中国科学技术大学
University of Science and Technology of China

GAMES 301：第9讲

基于调和映射的高质量形变

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无翻转光滑映射

1. 基于光滑基函数的光滑映射

- RBF
- 广义重心坐标
- 调和映射
- 样条 (B-Spline)

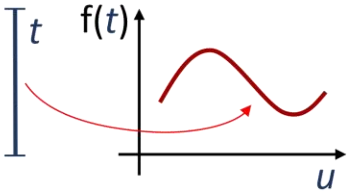
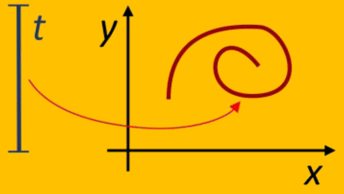
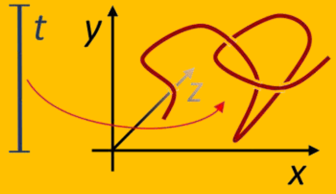
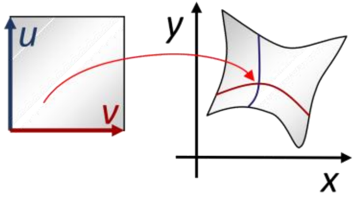
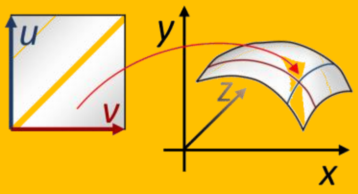
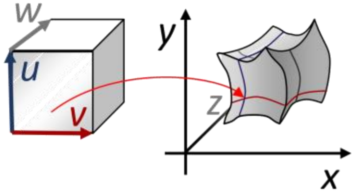
2. 无翻转光滑映射

- 样条基的凸包性质
- Lipschitz连续性

第13讲: 参数化应用3 – 高阶网格生成、曲面对应

第9讲:基于调和映射的高质量形变

Parameterization， Deformation & Mappings

	Output: 1D	Output: 2D	Output: 3D
Input: 1D	 Function graph	 Plane curve	 Space curve
Input: 2D		 Plane warp	 Surface
Input: 3D			 Space warp

Application – keyframe animations

- Model key poses/frames

Shape deformation

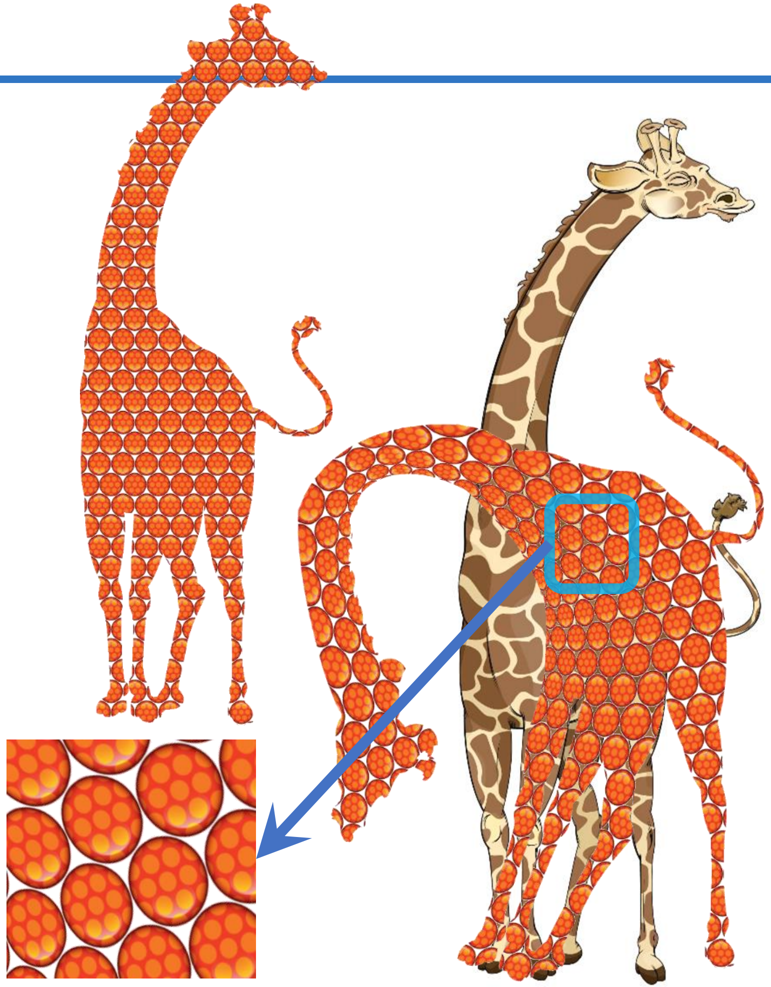
- Fill in between key poses

Shape interpolation

Shape deformation

- ✓Intuitive user-interface
  - ✓Drag and drop
- ✓Fast computation
  - ✓Interactive
- ✓High quality
  - ✓Smooth
  - ✓Locally injective (no foldovers)
  - ✓Bounded distortion

Good Map



Poranne & Lipman 2014, Provably Good Planar Maps

Deformation - Previous work

- Mesh-based
  - **Extremal quasiconformal** maps [Weber *et al.* 2012]
  - **Bounded distortion** mapping spaces [Lipman 2012]
  - **Locally injective** mappings [Schüller *et al.* 2013]
  - **Locally injective** parameterization [Weber & Zorin 2014]
  - Planar shape interpolation with **bounded distortion** [Chen *et al.* 2013]

P.W.L. → ✘ not smooth

- Meshless

✔ smooth

  - Generalized barycentric coordinates

✘ not locally injective
✘ no distortion bounds

  - Controllable **conformal** maps [Weber & Gotsman 2010]

✘ no positional constraints

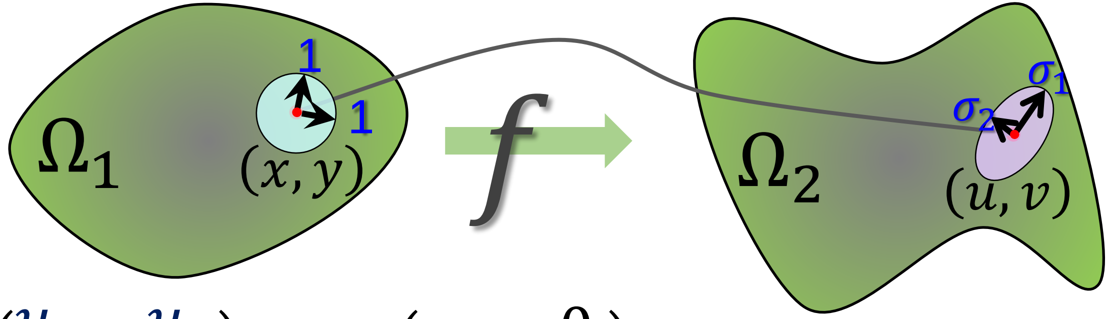
  - Provably good planar maps [Poranne & Lipman 2014]

R.B.F. → ✘ not shape aware

Planar map - notations

$$f: \Omega_1 \to \Omega_2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} u = u(x,y) \\ v = v(x,y) \end{pmatrix}$$



$$J = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix} = U \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} V$$

$$\sigma_1 \geq \sigma_2 \geq 0$$

$$f(z) = f(x + iy) \qquad f: \mathbb{C} \to \mathbb{C}$$

$$f_z = \frac{1}{2}\left(u_x + v_y + i\left(v_x - u_y\right)\right)$$

$$f_{\bar{z}} = \frac{1}{2}\left(u_x - v_y + i\left(v_x + u_y\right)\right)$$

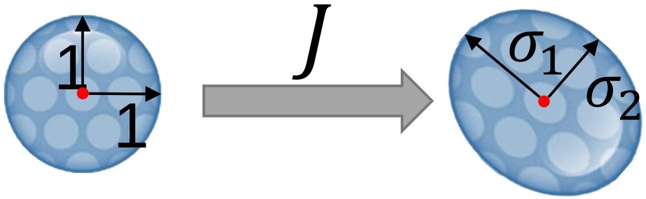
$$\sigma_1 = |f_z| + |f_{\bar{z}}|$$

$$\sigma_2 = \big||f_z| - |f_{\bar{z}}|\big|$$

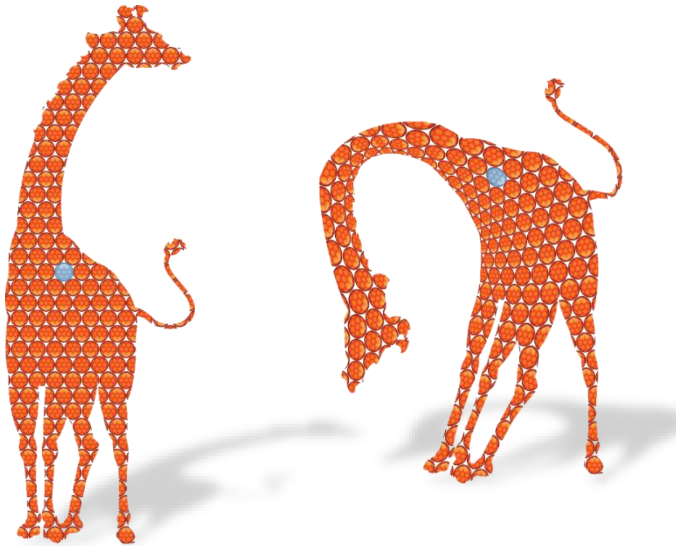
$$\sigma_2^s = |f_z| - |f_{\bar{z}}|$$

Distortions

$$\sigma_1 = |f_z| + |f_{\bar{z}}|$$
$$\sigma_2 = \big||f_z| - |f_{\bar{z}}|\big|$$



- Conformal
  - $k_f = \sigma_1 - \sigma_2 \searrow 0$
- Isometric
  - $\tau_f = \max(\sigma_1, {}^1/_{\sigma_2}) \searrow 1$
  - $\sigma_1^2 + \sigma_2^2 + \sigma_1^{-2} + \sigma_2^{-2} \searrow 4$



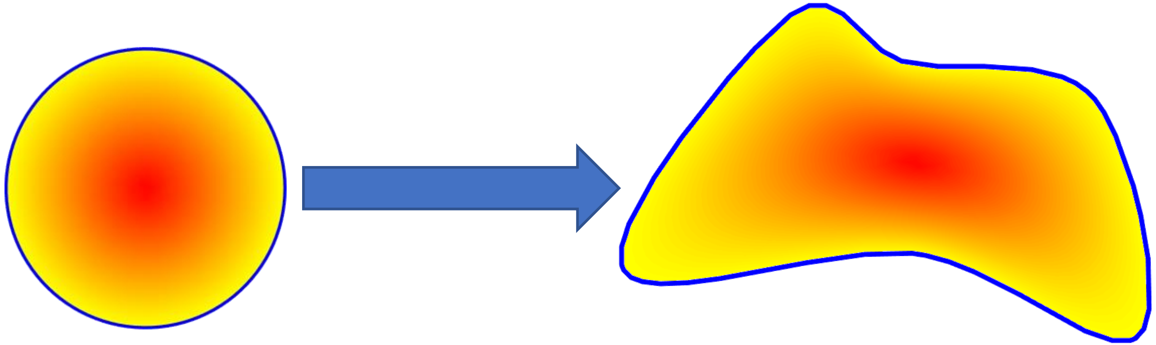
Planar Harmonic Deformation

Harmonic planar mapping

$$f(x,y) = \big(u(x,y), v(x,y)\big) \qquad f: \Omega \to \mathbb{R}^2$$

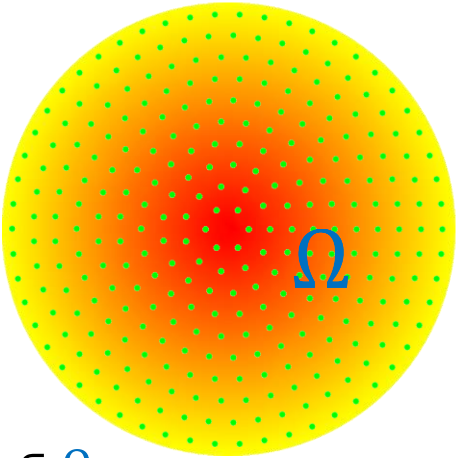
$$\Delta u = 0, \qquad \Delta v = 0$$

- Smooth
- Maximum/minimum principle
- Uniquely determined by boundary condition



Bounded distortion mapping

Bound the distortion at every point

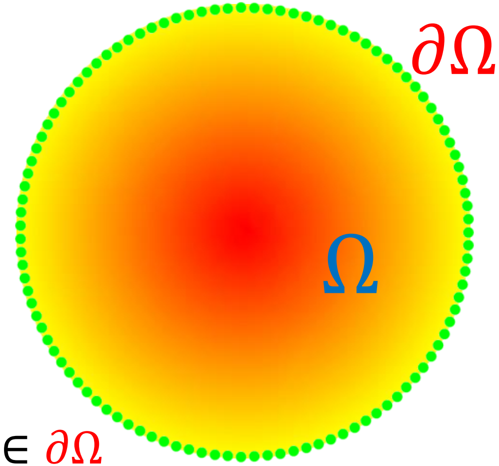


$\forall z \in \Omega$

$$k_f(z) \leq k$$

$$\tau_f(z) \leq \tau$$

Harmonic - Boundary only?



