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RESEARCH

A hybrid optical feedback method for narrowing and frequency-stabilizing diode lasers

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Abstract
Lasers with narrow linewidths and long-term frequency stability are required in various applications such as precision measurement and optical frequency reference. Here, we propose a hybrid method that combines techniques of optical feedback and optical heterodyne modulation locking to an external Fabry-Perot cavity to reduce the linewidth and frequency drift of the laser. The method is demonstrated on a distributed feedback laser with a free-running linewidth of 2 MHz. The frequency noise power density spectrum shows a reduction of 50 dB in the low-frequency range and 30 dB for white noise, and the linewidth has been reduced to 20 kHz. The lock can be maintained for days. This method can be applied to various lasers of different wavelengths.

1 Introduction

Frequency-stabilized kilohertz-linewidth lasers are of great interest to both fundamental studies and industry. Applications include coherent optical communication protocols [1], precision spectroscopy [2], optical measurements [3] and frequency standards [4]. Robust, compact, low-cost, and single-mode diode lasers with sub-MHz linewidths are preferred in many applications. Narrow-linewidth external cavity diode lasers (ECDLs) meet most application requirements, but have drawbacks of relatively large size, high price, and occasional mode-hopping. Commercial distributed feedback (DFB) lasers have the above advantages except for linewidth. The typical free-running linewidth of a DFB laser is in the MHz range due to spontaneous emission [5, 6]. The frequency noise in diode lasers could be suppressed by the optical feedback method first proposed by Lang et al. [7, 8]. They applied optical feedback to the laser from the external mirror and found that the linewidth was sensitive to the distance between the laser output facet and the external mirror. Tkach et al. [9] demonstrated that the linewidth can be reduced if the feedback strength is within a small range of around -40 dB. Exceeding this small range may increase the possibility of mode hopping or coherence collapse. There are also studies [10] showing that linewidth narrowing could be realized under a feedback strength above – 10 dB, where the feedback is irrelevant to the optical phase and the laser output facet needs to be anti-reflective coated.

Feedback from an external Fabry-Pérot cavity can considerably reduce the linewidth of a diode laser. The physical mechanism of the narrowing effect can be described as follows. When the laser is coupled to an external cavity, a portion of the light emitted from the external cavity feeds back into the laser cavity and suppresses the frequency noise. Here the external cavity can be considered as a narrow-bandwidth optical filter, and the internal laser cavity acts as a broadband amplifier. A typical application of optical feedback is self-locking between the laser and a cavity mode, which can maintain resonance at the cavity resonant frequency for several seconds without deliberately controlling the laser frequency [11, 12]. The instantaneous linewidth can be compressed to as low as 12 Hz [10, 13]. By using a stabilized V-type cavity, frequency stabilization was achieved by optical feedback of resonant light without the direct reflection from the first mirror of the cavity [14–16]. Zhao et al. [13, 17] show that the directly reflected light (or non-resonant light) does not induce instability of the laser in the self-locking region. The optical feedback technique has also been

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combined with various spectroscopic methods for sensitive detection applications, such as the optical-feedback cavity-enhanced absorption spectroscopy [12, 14, 15], optical-feedback cavity ring-down spectroscopy [18–20], and optical-feedback cavity-enhanced Raman spectroscopy [21, 22].

The cavity resonance maintained by optical feedback depends on the frequency stability of the optical cavity and the slow drift of the laser source. Note that the above optical feedback effect usually only works within the self-locking region [14, 23]. If the frequency drifts out of the region, which may reach hundreds of MHz or more [17, 23], both the lock and the narrowing will be lost. The long-term drift of laser frequency can be stabilized through optimization of the cavity with thermal structural design and expensive ultra low expansion materials [24]. An alternative is electronic feedback, which enables direct current control to achieve frequency stability. Our work presents a hybrid optical feedback method that integrates an optical heterodyne modulation loop with an optical feedback servo. This method discriminates the frequency shift and feeds it back to modulate the laser current. The linewidth of a commercial DFB diode laser can be reduced from several MHz to a few kHz using this method, and the lock can be maintained for days of operation without requiring extra control stabilization of the environment, such as temperature and vibration. This is a significant advantage, as the locking can be maintained for a long time, even if the cavity and laser drift out of the self-locking region. This method is suitable for applications that require long-term laser frequency locking, such as precision spectroscopy, optical clocks, and frequency references.

