中国科学技术大学
University of Science and Technology of China

计算机图形学
Computer Graphics

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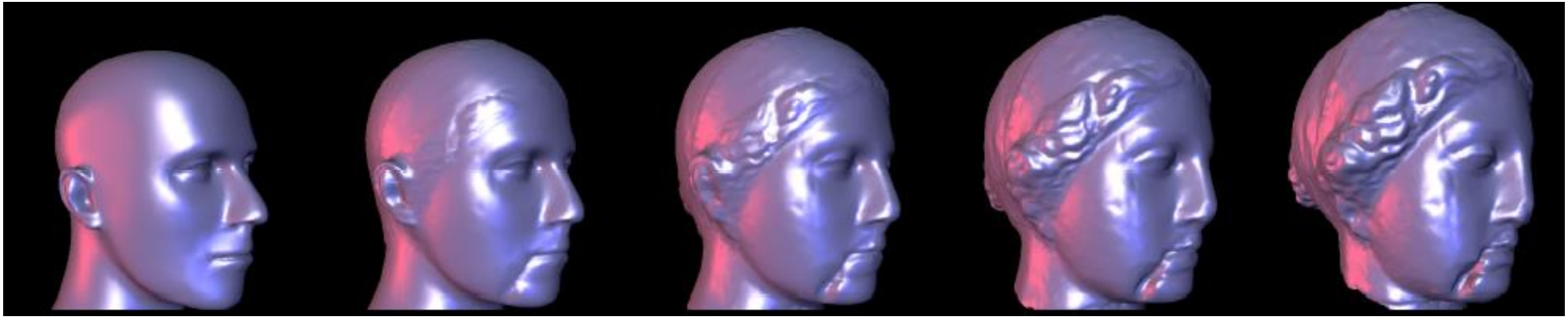
Mesh Morphing

Morphing

- Given two objects, produce sequence of intermediate objects that gradually evolve from one object to the other
  - Interpolate object shapes
  - Interpolate object attributes
    - Color, texture, normal, etc.



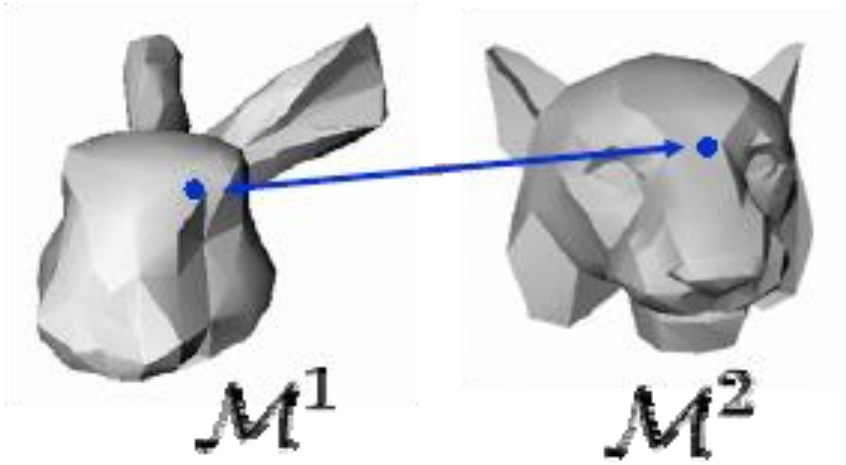
Terminologies

- Morphing
- Metamorphosis
- Shape blending
- Shape averaging
- Shape interpolation
- Shape transition

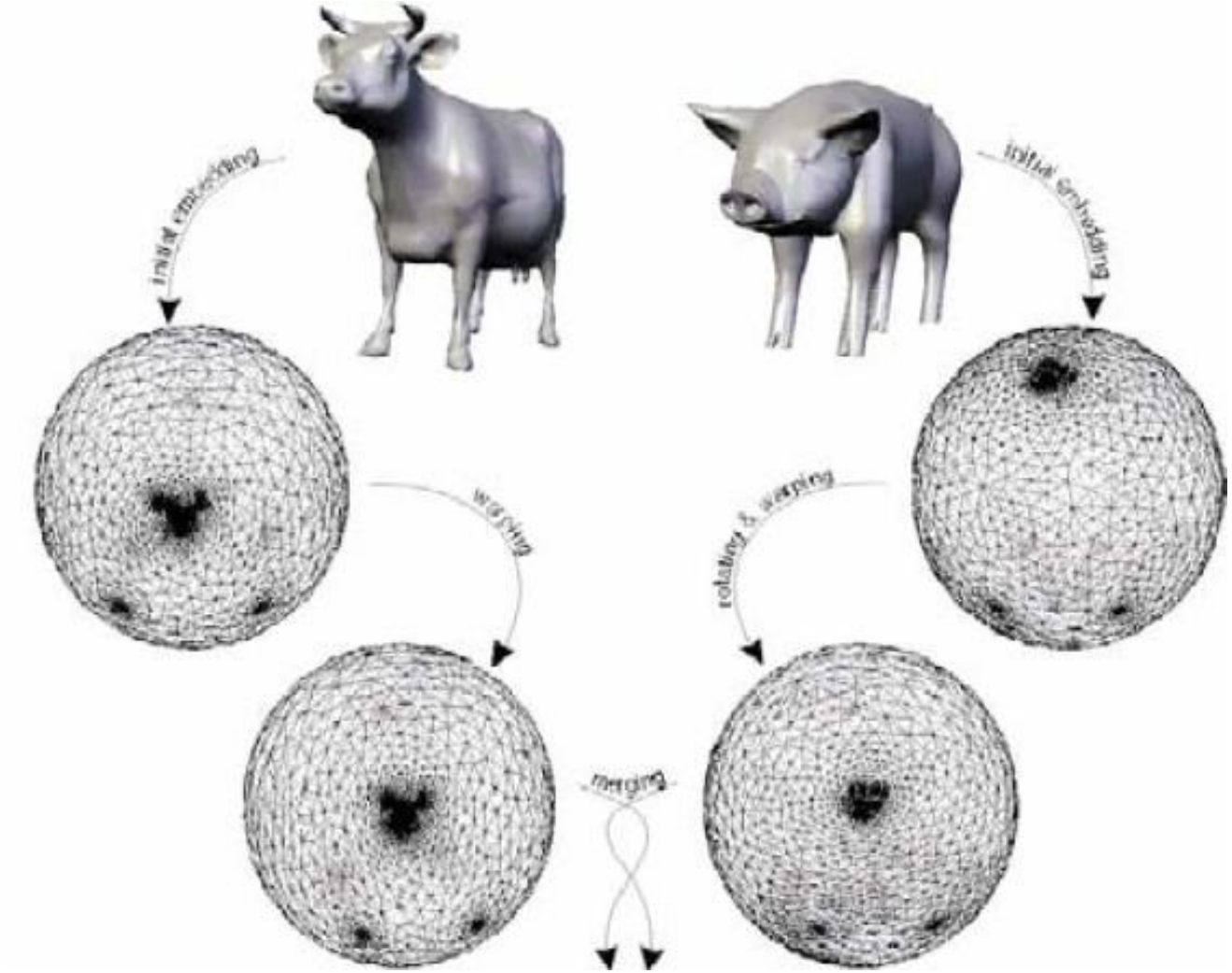
Morphing: Sub-Problems

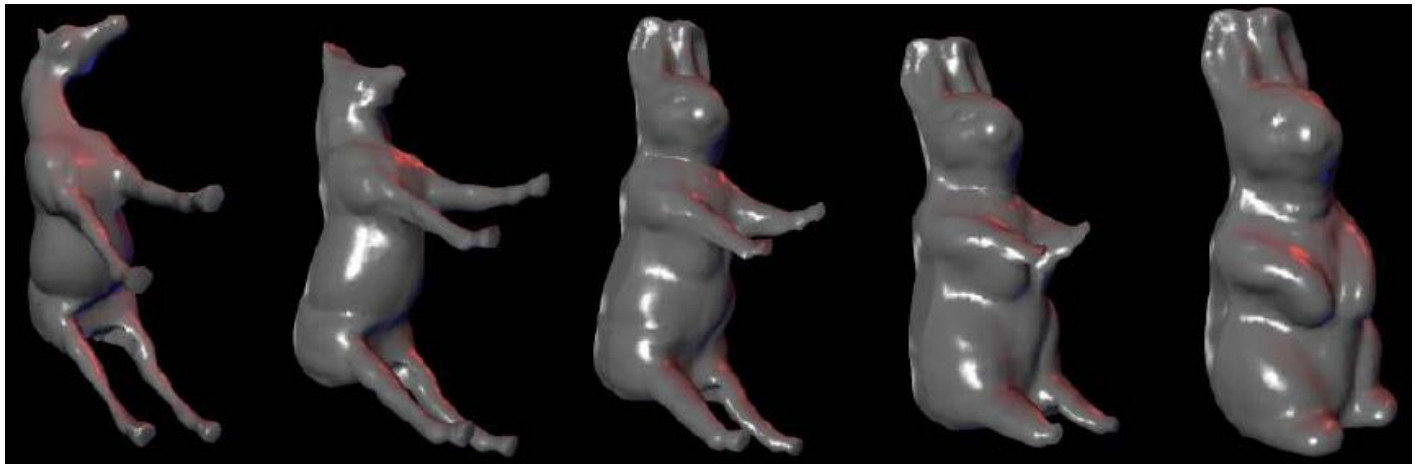
- Correspondence problem
  - For each point on source mesh M^1, find corresponding point on the target mesh M^2 (Parameterization)