$\forall w \in \partial\Omega$

$$k_f(w) \leq k$$

$$\tau_f(w) \leq \tau$$

Key theorem on multiply-connected domain

Harmonic mapping f has bounded distortion (k, τ) on multiply-connected domain Ω, iff

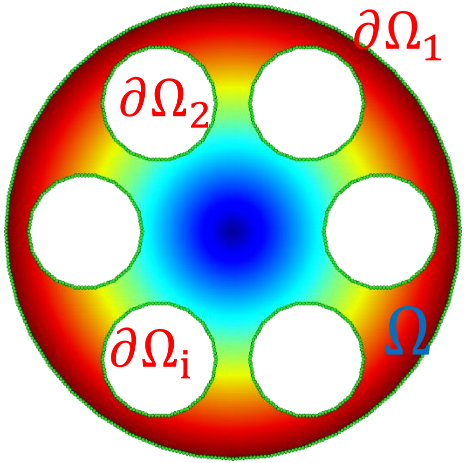
k_f, τ_f maximum principal $\Leftarrow$

$$\sum_i \oint_{\partial\Omega_i} \frac{f_z'(z)}{f_z(z)} dz = 0 \quad \Leftrightarrow f_z \neq 0, \forall z \in \Omega$$

$$k_f(w) \leq k$$
$$\tau_f(w) \leq \tau \qquad \forall w \in \partial\Omega$$

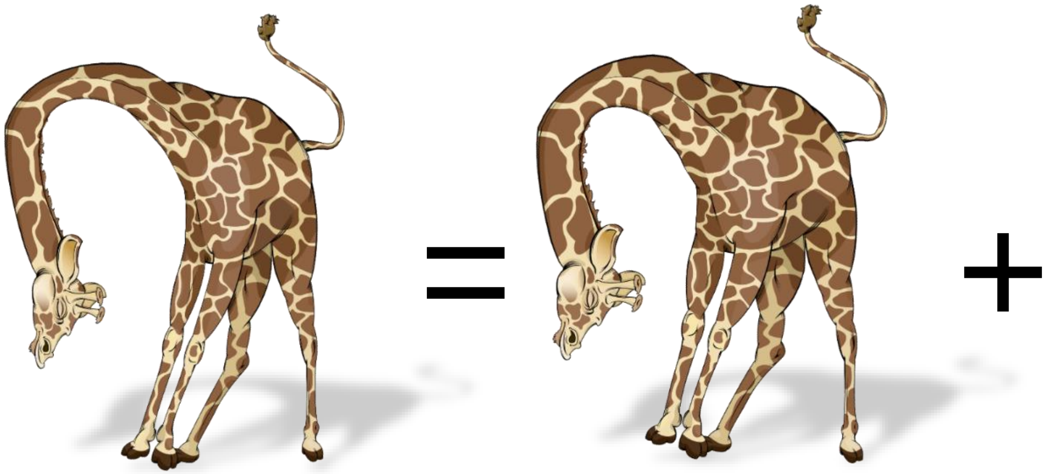
✘ Non-harmonic

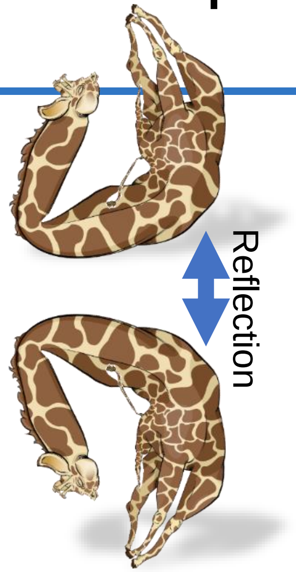
✘ Non-smooth



Chen & Weber, 2015, 2017

Harmonic mapping space



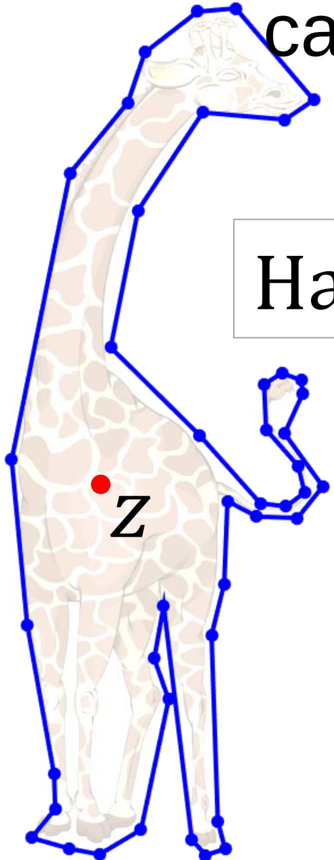


$$f = \Phi + \overline{\Psi}$$

- f: Harmonic
- Φ: Holomorphic (complex analytic)
- $\overline{\Psi}$: Anti-Holomorphic

Cauchy coordinates [Weber *et al*. 2009]

Cauchy complex barycentric coordinate



$$g(z) = \sum_i C_i(z) u_i = C(z) u$$

$$h(z) = C(z) v$$

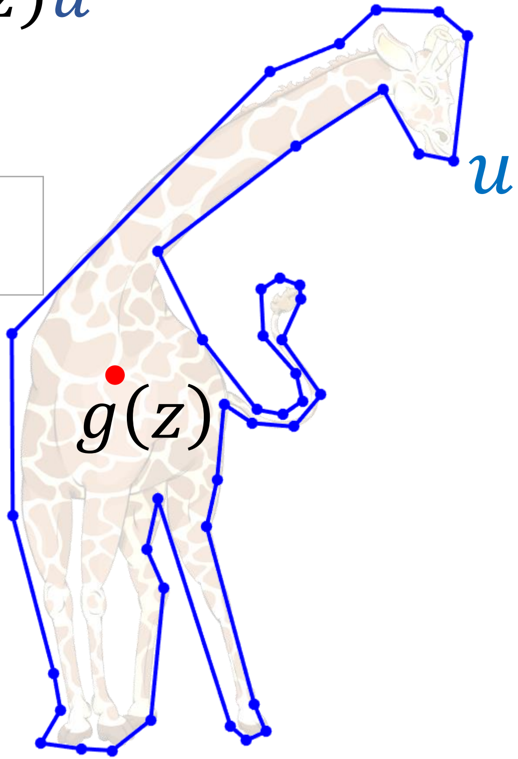
Harmonic: $f(z) = g(z) + \overline{h}(z)$

$$\Leftrightarrow (u, v)$$

$$g'(z) = \sum_i D_i(z) u_i = D(z) u$$

$$f_z = g'(z) = D(z) u$$

$$\overline{f_{\bar{z}}} = h'(z) = D(z) v$$



Bounded distortion harmonic deformation

- Input:
  - User prescribed bounds (k, τ)
  - Positional constraints
    - $\{p_i \to q_i, i = 1 \ldots n\}$
- Output
  - Locally injective harmonic mapping
  - Bounded conformal distortion
  - Bounded isometric distortion

$$\underset{f}{\text{minimize}}\, E_{ARAP}(f) + \lambda E_{p2p}(f)$$

$$s.t. \quad f \text{ is harmonic}$$

$$\oint_{\partial\Omega} \frac{f_z'(z)}{f_z(z)} dz = 0$$

$$\forall w \in \partial\Omega, \quad \begin{matrix} k_f(w) \le k \\ \tau_f(w) \le \tau \end{matrix}$$

Convexification [Lipman 2012]

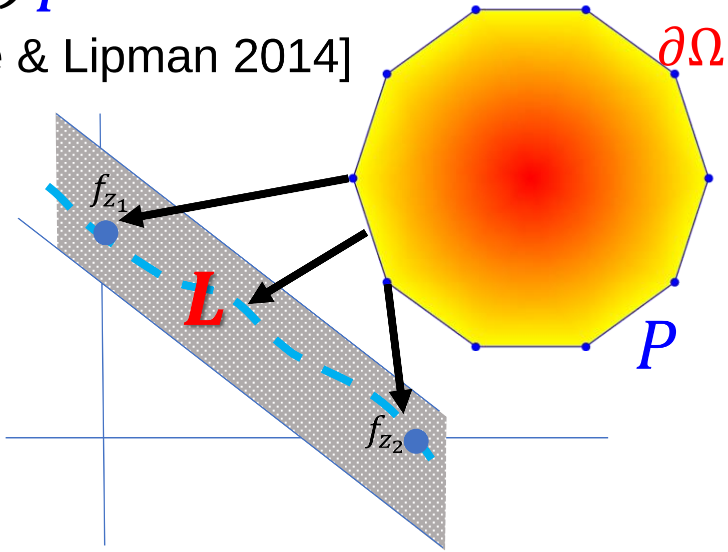
Global bound certification

$$(k,\tau)_{\partial\Omega} \leftarrow (k,\tau)_P$$

[Poranne & Lipman 2014]

$$k = \frac{|f_{\bar{z}}|}{|f_z|}$$

$$\tau = \max(\sigma_1, {}^1/_{\sigma_2}) = \max\left(|f_z| + |f_{\bar{z}}|, \frac{1}{||f_z| - |f_{\bar{z}}||}\right)$$

