

# Precision spectroscopy of the helium atom

Shui-ming HU (胡水明)<sup>1</sup>(✉), Zheng-Tian LU (卢征天)<sup>2,3</sup>, Zong-Chao YAN (严宗朝)<sup>4</sup>

<sup>1</sup> Hefei National Laboratory for Physical Sciences at the Microscale, University of Science and Technology of China, Hefei 230026, China

<sup>2</sup> Physics Division, Argonne National Laboratory, Cass Avenue, Argonne, IL 60439, USA

<sup>3</sup> Department of Physics and Enrico Fermi Institute, University of Chicago, 5801 South Ellis Avenue, Chicago, Illinois 60637, USA

<sup>4</sup> Department of Physics, University of New Brunswick, 8 Bailey Drive, Fredericton, NB E3B 5A3, Canada  
E-mail: smhu@ustc.edu.cn

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Persistent efforts in both theory and experiment have yielded increasingly precise understanding of the helium atom. Because of its simplicity, the helium atom has long been a testing ground for relativistic and quantum electrodynamic effects in few-body atomic systems theoretically and experimentally. Comparison between theory and experiment of the helium spectroscopy in  $1s2p^3P_J$  can potentially extract a very precise value of the fine structure constant  $\alpha$ . The helium atom can also be used to explore exotic nuclear structures. In this paper, we provide a brief review of the recent advances in precision calculations and measurements of the helium atom.

**Keywords** precision spectroscopy, helium atom, fine structure constant

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to build  $r_{12}$  in helium wave functions so that they can yield ever increasing degrees of accuracy. Some milestones along this line include the work of Kinoshita [2], Frankowski and Pekeris [3], Drake [4], Goldman [5], Korobov [6], Schwartz [7], and Nakashima and Nakatsuji [8]. Convergence to 20 or more significant digits can now be readily obtained variationally using the Hylleraas method or its variants. The work of Schwartz [7] and of Nakashima and Nakatsuji [8] indicates that it is practically feasible to solve the helium energy eigenvalue problem to any desired accuracy. After 80 years of Hylleraas work, the nonrelativistic problem for helium can be solved essentially exactly for all practical purposes and helium has now become a fundamental testing ground for relativistic and QED effects.

Figure 1 shows the energy diagram of the helium atom. The relativistic and QED corrections to the energy can be expressed as a double series [9] in terms of the fine structure constant  $\alpha : 1/137$ , and the mass ratio between the electron and the helium nucleus  $\mu/M : 10^{-4}$ . So far, the corrections up to order  $\alpha^4 R_y$  have been established by Pachucki for an arbitrary state of helium [10]. However, there exists a significant disagreement between Pachucki's theory and the experiment for the ionization energy of the singlet state  $1s2p^1P_1$ . An improved mea-

## 1 Precision theory for helium

Due to its few-electron nature, the starting point for the atomic helium problem is to solve the nonrelativistic Schrödinger equation precisely, including the finite nuclear mass correction. After obtaining the wave function, one can calculate relativistic and quantum electrodynamic (QED) effects perturbatively. The main stream of development in solving the nonrelativistic Schrödinger equation for helium began with the pioneer work of Hylleraas [1] who introduced the inter-electronic distance  $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$  explicitly in his variational basis set. Since then, intensive studies have been performed on how

surement for the  $1s2p^1P_1-1s3d^1D_2$  transition frequency would be very important towards resolving this discrepancy and thus in testing QED theory at the level of  $\alpha^4$ . The most important problem in helium [11, 12] is the fine structure splittings in the  $1s2p^3P_J$  manifold, where a comparison between theory and experiment could render a precise determination of the fine structure constant, provided both theory and experiment can be carried out to sufficiently high precision. Theoretical calculation of the  $1s2p^3P_J$  fine structure includes the spin-dependent part of relativistic and QED corrections of orders  $\alpha^2$ ,  $\alpha^3$ ,  $\alpha^4$ ,  $\alpha^5 \ln \alpha$ , and  $\alpha^5$ , as well as some nuclear recoil terms. All of these terms except the last one have been confirmed theoretically by at least two independent groups using different approaches. The  $\alpha^5$  term has been derived by Zhang [13–17] using Bethe–Salpeter formalism and by Pachucki [18–22] using an effective Hamiltonian approach, but the results from these two approaches are not in complete agreement. It is therefore highly desirable to re-examine these terms and their numerical evaluation before a precise value of  $\alpha$  can be derived from the  $1s2p^3P_J$  fine structure.

