中国科学技术大学
University of Science and Technology of China

GAMES 301：第12讲

锥奇异点参数化应用

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Content

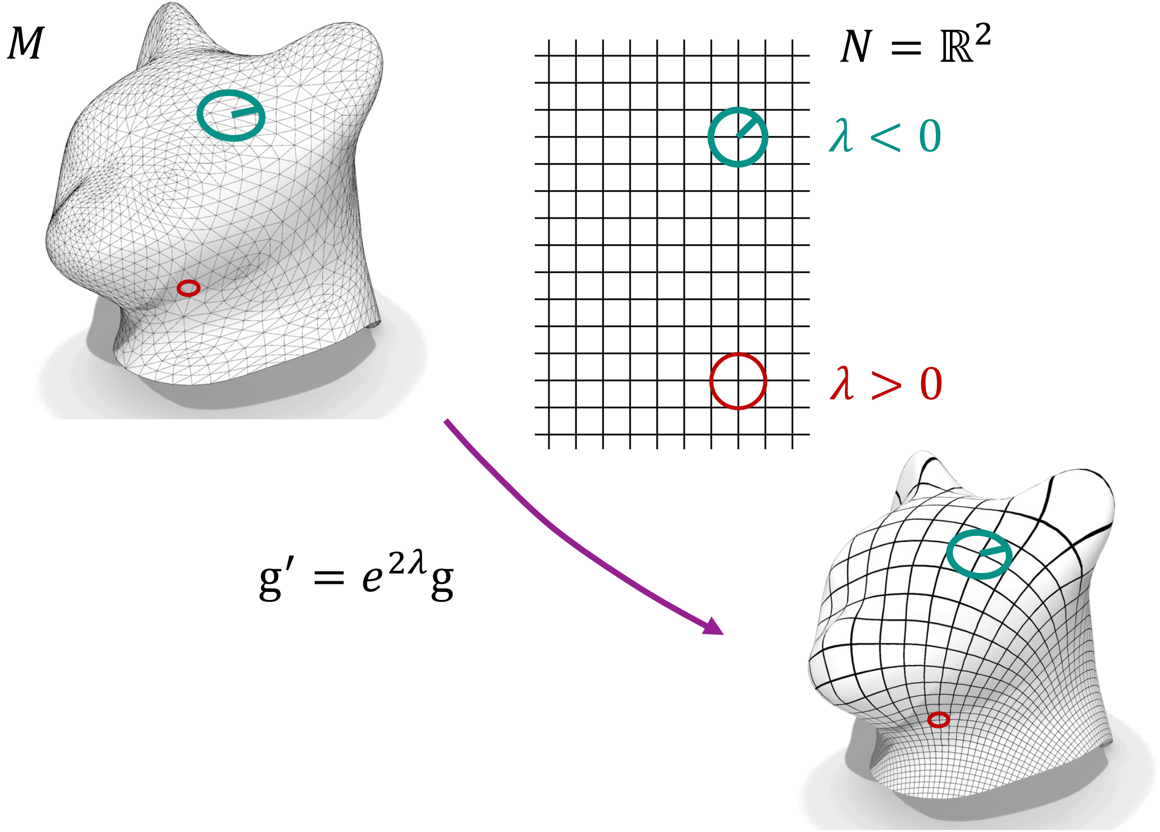
1. Introduction to cone parameterizations
2. Cone generation: heuristic methods
3. Cone generation: optimization-based methods
4. Applications related to cones

1

Introduction to cone parameterizations

Cone parameterizations

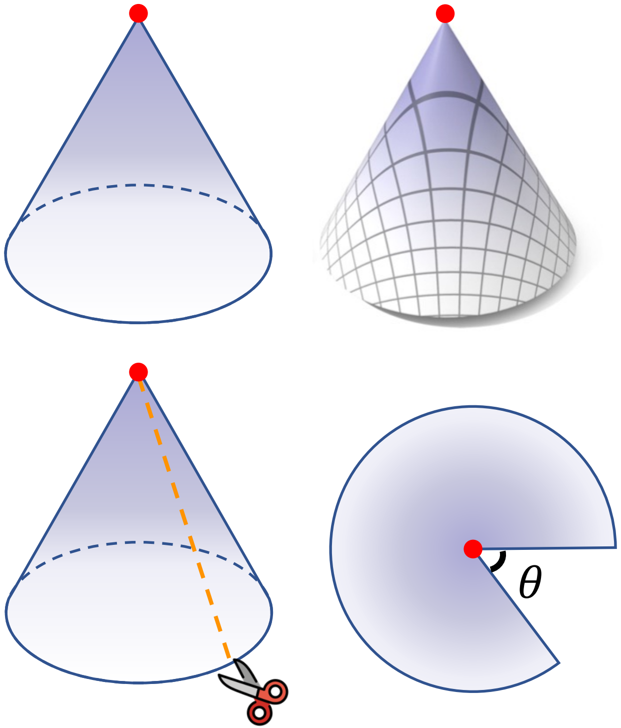
- High area distortion

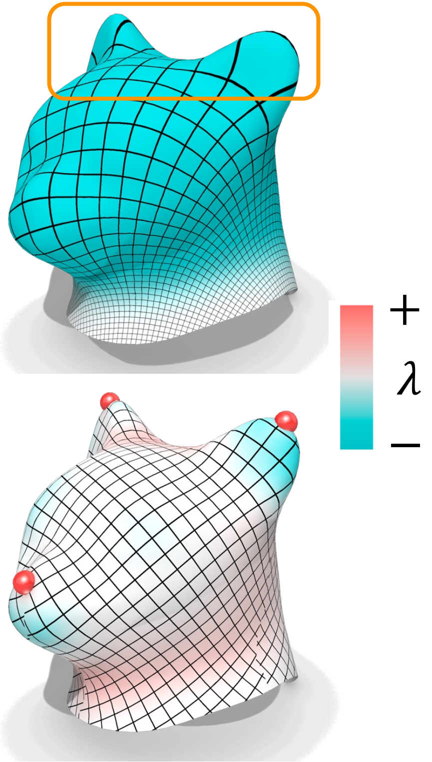


Cone parameterizations

- High area distortion
- Curvature → cones

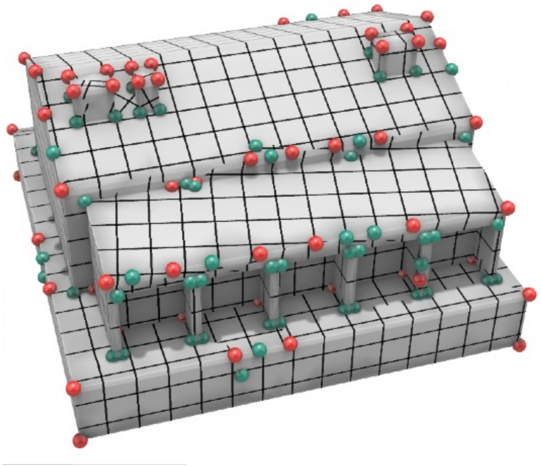
$$K = 2\pi - \theta$$



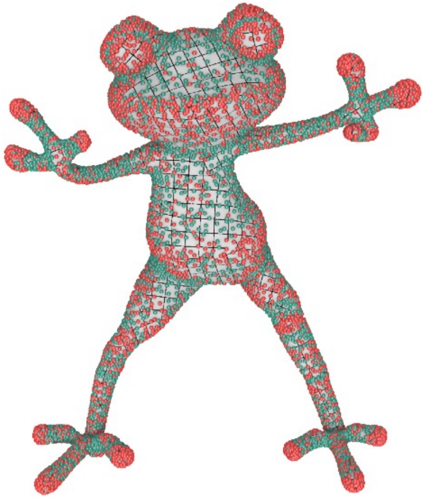


Cone parameterizations

- High area distortion
- Curvature → cones



$n = 130$

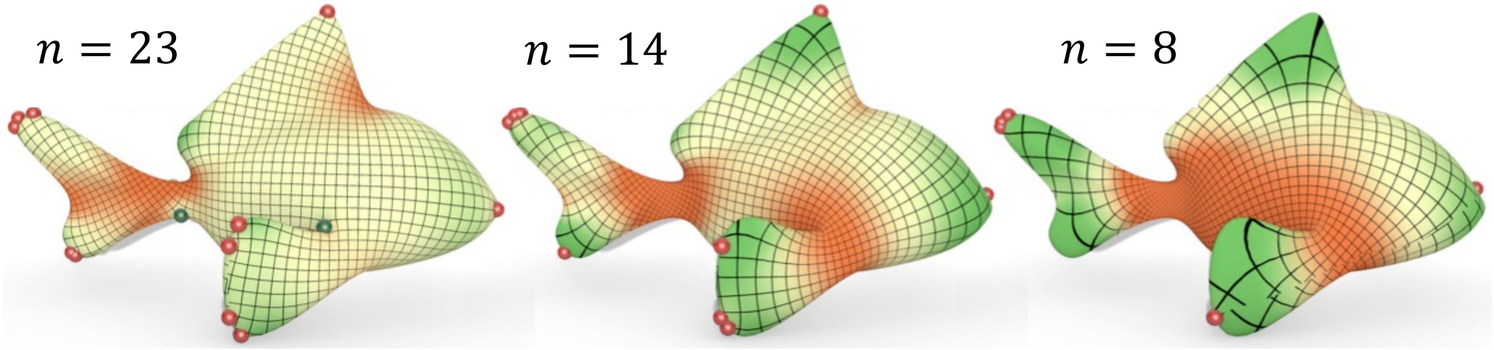


$n = 9982$

Cone parameterizations

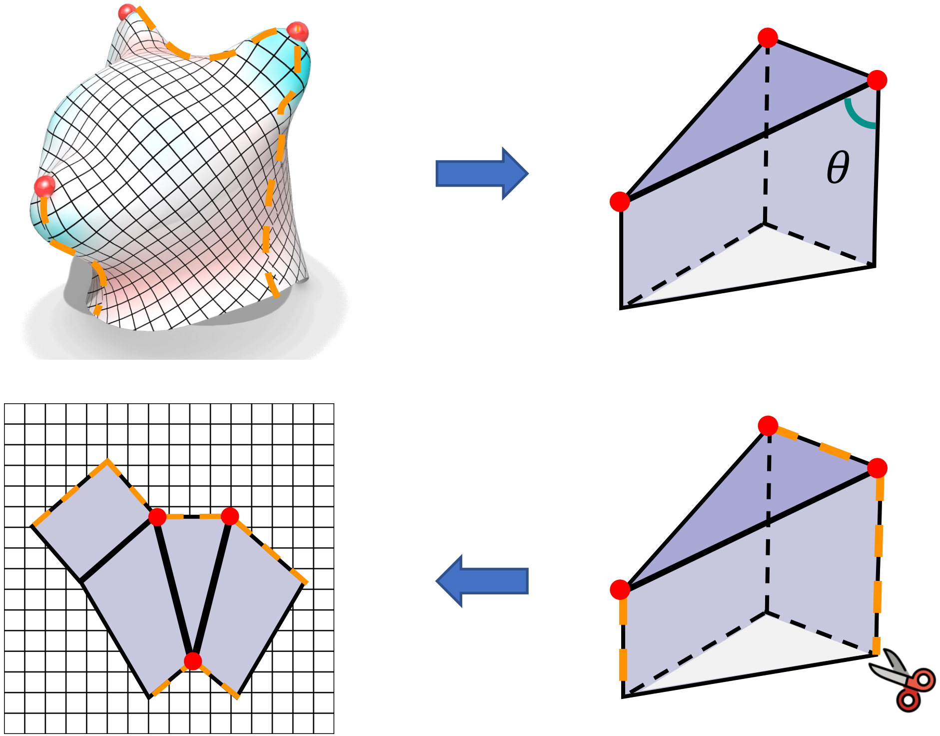
- High area distortion
- Curvature → cones