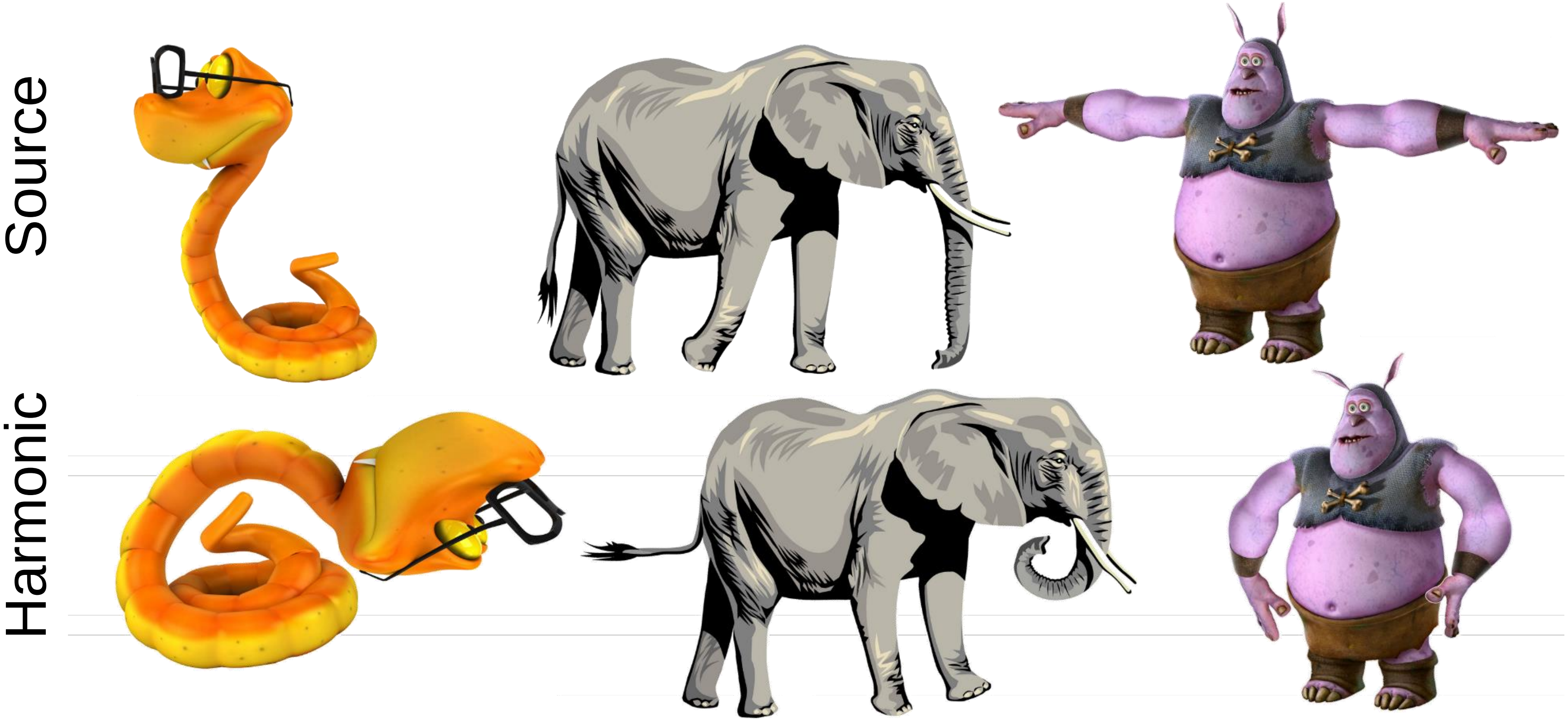


$f_z, f_{\bar{z}}$ are **Lipschitz** continuous

$$\forall z, w, |f_z(z) - f_z(w)| \leq L|z - w|$$

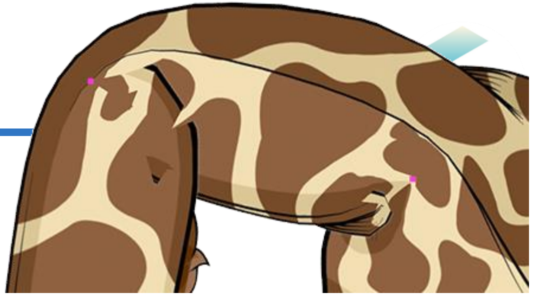
$$\Rightarrow \begin{cases} |f_z|_{min} \leq |f_z| \leq |f_z|_{max} \\ |f_{\bar{z}}|_{min} \leq |f_{\bar{z}}| \leq |f_{\bar{z}}|_{max} \end{cases}$$

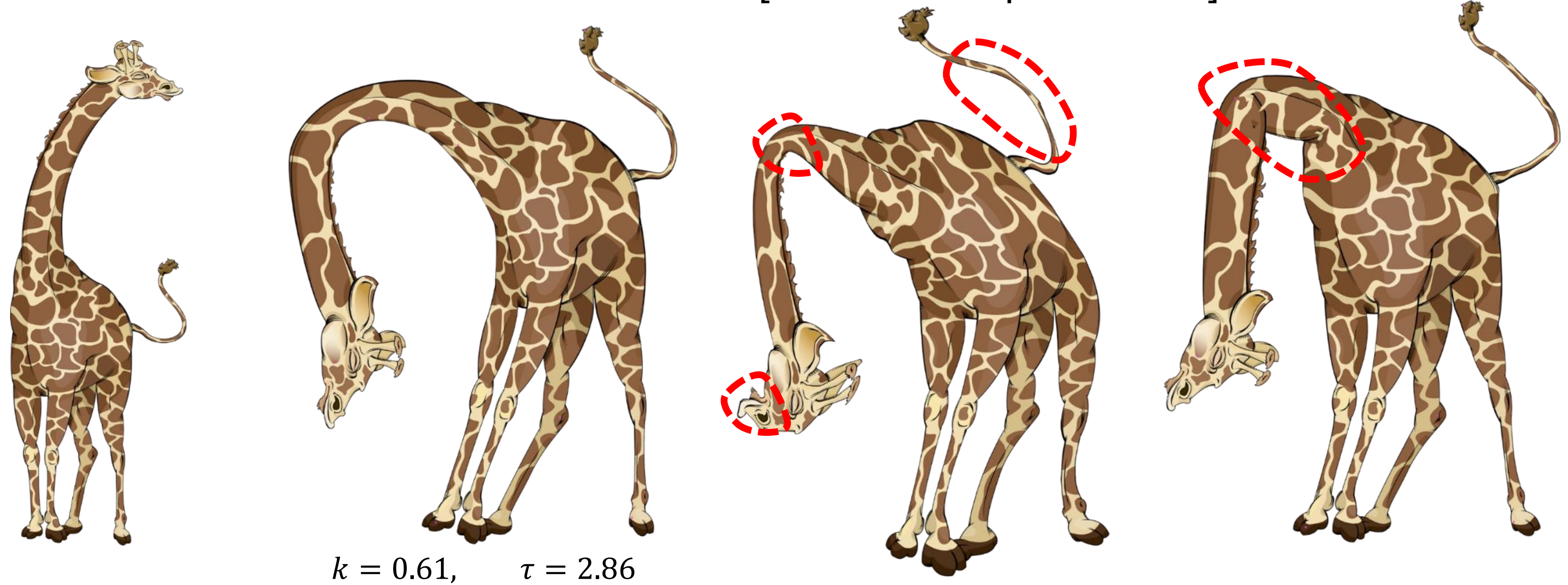
Results



$k = 0.2, \tau = 1.69$ $k = 0.27, \tau = 2.5$ $k = 0.11, \tau = 1.72$

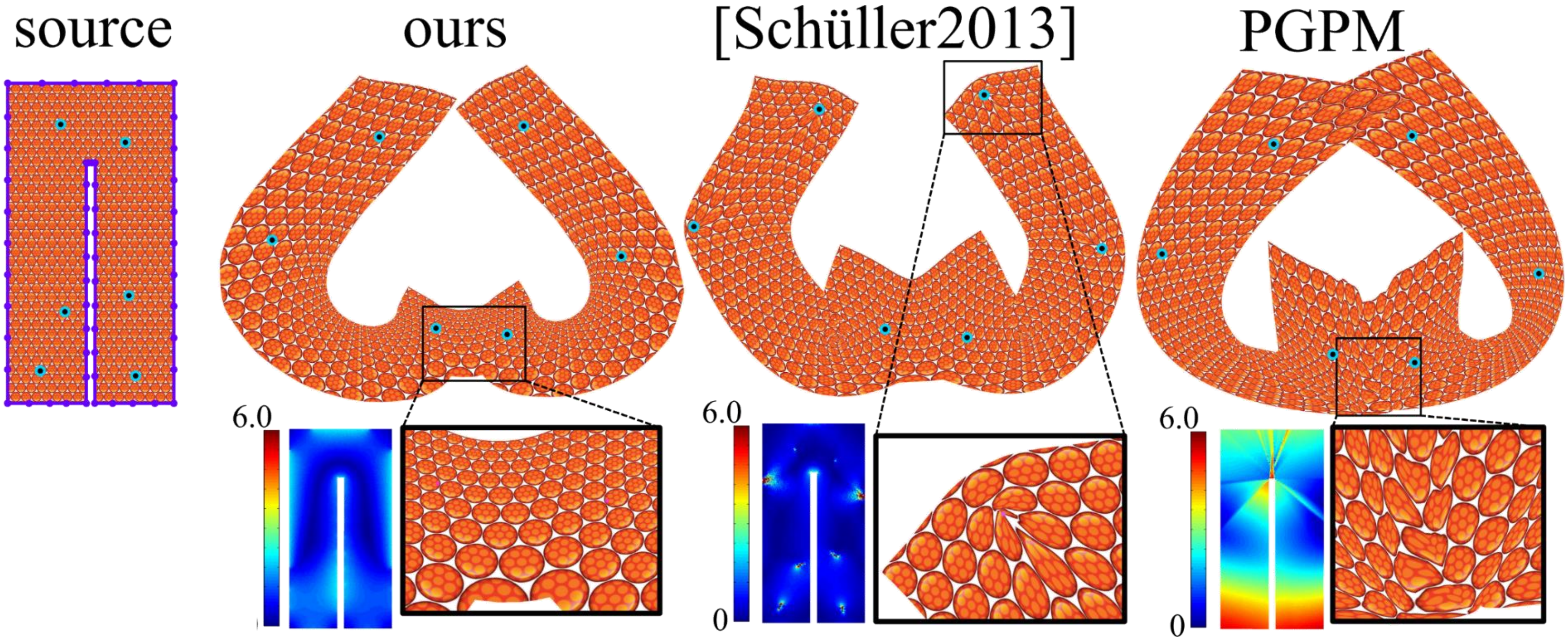
Results - comparisons





$k = 0.61, \qquad \tau = 2.86$

Results - comparisons



Limitations of BDHM

- Feasibility
  - Bounded distortion constraints
  - *Soft* positional constraints

- Non-convex optimization
  - Interactive, not real-time

Deformation – optimization

[Chen & Weber 2015]

- Iterative convexification
  - Conic optimization
- User-specified bounds
  - Feasibility

$$\min_{f} \ \mathrm{E}_{\mathrm{ARAP}}(f) + \lambda \mathrm{E}_{\mathrm{p2p}}(f)$$
$$s.t. \quad f \ is\ harmonic$$
$$\oint_{\partial\Omega} \frac{f_z'(z)}{f_z(z)} dz = 0$$
$$\forall w \in \partial\Omega, \quad \sigma_1^f(w) \le \boldsymbol{\sigma_1}, \quad \sigma_2^f(w) \ge \boldsymbol{\sigma_2}$$

Convexification [Lipman 2012]

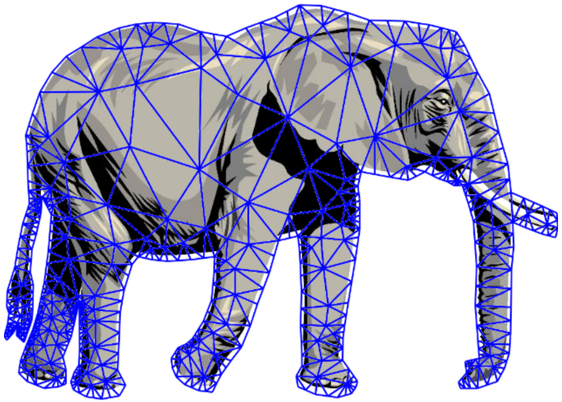
[Chen & Weber 2017]

- Newton's method
  - GPU acceleration
- Smooth isometric energy
  - Automatic distortion bounds
  - Unconstrained optimization

GPU Accelerated locally injective shape deformation

Piecewise linear mapping

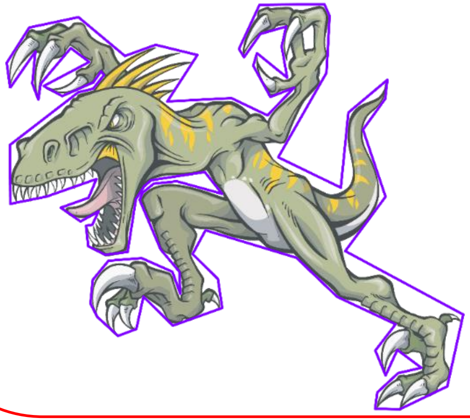
- Triangular mesh
- Pointwise (face-wise) constraints
- Sparse linear algebra
- Orientation preserving



Harmonic mapping

- Boundary element
- Boundary constraints
- Dense (small) linear algebra
- Locally injective

GPU friendly



Newton's method

$$\color{red}{Obj}: \text{ minimize } E(x)$$

- Taylor series

$$E(x) = \frac{1}{2}x'Hx + Gx + \cdots$$

- Iterative update

$$H(x)p = -G(x)$$
$$x_{n+1} = x_n + p$$

- Quadratic convergence

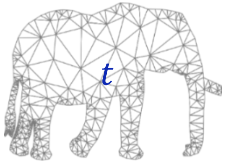
- Per-element modification

$$H^{+} = E\Lambda^{+}E^{T}$$

$$H > 0 \Rightarrow E(x_{n+1}) < E(x_n)$$

$$E(x) = \sum_t E_t(x) \Rightarrow H = \sum_t H_t$$

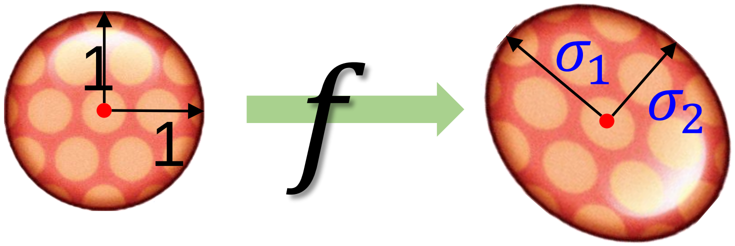
$$H^{+} \sim \sum_t H_t^{+}$$



Isometric energy

$$\mathrm{E_{iso}}(\sigma_1, \sigma_2)$$

$$\sigma_1 = |f_z| + |f_{\bar{z}}|, \ \sigma_2 = |f_z| - |f_{\bar{z}}|$$



$\sigma_1 \geq \sigma_2 \geq 0$

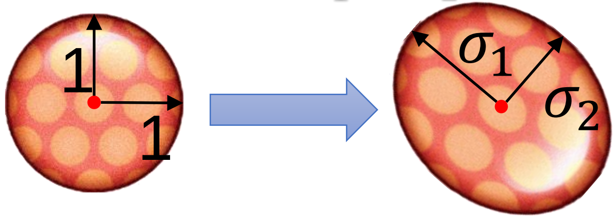
- Capture Rigidity
  - $\mathrm{E_{iso}}(1, 1) = 0$
- Barrier function
  - $\mathrm{E_{iso}}(\sigma_1, 0) = \infty$
  - Local injectivity
- Smooth, differentiable

