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ABSTRACT

The measurement and control of mechanical resonators are critical for cavity magnomechanics, which has emerged as an important frontier for hybrid quantum systems based on magnonics. Traditional microwave-based measurements require handling high-frequency signals and cannot achieve field distribution detection. Here, we demonstrate a method for optically measuring and manipulating a ferromagnetic mechanical resonator. This technique allows for direct observation of the response and field distribution of mechanical oscillations during magnomechanically induced transparency/absorption processes, confirming that the mechanical mode $S_{1,2}$ is coupled with the magnon. The optical measurement not only serves to validate the magnomechanical coupling theory but also reduces the necessity for high-frequency measurements. Furthermore, we utilize the instantaneously measured results to implement feedback control of the self-oscillating mechanical resonator to overcome the dynamical back-action limit, achieving a threefold enhancement of phonon lasing amplitude. This feedback control lays the foundation for the study of quantum cavity magnomechanics, such as the feedback cooling of a magnomechanical resonator.

In analogy to cavity optomechanics,[1–3] cavity magnomechanics has emerged as an important new frontier for hybrid quantum systems based on magnonics.[4–9] The magnetostrictive forces could be used to couple the mechanical degrees of freedom and magnon oscillator, which can be flexibly adjusted through the magnetic field.[10–12] In particular, the cavity magnomechanical system consisting of magnons and phonons in a single-crystal yttrium iron garnet (YIG) sphere has made remarkable progress recently. Many applications have been promoted based on this platform ranging from magnomechanically induced transparency/absorption (MMIT/MMIA),[13–15] magnetometry,[16,17] mechanical bistability,[18] tunable microwave-to-optical conversion,[19,20] and magnonic frequency comb[21–24] to long-lifetime coherent storage for microwave photons.[25–27] Among these applications, precise measurement and control of mechanical resonators play a critical role, especially for the self-oscillating mechanical resonators with larger amplitudes that can be harnessed to broaden magnonic frequency comb, minimize phase noise of microwave oscillators, probe the limits of magnomechanical sensors, and study nonlinear dynamics.

Although magnomechanical coupling allows for the measurement and control of mechanical resonators using microwave fields, mismatches in frequencies exceeding GHz between magnons and mechanical oscillators require high-frequency microwave measurements, or the use of high-frequency microwave sources as local oscillators to compensate for this frequency mismatch through homodyne measurement.[14] Additionally, the microwave measurements applied to magnons are unable to map the field distribution of the mechanical field due to its own wavelength limitation, which hinders the comprehensive characterization and performance optimization of magnomechanical resonators. At the same time, the further development of applications reliant on self-oscillation in magnomechanical systems is impeded by the constrained maximum displacement, arising from the dynamical back-action limit.[28]

Here, we demonstrate a method for optically measuring and manipulating the mechanical resonator of a YIG microsphere using optical heterodyne detection.[29–33] This technique allows for highly accurate retrieval of vibration information from the YIG sphere, including center frequency, amplitude, and linewidth. Our optical measurements have good agreement with microwave measurements, while also providing spatial resolution by mapping the distribution of mechanical vibrations along the equator of the YIG sphere. The experimental results confirm that the mechanical mode $S_{1,2}$ is coupled with the magnon, rather than the more common $S_{1,0}$ mode in the optomechanical resonator. Furthermore, when the mechanical-induced frequency shift of the magnon oscillator surpasses the magnon linewidth, the amplitude of the self-oscillating mechanical resonator tends to saturate. To overcome this limitation, we implement a measurement-based feedback control of the self-oscillation using this optical measurements, achieving a threefold enhancement of phonon lasing amplitude. Our results lay the foundation for fully implementing bandwidth-friendly measurement and feedback cooling of a magnomechanical resonator.

