 0.16, and (c) 0.15. Here we take $\gamma=0.05E_F$.

tions lie on the same (approximately) universal curve, with substantially higher T^* (by a factor of 2 or so).

We now turn to a first scenario for elucidating the exotic nonsuperfluid phase at high polarizations [5] by considering the possibility [19] that this state is a Fermi gas or liquid. The loss of superfluidity would be due to a destabilization (arising from more benign Hartree-like corrections not included here) in the competing normal Fermi gas phase. This scenario is not compatible with a zero-temperature pseudogap phase (since the presence of an excitation gap for fermions means that it is not in a Fermi gas or Fermi liquid state). Nevertheless, this scenario would give rise to a single, nearly symmetric RF peak at low temperatures and high polarizations, similar to that observed experimentally, albeit associated with an atomic rather than a pairing peak.

Figure 2 plots the RF spectra at very low temperatures $T=0.01T_F$ in the unitary (left) and the noninteracting limit (right panels), assuming state $|2\rangle$ is the majority (red dashed) and minority (black solid lines), respectively. The top two panels correspond to high polarizations δ=0.7 and 0.8. The bottom panel presents a comparison with an unpolarized gas. This low-temperature phase corresponds to superfluidity in all cases in the left column, since that is what is found in our calculations [9,16]. The two high polarizations correspond to phase separation. In the noninteracting gas case (right column), the results are very simple. We find, as expected, only atomic peaks in the majority and minority curves. They are located at precisely the same position—at the zero of our frequency scale. Comparing the two curves in Figs. 2(a) and 2(b) with those in Figs. 2(d) and 2(e), one sees that with future majority spectra there is a simple way to rule out this particular Fermi gas scenario. At low T the majority curves in Figs. 2(a) and 2(b) (unlike the minority) have atomic peaks as well as pairing peaks. The larger atomic peaks of the majority plots are associated with the fact that the majority has a much larger noninteracting gas tail in its particle density profile. By contrast for the minority curves on the left, all fermions are paired at these low T and we see only a single pairing peak.

When comparing with existing experiments, it should be noted that if the single peak in the zero-temperature pseudogap phase were an atomic peak as in the calculations of Ref. [19], there would be a shift in its position (relative to that computed here) though probably not large enough to match the experimental presumed pairing peak. In summary, this figure shows that the combined measurement of both majority and minority curves can serve to establish whether a single peak is coming from paired atoms or noninteracting atoms. In this way it can address the scenario [20] which associates the nonsuperfluid state at high polarizations with a Fermi gas phase.

In Fig. 3, we turn to another possible scenario for the mysterious phase in which there is pairing without superfluidity [5]: namely, that it is associated with a finite-temperature and normal-pseudogap state which arises in the Sarma portion of the phase diagram [9]. Here we consider higher polarization, δ=0.95, which allows us to access a normal but paired phase at relatively low T. The figure shows the RF spectra for this system at intermediate T, varying from $T=0.15T_F$ to $0.17T_F$ as we go from the bottom to the top panels. Importantly, we see from the figure that a two-peaked structure is clearly visible at the lowest T of this intermediate temperature scale, $0.15T_F$. It will be even better resolved at somewhat lower polarizations, as studied experimentally. The two peaks start to merge into a single atomic peak at higher T, around $0.17T_F$. The observation of two peaks in this figure at intermediate temperatures, in contrast with experiment, suggests that the experiments [4,5] were conducted at sufficiently low T.

A third possible scenario for the observed zero-temperature pseudogap phase follows from BdG-based calculations [15] which suggest that the ground state is, indeed, a superfluid, but with a (hard to detect) Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) ordering [21]. We have conducted a finite-temperature study (importantly including noncondensed pairs) of the simplest such state [22] which suggests that this oscillatory order parameter phase rapidly becomes unstable with increasing temperature. Because it is not sufficiently robust, we argue that the FFLO phase is not likely to be a candidate for the exotic ground state. Indeed, experiments from both Partridge *et al.* and Shin *et al.* seem to support phase separation [6,8] as found in LDA-based theories. We stress that the phase separation which we find [9,16] is not associated with pairing without condensation and that our theoretical phase diagram is closer to that of Ref. [6]. Moreover, at high polarization (where the zero-temperature pseudogap state is purported to exist) we find a very small self-consistently determined pairing gap as seen in Fig. 3 at low T. In a related fashion, we find quite generally that, for polarized gases, $T_c=0$ phases arise from the breakdown of pairing.

Since at $T=0$ bosons (or presumably Cooper pairs) tend to either condense or become insulating, this leads to a fourth scenario for a $T_c=0$ with $T^* \neq 0$ phase, in which there is a frustration of pair mobility and associated localization of pairs. We have found this phase in other theoretical contexts [23], and it also appears to exist in high-T_c superconductors [2]. Within the present theoretical framework, however, based on the BCS Leggett-like ground state, we find that high polarization breaks pairs apart and thus destroys T^* at the same time as it destroys T_c.

In summary, we have examined four different scenarios for the purported zero-temperature pseudogap phase: that the state is in reality (i) a (nonpaired) Fermi gas, (ii) a finite-temperature pseudogap phase, (iii) an exotic FFLO superfluid, and (iv) a bosonic insulating phase with localized pairs. We have not yet found support for scenarios (ii) and (iii) and conclude that future RF experiments are needed to rule out (i) by providing majority spectra which can confirm the presence of a pairing peak, as contrasted with an atomic peak. Hopefully, such measurements will also reduce the unexpectedly high values assigned to $T^* \gtrsim T_F$ in order to be consistent with other experimental [3] and theoretical estimates. The bosonic insulating state of (iv) seems to be ruled out because we find that polarization effects simultaneously diminish both T^* and T_c.

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