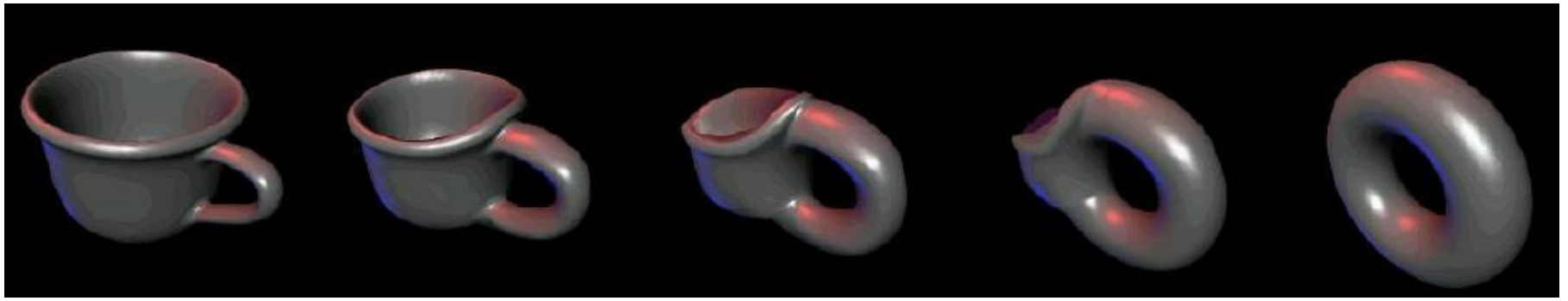
- Path problem
  - Specify trajectory in time for each point
  - For mesh – specify vertex trajectory

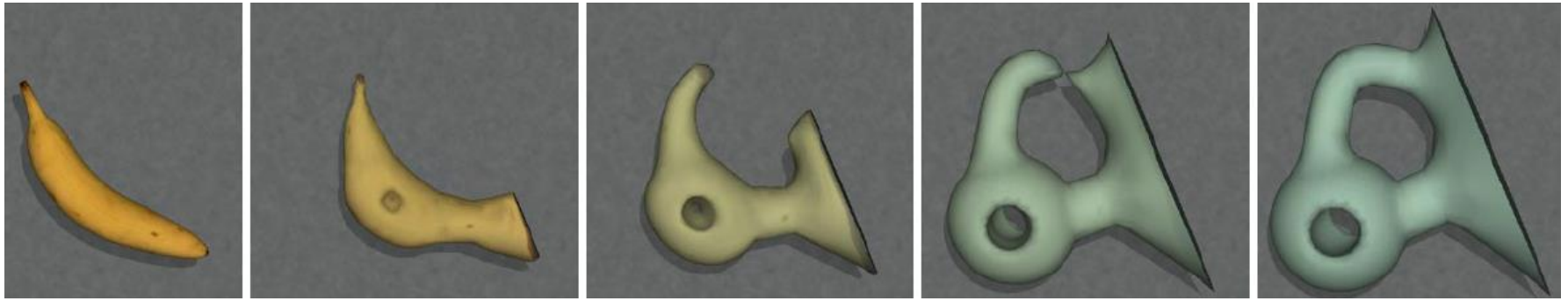


Vertex Correspondence



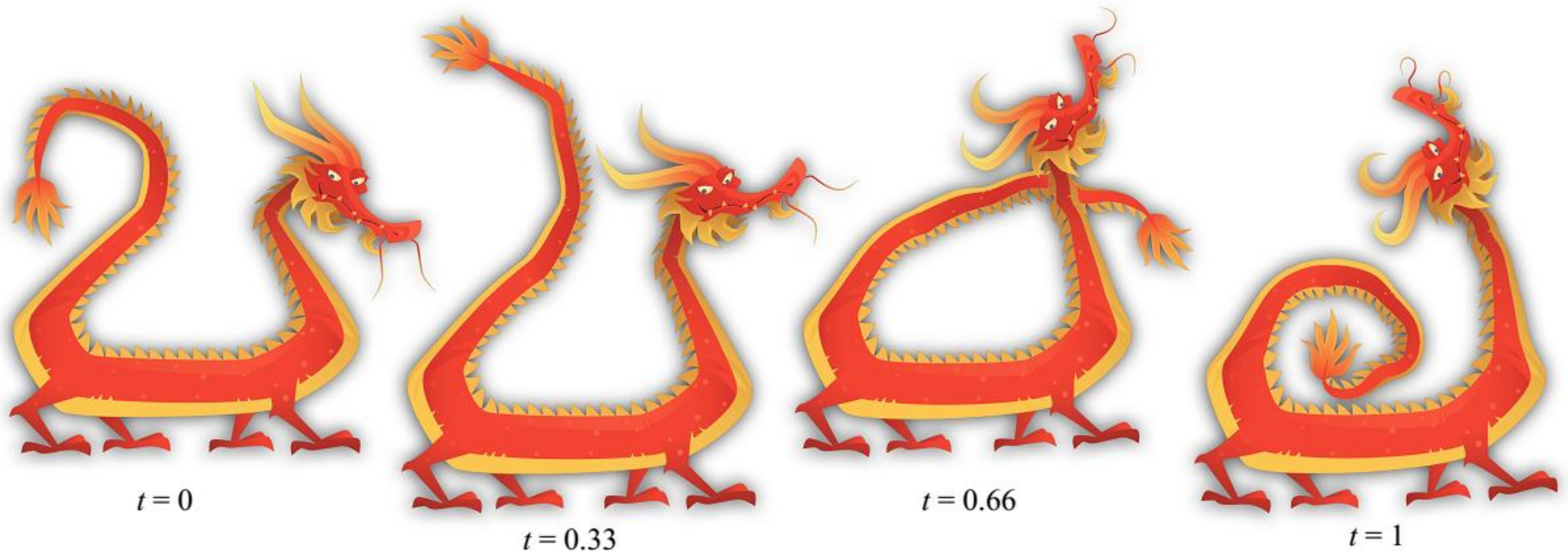




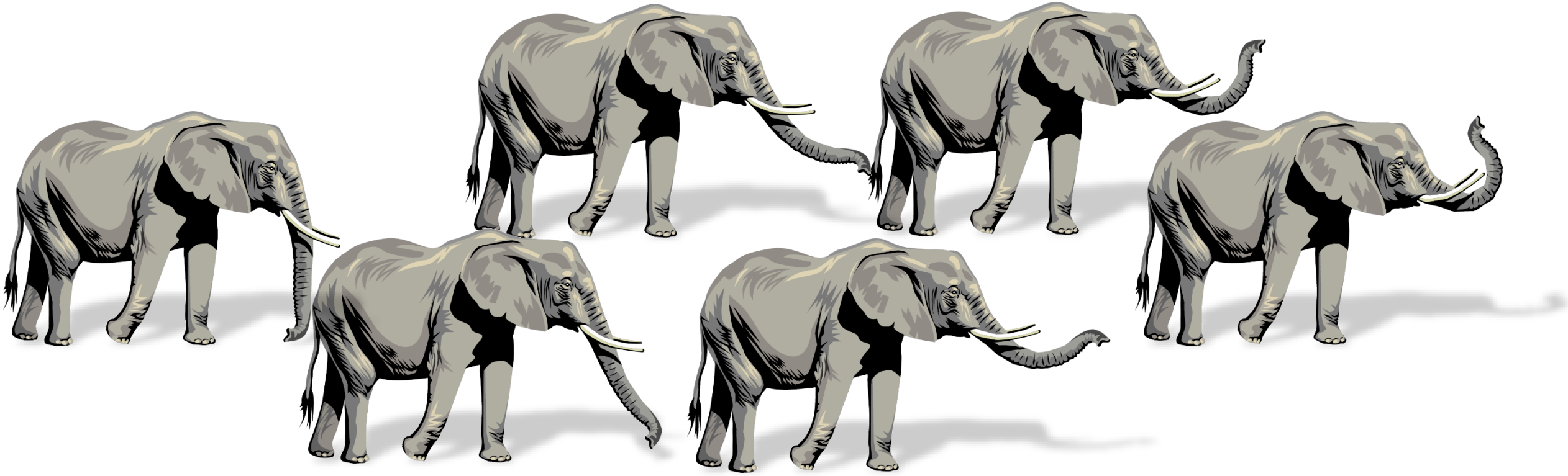


Planar Shape Interpolation with Bounded Distortion

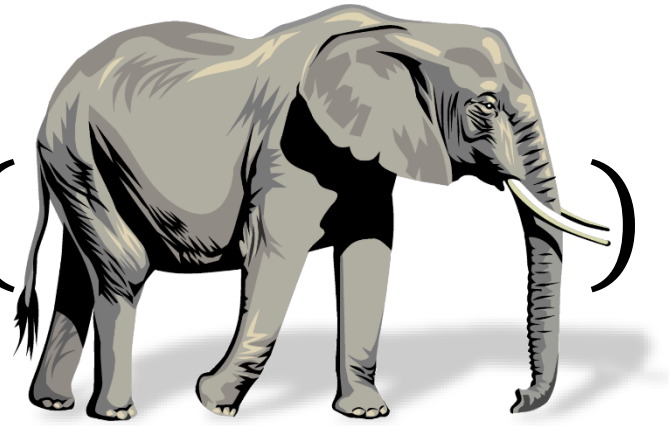
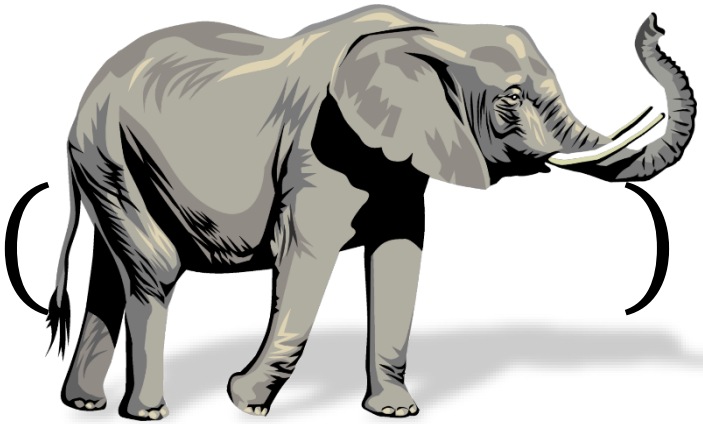
SIGGRAPH 2013