1. $\mathrm{E_{ARAP}} = (\sigma_1 - 1)^2 + (\sigma_2 - 1)^2$ ✗ [Igarashi et al. 2007]
2. $\tau = \max\left(\sigma_1, \frac{1}{\sigma_2}\right)$ ✗ [Sorkine et al. 2002]
3. Symmetric Dirichlet $\mathrm{E_{iso}} = \sigma_1^2 + \sigma_2^2 + \sigma_1^{-2} + \sigma_2^{-2}$ ✓ [Smith & Schafer 2015]
4. $\mathrm{E_{exp}} = e^{s\mathrm{E_{iso}}}$ ✓ [Rabinovich et al. 2017]
5. $\mathrm{E_{AMIPS}} = e^{s\left(\frac{\sigma_1}{\sigma_2}+\frac{\sigma_2}{\sigma_1}\right)+\sigma_1\sigma_2+\frac{1}{\sigma_1\sigma_2}}$ ✓ [Fu et al. 2015]

Locally injective harmonic deformation

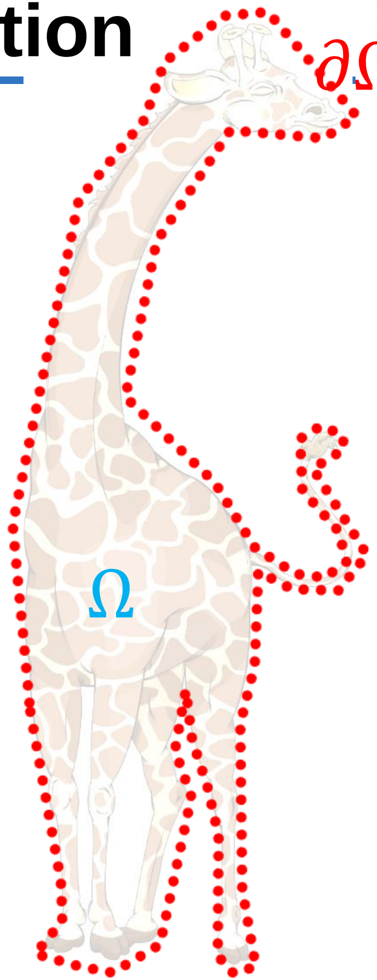
$\sigma_1 \geq \sigma_2 \geq 0$

$$\mathrm{E_{iso}} = \sigma_1^2 + \frac{1}{\sigma_1^2} + \sigma_2^2 + \frac{1}{\sigma_2^2}$$



$$\text{minimize } \mathrm{E}_{\mathrm{iso}}^{f} = \int_{\partial\Omega} \mathrm{E_{iso}}$$

$$s.t.\ J(z) > 0, \forall z \in \partial\Omega$$



Locally injective harmonic deformation

$$\text{minimize } \mathrm{E}_{\mathrm{iso}}^{f} + \lambda \mathrm{E}_{\mathrm{P2P}}$$
$$s.t.\ J(z) > 0, \forall z \in \partial\Omega$$

$$\mathrm{E}_{\mathrm{iso}}^{f} = \sum_{z \in P \subset \partial\Omega} \mathrm{E}_{\mathrm{iso}}(\mathrm{z})$$

- Positional constraints
  - $\mathrm{E}_{\mathrm{P2P}} = \|f(x) - y\|^2$
- Newton's method
  1. $G = \nabla E,\ H = \nabla^2 E,\ H^+ \to H$
  2. $\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = -H^{-1} G$
  3. $\begin{pmatrix} u \\ v \end{pmatrix} \leftarrow \begin{pmatrix} u \\ v \end{pmatrix} + t \begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix}$
- Local injective barrier

$$J(z) = \sigma_1 \sigma_2 \searrow 0^+ \Rightarrow \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} \nearrow \infty \quad \Rightarrow \exists t > 0, s.t. J(z) > 0, \forall z$$

Per-element SPD Hessian

$$\underbrace{H^+}_{4n\times 4n} \sim \sum_{z\in\partial\Omega} H(z)^+$$

$$H(z)^+ = E\Lambda^+E^T$$

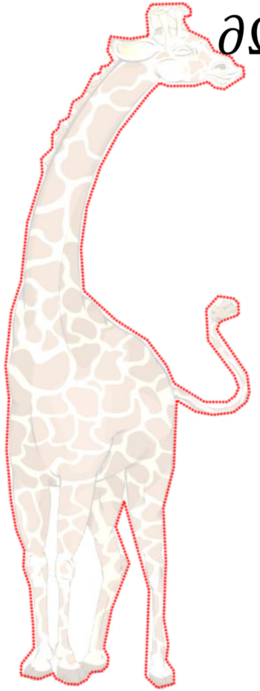
$$\mathrm{E}_{\mathrm{iso}}(\sigma_1,\sigma_2) = \mathrm{E}(|f_z|^2, |f_{\bar z}|^2) \qquad \begin{cases} f_z = D(z)u \\ \overline{f_{\bar z}} = D(z)v \end{cases}$$

$$\Rightarrow \underbrace{H(z)}_{4n\times 4n} = \nabla^2\mathrm{E}_{\mathrm{iso}} = M^T \times \underbrace{K}_{4\times 4} \times M \qquad M = \begin{pmatrix} D & 0 \\ 0 & D \end{pmatrix}$$

$$\boxed{MM^T = rI} \quad \Rightarrow \quad \boxed{H(z)^+ = M^T K^+ M}$$

Analytical 4x4 SPD projection: $K^+ = E\Lambda^+E^T$

$\partial\Omega$



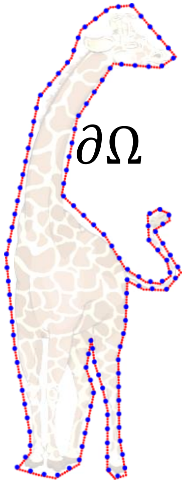
Hessian approximation

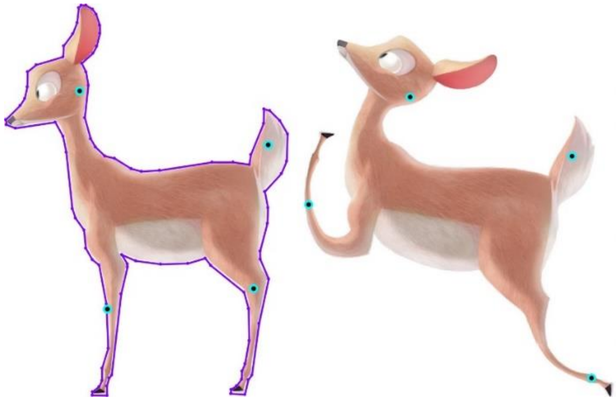
$$\mathrm{E}_{\mathrm{iso}}^{f} = \int_{\partial\Omega} \mathrm{E}_{\mathrm{iso}}(\mathrm{z}) \approx \sum_{z \in P \subset \partial\Omega} \mathrm{E}_{\mathrm{iso}}(z)$$

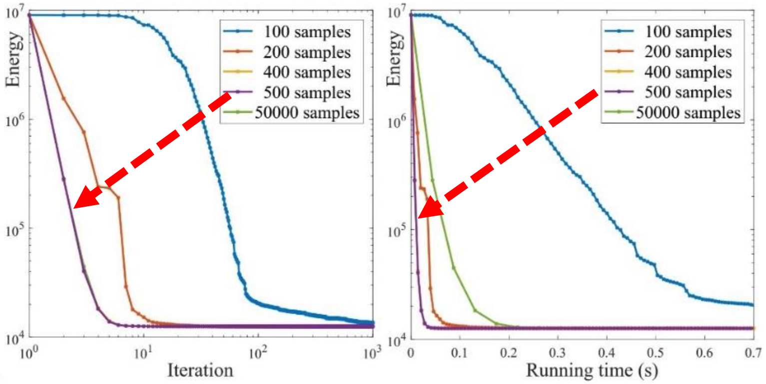
$$\Rightarrow \begin{matrix} g^{f} \approx \sum_{z \in P} g(z) \\ H^{f} \approx \sum_{z \in P' \subset P} H(z) \end{matrix}$$

$H(z) \gg \mathrm{g}(\mathrm{z})$

Complexity







Newton iteration on GPU

$$\mathrm{E_{iso}} = \sum_z \mathrm{E}(|f_z|^2, |f_{\bar{z}}|^2)$$

$$\begin{cases} f_z = Du \\ \overline{f_{\bar{z}}} = Dv \end{cases}$$

1. $g = \nabla \mathrm{E},\ H = \nabla^2 \mathrm{E},\ H^+ \rightarrow H$

$$g = D^T D(u \quad \mathrm{v})$$

$$H^+ = M^T K^+ M$$

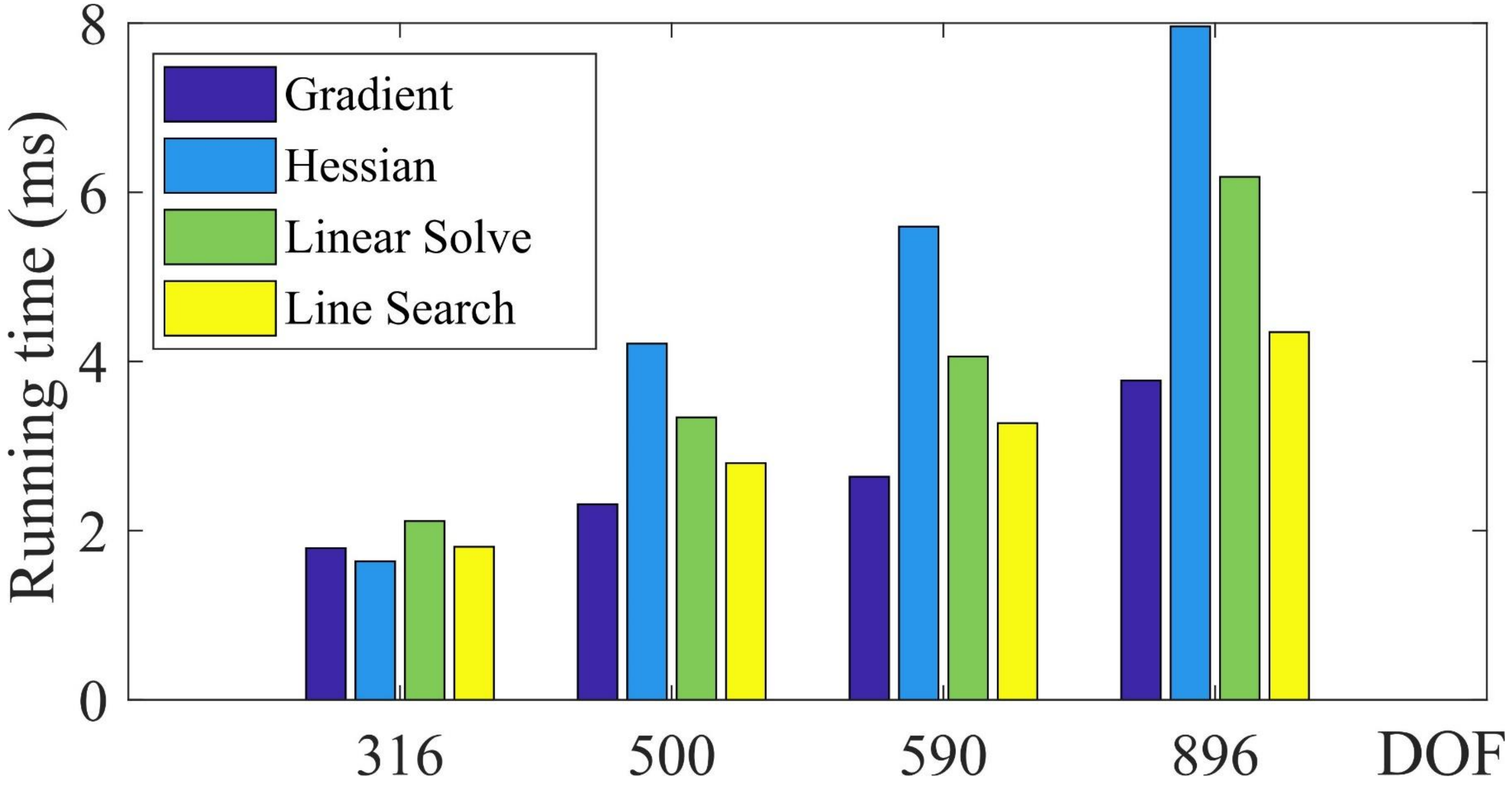
2. $\Delta = -H^{-1} g$

3. $\begin{pmatrix} u \\ v \end{pmatrix} \leftarrow \begin{pmatrix} u \\ v \end{pmatrix} + t\Delta$