Trade-off



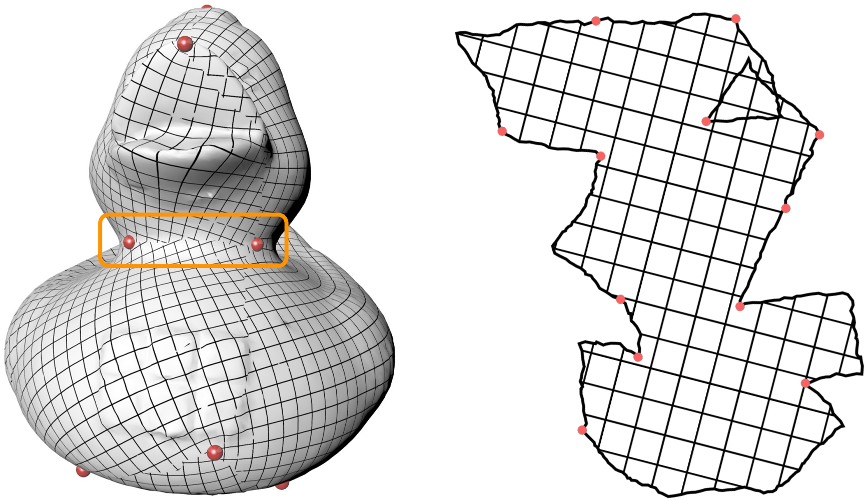
Cone parameterizations

- High area distortion
- Curvature → cones
- Lower distortion & fewer cones
  - Parameterization cuts



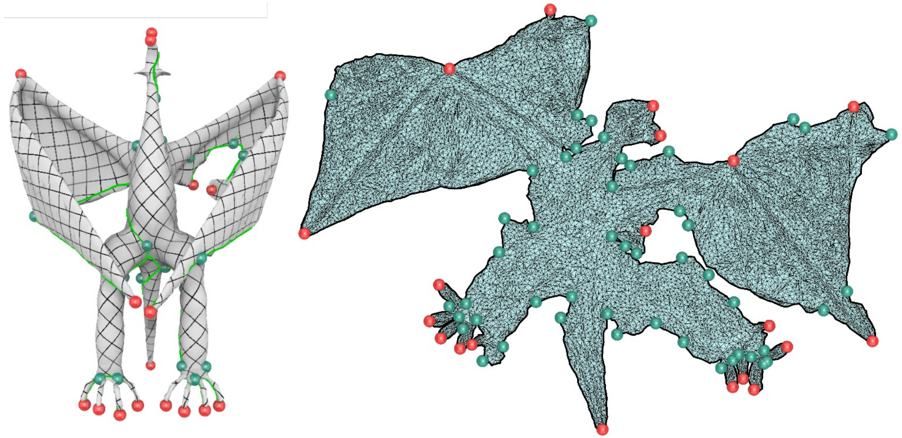
Cone parameterizations

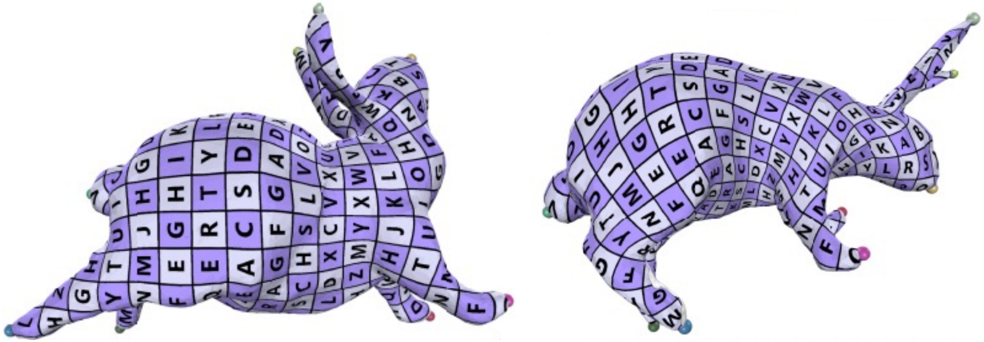
- High area distortion
- Curvature → cones
- Lower distortion & fewer cones
  - Parameterization cuts



Cone parameterizations

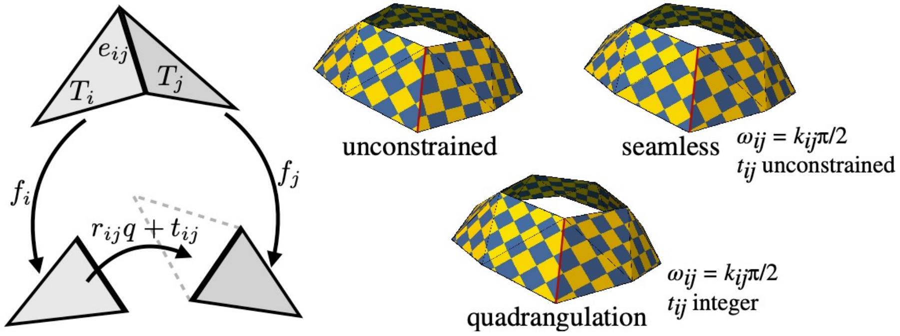
- High area distortion
- Curvature → cones
- Lower distortion & fewer cones
  - Parameterization cuts
  - As landmarks

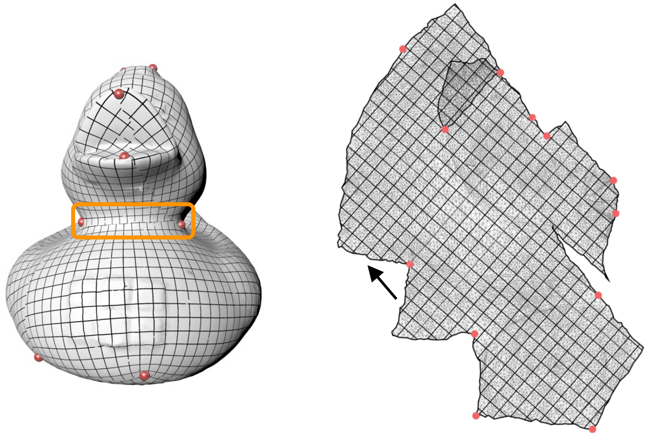




Cone parameterizations

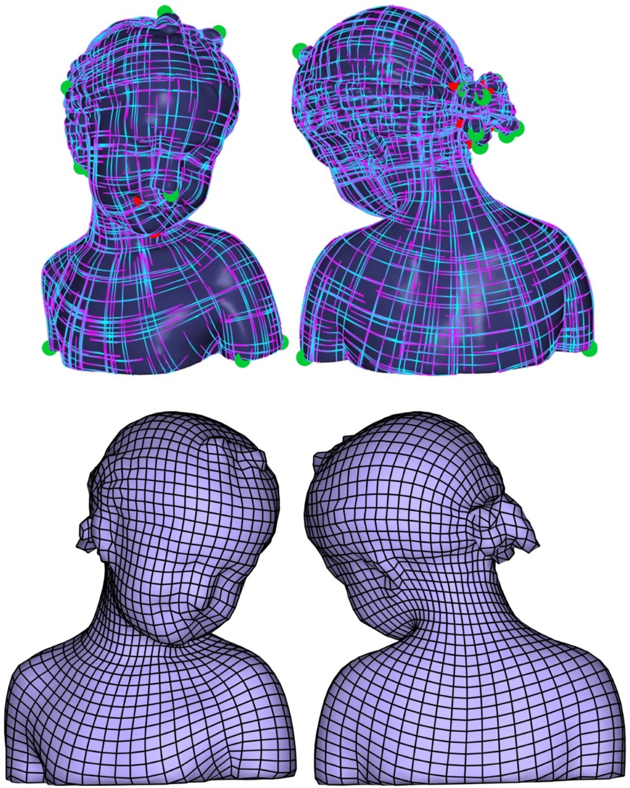
- High area distortion
- Curvature → cones
- Lower distortion & fewer cones
  - Parameterization cuts
  - As landmarks
  - $K = \frac{\pi}{2}\mathbb{Z}$
    - Rotational seamless parameterizations





Cone parameterizations

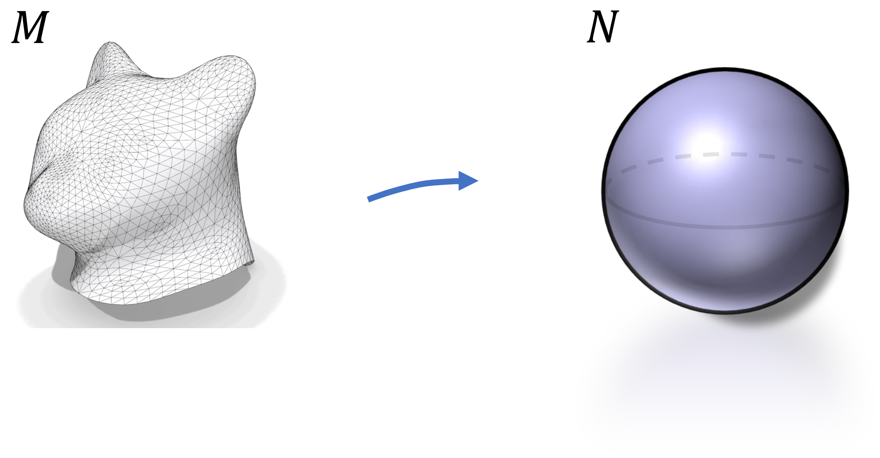
- High area distortion
- Curvature → cones
- Lower distortion & fewer cones
  - Parameterization cuts
  - As landmarks
  - $K = \frac{\pi}{2}\mathbb{Z}$
    - Rotational seamless parameterizations
    - Cross fields & quad meshing



Cone parameterizations

- Yamabe equation:

$$K' = e^{-2\lambda}(K - \Delta_g \lambda)$$



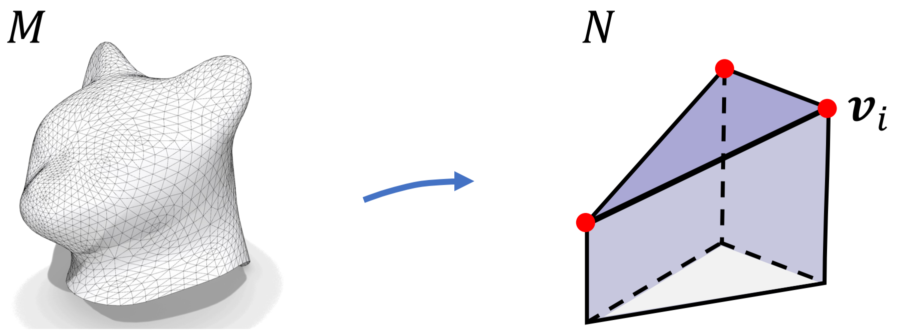
Cone parameterizations

- Yamabe equation:

$$K' = e^{-2\lambda}(K - \Delta_g \lambda)$$

- Cones distribution:

$$K'(\boldsymbol{v}) = \sum_i K_i' \delta_{\boldsymbol{v}_i}(\boldsymbol{v})$$



$$\delta_{\boldsymbol{v}_i}^{\epsilon}(v) = \begin{cases} \frac{1}{\pi\epsilon^2}, & dist(\boldsymbol{v}, \boldsymbol{v}_i) \le \epsilon \\ 0, & otherwise \end{cases}$$