The experimental setup for optical measurement and feedback control of the YIG microsphere is illustrated in Fig. 1(a). For the heterodyne measurement part, a laser beam is divided into two parts via a polarization beam splitter (PBS). One is the detection beam and the other is the local oscillator, which is blue-shifted 100 MHz by an acousto-optic modulator (AOM) to avoid the low-frequency noise.[34] The detection beam is focused on the surface of the YIG microsphere through the objective lens. When the mechanical resonator is excited, the vibration information of the mechanical mode is carried by the photons reflected from the surface of the YIG sphere and measured through the interference between the reflected photons and local oscillator. The beating signal is around the vicinity of $100\,\text{MHz}\pm\omega_m$, where $\omega_m \approx 8$ MHz is the mechanical frequency of the YIG sphere. The beating signal can be demodulated and amplified by the lock-in amplifier (LIA).[35]

First, our approach is used to directly measure the response of the mechanical resonator during MMIT and MMIA processes. This measurement not only serves to validate the magnomechanical coupling theory but also allows for comparison of results with microwave measurements, thus corroborating the efficacy of optical measurements. The MMIT/MMIA processes are derived from the cavity magnomechanical properties of the YIG microsphere, by analogy with optomechanical systems.[11,36–40] When an external magnetic field exists, the YIG microsphere contains a uniform magnon mode with frequency of ω_{magnon} that can be excited by an antenna placed near the YIG microsphere. The magnon mode can couple with the mechanical mode through magnetostrictive interaction by introducing a strong pump microwave. When the frequency of the pump field satisfies $\omega_{pump} + \omega_m = \omega_{magnon}$, corresponding to the red detuning and MMIT process as shown in Fig. 1(b), the interaction between the magnon mode and the mechanical mode can be described with a linear beam splitter model, where a weak probe microwave ($\sim\omega_{probe}$) resonant with the magnon mode is converted into mechanical vibrations. This conversion leads to the observation of a transparent window in the reflection spectrum of the probe when sweeping across the magnon mode using a network analyzer, as shown in Fig. 1(c). In our experiment, the dissipation rate of the mechanical and magnon modes are about $\kappa_b/2\pi = 500$ Hz and $\kappa_m/2\pi = 3.8$ MHz respectively. The single magnon-phonon coupling strength is described by $g/2\pi = 6.5$ mHz.

During the sweep of the probe across the magnon mode and occurrence of the MMIT process, the corresponding vibration information is directly detected through optical measurements and constitutes the mechanical mode in the frequency domain. By varying the intensity of the pump microwave, the vibration amplitude and the properties of the mechanical resonator change as well. Figure 2(a) shows the measured mechanical modes with different pump intensities from 5 to 11 dBm. It clearly indicates that, with the intensity of pumping microwave increasing, the linewidth of the mechanical mode is widened, as depicted in Fig. 2(b). In addition, the mechanical mode frequency shifted toward low frequency, and the electrical measurement results from the network analyzer are also shown in Fig. 2(b). These results are consistent with the heterodyne measurement. Furthermore, when the pump field ω_{pump} is blue-detuned from the magnon resonance ω_{magnon} by one mechanical frequency, the magnomechanical interaction is converted into the parametric oscillator model, corresponding to MMIA process.[13] The response of mechanical resonator is illustrated in Fig. 2(c), showing different trends for the