cuBLAS

cuSolver

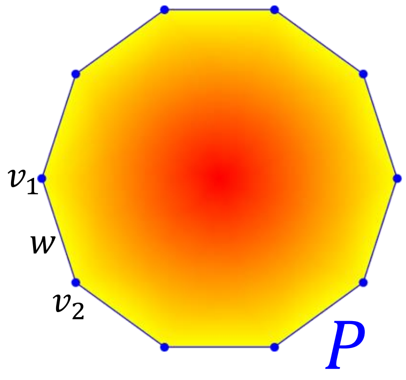
GPU Balance



Gradient
Hessian
Linear Solve
Line Search
Running time (ms)
0
2
4
6
8
316
500
590
896
DOF

Local Injectivity with Lipschitz Continuity

$$|J| = \sigma_1 \sigma_2 > 0 \Leftrightarrow \sigma_2 = |f_z| - |f_{\bar{z}}| > 0$$



$$|f_z|_{\min} = \frac{|f_z(v_1)| + |f_z(v_2)| - L_{f_z} l}{2} \leq \min_{w \in [v_1, v_2]} |f_z(w)|$$

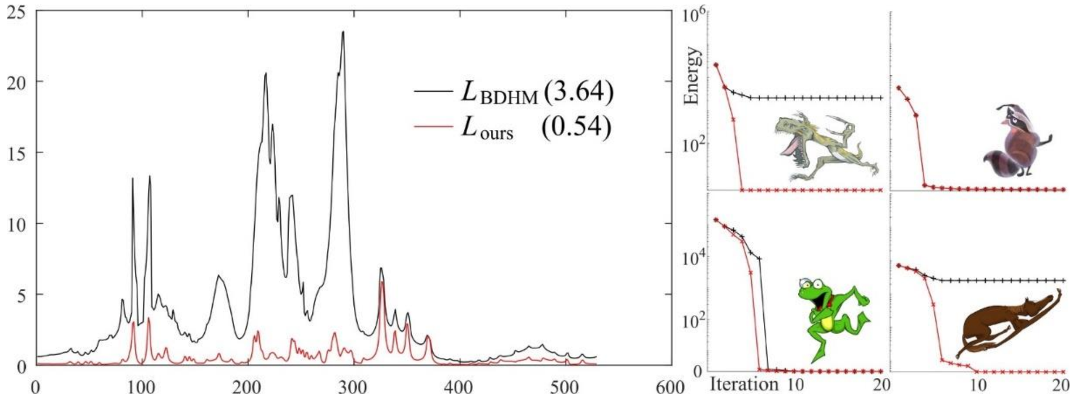
$$|f_{\bar{z}}|_{\max} = \frac{|f_{\bar{z}}(v_1)| + |f_{\bar{z}}(v_2)| + L_{f_{\bar{z}}} l}{2} \leq \min_{w \in [v_1, v_2]} |f_{\bar{z}}(w)|$$

$$\Rightarrow \sigma_2(w) \geq |f_z|_{\min} - |f_{\bar{z}}|_{\max} > 0 \Leftrightarrow \sigma_2(v_1) + \sigma_2(v_2) \geq \left(L_{f_{\bar{z}}} + L_{f_z}\right) l$$

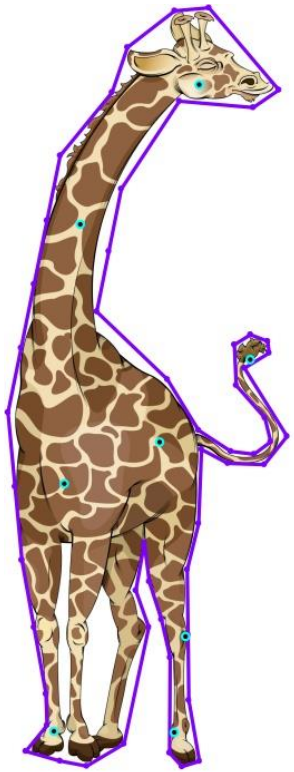
BDHM [Chen&Weber 15]: $L_{f_z} \geq |f_z'|_{\max}$

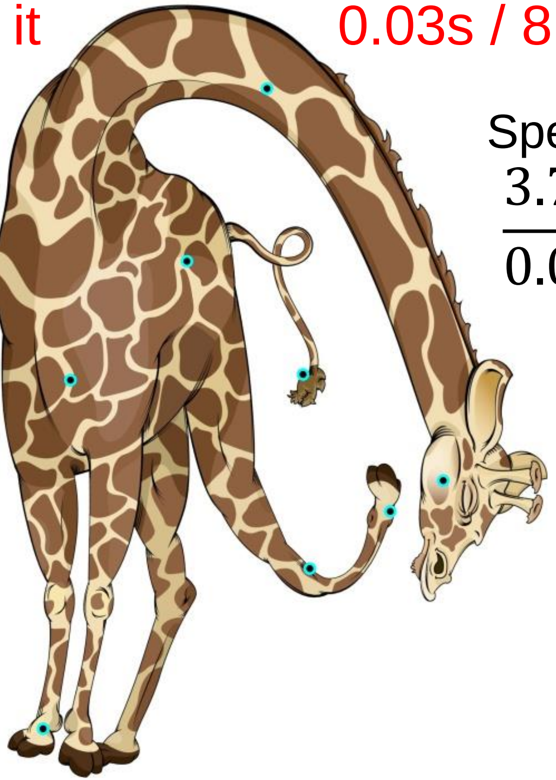
GLID [Chen&Weber 17]: $L_{f_z'} \geq |f_z''|_{\max}$

$$\Rightarrow L_{f_z} = \frac{|f_z'(v_1)| + |f_z'(v_2)| + L_{f_z'} l}{2}$$



Results & Comparison

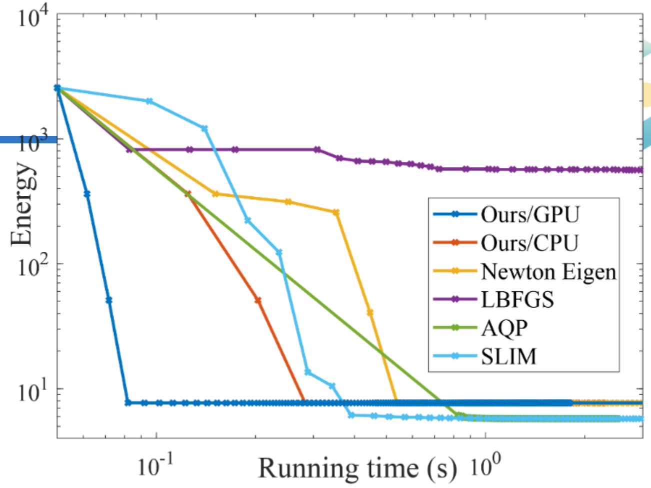


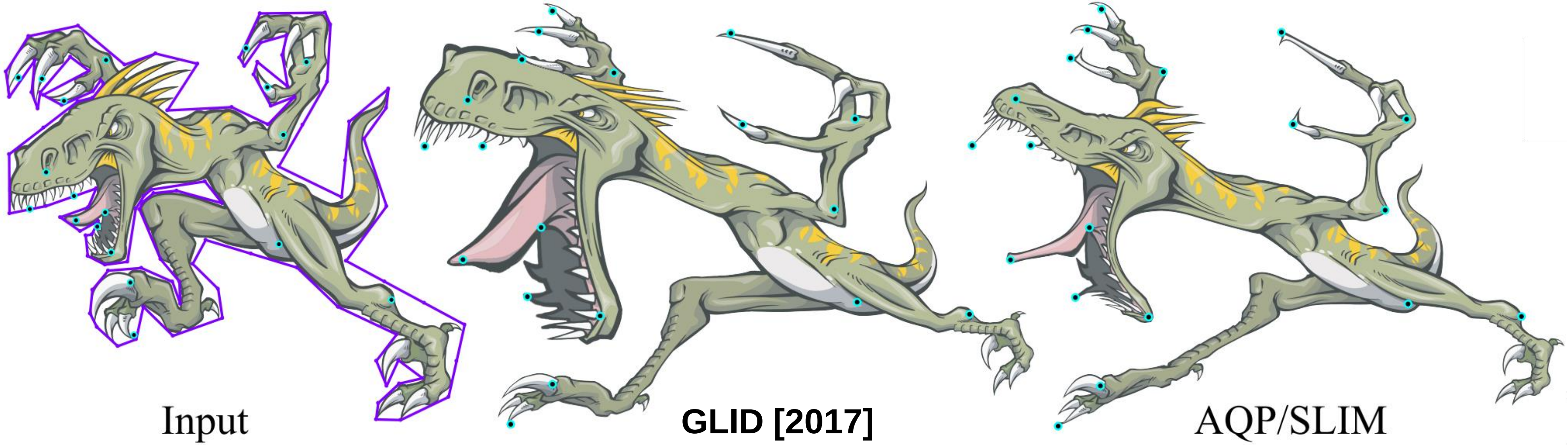


Speedup factor:

$$\frac{3.71}{0.03} = 125\times$$

Results & Comparison





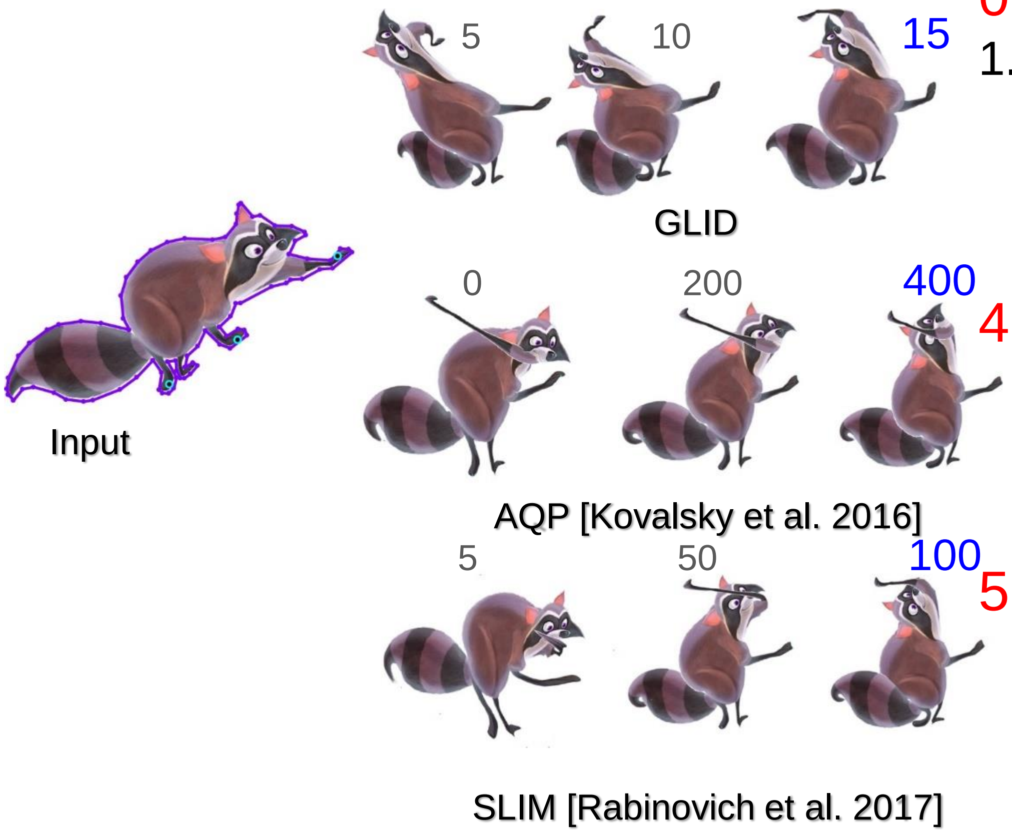
GLID [2017]