Cone parameterizations

- Yamabe equation:

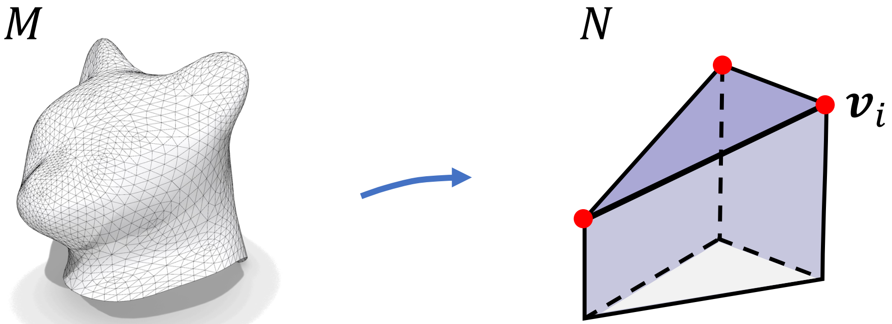
$$K' = e^{-2\lambda}(K - \Delta_g \lambda)$$

- Cones distribution:

$$K'(\boldsymbol{v}) = \sum_i K_i' \delta_{\boldsymbol{v}_i}(\boldsymbol{v})$$

- Linearization ([Bunin 2008]):

$$\sum_i K_i' \delta_{\boldsymbol{v}_i}(\boldsymbol{v}) = (K - \Delta_g \lambda)$$



$$\delta_{\boldsymbol{v}_i}^{\epsilon}(v) = \begin{cases} \frac{1}{\pi\epsilon^2}, & dist(\boldsymbol{v}, \boldsymbol{v}_i) \le \epsilon \\ 0, & otherwise \end{cases}$$

Bunin, G. (2008). A continuum theory for unstructured mesh generation in two dimensions. Computer Aided Geometric Design.

Cone parameterizations

- Linear Yamabe equation:

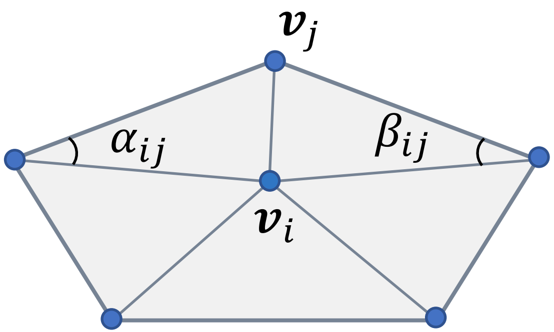
$$\sum_i K_i' \delta_{v_i}(\boldsymbol{v}) = (K - \Delta_g \lambda)$$

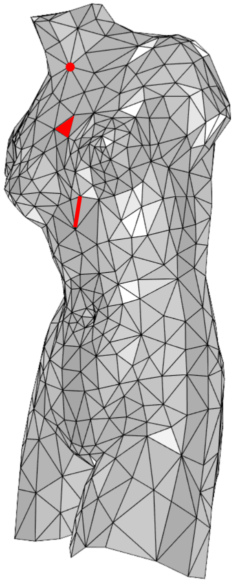
- FEM discretization ([Ben-Chen et al. 2008]):

$$\boldsymbol{K}' \cong (\boldsymbol{K} - \Delta \boldsymbol{\lambda})$$

$$V = \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{N_v}\}, \boldsymbol{K} = (\boldsymbol{k}_1, \ldots, \boldsymbol{k}_{N_v})$$

$$\Delta_{ij} = \begin{cases} -\frac{1}{2}\left(\cot \alpha_{ij} + \cot \beta_{ij}\right) & i \neq j \\ \sum_{v_k \in \Omega(v_i)} L_{ik} & i = j \end{cases}$$





Ben-Chen, M. et al. (2008). Conformal flattening by curvature prescription and metric scaling. Computer Graphics Forum.

Cone parameterizations

- Linear Yamabe equation:

$$\sum_i K_i' \delta_{v_i}(\boldsymbol{v}) = (K - \Delta_g \lambda)$$

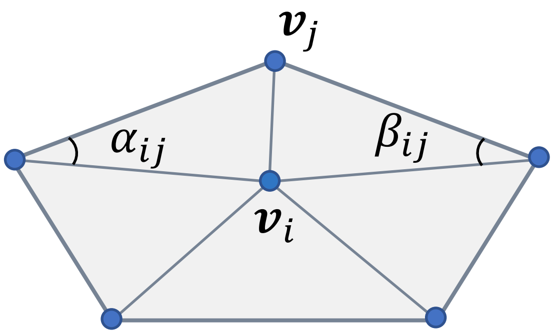
- FEM discretization:

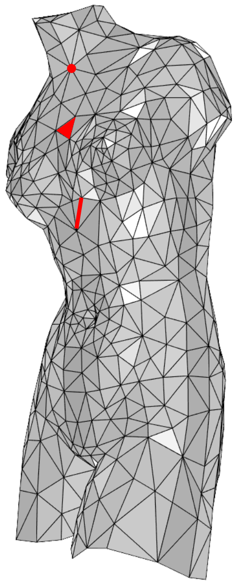
$$\boldsymbol{K}' \cong (\boldsymbol{K} - \Delta\boldsymbol{\lambda})$$

- Trade-off:
  - Area distortion
  - The number of cones

$$V = \{\boldsymbol{v}_1, \dots, \boldsymbol{v}_{N_v}\}, \boldsymbol{K} = (\boldsymbol{k}_1, \dots, \boldsymbol{k}_{N_v})$$

$$L_{ij} = \begin{cases} -\frac{1}{2}(\cot\alpha_{ij} + \cot\beta_{ij}) & i \neq j \\ \sum_{\boldsymbol{v}_k \in \Omega(\boldsymbol{v}_i)} L_{ik} & i = j \end{cases}$$



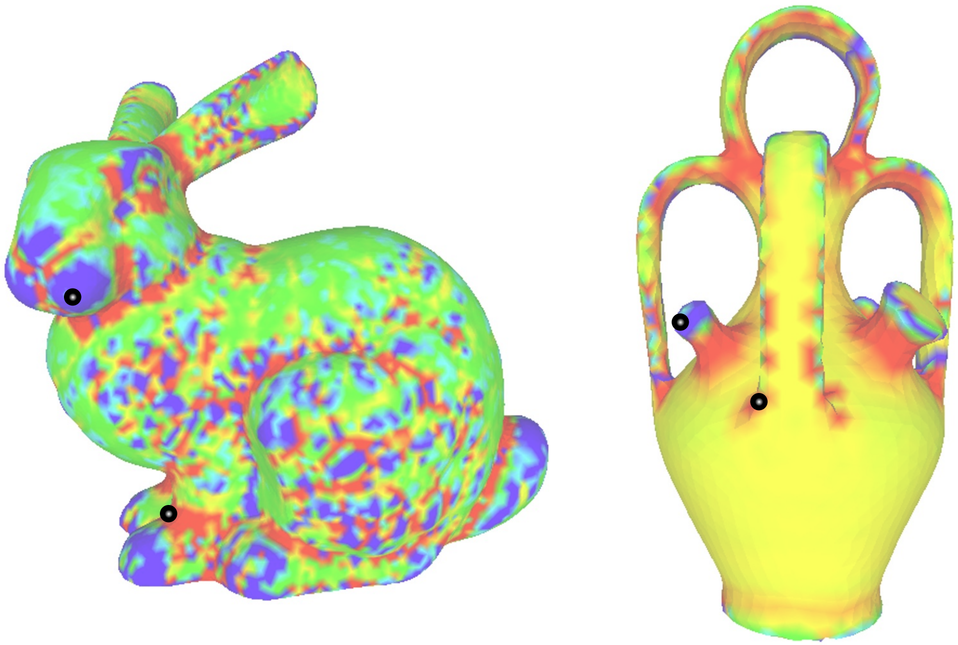


2

Cone generation: heuristic methods

Heuristic methods

- Placement
  - Curvature

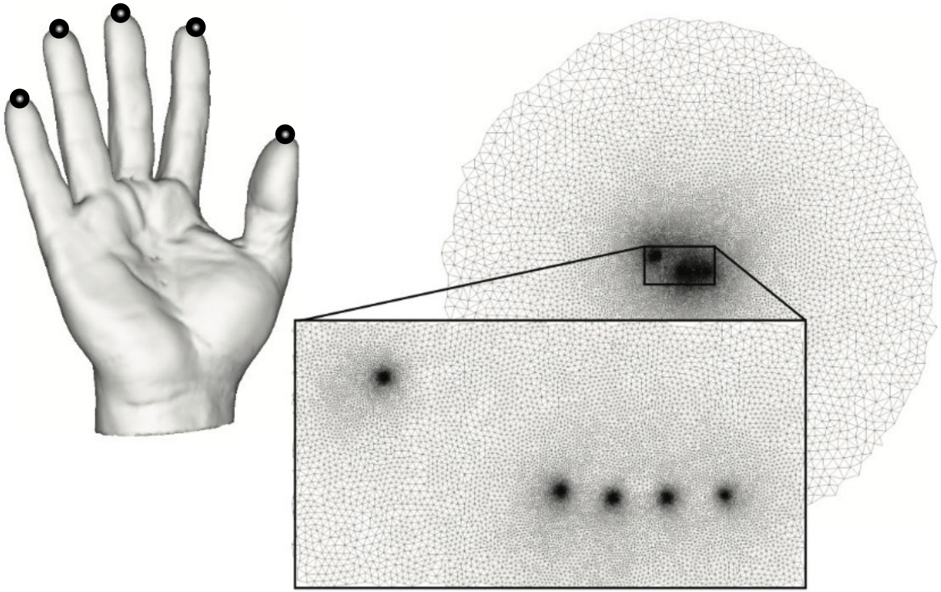


Ben-Chen, M. et al. (2008). Conformal flattening by curvature prescription and metric scaling. Computer Graphics Forum.

Heuristic methods

- Placement
  - Curvature
  - Log conformal factor

$$K' \cong (K - \Delta\lambda)$$



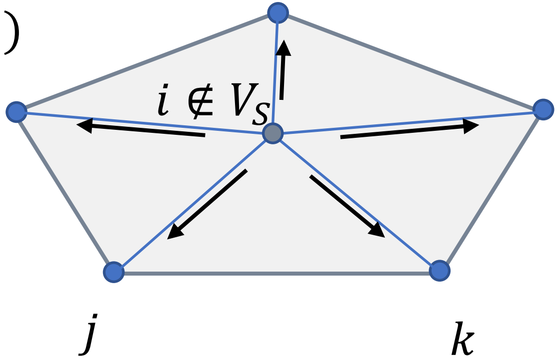
Ben-Chen, M. et al. (2008). Conformal flattening by curvature prescription and metric scaling. Computer Graphics Forum.