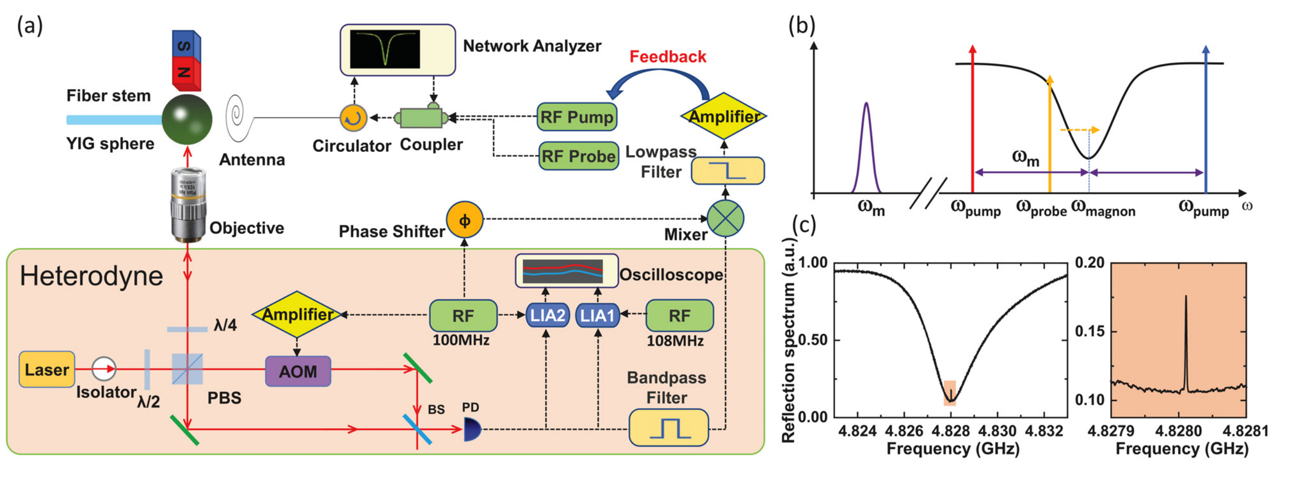

FIG. 1. (a) Schematic of the experimental setup of mechanical vibration measurement. PBS: polarization beam splitter; BS: beam splitter; PD: photodetector; AOM: acoustic optical modulator; LIA: lock-in amplifier; All microwave sources have been synchronized. (b) Spectral positions for the pump and probe signals used in magnomechanically induced transparency/absorption experiment, while the mechanical mode exists. (c) The typical spectrum showing a transparency window when the probe signal is scanning across the magnon mode.

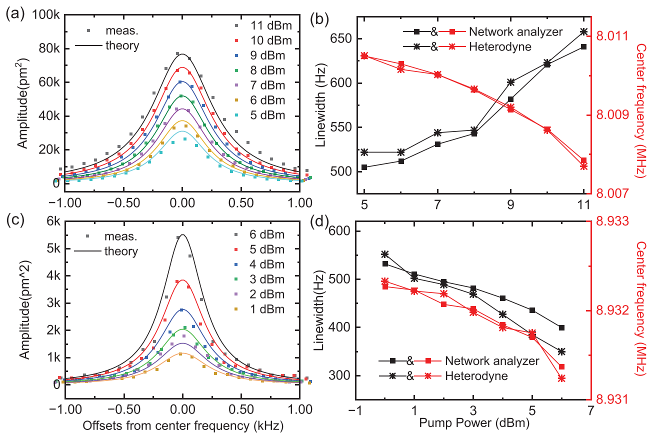

FIG. 2. (a,c) The points represent the squared YIG vibration amplitudes obtained from measurements with different pump intensities at red or blue detuning, respectively. The Lorentz lines are the corresponding theoretical results of squared amplitudes of the mechanical fields. (b,d) The center frequencies and linewidths of the mechanical mode with different pump intensities with heterodyne method from (a,c) or network analyzer, respectively.

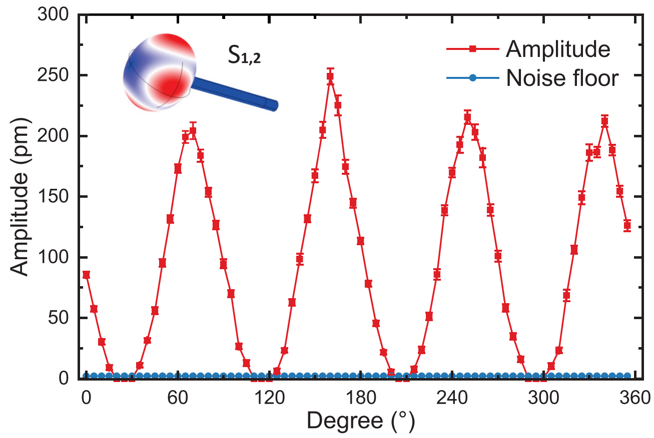

FIG. 3. Measured amplitude distribution along the equatorial line of YIG microsphere (red line) and the corresponding noise level (blue line). The inset shows the COMSOL simulation of the amplitude distribution of the mechanical mode.