2 Method and experiment

The schematic diagram of our experiment is shown in Fig. 1a. The effective optical length of the laser source is L_D. The external Fabry-Pérot (FP) cavity consists of two concave mirrors with a reflectivity of $R_m (R_m = r_m{}^2)$ and separated by a distance of L_c. The distance between the laser output facet and the first FP mirror is L_a. The feedback optical power attenuation ratio is γ, which could be manually adjusted. The feedback light consists of two parts, one is resonant with the FP cavity, and the other one is non-resonant and reflected from the first FP cavity mirror. Under the conditions of the linear cavity and weak optical feedback ($\gamma \ll 1$), the relationship between the laser frequency coupled to the FP cavity (ω) and the free-running laser frequency (ω_{free}) was given by the formula presented by Zhao et al. [13, 17]:

$$\omega_{\text{free}} = \omega + K_1 \frac{\sin[\omega(\tau_a + \tau_c) + \theta] - R_m \sin[\omega\tau_c + \theta]}{1 + F^2 \sin^2(\omega\tau_c/2)} - K_2 \sin(\omega\tau_a + \theta) \quad (1)$$

where $K_1 = K r_m/(1 - r_m{}^2)$, $K_2 = K r_m$, $F = 2r_m/(1 - r_m{}^2)$, $\theta = \arctan(\alpha_H)$, $K = \sqrt{\gamma(1 + \alpha_H{}^2)}\tau_d/2$, α_H is the Henry's factor of the DFB laser, τ_d is the phonton life time in the diode laser cavity [32].

From the simulation results of the feedback model in [12, 17], when the free-running laser frequency is far away from a longitudinal cavity mode, the non-resonant feedback of the linear cavity will cause a sinusoid frequency shift during the laser frequency sweep. When the laser frequency is close to a longitudinal mode of the FP cavity, the feedback from the resonant part will have a significant influence on the field inside the laser cavity. Consequently, the resulting

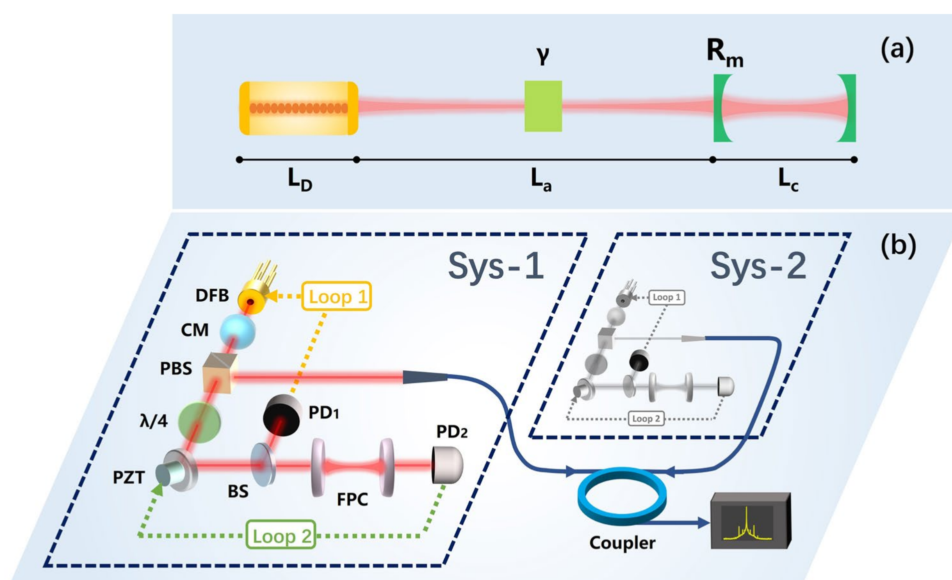

Fig. 1 **a** Schematic diagram of the hybrid external-cavity optical feedback method. **b** Experimental setup. Abbreviations: DFB, a TO-can distributed feedback laser diode; CM, collimator lens; PBS, polarization beamsplitter; λ/4, quarter-wave plate; PZT, piezo-electric transducer; BS, beamsplitter; PD, photodetector; FPC, a single-layer quartz glass Fabry-Pérot cavity; ESA, electrical spectrum analyzer

laser frequency changes with a very small slope and appears to be "locked" to the FP cavity mode. When the ratio L_a/L_c is close to an integer, the coupled laser frequency could be locked to the cavity modes one by one during the laser frequency sweep [11–13, 17]. The locking range at each resonant point is identical and is related to the fineness of the optical cavity and the feedback strength.