Heuristic methods

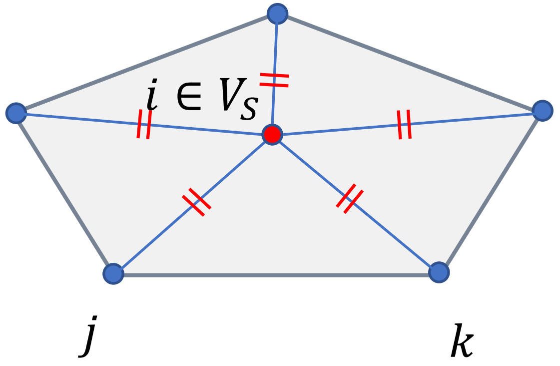
- Placement
  - Curvature
  - Log conformal factor
- Cone angle : random walk
  - $\boldsymbol{K} \rightarrow \boldsymbol{K}'$
  - $\sum_{v_i} \boldsymbol{K}_i = \sum_{v_i} \boldsymbol{K}'_i = 2\pi\chi$

$$P_{ij} = \begin{cases} w_{ij}, & j \in N(i) \\ 0, & else \end{cases}$$

$$\sum_{j \in N(i)} w_{ij} = 1$$



$$P_{ij} = \begin{cases} 1, & j = i \\ 0, & else \end{cases}$$



Ben-Chen, M. et al. (2008). Conformal flattening by curvature prescription and metric scaling. Computer Graphics Forum.

Heuristic methods

- Flattening point set: $\boldsymbol{K}' = \boldsymbol{K}$

$$\mathcal{F}_\epsilon(\boldsymbol{K}') = \{\boldsymbol{v}_i \in V, |\boldsymbol{K}'_i| < \epsilon\}$$

ϵ increase, $\mathcal{F}_\epsilon(\boldsymbol{K}') \nearrow V$

Myles, A., & Zorin, D. (2012). Global parametrization by incremental flattening. ACM Transactions on Graphics.

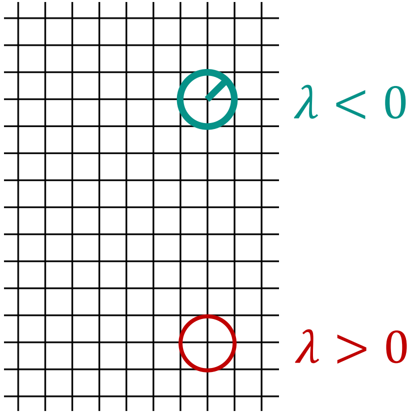
Heuristic methods

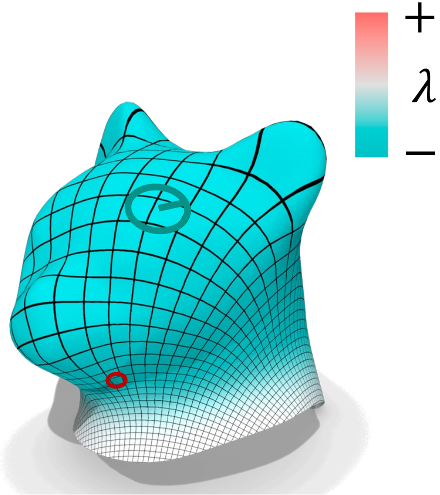
- Flattening point set: $\boldsymbol{K}' = \boldsymbol{K}$

$$\mathcal{F}_\epsilon(\boldsymbol{K}') = \{\boldsymbol{v}_i \in V, |\boldsymbol{K}'_i| < \epsilon\}$$

- Curvature $\boldsymbol{K}'_i,\ v_i \notin \mathcal{F}_\epsilon(\boldsymbol{K}')$
  - Area distortion $\min \mathcal{A}\,(\boldsymbol{\lambda})$

$$\mathcal{A}\,(\boldsymbol{\lambda}) \triangleq \int \lambda^2 dA \cong \sum_i A_i \lambda_i^2$$





Myles, A., & Zorin, D. (2012). Global parametrization by incremental flattening. ACM Transactions on Graphics.

Heuristic methods

- Flattening point set: $\boldsymbol{K}' = \boldsymbol{K}$

$$\mathcal{F}_\epsilon(\boldsymbol{K}') = \{\boldsymbol{v}_i \in V, |\boldsymbol{K}_i'| < \epsilon\}$$

- Curvature $\boldsymbol{K}_i'$, $v_i \notin \mathcal{F}_\epsilon(\boldsymbol{K}')$
  - Area distortion $\min \mathcal{A}\,(\boldsymbol{\lambda})$

$$\mathcal{A}\,(\boldsymbol{\lambda}) \triangleq \int \lambda^2 dA \cong \sum_i A_i \lambda_i^2$$

  - Linear Yamabe constraint

$$(\Delta \boldsymbol{\lambda})\Big|_{\mathcal{F}_\epsilon(\boldsymbol{K}')} = \boldsymbol{K}\,\Big|_{\mathcal{F}_\epsilon(\boldsymbol{K}')}$$

1. Initialize： $\epsilon = \epsilon_0, \boldsymbol{K}' = \boldsymbol{K}$.
2. Compute $\mathcal{F}_\epsilon(\boldsymbol{K}')$.
3. Solve constrained LSQ and Update $\boldsymbol{K}'$.
4. If $V \backslash \mathcal{F}_\epsilon(\boldsymbol{K}')$ contains non-isolated vertices, $\epsilon = \epsilon + \Delta\epsilon$; else, terminate.
5. Repeat 2-4.

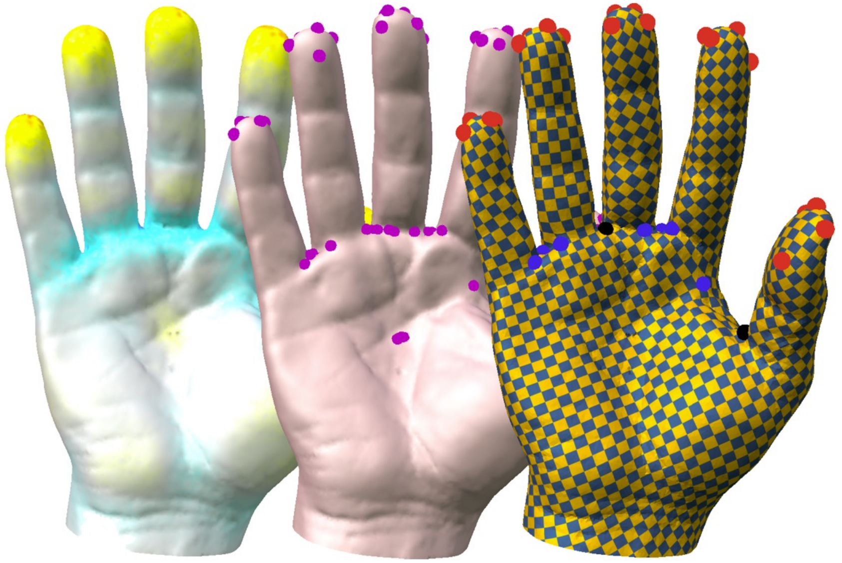
Myles, A., & Zorin, D. (2012). Global parametrization by incremental flattening. ACM Transactions on Graphics.

Heuristic methods

- Rounding set $\Re(\boldsymbol{K}')$
  - $\boldsymbol{K}' = (0, \dots, 0.48\pi, \dots, -1.11\pi, \dots, 0)$
  - $\boldsymbol{TK}' = (0, \dots, 0.50\pi, \dots, -1.11\pi, \dots, 0)$

$$\min \mathcal{A}(\boldsymbol{\lambda}) = \sum_i A_i \lambda_i^2 ,$$

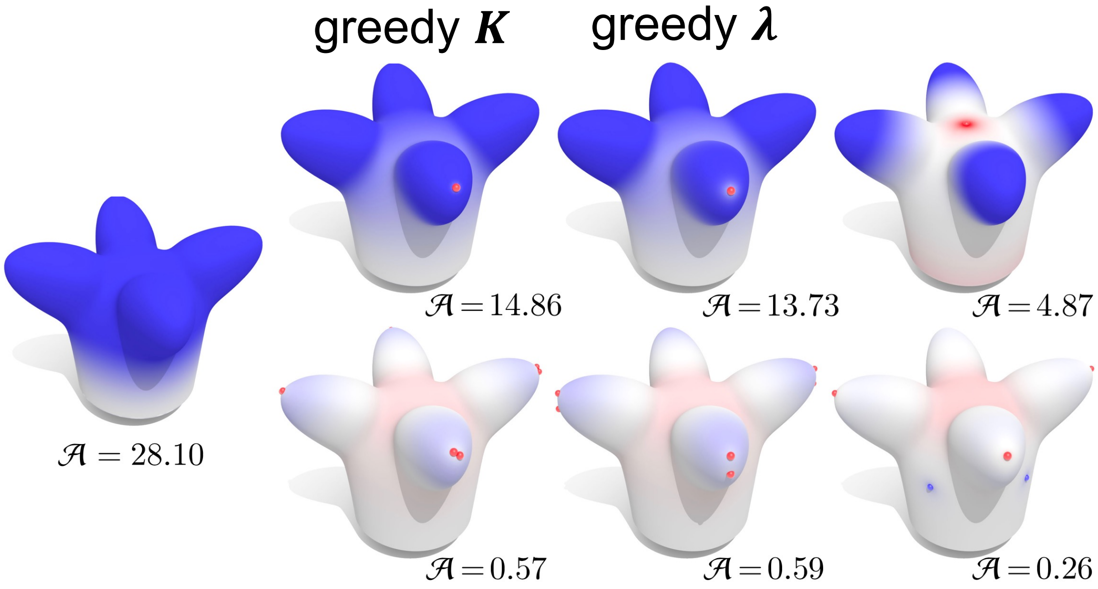
$$s.t. \begin{cases} (\Delta\boldsymbol{\lambda})\big|_{\mathcal{F}_\epsilon(\boldsymbol{K}')} = \boldsymbol{K}\big|_{\mathcal{F}_\epsilon(\boldsymbol{K}')} \\ (\Delta\boldsymbol{\lambda})\big|_{\Re(\boldsymbol{K}')} = \boldsymbol{K}\big|_{\Re(\boldsymbol{K}')} - \boldsymbol{TK}'\big|_{\Re(\boldsymbol{K}')} \end{cases}$$



Myles, A., & Zorin, D. (2012). Global parametrization by incremental flattening. ACM Transactions on Graphics.