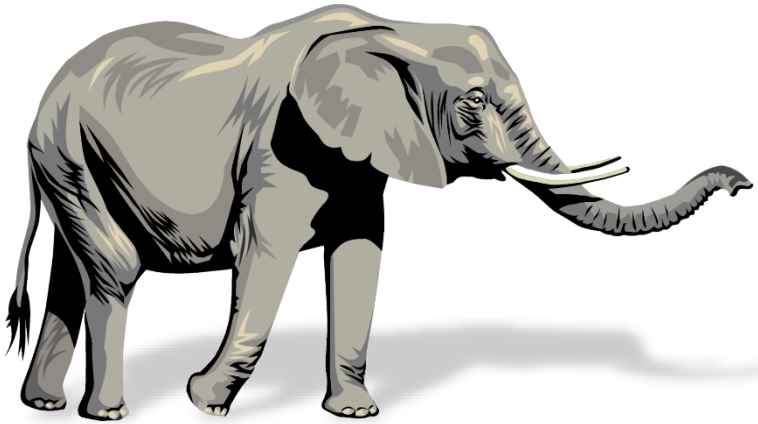


Motivation

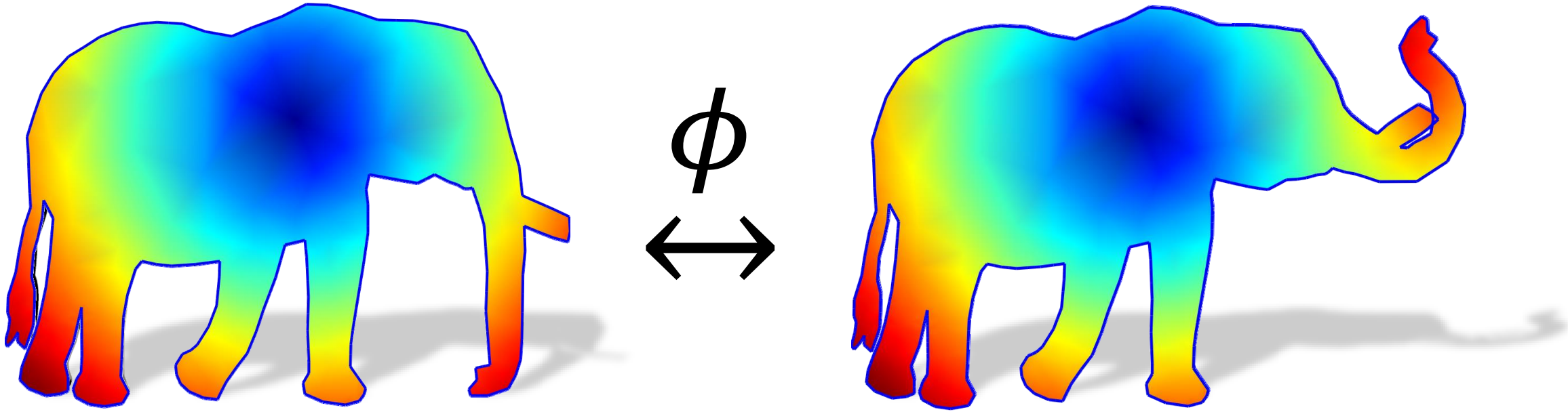


Shape interpolation - the problem

Input: $(1-t)$  $+ t$ 

Output: $=$ 

Shape interpolation - input

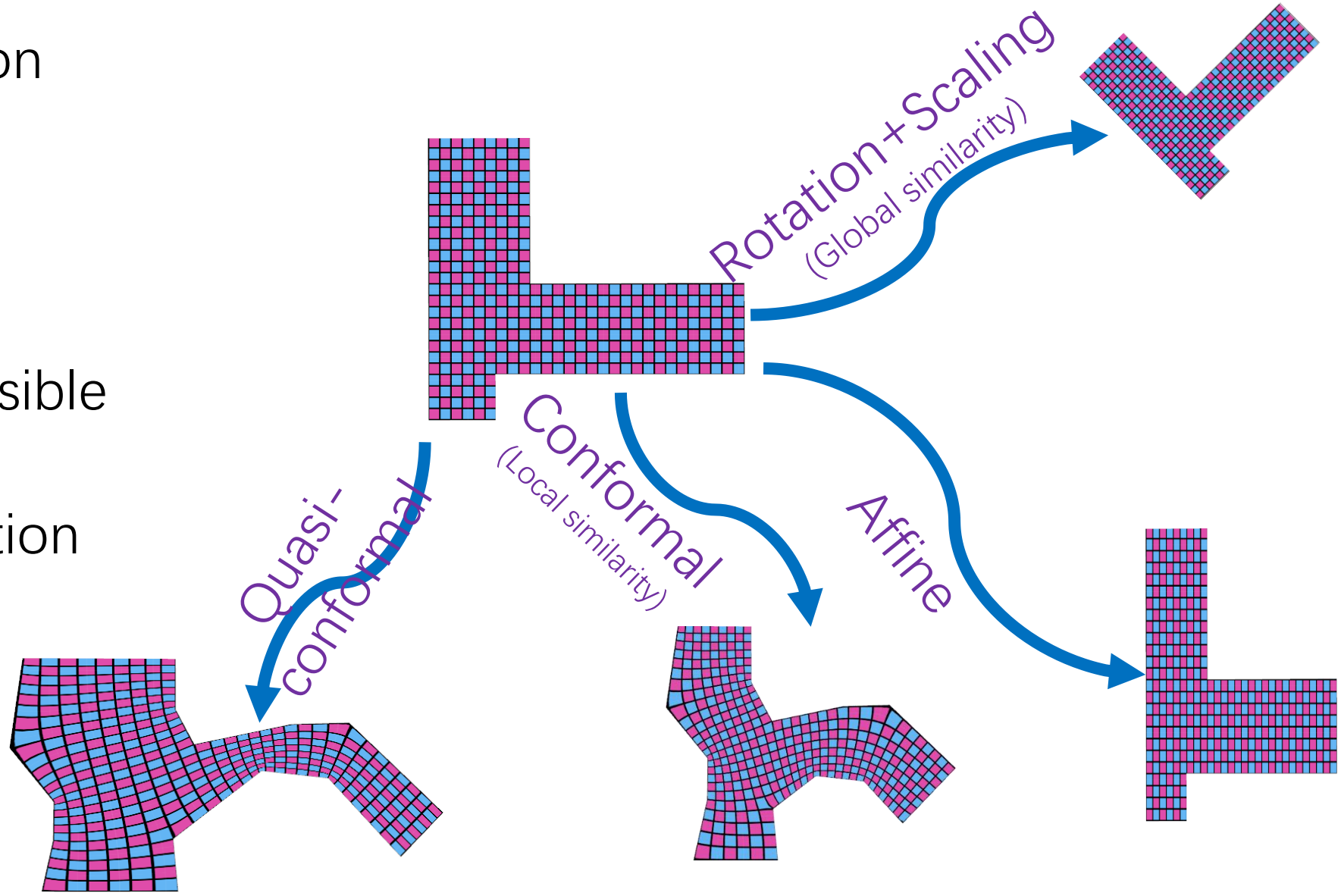


Source

Target

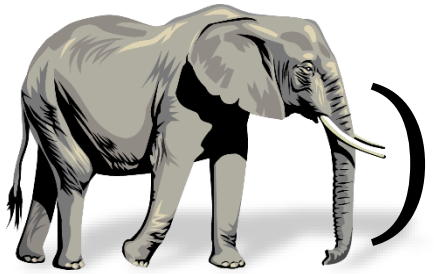
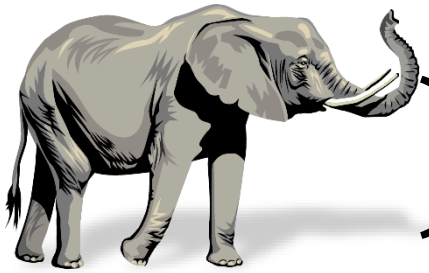
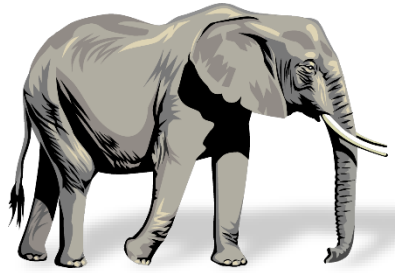
Shape interpolation – guidelines

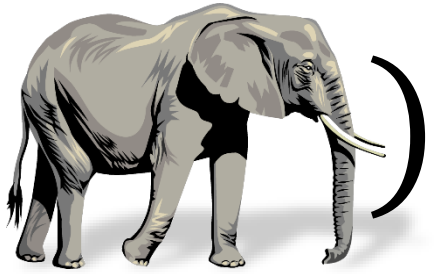
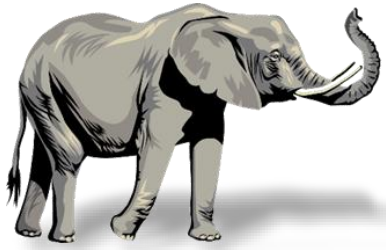
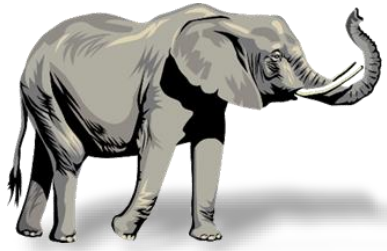
- Simple deformation
  - Rotation
  - Scaling
  - Conformal
  - Affine
- As Similar as possible
  - Quasi-conformal
- Smooth interpolation