As mentioned above, the laser frequency may gradually drift out of the lock region, resulting in the loss of the optical feedback lock. To solve this problem, we introduce an additional feedback loop based on the optical heterodyne modulation, which actively locks the laser frequency to a longitudinal mode of the FP cavity. This locking loop is similar to the conventional PDH method [25], while the difference is that the feedback servo here mainly deals with drift in the very low frequency domain. The hybrid method, which combines the optical feedback and the PDH-like heterodyne locking servo, can provide robust locking of the laser to the external FP cavity. The PDH-like loop compensates for the slow drift of the FP cavity, and the linewidth narrowing effect resulting from the optical feedback reduces the feedback bandwidth requirement in PDH locking.

The optical layout is shown in Fig. 1b. The laser source is a continuous-wave TO-can DFB laser (Eblana Photonics) operating in the wavelength range of 1640–1670 nm with an output power of up to 13 mW, and a linewidth of 2 MHz according to the manufacturer. The diode laser chip is placed in a homemade mount and an aspherical lens is used to collimate the output laser beam. The temperature of the chip is controlled by a Thorlabs TED200C controller, while the diode current is supplied by a homemade current source equipped with DC and AC modulation channels. The feedback coefficient γ is controlled to be about – 50 dB in our experiment using the combination of an adjustable optical isolator (not shown in Fig. 1b), a polarized beam splitter (PBS), and a quarter-wave plate. The laser passes through a set of mode-matching lenses and a PZT-mounted mirror for optical phase adjustment, and then couples into the FP cavity. The cavity length is about 40 mm and the free spectral range (FSR) is 3.75 GHz. It consists of two planoconcave mirrors with a reflectivity of 99.92% and a radius of curvature of 1 m, corresponding to a finesse of about 4000 and a cavity mode width of 995 kHz. Part of the feedback light is sent through a beam splitter (BS) to a photodetector (PD1), while the light transmitted from the cavity is collected by another photodetector (PD2).

Here we use two electrical servo loops. The PDH-like optical heterodyne locking loop (Loop 1) is used to lock the laser frequency to a cavity mode. The heterodyne modulation is introduced by adding a sinusoidal wave with a frequency of 30 MHz to the current driving the DFB. The error signal is derived by mixing the modulated signal obtained from PD1 and the 30 MHz local reference and is sent to a proportional-integral-derivative (PID) controller. Note that the standard PDH method typically has a wide feedback bandwidth of over 10^5 Hz, while the heterodyne locking servo used here requires a bandwidth of just under 100 Hz. Loop 2 controls the optical phase in the optical feedback servo. It is implemented by adjusting the optical path length with sub-wavelength precision. A sinusoidal voltage signal is applied to the PZT mounted on the mirror in front of the FP cavity to modulate the optical path length and the optical phase. The signal from PD2 is sent to a lock-in amplifier (LIA), and the first-order harmonic (1f) output of the LIA is fed back to the PZT through a PID loop. Since Loop 1 locks the laser frequency near a specific mode of the FP cavity and Loop 2 compensates for the optical phase shift due to ambient noise, we no longer need the ratio L_a/L_c to be close to an integer.

3 Results and discussion

The locking of the laser could be monitored by the light transmitted from the FP cavity. We measured the cavity transmission signals under three conditions: with optical heterodyne feedback only, with optical feedback only, and with both feedback servos. The results are illustrated in Fig. 2a. When only the optical heterodyne feedback servo is on, the cavity transmittance signal is very low (blue trace in Fig. 2a). Due to the limited bandwidth of the servo loop, the servo can hardly lock the laser (linewidth ~ 2 MHz) to a rather narrow (995 kHz) cavity mode. If only the optical feedback ($\gamma = 6 \times 10^{-6}$) is turned on, we can occasionally observe a high transmission signal (orange curve) when the laser frequency is resonant with the FP cavity. However, the resonance can hardly be maintained for a few seconds due to the optical phase drift. When both servos are established, the transmission signal (green trace in Fig. 2a) is held at a high level, equal to the maximum of the orange trace, and the lock could be maintained for many hours (as shown in Fig. 2b) if both feedback servos are properly adjusted.