Heuristic methods

- Sub-optimal



Heuristic methods

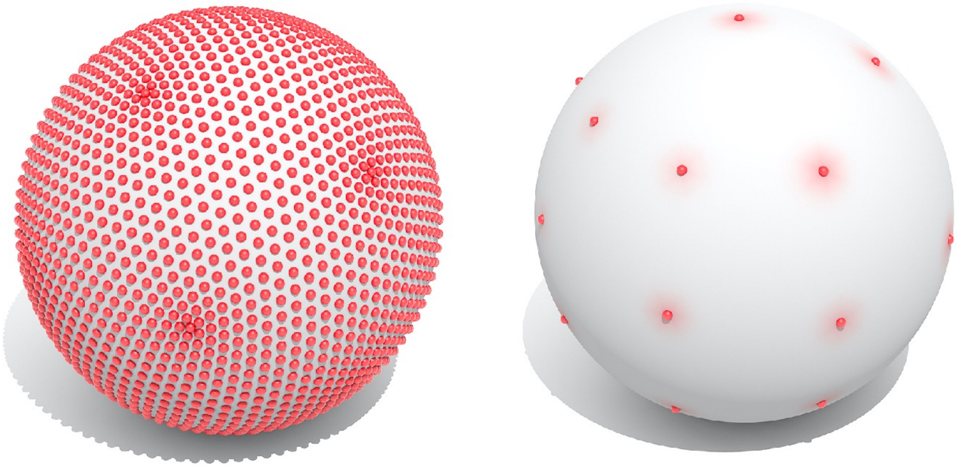
- Sub-optimal
- Constraint

$$\boldsymbol{K}^{'} = (\boldsymbol{K} - \Delta\boldsymbol{\lambda})$$

- Aim
  - Low area distortion

$$\mathcal{A}(\boldsymbol{\lambda}) = \sum_i A_i \lambda_i^2$$

  - Sparse cones



$$\int \left|\sum_i \boldsymbol{K}_i \delta_{\boldsymbol{v}_i}(\boldsymbol{v})\right| dA = \sum_{\boldsymbol{v}_i \in V} |\boldsymbol{K}_i|$$

$$= \sum_{\boldsymbol{v}_i \in V} \boldsymbol{K}_i = 2\pi\chi$$

?

3

Cone generation: optimization-based methods

Optimization-based methods

- Cones distribution : $K(v) = \sum_i K_i \delta_{v_i}(\boldsymbol{v}) \rightarrow \sum_i K_i \mu(\boldsymbol{v}_i)$
- Measure norm: $||\mu||_M = \sup_{f \in C(M)} \{\int_M f\, d\mu : |f(v)| \leq 1 \ \forall\ x \in M\}$

Fenchel-Rockafellar duality

$$\min_{\lambda,\mu} \int \lambda^2 dA + \alpha ||\mu||_M,$$
$$s.t.\ \mu = (K - \Delta\lambda)$$

$\Rightarrow$

$$\min_{\lambda, f \in C(M)} \int \lambda^2 dA - \int K\, f dA,$$
$$s.t.\ \Delta f = \lambda \ \&\ |f(v)| \leq \alpha, \forall\ v \in M$$

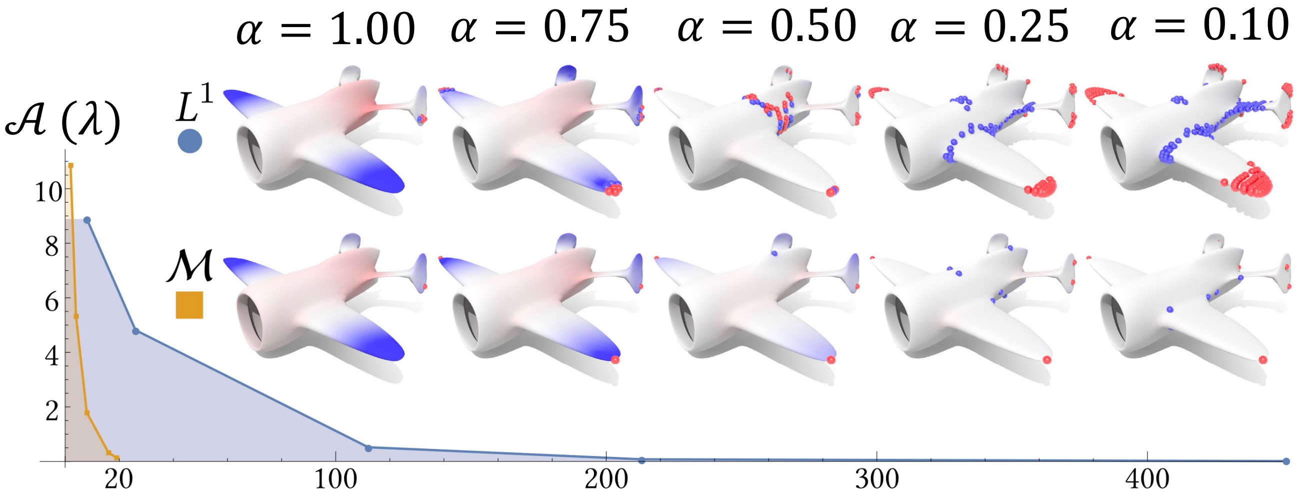
ADMM or DR Splitting!

Soliman, Y. et al. (2018). Optimal cone singularities for conformal flattening. *ACM Transactions on Graphics*.

Optimization-based methods

- Cones distribution : $K(v) = \sum_i K_i \delta_{\boldsymbol{v}_i}(\boldsymbol{v}) \to \sum_i K_i \mu(\boldsymbol{v}_i)$

- Measure norm: $\|\mu\|_M = \sup_{f \in C(M)} \{\int_M f\, d\mu : |f(v)| \le 1\ \forall\ x \in M\}$



Soliman, Y. et al. (2018). Optimal cone singularities for conformal flattening. *ACM Transactions on Graphics*.

Optimization-based methods

- Cones distribution : $K(v) = \sum_i K_i \delta_{\boldsymbol{v}_i}(\boldsymbol{v}) \rightarrow \boldsymbol{K} = (\boldsymbol{K}_1, \dots, \boldsymbol{K}_{N_v})$
- L0 norm : $\|\boldsymbol{K}\|_0$
- Reformulation

$$\min_{\boldsymbol{\lambda},\boldsymbol{K}'} \sum_i A_i \boldsymbol{\lambda}_i^2 + \alpha \|\boldsymbol{K}'\|_0 , \quad s.t.\ \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \boldsymbol{K}'$$

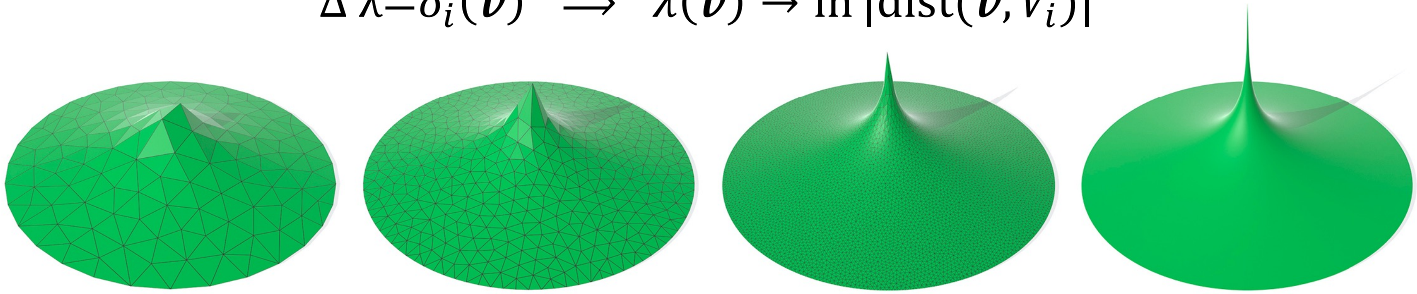
$$\Longleftrightarrow$$

$$\min_{\boldsymbol{\lambda},\boldsymbol{K}'} \|\boldsymbol{K}'\|_0 , s.t. \begin{cases} \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \boldsymbol{K}' \\ \left(\sum_i A_i \boldsymbol{\lambda}_i^2\right)^{\frac{1}{2}} \le \beta \end{cases}$$

Fang, Q. et al. (2021). Computing sparse cones with bounded distortion for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

$$\Delta\lambda = \delta_i(\boldsymbol{v}) \implies \lambda(\boldsymbol{v}) \to \ln|\mathrm{dist}(\boldsymbol{v}, V_i)|$$



$$\left(\int \lambda^p dA\right)^{\frac{1}{p}} \to \|\lambda\|_\infty, \text{ as } p \to \infty$$

$$\min_{\lambda, \boldsymbol{K}'} \left(\sum_i A_i \boldsymbol{\lambda}_i^{p}\right)^{\frac{1}{p}} + \alpha \|\boldsymbol{K}'\|_0, \quad s.t.\ \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \boldsymbol{K}'$$

$$\Longleftrightarrow$$

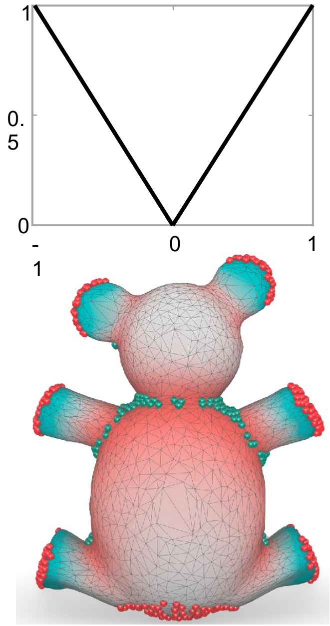
$$\min_{\lambda, \boldsymbol{K}'} \|\boldsymbol{K}'\|_0, s.t. \begin{cases} \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \boldsymbol{K}' \\ \left(\sum_i A_i \boldsymbol{\lambda}_i^{p}\right)^{\frac{1}{p}} \le \beta \end{cases}$$

Approximate projection!

Fang, Q. et al. (2021). Computing sparse cones with bounded distortion for conformal parameterizations. *ACM Transactions on Graphics*.

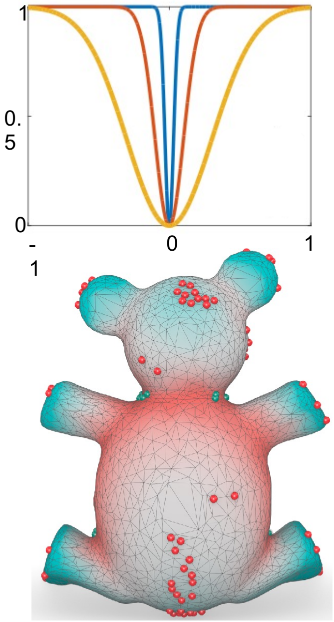
Optimization-based methods

L1



$n = 505$

Smooth L0

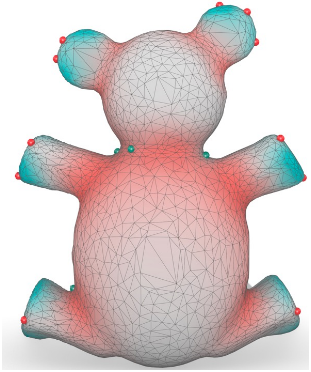


$n = 113$

Reweighted L1

$$\sum w_i^{(m)} |\boldsymbol{K}'_i|$$

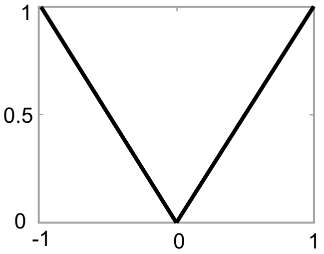
$$m = 1, \dots, n_m$$



$n = 24$

Fang, Q. et al. (2021). Computing sparse cones with bounded distortion for conformal parameterizations. *ACM Transactions on Graphics*.