Shape interpolation – desirable properties

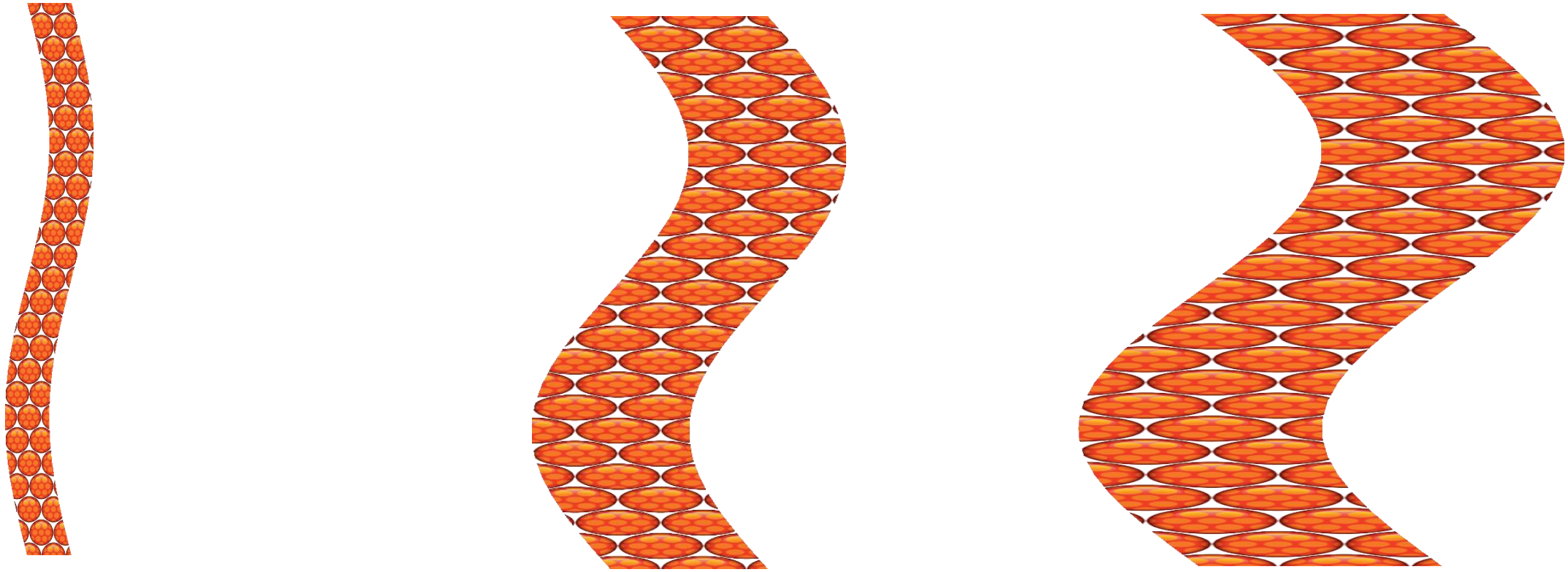
- Source/Target reproduction

$1($  $) + 0($  $) =$ 

$0($  $) + 1($  $) =$ 

Shape interpolation – desirable properties

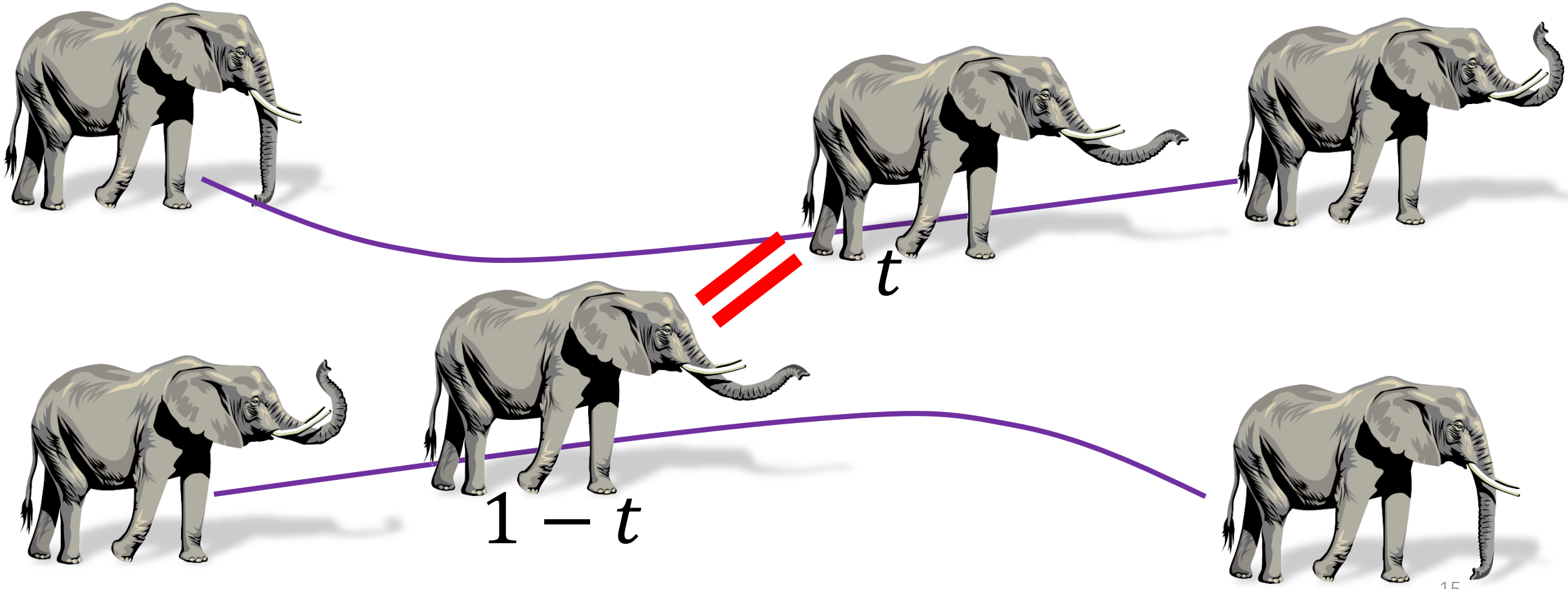
- Affine reproduction



Source t Target

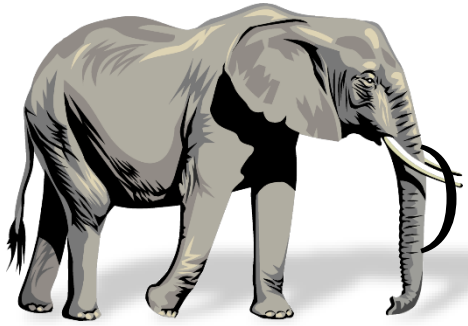
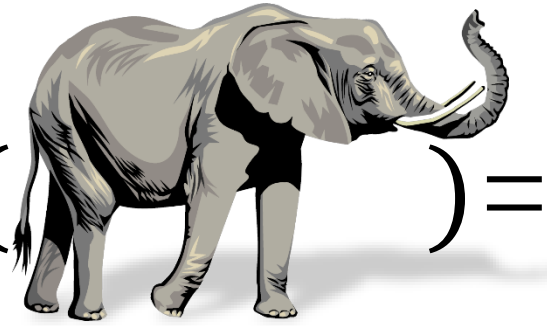
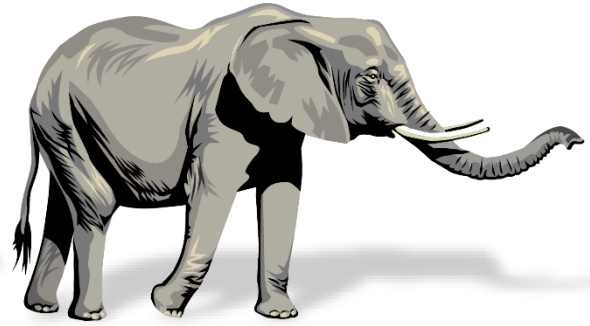
Shape interpolation – desirable properties

- Symmetry

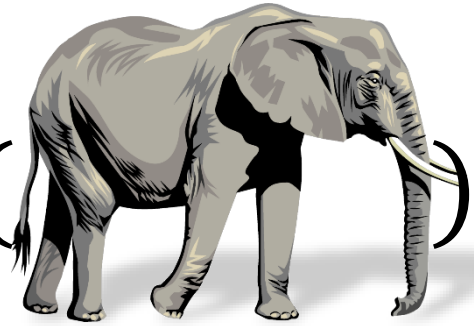
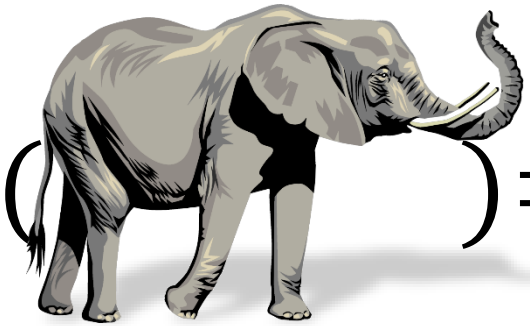
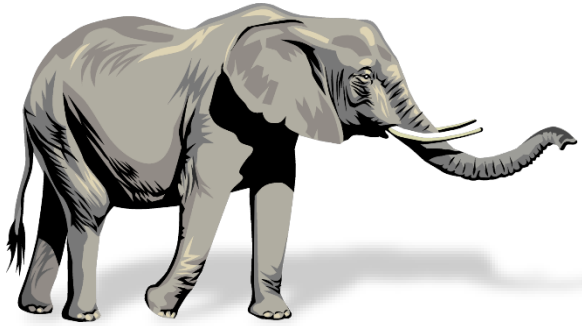