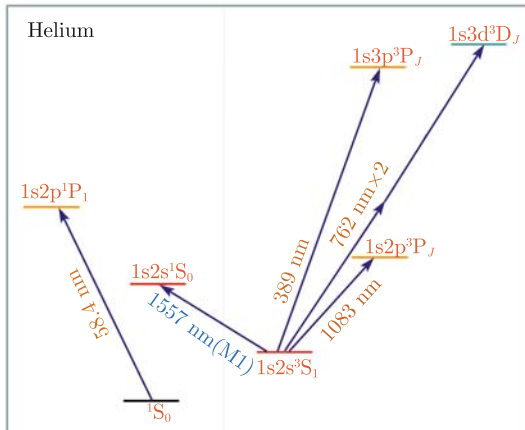


Fig. 1 Energy diagram of the helium atom.

## 2 Experimental progress

### 2.1 The fine structure constant $\alpha$

The most accurate value of the fine structure constant  $\alpha = e^2/(\hbar c)$  was derived from a comparison between the QED calculation and the Penning-trap based measurement of the anomalous magnetic moment ( $g - 2$ ) of the electron [23] with a precision of 0.37 ppb. It is desirable to determine  $\alpha$  from an independent system involving very different physical effects in order to check the consistency of the QED theory [24]. In 1964, Schwartz [25] pointed out that the fine structure constant  $\alpha$  can potentially be derived from the fine structure splittings of the  $1s2p^3P_J$  ( $J = 0, 1, 2$ ) manifold. Since then, signif-

icant progress has been made in studying the  $1s2p^3P_J$  fine structure both theoretically and experimentally. The fine structure of  $1s2p^3P_J$  of  $^4\text{He}$  is shown in Fig. 3. Although the fine structure of the hydrogen atom can also be calculated and measured, the fine structure of  $1s2p^3P_J$  of the  $^4\text{He}$  atom has three advantages over hydrogen, i.e., its larger fine structure splittings, narrower transition linewidth, and the lack of hyperfine structure. A 1 kHz uncertainty in comparison between theory and experiment for the  $2^3P_0-2^3P_1$  interval of 29.6 GHz corresponds to a 17 ppb uncertainty in the determination of  $\alpha$ . In recent years, measurements at the kHz level have been performed on either direct microwave transitions within the manifold [26, 27] or frequency differences of the optical transitions into the manifold [28–31].

Hessels *et al.* measured the  $2^3P_1-2^3P_2$  [26] and  $2^3P_1-2^3P_0$  [27] microwave transitions at 2.29 GHz and 29.6 GHz, respectively. In their measurements, a thermal beam of metastable helium atoms in the  $2^3S_1$  states was extracted from a DC discharge. The  $2^3S_1$  state with  $m = 0$  was first emptied by a 1083 nm laser beam tuned to the resonance of  $2^3S_1-2^3P_1$ . Then this state was repopulated by the microwave induced transition and its population was detected with a laser induced fluorescence. The transition frequencies were determined to be  $2\,291\,174.0 \pm 1.4$  kHz and  $29\,616\,950.9 \pm 0.9$  kHz for these two fine structure intervals, respectively.

In a different experiment performed by Castilleja *et al.* [30], also starting from a thermal metastable helium beam, the optical transitions  $2^3S_1-2^3P_J$  ( $J = 1, 2$ ) were induced to determine the  $2^3P_1-2^3P_2$  interval. The experimental design was similar to the Rabi's atomic beam magnetic resonance method, but with the magnetic resonance part being replaced by a depolarizing laser-induced transition resonance. Their uncertainty was 1.0 kHz, which mainly came from the line shape fitting and the first-order Doppler shift.