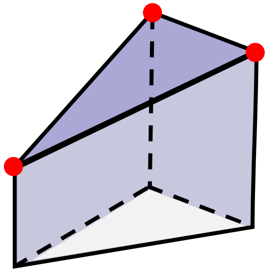
Optimization-based methods



$\|\boldsymbol{K}'\|_0$

ADMM or DR Splitting!

$$\min_{\boldsymbol{\lambda},\boldsymbol{K}'} \sum w_i^{(m)} |\boldsymbol{K}_i'|, \quad s.t. \begin{cases} \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \boldsymbol{K}' \\ \sum_i A_i \boldsymbol{\lambda}_i^2 \leq \beta \end{cases}$$



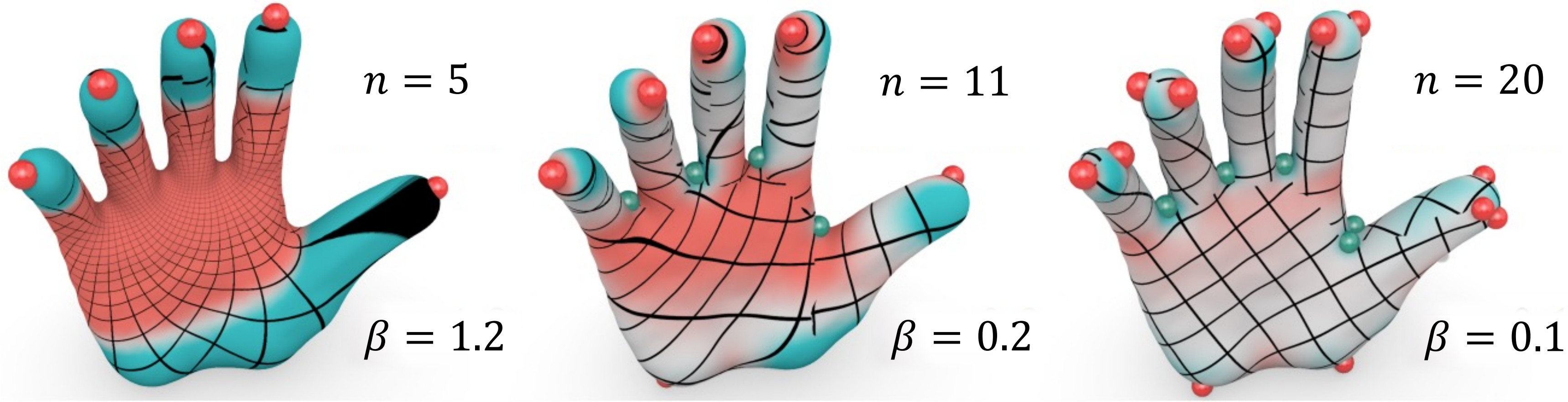
$$w_i^{(m+1)} = \frac{1}{\left|\boldsymbol{K}_i'^{(m)}\right| + \epsilon^{(m)}}$$

$w_i^{(0)} = 1 \Downarrow \boldsymbol{\lambda}^{(0)}, \mathbf{K}'^{(0)}$ → … $w_i^{(m)} \Downarrow \boldsymbol{\lambda}^{(m)}, \mathbf{K}'^{(m)}$ → $w_i^{(m+1)} \Downarrow \boldsymbol{\lambda}^{(m+1)}, \mathbf{K}'^{(m+1)}$ → … Convergence $\mathbf{K}'^{(m+1)} = \mathbf{K}'^{(m)}$

Fang, Q. et al. (2021). Computing sparse cones with bounded distortion for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

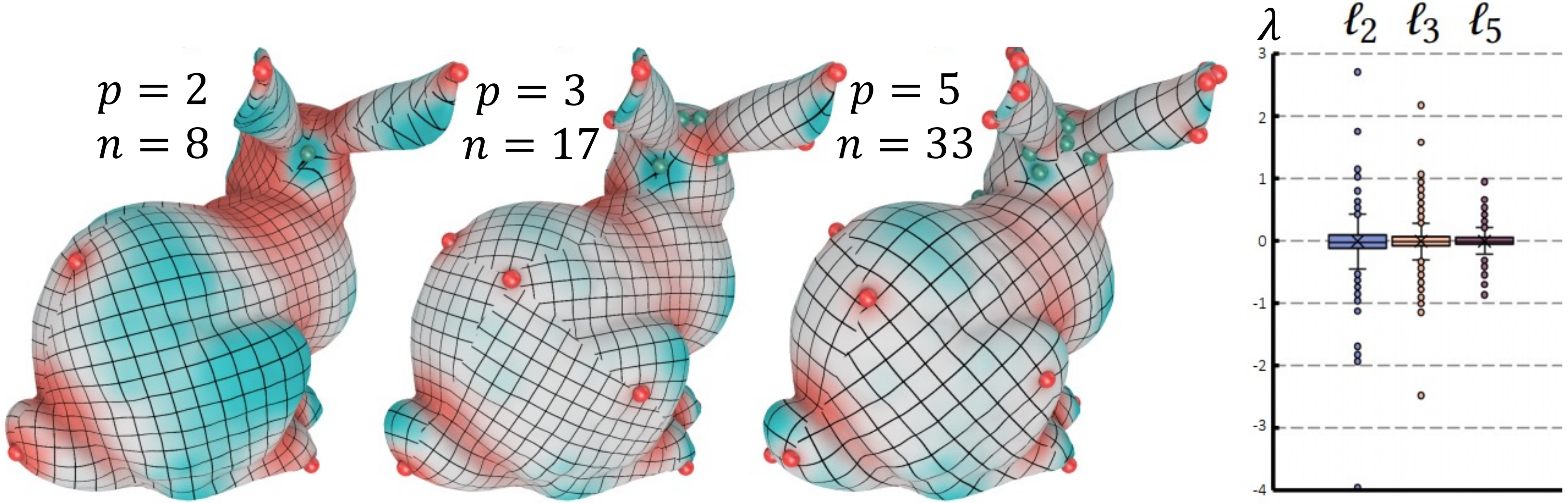
- Area distortion: different bound (L2 norm)



Fang, Q. et al. (2021). Computing sparse cones with bounded distortion for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

- Area distortion: different bound (L2 norm)
- Area distortion: different norm ($\beta = 0.2$)



Fang, Q. et al. (2021). Computing sparse cones with bounded distortion for conformal parameterizations. *ACM Transactions on Graphics*.

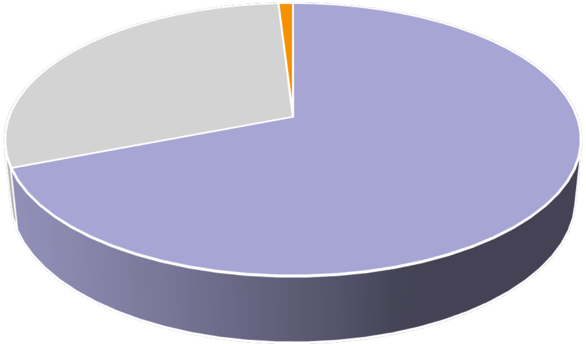
Optimization-based methods

- Compare to measure norm

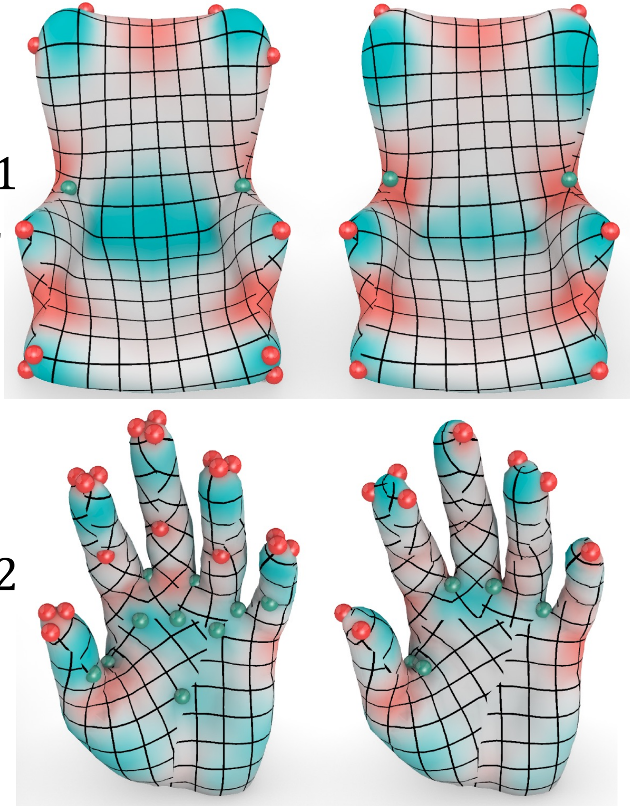
$n < n_c$ 69.09%

$n = n_c$ 29.96%

$n > n_c$ 0.95%



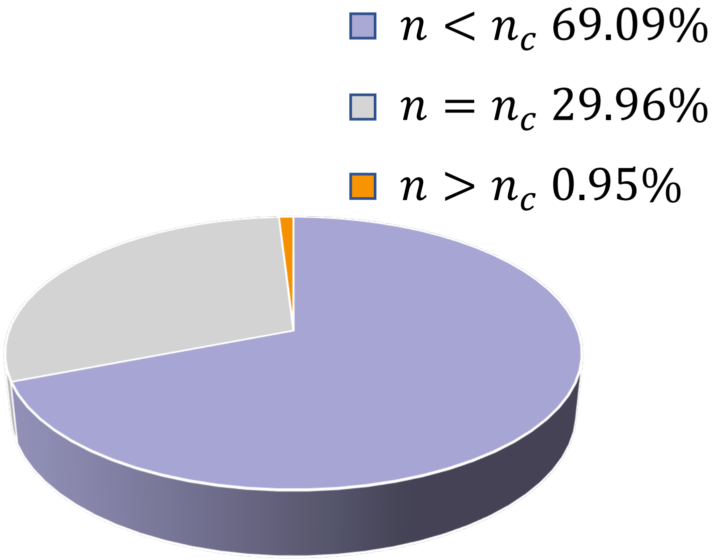
Dataset (3885 models)



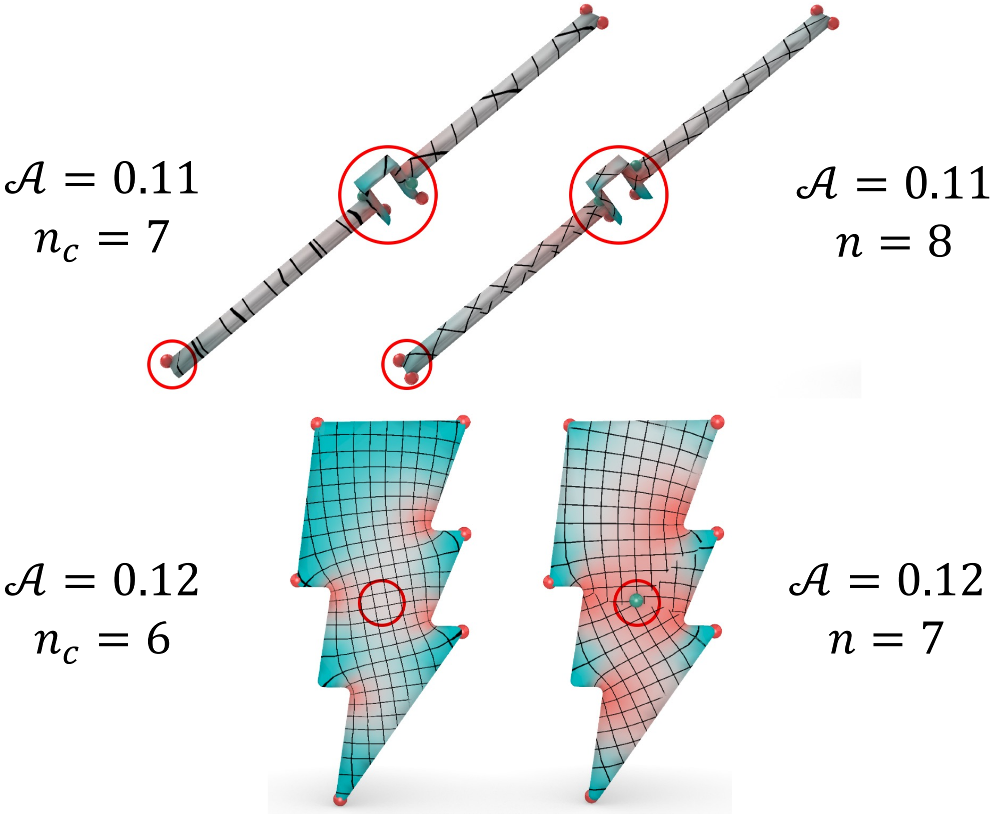
Fang, Q. et al. (2021). Computing sparse cones with bounded distortion for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