Shape interpolation – desirable properties

- Smoothness

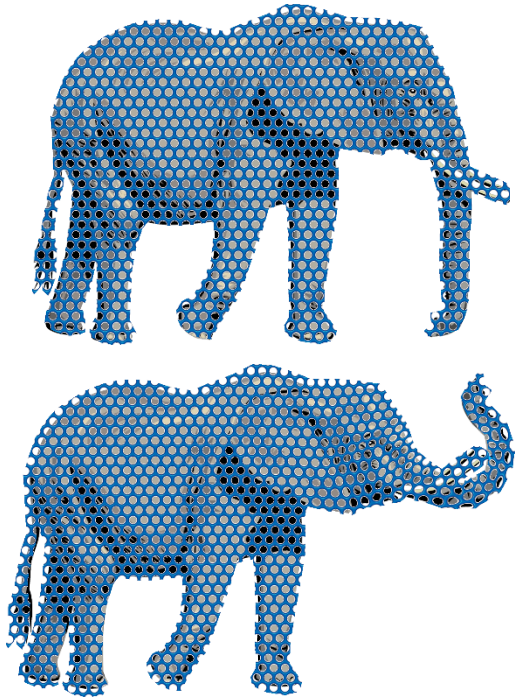
$(1-t)($  $)+t($  $)=$ 

$t \approx s$ ⟶ $\approx$

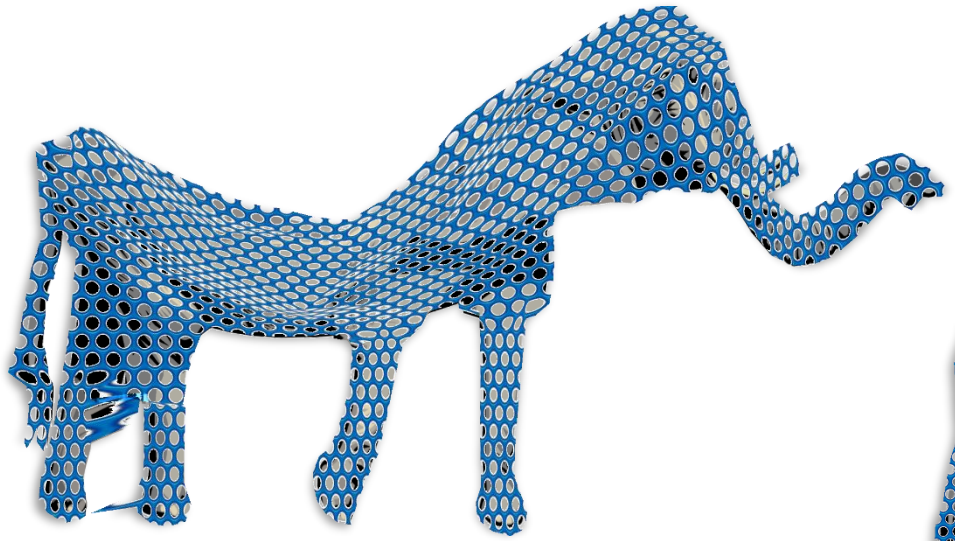
$(1-s)($  $)+s($  $)=$ 

Shape interpolation – desirable properties

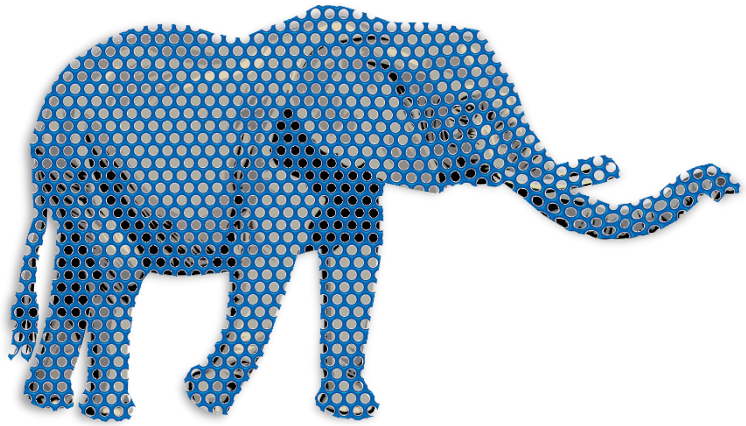
- Pointwise bounded distortion
  - Conformal/angular distortion