AQP


SLIM


Ours

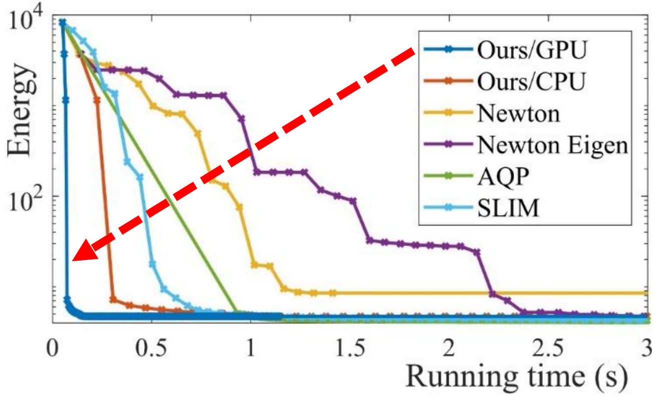
Results & Comparison



0.11s GPU
1.18s CPU

4.32s

5.49s



AQP

SLIM

Ours

Runtime/iteration

ms	DOF	TitanX Ours/GPU	6 cores 12 threads Ours/CPU	Newton Eigen	LBFGS	SLIM	AQP / Initialization	
deer	364	8.87	38.8	49.6	54.4	69.3	7.84	1239.7
archery	596	15.31	71.2	95.1	88.3	59.4	9.27	1266.3
giraffe	624	14.05	58.1	86.8	54.9	67.0	8.27	1182.9
rex	392	11.7	61.6	77.6	49.5	72.8	8.52	1141.0
racoon	320	7.25	34.4	39.9	68.1	70.4	9.01	904.6
raptor	416	10.18	45.2	56.5	82.3	66.3	8.64	883.4

SLIM

Ours

Recap

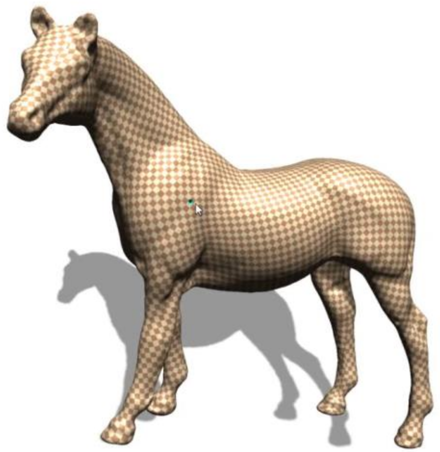
- Harmonic maps for deformation
  - Good mapping
    - C^∞ smooth
    - Bounded distortion theorem
  - Construction with Cauchy coordinates

- Newton's method
  - Isometric energies
  - Analytic Hessian + SPD modification
  - GPU acceleration

Harmonic Volumetric Deformation

Shape Deformation in 3D

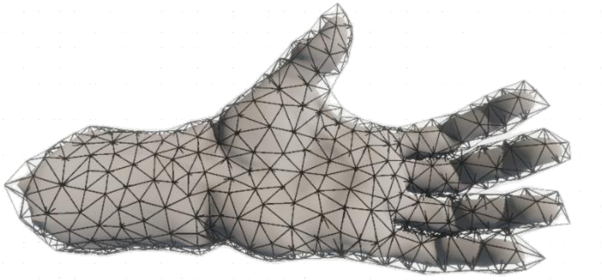
- ✓**Intuitive user-interface**
  - ✓Drag and drop
- ✓**Fast computation**
  - ✓Interactive
- ✓**High quality**
  - ✓Smooth (C^{∞})
  - ✓Locally injective (no foldovers)
  - ✓Low distortion



Locally Injective Volumetric Deformation

Piecewise linear map (PWL)

- Non-smooth
- Per-element constraints
- Sparse (**large**) linear algebra

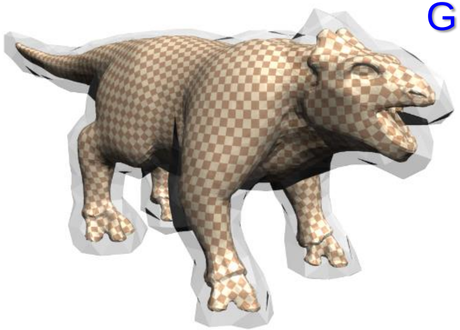


Tetrahedral mesh

Harmonic map

- Smooth
- Sparse sampling set constraints
- Dense (**small**) linear algebra

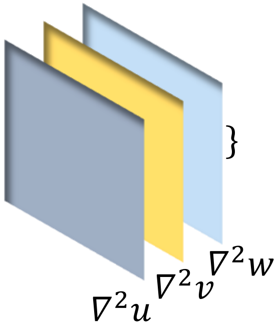
GPU friendly

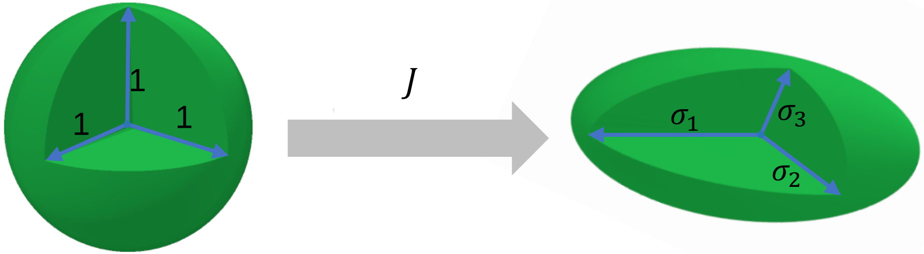


Volumetric Mapping - Notations

- $f(x,y,z) = (u(x,y,z), v(x,y,z), w(x,y,z))$ $\quad f: \Omega \to \mathbb{R}^3$

- $J = \nabla f = U \begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{pmatrix} V^T$

- $H = \nabla^2 f\{$  $\}$

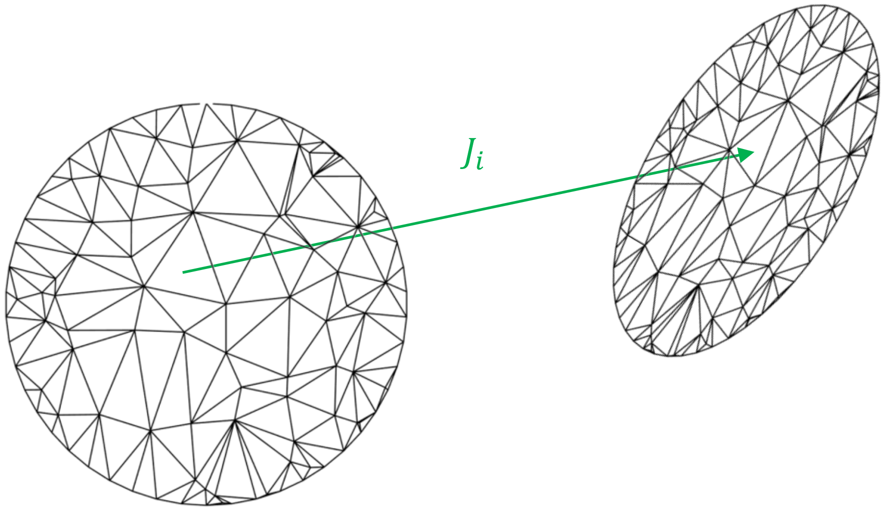


$\sigma_1 \geq \sigma_2 \geq \sigma_3$

Locally injective: $|J| > 0 \Leftrightarrow \sigma_3 > 0$

Locally Injective Certification

- Condition: $|J_i| > 0, \forall i$



Mesh-based

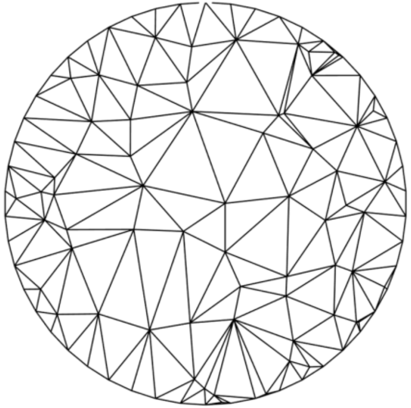
Locally Injective Certification

- Condition: $|J_i| > 0, \forall i$

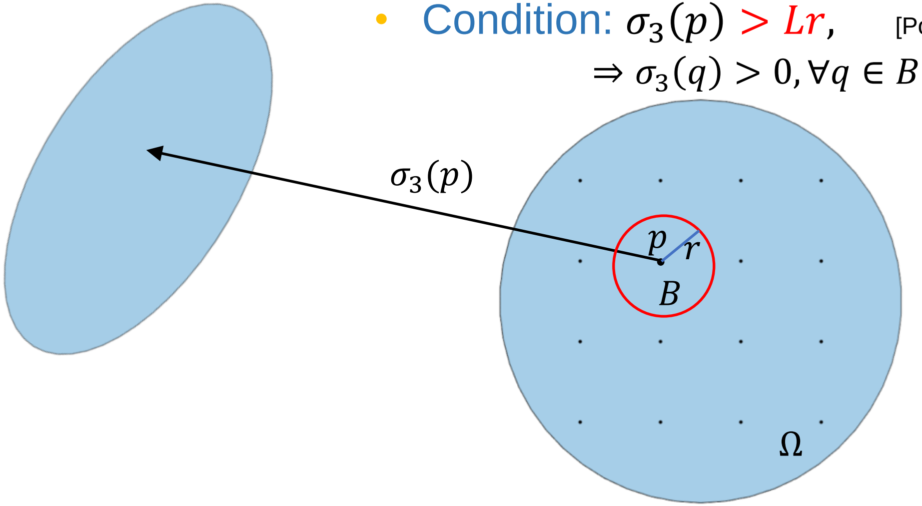
- Condition: $\sigma_3(p) > 0, \forall p \in \Omega$

Infeasible!

- Condition: $\sigma_3(p) > Lr$, [Poranne & Lipman 2014]
  $\Rightarrow \sigma_3(q) > 0, \forall q \in B$



Mesh-based

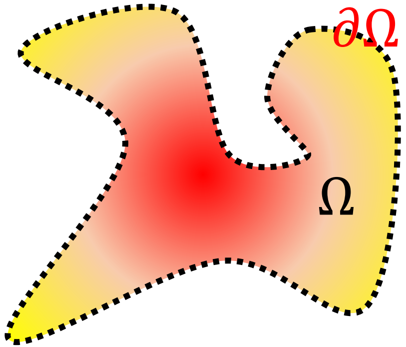


Meshless

Challenges in 3D

- Deriving a Lipschitz constant
  - Planar: $\sigma_1 = |f_z| + |f_{\bar{z}}|, \sigma_2 = |f_z| - |f_{\bar{z}}|$
  - $\mathbb{R}^3$: $\sigma_i =$?

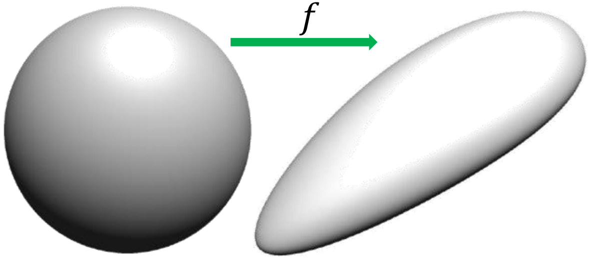
- Lack of bounded distortion theorem
  - Planar: $\min_{p\in\Omega} \sigma_2(p) \geq \min_{p\in\partial\Omega} \sigma_2(p)$
  - Volumetric: ?