- Compare to measure norm



Dataset (3885 models)



Fang, Q. et al. (2021). Computing sparse cones with bounded distortion for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

- Consider $\boldsymbol{K}' \in \frac{\pi}{2}\mathbb{Z}^V$

$$\min_{\boldsymbol{\lambda},\boldsymbol{K}'} \|\boldsymbol{K}'\|_0 \,, s.t. \begin{cases} \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \boldsymbol{K}' \\ \left(\sum_i A_i \boldsymbol{\lambda}_i^2\right)^{\frac{1}{2}} \leq \beta \end{cases}$$

$$\Downarrow$$

$$\min_{\boldsymbol{\lambda},\boldsymbol{K}'} \|\boldsymbol{K}'\|_0 \,, s.t. \begin{cases} \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \boldsymbol{K}' \\ \left(\sum_i A_i \boldsymbol{\lambda}_i^2\right)^{\frac{1}{2}} \leq \beta \\ \boldsymbol{K}' \in \frac{\pi}{2}\mathbb{Z}^V \end{cases}$$

Li, M. et al. (2022). Computing sparse integer-constrained cones for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

- Consider $\boldsymbol{K}' \in \frac{\pi}{2}\mathbb{Z}^V$
- Integer constraint to binary constraint

$$\boldsymbol{K}' \in \frac{\pi}{2}\mathbb{Z}^V \mapsto \boldsymbol{K}' \in \frac{\pi}{2}\{-2^\tau, -2^\tau + 1, \dots, 2^\tau - 1\}$$

$$\boldsymbol{K}' = \frac{\pi}{2}(\boldsymbol{c}^t\boldsymbol{x} - 2^\tau\boldsymbol{e}), \quad \begin{cases} \boldsymbol{c} = (2^0, 2^1, \dots, 2^\tau) \\ \boldsymbol{x} \in \{0,1\}^{\tau+1} \end{cases}$$

$$\min_{\boldsymbol{\lambda},\boldsymbol{x}} \|\boldsymbol{c}^t\boldsymbol{x} - 2^\tau\boldsymbol{e}\|_0,$$

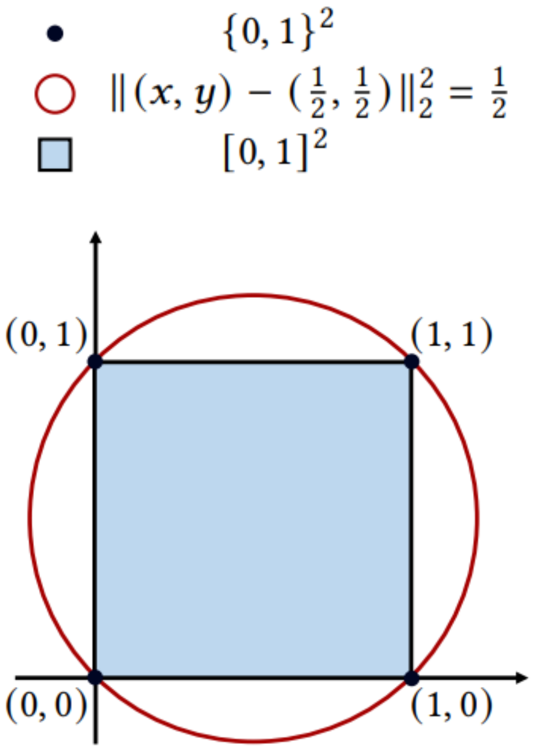
$$s.t. \begin{cases} \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \frac{\pi}{2}(\boldsymbol{c}^t\boldsymbol{x} - 2^\tau\boldsymbol{e}) \\ \left(\sum_i A_i \boldsymbol{\lambda}_i^2\right)^{\frac{1}{2}} \leq \beta \\ \boldsymbol{x} \in \{0,1\}^{\tau+1} \end{cases}$$

Li, M. et al. (2022). Computing sparse integer-constrained cones for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

- Consider $\boldsymbol{K}' \in \frac{\pi}{2}\mathbb{Z}^V$
- Integer constraint to binary constraint
- Binary constraint to the intersection of box and sphere

$$\{0,1\}^{\tau+1} = [0,1]^{\tau+1} \cap \partial B_{r=\frac{\sqrt{\tau+1}}{2}}\left(\left\{\frac{1}{2}\right\}^{\tau+1}\right)$$



Case: $\tau = 1$

Li, M. et al. (2022). Computing sparse integer-constrained cones for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

- Consider $\boldsymbol{K}' \in \frac{\pi}{2}\mathbb{Z}^V$
- Integer constraint to binary constraint
- Binary constraint to the intersection of box and sphere

$$\min_{\boldsymbol{\lambda},\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \|\boldsymbol{c}^t\boldsymbol{x} - 2^\tau\boldsymbol{e}\|_0\,, s.t. \begin{cases} \Delta\boldsymbol{\lambda} = \boldsymbol{K} - \frac{\pi}{2}(\boldsymbol{c}^t\boldsymbol{x} - 2^\tau\boldsymbol{e}), \boldsymbol{x} = \boldsymbol{y}, \boldsymbol{x} = \boldsymbol{z} \\ \left(\sum_i A_i\boldsymbol{\lambda}_i^2\right)^{\frac{1}{2}} \leq \beta \\ \boldsymbol{y} \in [0,1]^{\tau+1}, \boldsymbol{z} \in \partial B_{r=\frac{\sqrt{\tau+1}}{2}}\left(\left\{\frac{1}{2}\right\}^{\tau+1}\right) \end{cases}$$

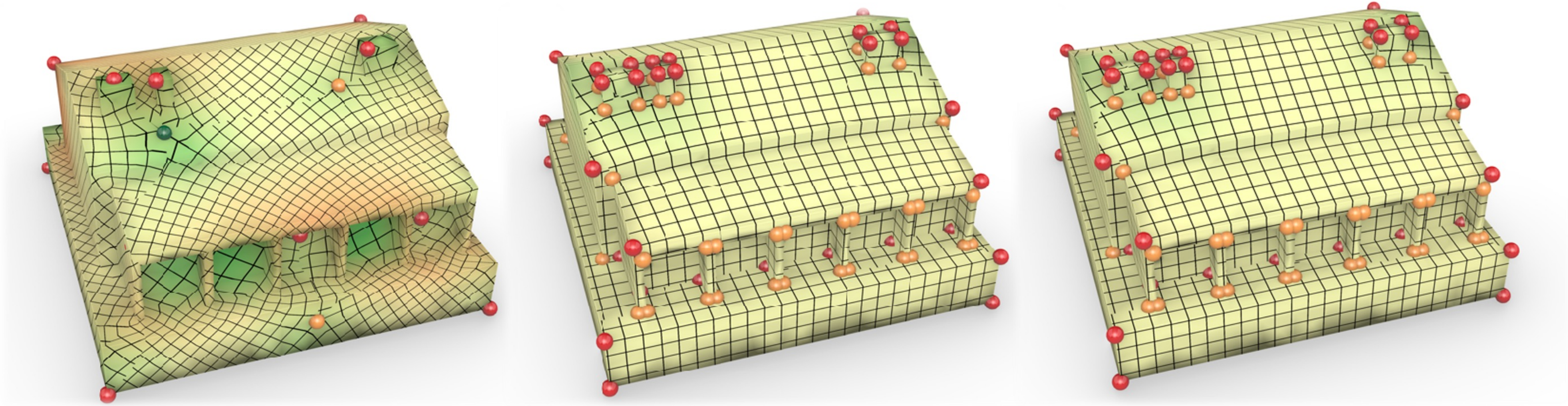
ADMM or DR Splitting!

Li, M. et al. (2022). Computing sparse integer-constrained cones for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

- Feasibility: $\boldsymbol{K}' \in \frac{\pi}{2}\mathbb{Z}^V \Leftrightarrow \mathcal{A} \le \beta \quad \left(\Delta\boldsymbol{\lambda} = \boldsymbol{K} - \frac{\pi}{2}\mathbb{Z}^V\right)$

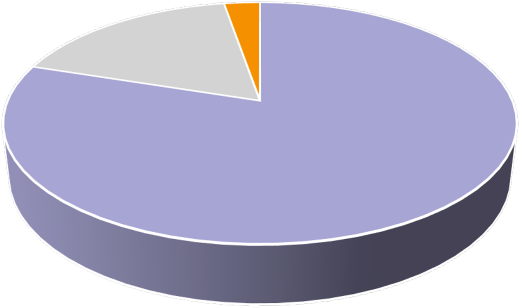
$\beta = 0.2$	$\beta = 0.1$	$\beta = 0.001$
$\mathcal{A} = 0.2$	$\mathcal{A} = 0.05$	$\mathcal{A} = 0.05$
$n = 15$	$n = 88$	$n = 88$



Li, M. et al. (2022). Computing sparse integer-constrained cones for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

- Feasibility: $\boldsymbol{K} \in \frac{\pi}{2}\mathbb{Z}^V \Leftrightarrow \mathcal{A} \le \beta$
- Compare to rounding strategy



Dataset (3885 models)

$\mathcal{A} < \mathcal{A}_c, n < n_c$ $\mathcal{A} = \mathcal{A}_c, n < n_c$ $\mathcal{A} < \mathcal{A}_c, n = n_c$	79.98%
$\mathcal{A} = \mathcal{A}_c, n = n_c$ $\mathcal{A} > \mathcal{A}_c, n < n_c$ $\mathcal{A} < \mathcal{A}_c, n > n_c$	17.32%
$\mathcal{A} > \mathcal{A}_c, n > n_c$ $\mathcal{A} > \mathcal{A}_c, n = n_c$ $\mathcal{A} = \mathcal{A}_c, n > n_c$	2.7%

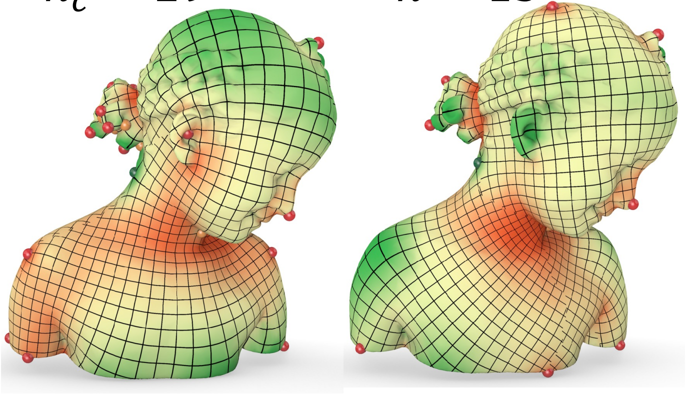
Li, M. et al. (2022). Computing sparse integer-constrained cones for conformal parameterizations. *ACM Transactions on Graphics*.