Input



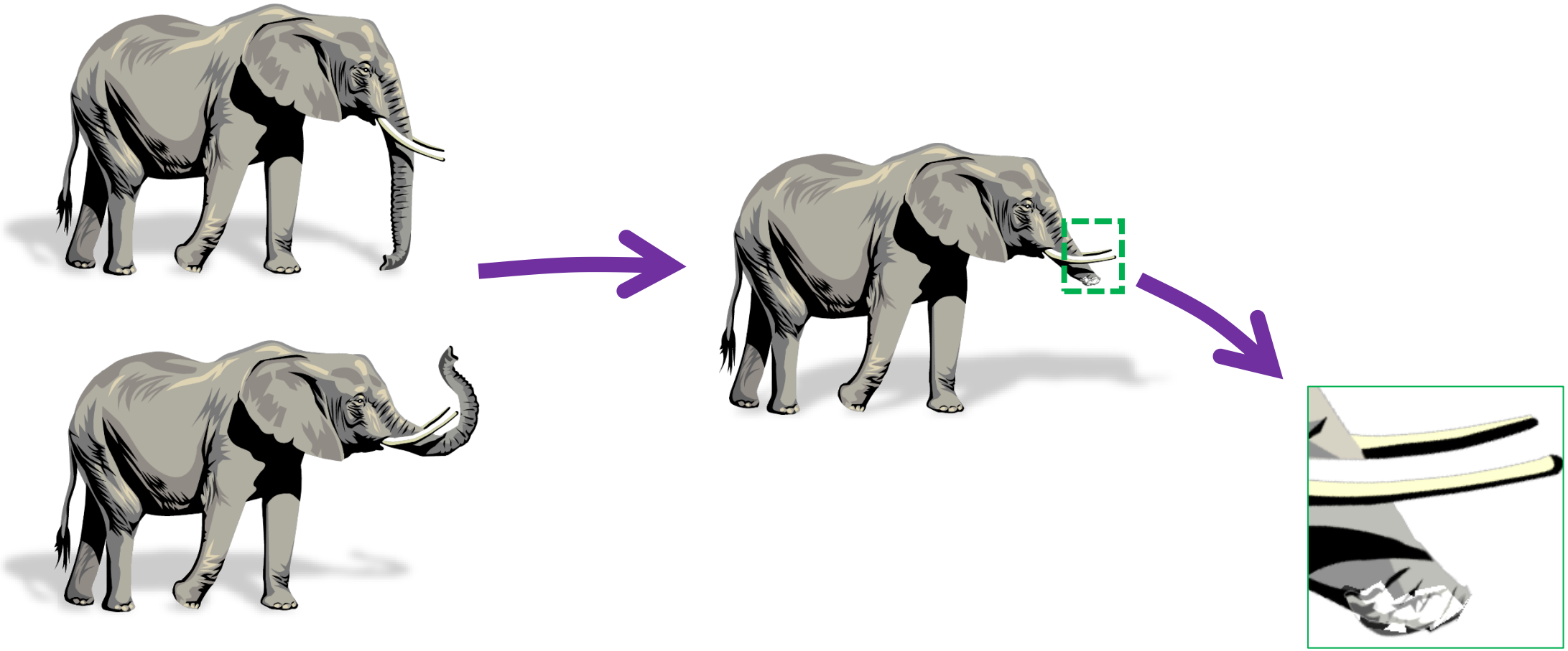
Result with
Unbounded distortion



Result with
bounded distortion

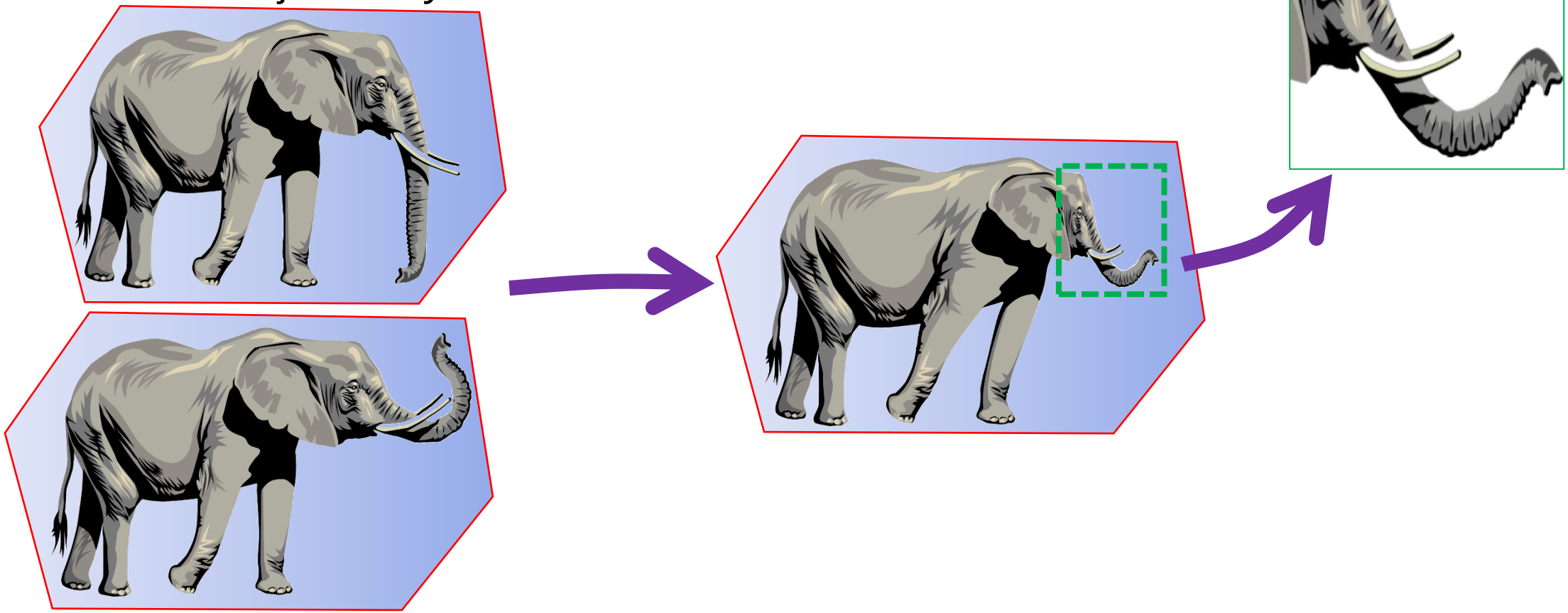
Shape interpolation – previous work

- Linear coordinates interpolation



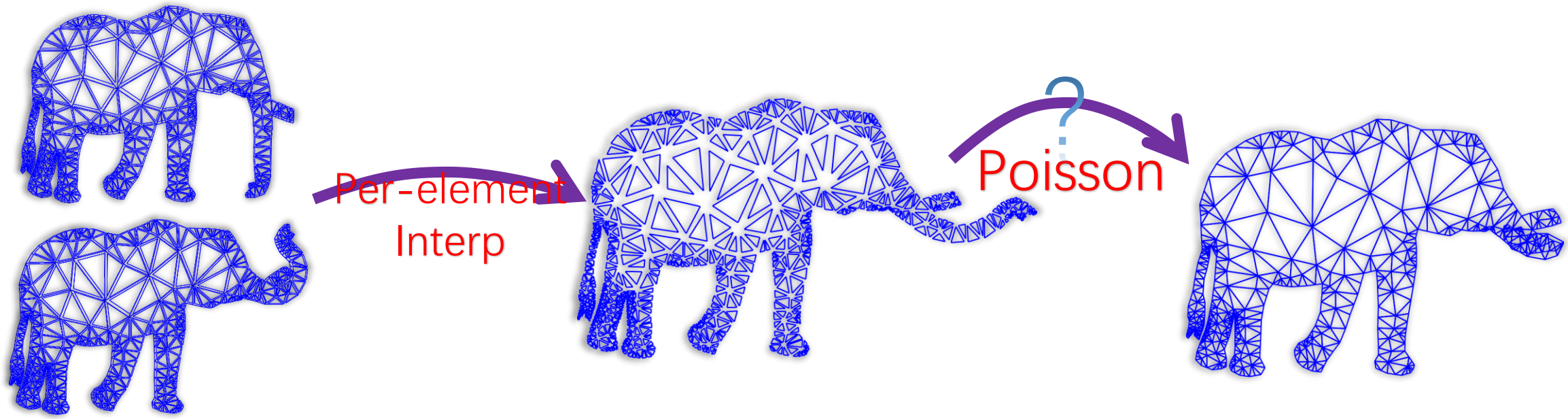
Shape interpolation – previous work

- Positive barycentric coordinates
  - Convex boundary
  - Guaranteed bijectivity

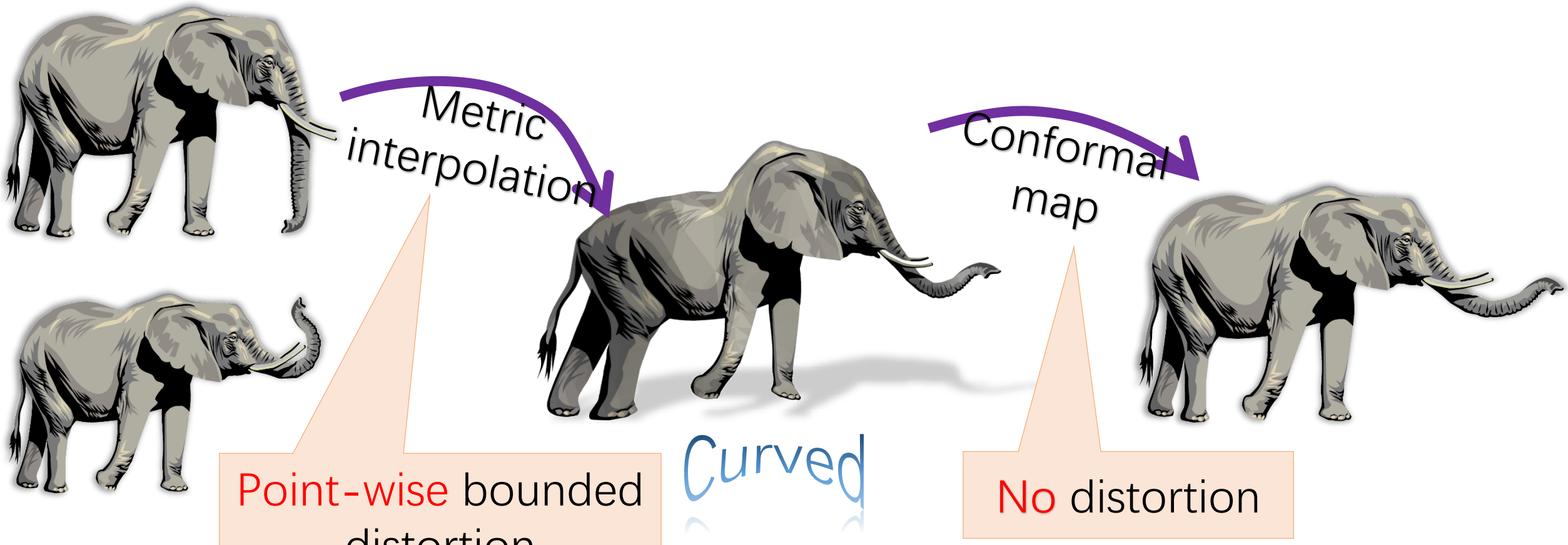


Shape interpolation – previous work

- Independent per-element(edge/face/triangle pair…) interpolation + Poisson
  - **ARAP interpolation [Alexa *et. al*. 2000]**
  - ARAP local/global [Chao *et. al*. 2010, Liu *et. al*. 2008]
  - Free-form motion processing [Kircher and Garland 2008]