The hybrid method has a large locking range. We intentionally heated the FP cavity to increase the cavity length and measured the laser frequency using a wavemeter with an accuracy of 30 MHz (HighFinese WS8–30). We found that the locking was maintained during the process and the laser frequency followed the change of the cavity length, as shown in Fig. 2c. When the heater voltage is high, the cavity length changes rapidly, leading to a rapid variation in laser frequency. When the heater voltage is zero, the variation slope of cavity length and laser frequency also close to zero. The laser was kept locked to the cavity with a frequency change of several hundred MHz. Note that the amplitude of the drift is already larger than the capture range of the optical feedback locking. We also tried to let the cavity expand

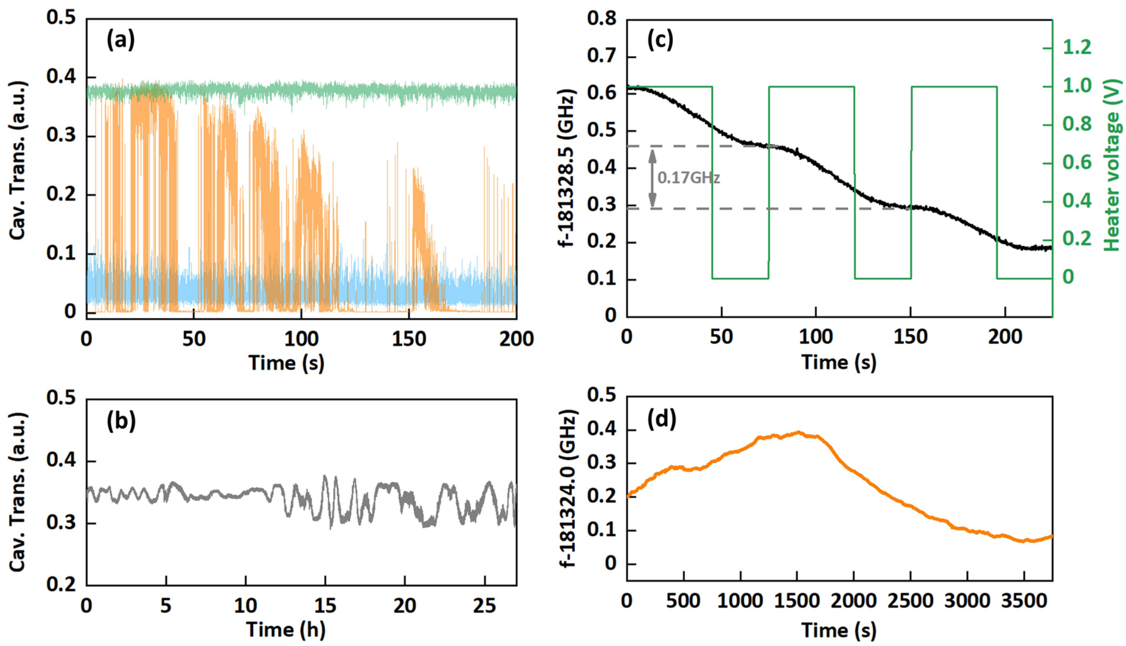

Fig. 2 **a** Cavity transmittance under three different conditions: with only optical heterodyne feedback (blue), with only optical feedback of $\gamma = 6 \times 10^{-6}$ (orange), and with both electrical and optical feedback of $\gamma = 6 \times 10^{-6}$ (green). **b** A 1-day-long record of the cavity transmittance when the hybrid locking is on. **c** Laser frequency (black) followed the change of the FP cavity length when the cavity was purposely heated by applying a heating current (green). **d** Laser frequency followed the change of the FP cavity length when the temperature of the cavity drifted with the ambient air

freely with the ambient temperature. In a test of about one hour, the locking was maintained when the laser frequency drifted in the range of about 350 MHz, as shown in Fig. 2d. The above test demonstrates that we can deliberately tune the laser frequency by controlling the external FP cavity, which is very useful in many applications. Note that a 30 MHz current modulation was applied to the laser diode for optical heterodyne locking. If sidebands induced by the current modulation are a concern in some applications, the electro-optic modulator could be used to avoid the sidebands in the laser output.