Zelevinsky *et al.* [31] studied the  $2^3P_J$  fine structure intervals by performing a saturation spectroscopy measurement on the  $2^3S_1-2^3P_J$  transition at 1083 nm with helium atoms in a continuously pumped gas cell. A sub-kHz precision was achieved after taking into account the systematic line shifts due to various effects, such as gas pressure, light power, and discharge power.

The most accurate experimental and theoretical fine structure intervals of  $1s2p^3P_J$  are collected in Table 1. Although the experimental results from different groups agree with each other at the 1 kHz level, there is an unexplained discrepancy between the calculated and experimental values. The disagreement in  $\nu_{12}$  is as large as 10 times the experimental accuracy. Note that there is also a large disagreement between the two different theoretical values so that further theoretical investigations are necessary to check the consistency. We believe

that further efforts in both theory and experiment can resolve these discrepancies, and will then provide the second most accurate determination of  $\alpha$ . This will be important because the QED calculation for the fine structure of helium is very different from the QED calculation of  $g - 2$  of the electron [24].

The fine structure in the  $1s3p^3P_J$  manifold is similar to that in  $1s2p^3P_J$  except that the intervals are smaller by approximately a factor of three. Given the existing discrepancy between experiment and theory on the fine structure intervals of  $1s2p^3P_J$ , it should be interesting and significant to examine whether a similar discrepancy also exists for the  $1s3p^3P_J$  levels. The most accurate experimental measurement of the  $1s3p^3P_J$  fine structure intervals was carried out by Mueller *et al.* [32]. The intervals were determined by probing each of the  $2^3S_1 - 3^3P_J$  ( $J = 0, 1, 2$ ) transitions with a 389 nm laser light and detecting the laser induced fluorescence from a beam of metastable helium atoms. The atom beam was produced in a RF driven discharge, and two-dimensional transverse cooling was used to increase the beam flux. The experimental and calculated fine structure intervals are presented in Table 2. The achieved precision is at tens of kHz. Obviously, an opportunity exists for further improvement.

**Table 1** Theoretical and experimental values for the fine structure intervals of  $1s2p^3P_J$  in helium, in kHz.

Reference	$\nu_{01}$	$\nu_{12}$
Experiment		
Hessels 2000,2001 [26, 27]	29 616 950.9(9)	2 291 174.0(14)
Shiner 2000 [30]	29 616 959(3)	2 291 175.9(10)
Inguscio 2005 [29]	29 616 952.7(10)	2 291 168(11)
Gabrielse 2005 [31]	29 616 951.66(70)	2 291 175.59(51)
Theory		
Drake 2002 [12]	29 616 946.42(18)	2 291 154.62(31)
Pachucki 2006 [22]	29 616 943.01(17)	2 291 161.13(30)

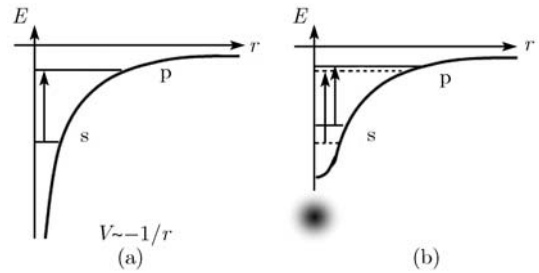
**Table 2** Theoretical and experimental values [32] for the fine structure intervals of  $1s3p^3P_J$  in helium, in MHz.

	$\nu_{01}$	$\nu_{12}$	$\nu_{02}$
Experiment	8113.714(28)	658.810(18)	8772.524(33)
Theory	8113.730(6)	658.801(6)	8772.531(6)

## 2.2 Nuclear effects: isotope shift and hyperfine structure

The isotopes of helium provide an excellent testing ground for the nuclear theory of few-body nuclei. In helium, the nucleus occupies a fractional volume at the order of  $10^{-13}$ , yet the minute perturbation on the atomic energy level due to the finite size of the nucleus can be precisely measured and calculated. Figure 2(a) shows the electrostatic potential of a hypothetical point nucleus of zero charge radius. The potential goes towards negative infinity as it approaches the nucleus at the origin. On the other hand, inside a real nucleus as shown in

Fig. 2(b), the charge is distributed over a finite volume and the potential deviates from the simple  $-1/r$  dependence. The actual potential within the nucleus is finite and higher than in the point-nucleus case. This effect lifts the energy levels of atomic states, which is particularly significant on the  $s$ -states where the electron wave functions do not vanish at the nucleus. For example, the frequencies of the  $2^3S_1 - 3^3P_J$  transitions are shifted down by a few MHz due to the finite nuclear charge radius of a few fm.