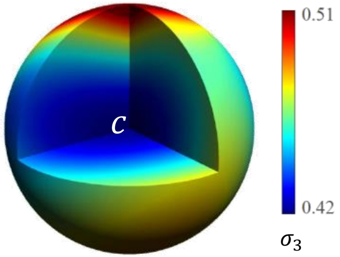


Bounded Distortion Theorem

- Maximum principle for σ_1 ✓
- Minimum principle for σ_3 ✗

counterexample

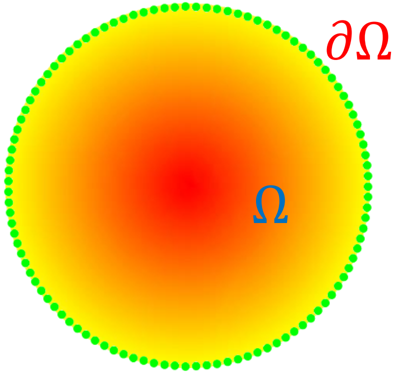




$$\sigma_3(c) < \min_{w \in \partial\Omega} \sigma_3(w)$$

Generalization to 3D

Planar

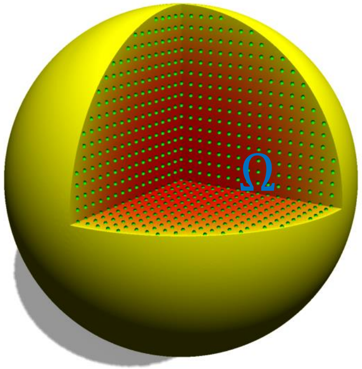


$\forall w \in \partial\Omega$

$\sigma_2(w) > 0$

Boundary constraints

Volumetric

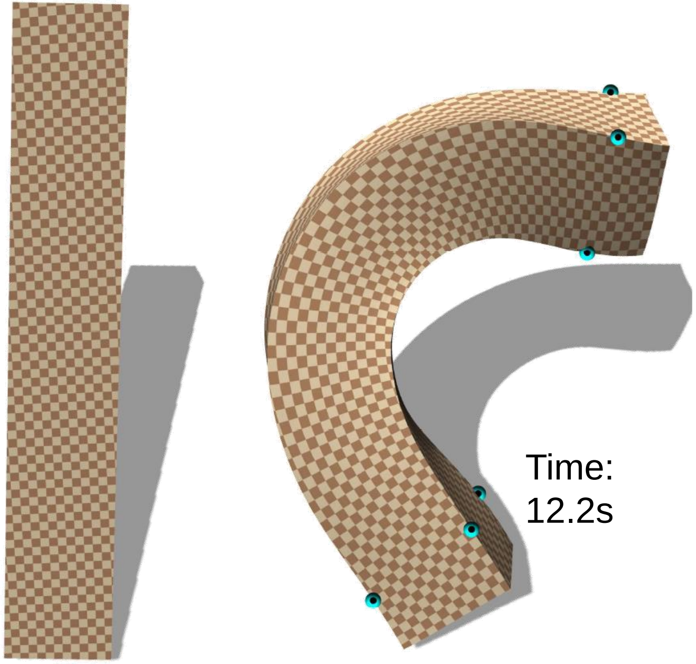


$\forall w \in \Omega$

$\sigma_3(w) > 0$

Interior constraints

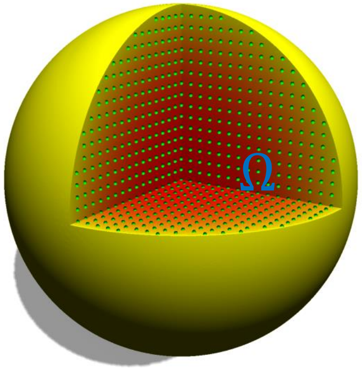
Generalization to 3D



Input

#s:197632

Volumetric



$\forall w \in \Omega$

$\sigma_3(w) > 0$

Interior constraints

Lipschitz Constant for σ_3

- Lipschitz continuous

  $g: \Omega \rightarrow$ is L-Lipschitz continuous if $\exists L,$ such that $\forall p, q \in \Omega: |g(p) - g(q)| \leq L|p - q|$

  $$L = \sup_{p \in \Omega} \|\nabla g(p)\|$$

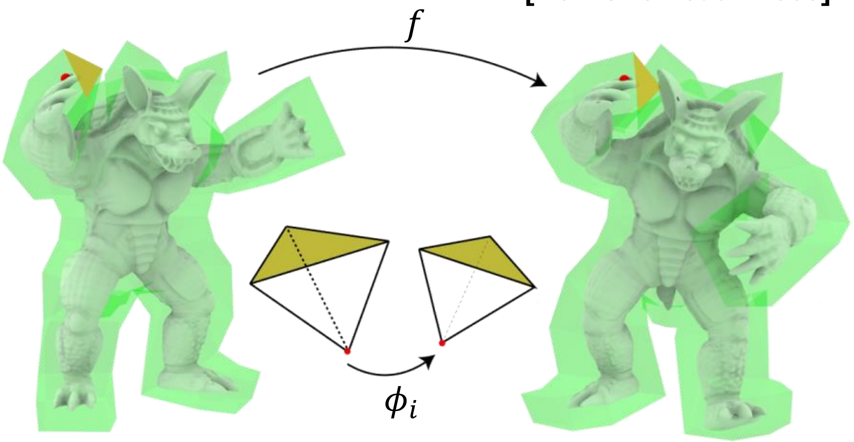
- Key theorem:

  Theorem: $\max_{v \in \Omega} \|H(v)\|$ is a Lipschitz constant for σ_3.

A Tight Bound for $\|H\|$

Volumetric harmonic mapping

[Ben-Chen et al. 2009]



$$f(p) = \sum \phi_i(p) x_i$$

$$J(p) = \sum J_{\phi_i}(p) x_i$$

$$H(p) = \sum H_{\phi_i}(p) x_i$$

↓

$$\|H(p)\| \leq \sum \|H_{\phi_i}(p)\| \|x_i\|$$

- ✘ do not vanish for global affine map
- ✘ variant to composition with rigid transformations

A Tight Bound for $\|H\|$

Volumetric harmonic mapping

$$f(p) = \sum \phi_i(p) x_i$$

$$J(p) = \sum J_{\phi_i}(p) x_i$$

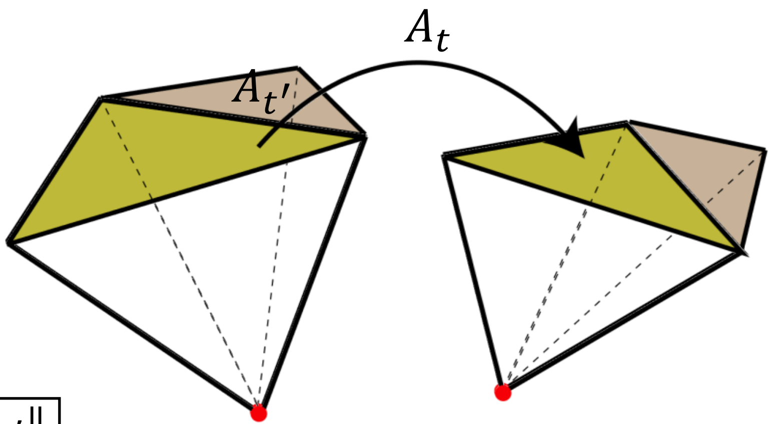
$$J(p) = \sum w_t(p) A_t$$

$$H(p) = \sum H_{\phi_i}(p) x_i$$

$$H(p) = \sum w_e(p)(A_t - A_{t'})$$

$$\|H(p)\| \le \sum \|w_e(p)\| \|A_t - A_{t'}\|$$

- vanishes for global affine map
- invariant to compositions with rigid transformations



Higher Order Estimation for $\|H\|$

- 2^{nd} order

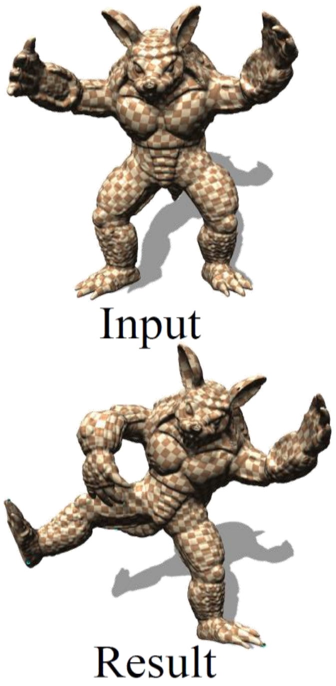
$$L = \max_{q\in\Omega} \|H(q)\|$$

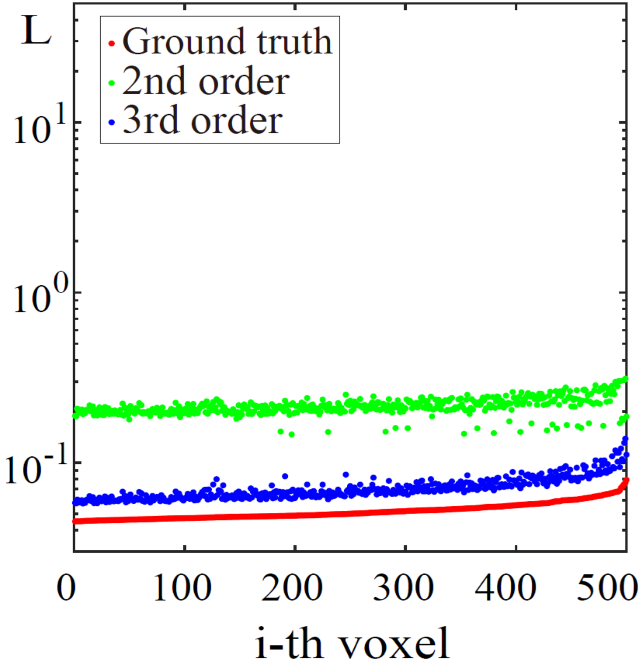
Only use 2^{nd} order derivative of f !

- 3^{rd} order

$$L = \|H(p)\| + \max_{q\in\Omega} \|\nabla H(q)\|\, r$$

Use 3^{rd} order derivative of f





Reduce the Optimal L

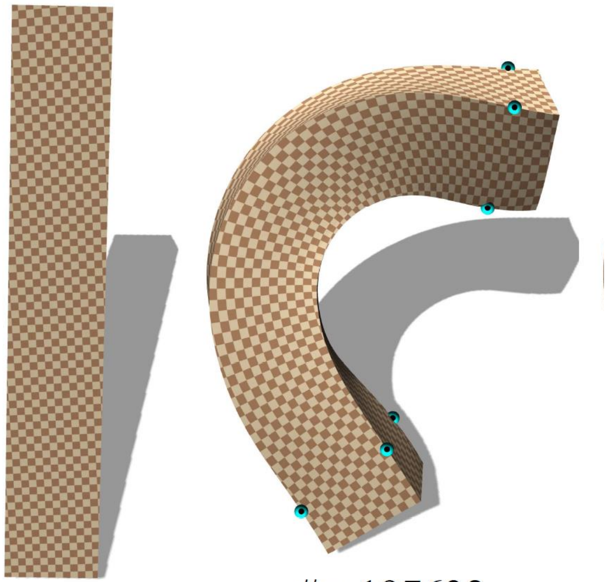
$$\min_{f} \mathrm{E_{dis}} + \lambda_1 \mathrm{E_{p2p}} + \lambda_2 \mathrm{E_{smooth}}$$

$$s.t. \;\; \sigma_3(p) > 0$$

$\mathrm{E_{dis}}$: distortion energy

$\mathrm{E_{p2p}}$: position constraints

$\mathrm{E_{smooth}} = \|H\|^2$



Input

#s: 197632
without $\mathrm{E_{smooth}}$

Time:12.2s

Optimization

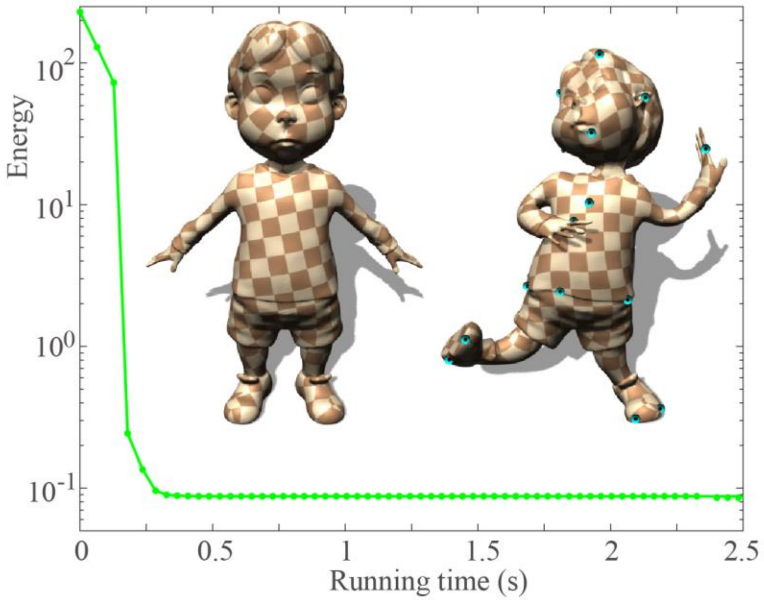
$$\min_{f} \mathrm{E}_{\mathrm{dis}} + \lambda_1 \mathrm{E}_{\mathrm{p2p}} + \lambda_2 \mathrm{E}_{\mathrm{smooth}}$$

$$s.t.\ \ \sigma_3(p) > 0, \forall p \in \Omega$$

Newton's method

1. $g = \nabla E, H = \nabla^2 E, H^+ \rightarrow H$ [Smith 2019]
2. $H\Delta = -g$
3. $x \leftarrow x + t\Delta$