To evaluate the linewidth narrowing effect of the hybrid optical feedback method, we built two similar systems, named "sys-1" and "sys-2" below, and the beat note power spectrum was analyzed [26] to determine the linewidths of both lasers. As shown in Fig. 1b, a portion of the laser output from each system is coupled into an optical fiber and measured by a photodetector connected to a spectrum analyzer. When both DFB lasers are free running, the beat note is shown in Fig. 3a. The peak was fit with a Lorentzian function and the width is 5.5 MHz, which is reasonably close to twice the stated linewidth of each laser. We also measured the beat note with only optical heterodyne locking (not shown in Fig. 3) and found no significant difference in linewidth compared to the free-running laser. By applying electrical and optical feedback servos, the linewidths of both systems could be significantly reduced. The resulting spectra of the beat signal obtained with different optical feedback strengths are shown in Fig. 3b-d. In these three measurements, the feedback strength for sys-1 was kept the same, $\gamma_1 = 6$ ppm, while that for sys-2 was different: $\gamma_2 = 5.4$ ppm (Fig. 3b), $\gamma_2 = 5.6$ ppm (Fig. 3c), and $\gamma_2 = 7.0$ ppm (Fig. 3d). The widths of the beat spectra were measured to be 70 kHz, 41 kHz, and 30 kHz, respectively. This indicates that the linewidths of both lasers were reduced by more than two orders of magnitude.

The laser frequency noise was measured by the correlated delayed self-heterodyne (DSH) method [27, 28]. Part of the laser beam was separated by a 3 dB beam splitter, passed through an AOM with 80 MHz frequency shift, and another beam passed through a 20 m delay fiber. The two beams were then combined and sent to a signal source analyzer (Rohde-Schwarz, FSWP) for phase noise measurement. FNPSD data obtained under three different conditions, free running, optical heterodyne locking with the external FP cavity, and with both electrical and optical feedback servos, are presented in Fig. 4 as gray, blue, and green lines, respectively. We used the β-separation line method [29] to estimate the corresponding linewidth. According to the dark gray trace shown in Fig 4, the free-running linewidth of the DFB laser was 3.2 MHz with an integration time of 0.01 s[30]. The blue curve obtained with only optical heterodyne locking applied shows little frequency noise suppression and the