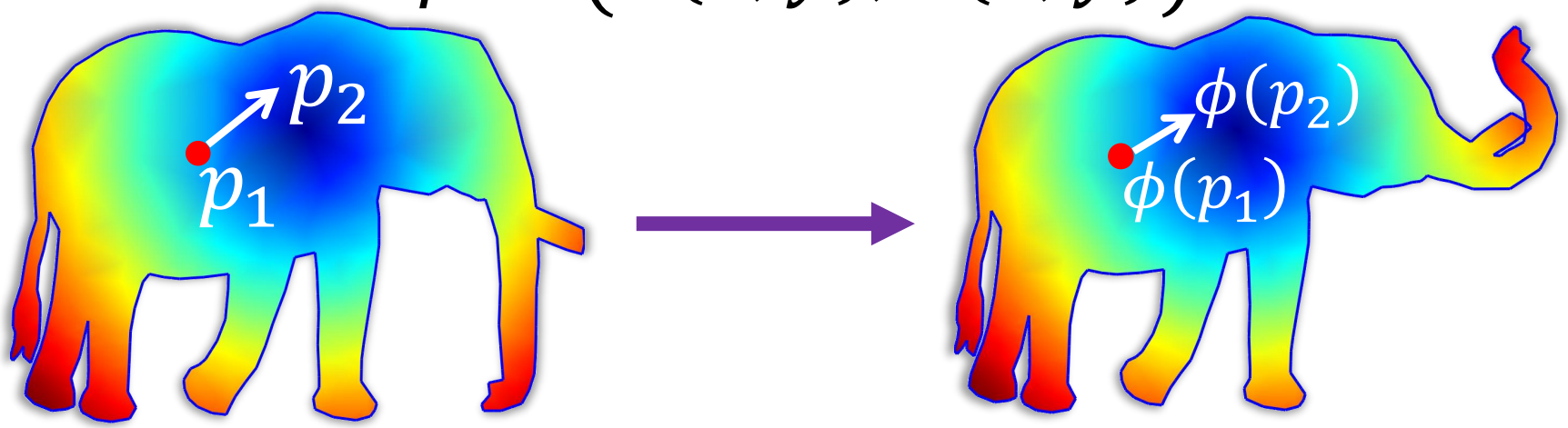


Our 2-step solution



Pullback metric

$$\phi = \big(u(x,y), v(x,y)\big)$$



The *Pullback* metric

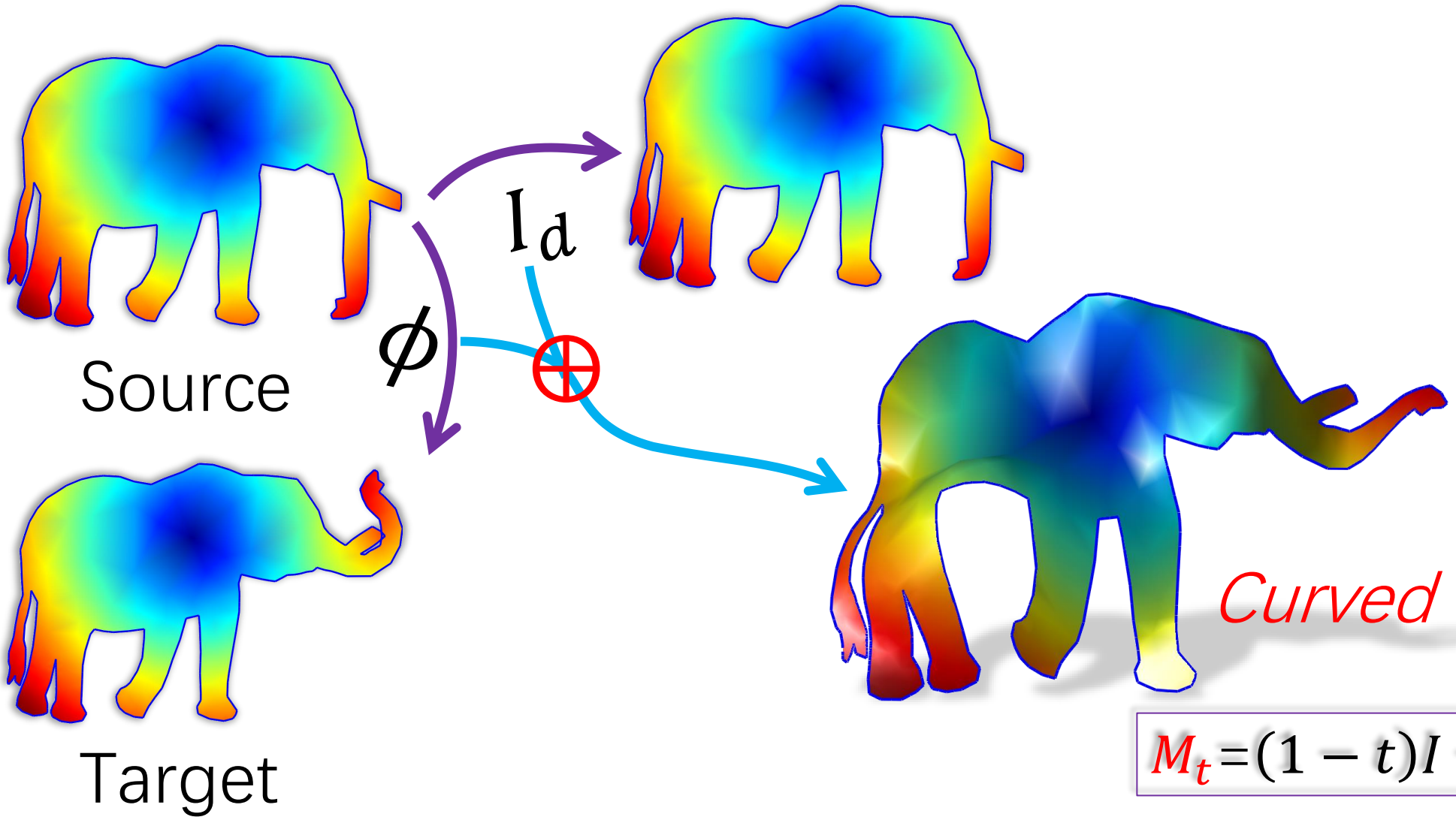
$$M = J^T J$$

$$J = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$$

$$\|\phi(p_1) - \phi(p_2)\| = (p_1 - p_2)^T M (p_1 - p_2)$$

Symmetric
Positive Definite

Bounded distortion interpolation – Step 1

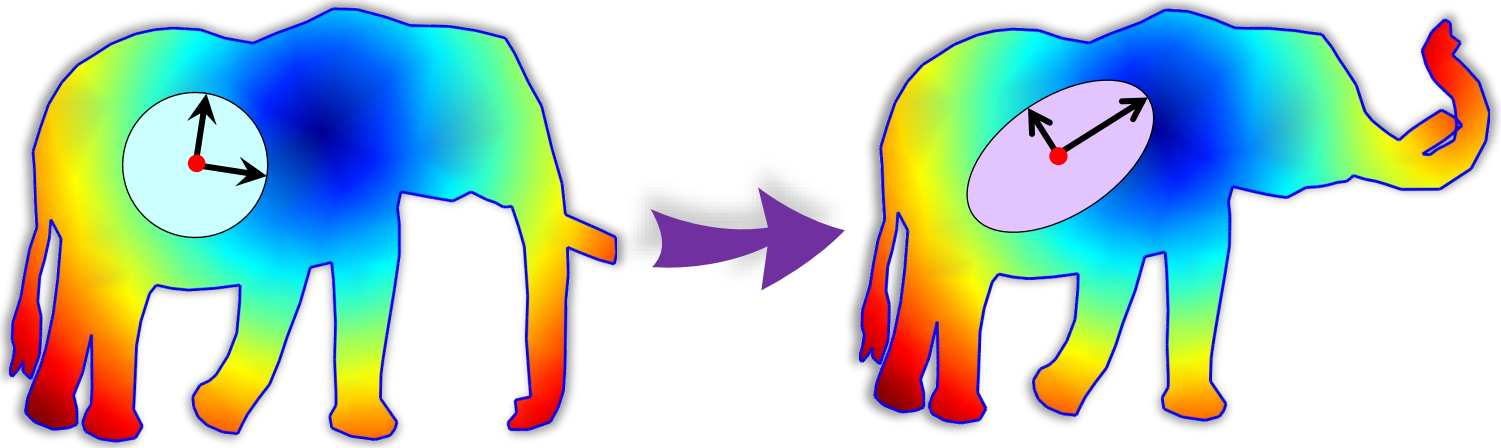


$$M_t = (1-t)I + tM(\phi)$$

Conformal distortion

Pullback metric:

$$M(\phi) = J^T(\phi)J(\phi)$$



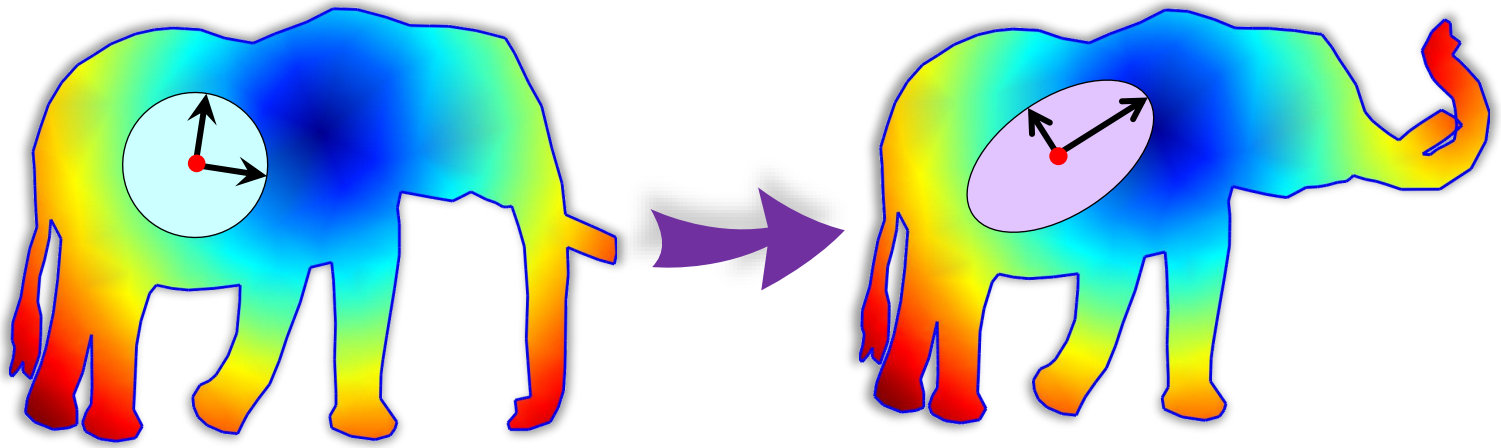
Conformal distortion: $K(M) = \sqrt{\frac{\lambda_1(M)}{\lambda_2(M)}}$

Eigen values

Key theorem

Pullback metric:

$$M(\phi) = J^T(\phi)J(\phi)$$

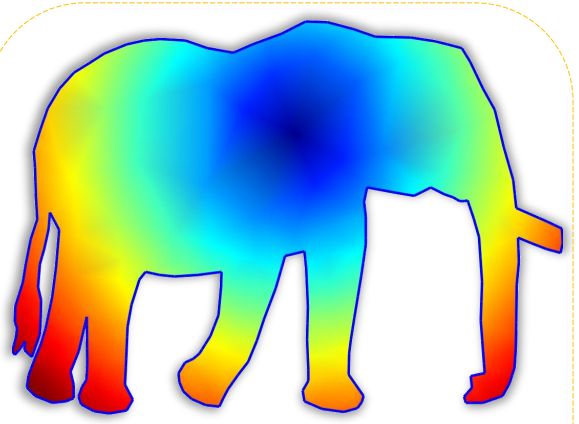


Given SPD matrices M_1, M_2

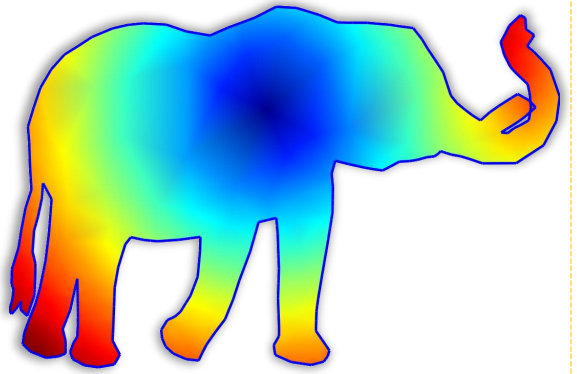
$$K_t \leq \max(K_1, K_2)$$

$$K_t = K\big((1-t)M_1 + tM_2\big)$$

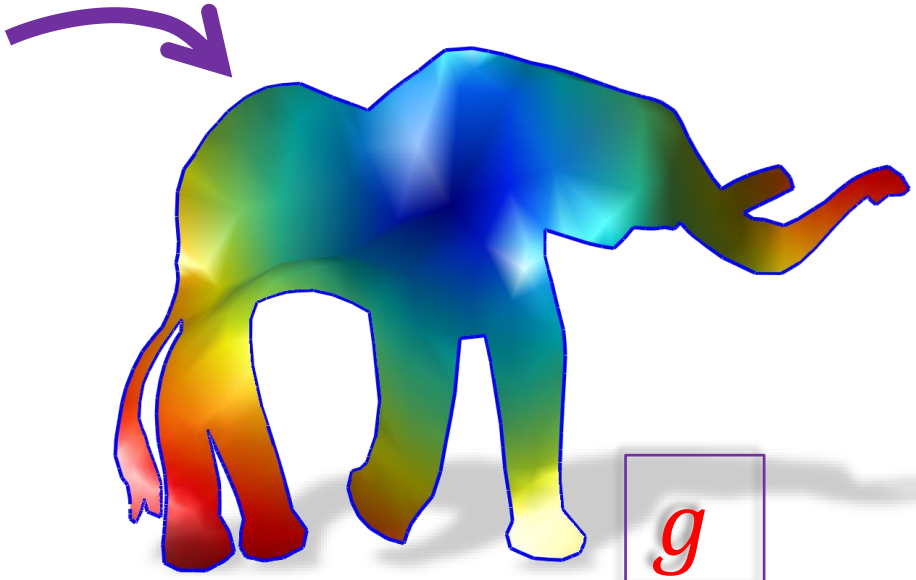
Bounded distortion interpolation – Step 2