linewidth with increasing pump power, as depicted in Fig. 2(d). However, the center frequency is shifted with the similar trends to MMIT case. Here, another YIG microsphere with $\omega_m \approx 8.93$ MHz is used for Figs. 2(c) and 2(d). Through optical measurements, we directly observed the optical spring effect of mechanical resonator and the changes in linewidth caused by dynamical back-action.[14] These observations are consistent with both electrical measurements and theoretical predictions, making the experimental study of this process more comprehensive.

While YIG microspheres share a similar structure to silica microspheres, the mechanical mode coupled with optical modes through radiation pressure in silica microspheres is the $S_{1,0}$ breathing mode, which has a uniform amplitude distribution on the equator. In contrast, theoretical analysis indicates that the mechanical mode $S_{1,2}$ couples with the Kittel magnon mode in YIG spheres,[13] characterized by a non-uniform amplitude distribution. Microwave measurements lack the capability to provide detailed pattern distribution of mechanical resonators, rendering direct verification challenging. Leveraging the enhanced spatial resolution of optical detection down to the spot size, we will next investigate the amplitude distribution along the equator to distinguish this mechanical mode. Here, the YIG sphere is continuously rotated with a step of 5° with the stem as the rotation axis, maintaining other conditions. The measured vibration amplitudes are shown in Fig. 3. Due to the noise floor, some experimental results with too small amplitude are overwhelmed, which will not materially affect the overall trend. Obviously, the vibration amplitude presents a periodic change with a period length of about 90°. COMSOL simulation of mechanical mode $S_{1,2}$ shown as an inset also exhibits four periods within 360°. The measurement results are consistent with theoretical analysis and show the advantages of optical measurements.

Our approach not only enables the precise characterization of mechanical resonators but also facilitates direct application in feedback control, achieving enhanced self-sustained mechanical oscillation, commonly referred to as a phonon laser.[41] When the microwave pump is blue-detuned from the magnon mode at the mechanical frequency, the effective dissipation rate of the mechanical resonator decreases and the magnon undergoes a frequency shift dependent on the phonon number. Upon surpassing a certain threshold, the effective dissipation rate can become zero, which means the amplitude of mechanical oscillation will be strongly excited and increase exponentially over time until saturated, when the instantaneous frequency shift of the magnon mode induced by the oscillations is comparable to the magnon linewidth. This indicates the pump has deviated significantly from the originally nominal blue detuning. Therefore, the principle of our feedback is to adjust the pump frequency based on the instantaneously measured mechanical oscillation to compensate for the mechanical-induced frequency shift of the magnon mode, and dynamically match the frequency difference between the pump and the magnon oscillator with that of the mechanical resonator. The experimental setup for the feedback section is introduced as shown in Fig. 1(a). The signal received by photodetector first passes through a bandpass filter to extract only one probe signal (with the frequency of $\omega_{ref} + \omega_{mech}$ or $\omega_{ref} - \omega_{mech}$), which contains vibration information. After that, it is mixed with ω_{ref} which has been phase shifted by φ. The original vibration signal is extracted out from a lowpass filter. Using this signal as the phase modulation input for the pump microwave source, the feedback control of phonon laser can be realized.[42]