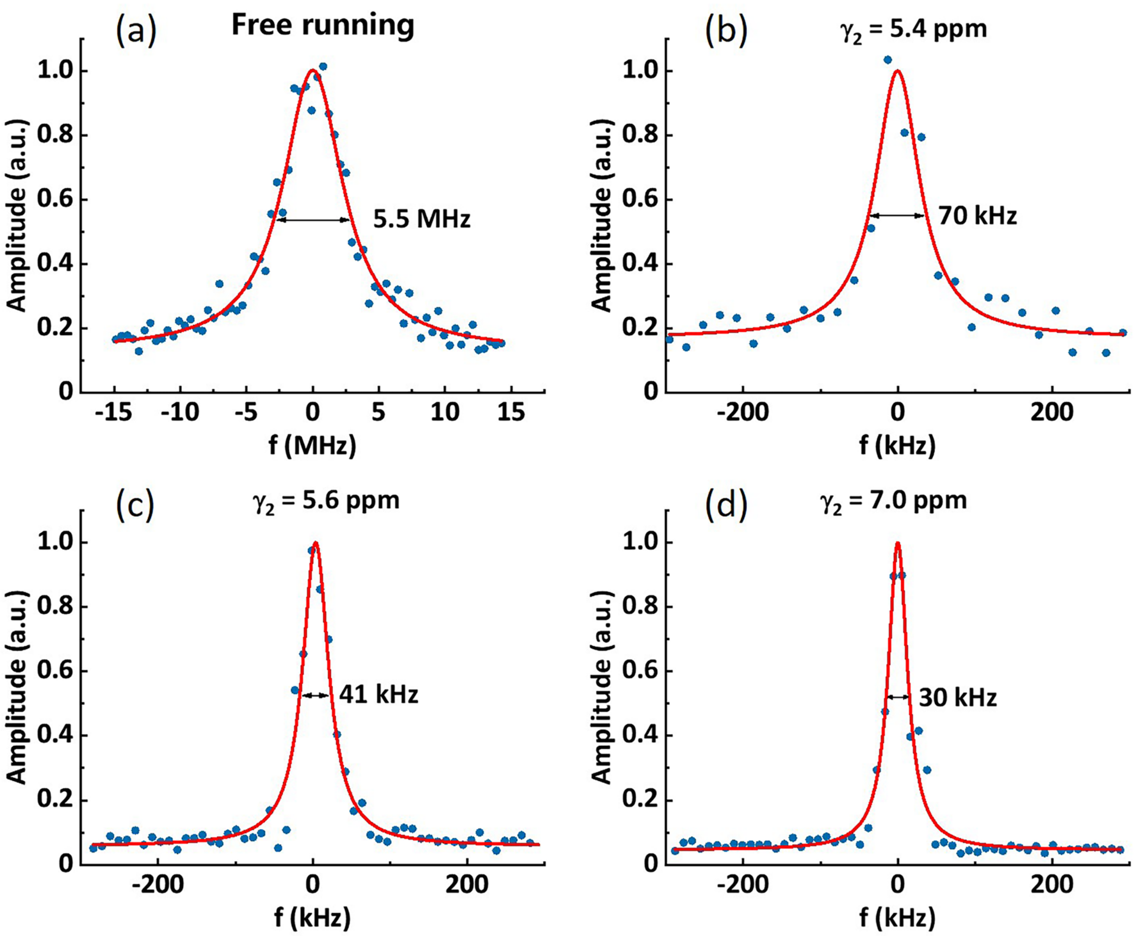

Fig. 3 Beat notes of two laser systems under different feedback conditions: **a** both free-running, **b–d** both optical heterodyne and optical feedback servo on, with the same $\gamma_1 = 6$ ppm but different γ_2 values. The sweeping time was 30 ms in **a** and about 500 ms in (**b–d**)

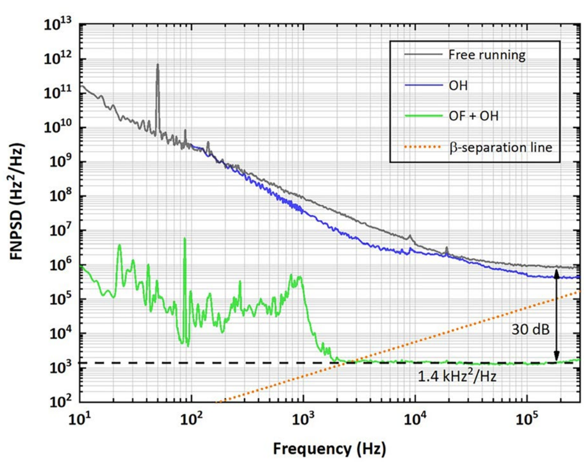

Fig. 4 Frequency noise power spectral density (FNPSD) data obtained under different conditions: free-running (dark grey), optical heterodyne-locked with the external FP cavity (blue), and with resonant optical feedback and optical heterodyne locking (green). The β -separation line is given by $S_\nu(f) = 8 \times \ln 2 \times f/\pi^2$ (dotted orange)

linewidth obtained by the integration is 2 MHz, close to that obtained under free running. It is well known that the frequency noise of a DFB laser can hardly be eliminated since the noise spectrum of the DFB laser is usually wider than the bandwidth of the electrical feedback servo [31]. However, the optical feedback can effectively reduce the linewidth of the laser. With both the electrical and optical feedback ($\gamma_1 = 6$ ppm) turned on in our system, the frequency noise was suppressed by about 30 dB in the frequency range of 2–100 kHz, and by almost 50 dB at frequencies below 100 Hz, as shown by the green curve in Fig. 4. The frequency noise is about 1.4 kHz^2/Hz at frequencies above 2 kHz, corresponding to an instantaneous linewidth of 4.4 kHz. The linewidth obtained by the β-line method under the same integration times is 28.4 kHz, which agrees well with that obtained by fitting the beat note spectrum (Fig. 3d). The residual frequency noise is mainly due to low-frequency 1/f noise and systematic electronic noise. Spikes in the frequency range of 10–1000 Hz may arise from the interference of electronic circuits and mechanical vibrations, since no temperature control or vibration isolation was used in our experiment.