Optimization-based methods

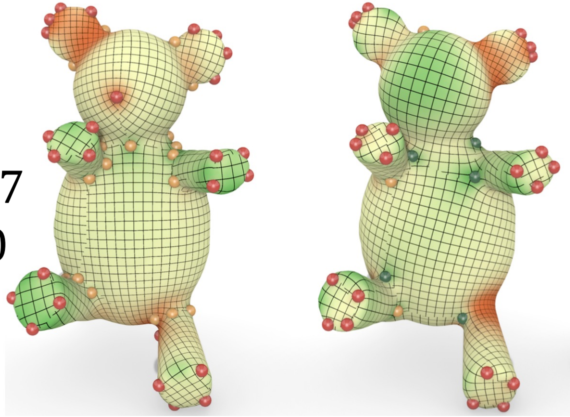
- Feasibility: $\boldsymbol{K} \in \frac{\pi}{2}\mathbb{Z}^V \Leftrightarrow \mathcal{A} \leq \beta$
- Compare to rounding strategy

$\mathcal{A} = 0.22$
$n_c = 29$

$\mathcal{A} = 0.22$
$n = 13$

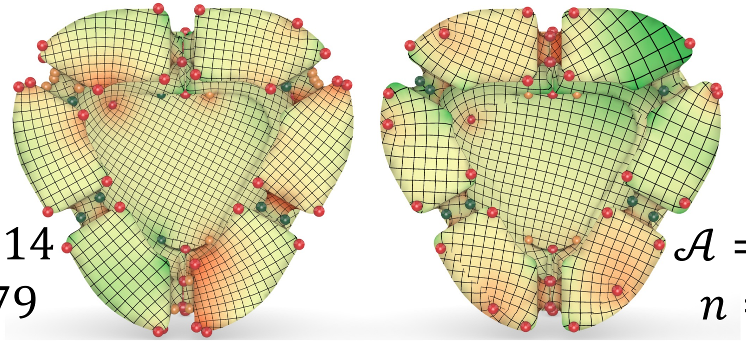


$\mathcal{A} = 0.17$
$n_c = 50$



$\mathcal{A} = 0.19$
$n = 35$

$\mathcal{A} = 0.14$
$n_c = 79$



$\mathcal{A} = 0.16$
$n = 82$

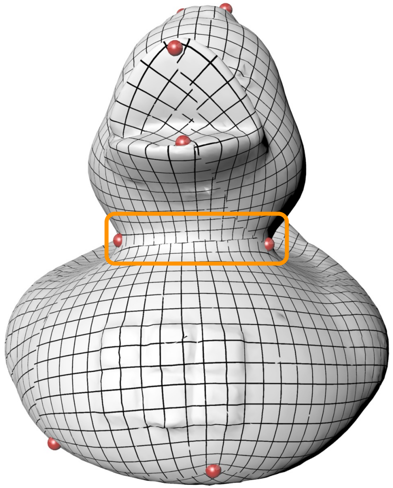
Li, M. et al. (2022). Computing sparse integer-constrained cones for conformal parameterizations. *ACM Transactions on Graphics*.

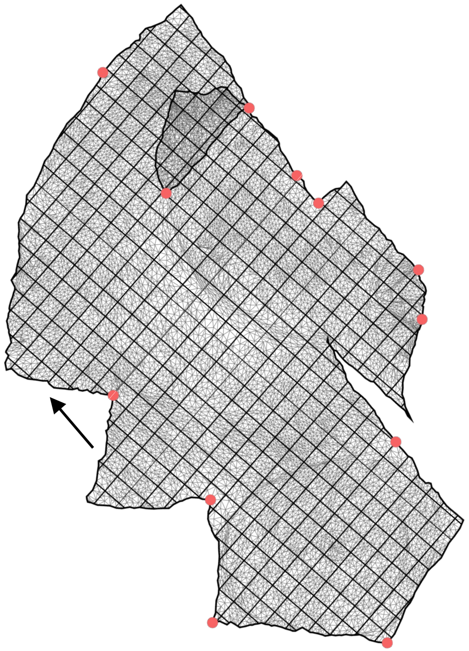
4

Applications related to cones

Seamless parameterizations

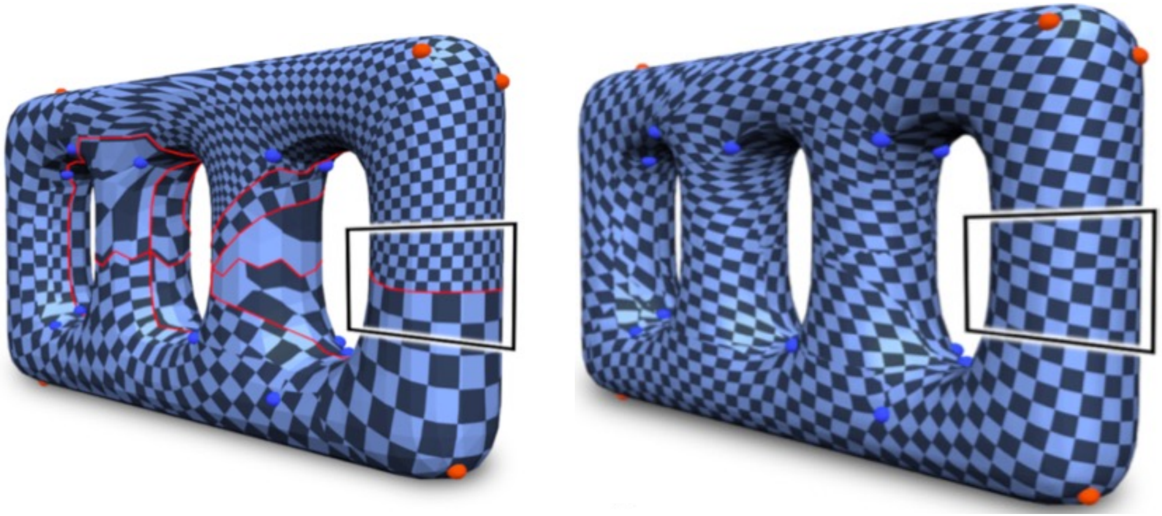
- Integer cones: $K' = \frac{\pi}{2}\mathbb{Z}$
  - Genus-0 mesh: global seamless





Seamless parameterizations

- Integer cones: $K' = \frac{\pi}{2}\mathbb{Z}$
  - Genus-0 mesh: global seamless
  - High genus mesh: rotationally seamless ⟶ global seamless

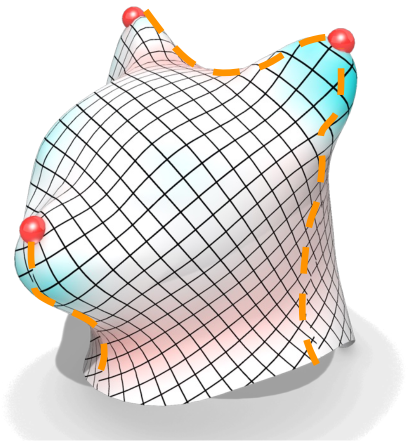


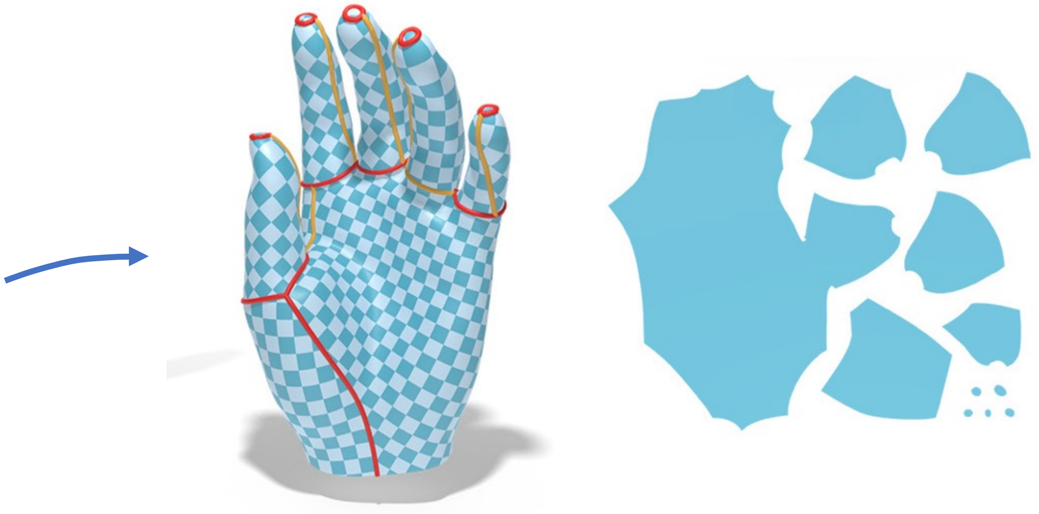
Post processing!

Piecewise parameterizations

- Cones ⟶ seam curves

$$\min_\lambda \mathcal{A}(\lambda) + \sum_i l(\partial P_i)\,, s.t.\;\; \Delta\lambda = -K \;\; in \;\cup P_i^{\circ}$$



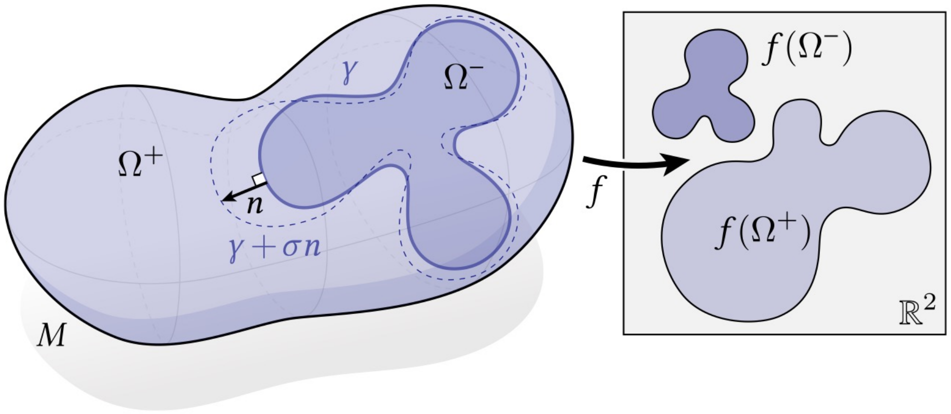


Piecewise parameterizations

- Cones ⟶ seam curves

$$\min_\lambda \mathcal{A}(\lambda) + \sum_i l(\partial P_i)\,, s.t.\ \ \Delta\lambda = -K\ \ in\ \cup P_i^\circ$$

- Level set revolution



High non-convex!

Sharp, N., & Crane, K. (2018). Variational surface cutting. *ACM Transactions on Graphics (TOG)*.

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谢 谢 ！

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