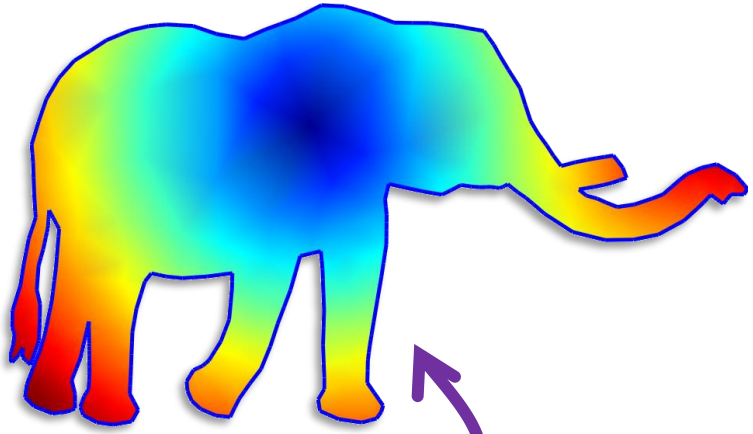
Source



Target



g

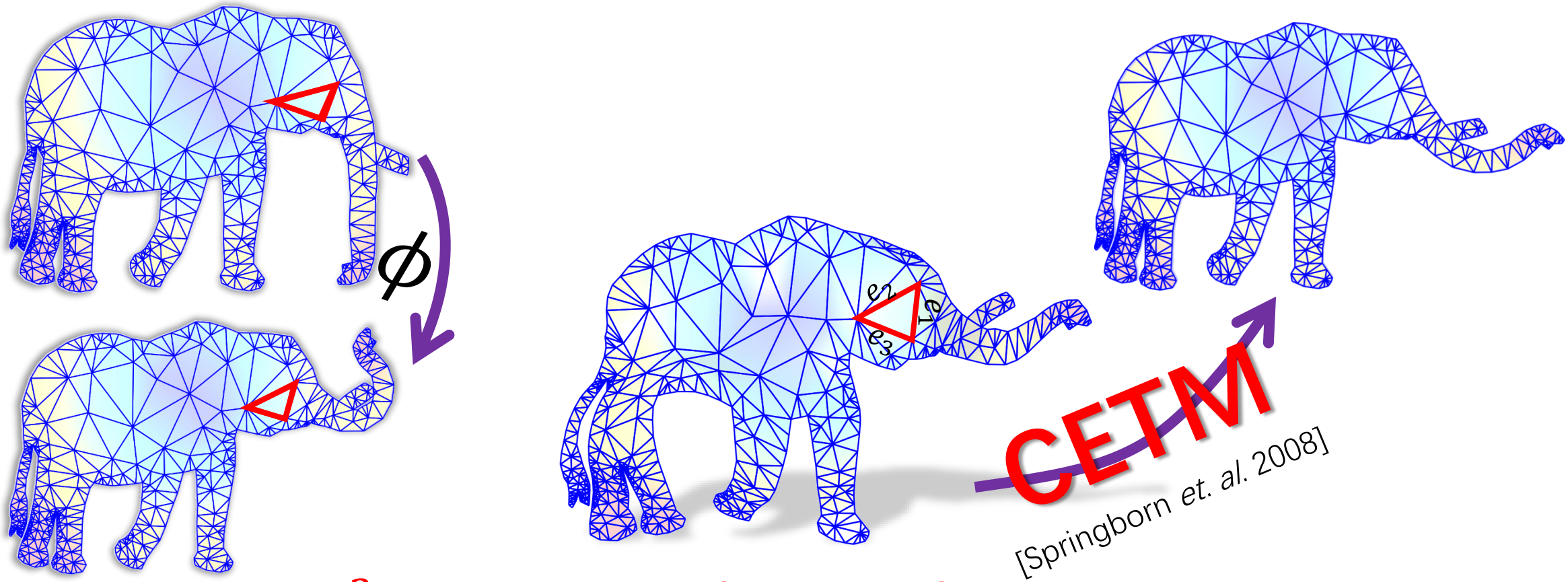


$\hat{g} = e^u g$

$\Delta u = k(g)$

u: conformal scaling factor
Δ: Laplace – Beltrami operator
k: Gaussian curvature

Discretization

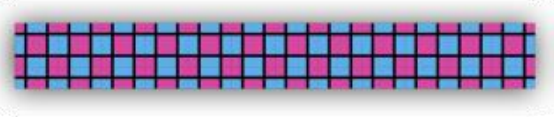


Input

$$\|e_i\|^2 = (1-t)\|e_{i0}\|^2 + t\|e_{i1}\|^2$$

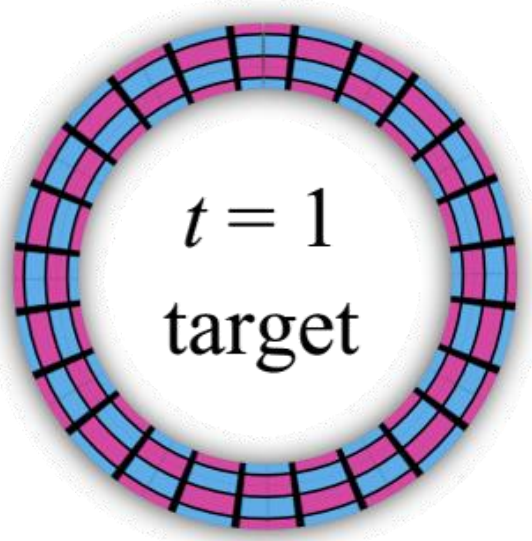
Square edge length interpolation

Results

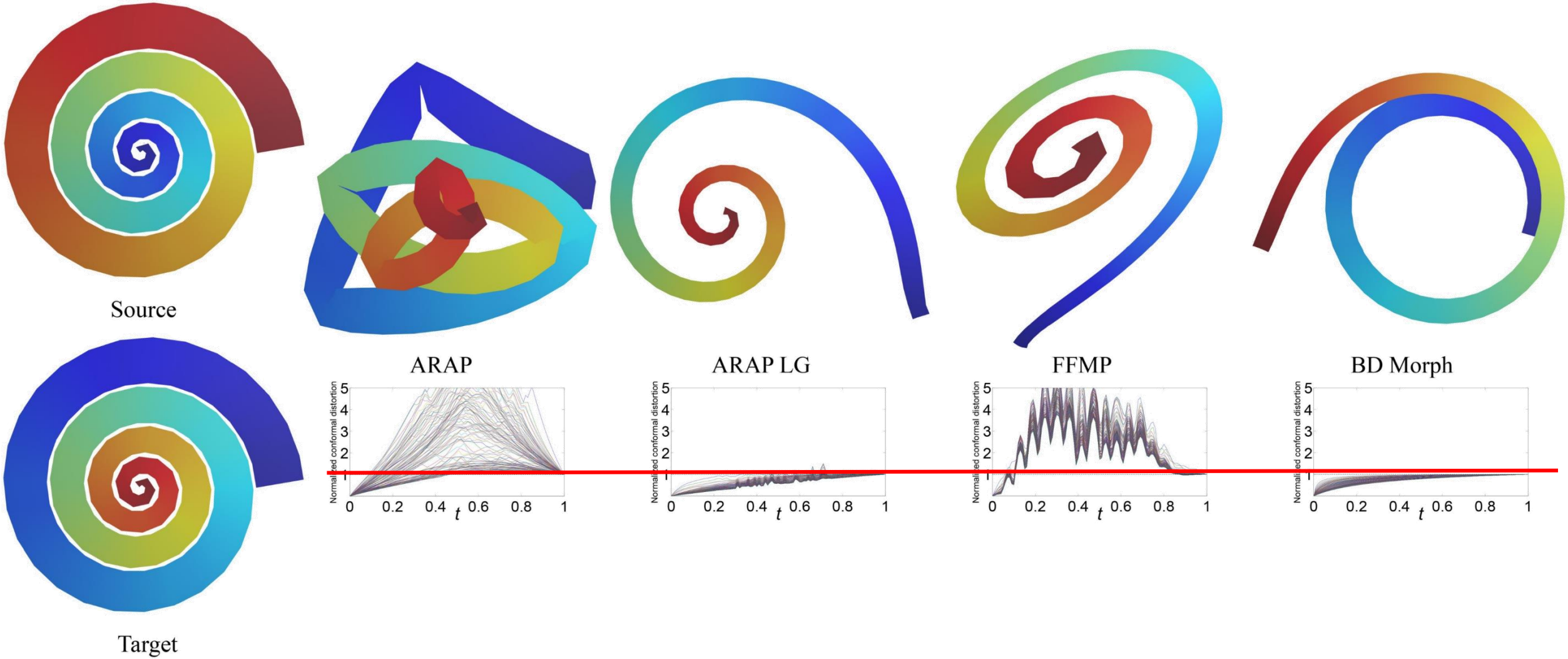


$t = 0$

source



Results – comparisons



Results

Source

Target

ARAP
[Alexa *et. al*. 2000]

FFMP
[Kircher and Garland 2008]

ARAP LG
[Chao *et. al*. 2010]

BD Morph

Results – multiple targets

Thank you!

Questions?