Theoretical predictions for the effect of optical feedback in narrowing the laser linewidth have been presented in Refs. [11, 32]:

$$\frac{\Delta\nu_{\text{locked}}}{\Delta\nu_{\text{free}}} = \frac{1}{\gamma'\left(1+\alpha_{\text{H}}^2\right)}\left(\frac{\tau_d}{\tau_{\text{FP}}}\right)^2 \tag{2}$$

where only the white noise contribution is considered, ν_{free} is the linewidth of the free-running laser, γ' is the total feedback strength taking into account the coupling coefficient of the feedback light and DFB [13, 32], α_{H} is the Henry's factor of the DFB laser, τ_d is the photon lifetime in the diode laser cavity [32], and τ_{FP} is the ring-down time of the external FP cavity. Taking the parameters in Eq. 2 according to those used in our experiment, $\tau_d = 4.6$ ps, $\tau_{\text{FP}} = 0.17\ \mu$s, $\alpha_{\text{H}} = 2$, $\gamma = 6 \times 10^{-6}$, and $\gamma' = 0.03\gamma$ [13], we estimated that the line width could be reduced by a factor of 8.4×10^{-4}. The observed experimental result is $\frac{4.4\ \text{kHz}}{3.2\ \text{MHz}} = 1.4 \times 10^{-3}$, which is in reasonable agreement with the calculated value.

Note that both the optical injection and the optical heterodyne modulation locking in this work are based on the same external FP cavity, which is different from the method in Ref. [33]. This method is convenient and could be applied in different wavelength regions. Since the self-locking range is much smaller than the FSR of the optical cavity and the optical cavity length is affected by the environmental temperature drift and mechanical vibrations, the pure optical feedback method can hardly maintain the lock for a long time, without carefully control of the external environment. In the hybrid locking scheme presented here, the electrical feedback servo keeps the laser frequency tracking the drift of the optical cavity mode, which makes the optical feedback immune to environmental drift. Moreover, our method does not require the value of L_a to be close to an integer multiple of L_c, which benefits experimental convenience and flexibility. It is also worth noting that the spectrum of a cavity mode could be distorted by optical feedback, as well as the error signal in the electrical locking servo. Therefore, we need to use a relatively low optical feedback strength in this hybrid method, typically – 50 dB in this work, which is much lower than that used in pure optical feedback methods (– 40 dB or even – 25 dB) [10, 13]. The conflict between the electrical and optical feedback loops is not significant because of the different capture ranges in the frequency domain of these two feedback mechanisms. We have observed that a narrower linewidth could be achieved by using a high-finesse optical cavity, but the system would be more sensitive to the control of the optical feedback strength.

4 Conclusion

We propose a hybrid method combining optical feedback and PDH-like locking based on optical heterodyne modulation to lock the frequency and reduce the linewidth of a DFB laser from several MHz to the kHz level by using an external Fabry-Pérot cavity. The optical feedback suppresses the frequency noise of the laser, and the heterodyne servo tracks the laser frequency with the FP cavity mode. A large dynamic locking range could be achieved by this hybrid method, which allows us to adjust the frequency of the narrow-linewidth laser by controlling the external FP cavity. The method avoids the constraints of the laser-to-cavity length ratio, but only needs to stabilize the phase of the weak optical feedback. We demonstrated the method using a TO-can DFB laser with 2 MHz free-running linewidth at 1650 nm and a single-layer quartz cavity with a finesse of 4000. The laser frequency could be locked to the FP cavity for days, and the linewidth was reduced from about 2 MHz to less than 20 kHz. The system is robust and can be further improved by using an integrated mechanical structure. The use of a more refined cavity with a narrower mode width can further reduce the linewidth of the laser if the electronic servo loop is also optimized. In general, this hybrid method is very suitable for applications where both narrow linewidth and long-term frequency stability of the laser are required. The method could be applied to various lasers with different wavelengths, such as quantum cascade lasers, distributed Bragg reflector lasers, and DFB semiconductor lasers in the near-infrared and mid-infrared regions. Such narrow linewidth and frequency-stabilized laser sources could be applied to various studies, such as precision spectroscopy, optical frequency standards, laser cooling, and trace detection.

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Data availability All data generated or analyzed during this study are included in this published article.

Declarations

Conflict of interest The authors declare no conflict of interest.

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