Figure 4(a) shows the max-hold spectrum of the phonon laser with the feedback on and off. The mechanical comb teeth with interval of ω_m are all enhanced. The effect of feedback is related to phase shift φ and modulation depth of microwave source. Figure 4(b) shows the influence of phase shift φ on the first sideband of the mechanical comb. The feedback is phase sensitive, with the maximum enhancement located near 140°. This value may vary depending on the experimental devices. Figure 4(c) shows the influence of the depth of phase modulation under optimal phase shift. With a modulation depth of about 2 rads, the feedback is close to saturation. The enhanced self-oscillation amplitude provides access to rich nonlinear optomechanical dynamics[43] and assist in observing nonlinear nanomechanical effects.[44]

In summary, we present an optical measurement approach for the detection of the mechanical resonators of the YIG microsphere, as

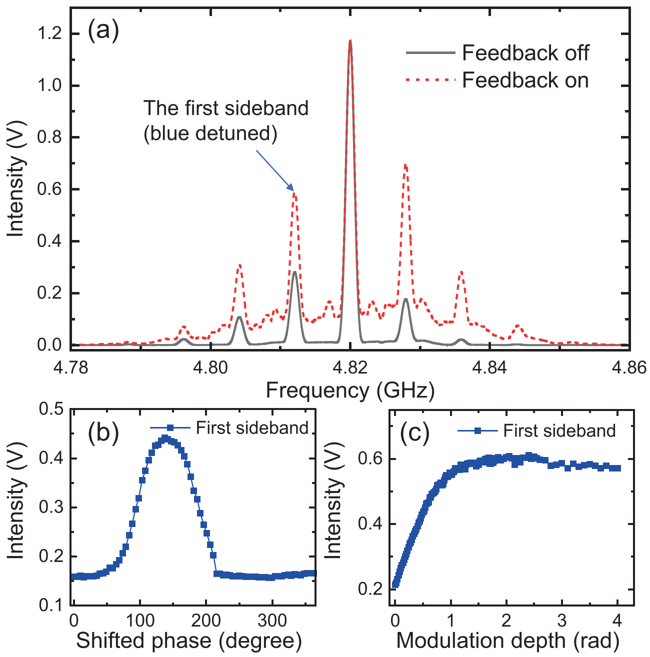

FIG. 4. (a) The max-hold spectrum of the phonon laser before and after the feedback is turned on. (b) Feedback enhancement dependence on phase shift. (c) The max feedback enhancement dependence on modulation depth.

well as the feedback control loop of the enhanced phonon lasing. In contrast to traditional electrical measurements, this scheme allows for the direct observation of the response and field distribution of mechanical resonators during MMIT and MMIA processes. This not only helps to verify the magnomechanical coupling theory but also reduces the necessity for high-frequency measurements. It also has the potential to be applied to infer the magnon population of ferromagnet[45] and realize the coupling between opto-magnonmechanical systems and atoms.[46] Currently, the excitation of mechanical vibrations still relies on the parametric process with high-frequency pumping, which could be replaced by laser pulse excitation to achieve fully bandwidth-friendly measurement.[47] Furthermore, we utilize the instantaneously measured results to implement feedback control of the self-oscillating mechanical resonator to overcome the dynamical back-action limit, achieving a threefold enhancement of phonon lasing amplitude. This result not only benefits the applications reliant on self-oscillation, such as generating frequency combs and squeezed states, but also lays the foundation for the study of quantum cavity magnomechanics, such as the feedback cooling of a magnomechanical resonator.[12,22]

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Hong-Yi Qiao: Data curation (lead); Formal analysis (equal); Investigation (equal); Writing – original draft (lead). **Guan-Ting Xu:** Investigation (equal); Methodology (equal). **Zhen Shen:** Conceptualization (equal); Funding acquisition (equal); Supervision (equal); Writing – review & editing (equal). **Yu Wang:** Methodology (equal). **Guang-Can Guo:** Resources (equal). **Shui-Ming Hu:** Resources (equal); Supervision (equal). **Chun-Hua Dong:** Funding acquisition (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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