Locally injective line search



Optimization Dimensionality Reduction

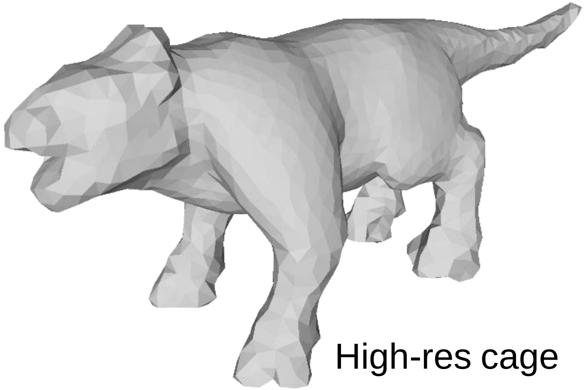
- **The harmonic mapping space**
  - $f(p) = \Phi x, \quad x \in R^{n \times 3}$

DOFs: large

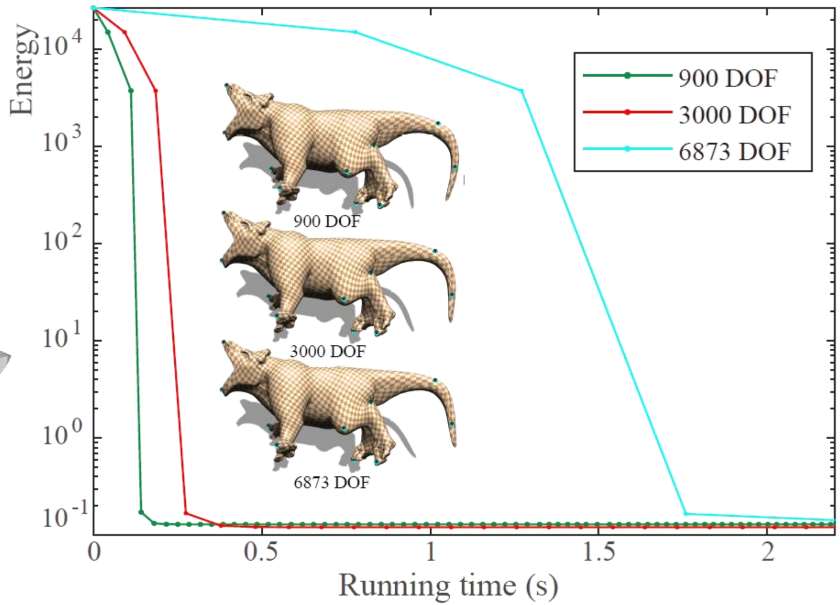
- **Reduce the dimension**
  - subspace construction

$x = \mathrm{P}\tilde{x}, \quad \tilde{x} \in R^{s \times 3}, \; s \ll n$

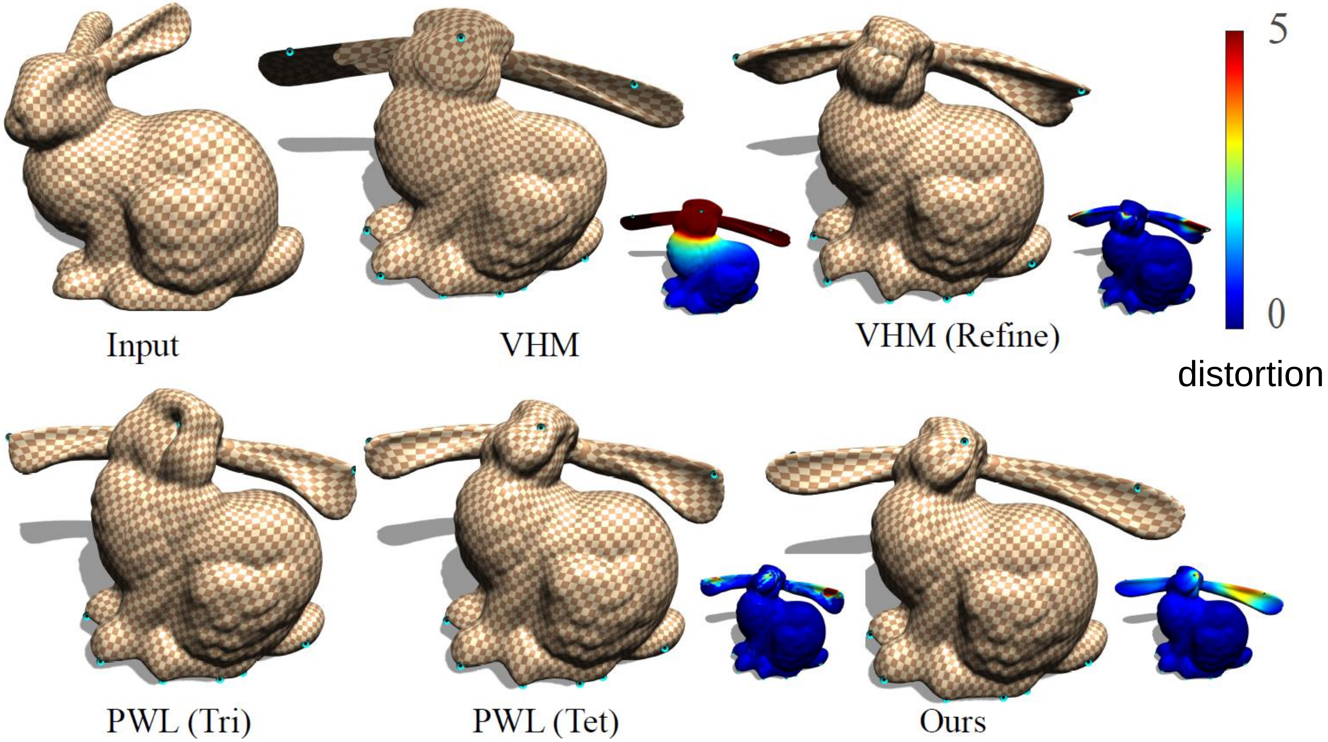
DOFs: small

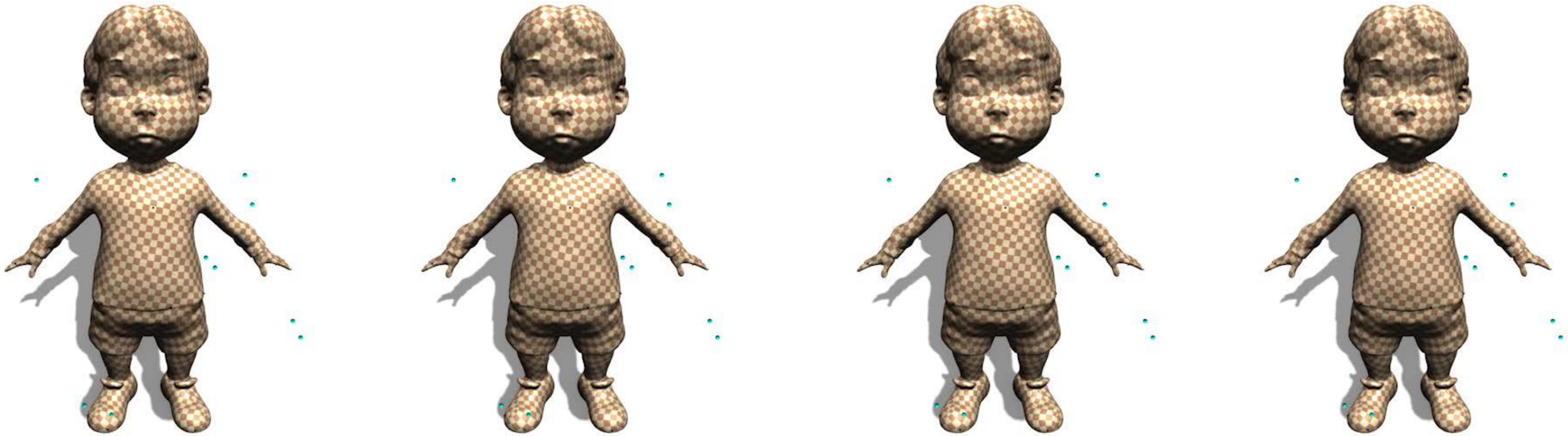


High-res cage



Results & Comparison

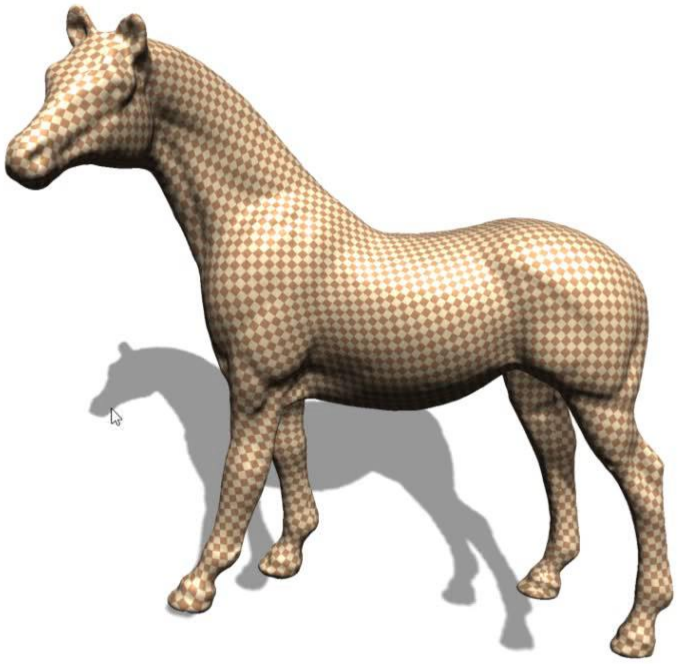




PWL(Tri) Levi & Gotsman 2015 | PWL(Tet) Smith et al. 2019 | VHM Ben-Chen et al. 2009 | Ours

FPS: 58

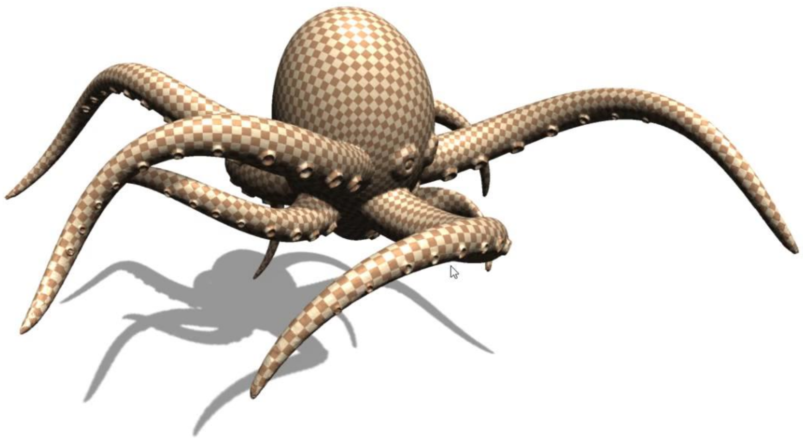
Interactive deformation



Recap

Locally injective volumetric deformation

- Harmonic mapping
- Real-time
- Low distortion



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谢 谢 ！

GAMES 301