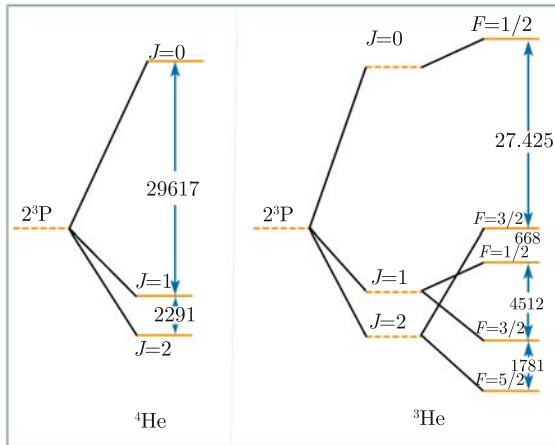


**Fig. 2** The electrostatic potential and energy of bound  $s$ - and  $p$ -electronic levels are illustrated in (a) for a hypothetical point nucleus, and in (b) for the real case of nucleus with a finite volume. The higher potential within the finite size nucleus causes the electrons to be less bound. This so called volume effect is most prevalent in  $s$  electrons.

Shiner *et al.* measured the  $^3\text{He} - ^4\text{He}$  isotope shift of the  $2^3S_1 - 2^3P_0$  transition (1083 nm) to be  $33\,668\,074 \pm 5$  kHz. Together with atomic theory and the known  $^4\text{He}$  nuclear charge radius, the nuclear charge radius of  $^3\text{He}$  was determined as 1.9506(14) fm [33]. Due to a controversy in the reported nuclear charge radius of  $^4\text{He}$ , we revise the nuclear charge radius of  $^3\text{He}$  to 1.951(8) fm. The result tests nuclear structure calculations in a simple three-nucleon system. More recently, isotope shift measurements were extended to the exotic isotopes  $^6\text{He}$  and  $^8\text{He}$ .  $^8\text{He}$  is the most neutron-rich helium isotope that can be synthesized on Earth: it consists of two protons and six neutrons, and remains stable for an average of 0.2 seconds. Unlike the abundant helium isotope  $^4\text{He}$  which has the two neutrons packed closely with two protons, the additional neutrons in  $^8\text{He}$  form a “halo” around a compact  $^4\text{He}$ -like core. A similar neutron-halo structure also exists in  $^6\text{He}$  whose lifetime is 1 s. Because of their intriguing properties,  $^6\text{He}$  and  $^8\text{He}$  have the potential to reveal new aspects of the fundamental forces among the constituent nucleons. Mueller *et al.* [34] have recently succeeded in laser trapping and cooling these exotic helium isotopes and have performed precision laser spectroscopy on individual trapped atoms. Based on atomic frequency differences measured along the isotope chain  $^4\text{He} - ^6\text{He} - ^8\text{He}$ , the nuclear charge radius of  $^8\text{He}$  has been determined for the first time to be 1.93(3) fm [34] and that of  $^6\text{He}$  to be 2.054(14) fm [35]. These results are in good agreement with recent predictions of state-of-the-art nuclear structure theories, and provide a critical test

of the present understanding of these loosely-bound halo nuclei.

The hyperfine structure of the  $2^3P_J$  state of  $^3\text{He}$  is illustrated in Fig. 3. It was measured by Prestage *et al.* [36] and was recently calculated by Wu and Drake [37]. The hyperfine splittings were found to be comparable to the fine structure. The situation is similar for the  $3^3P_J$  state of  $^3\text{He}$ . The ordinary  $LS$  coupling scheme is no longer applicable in this case. Sulai *et al.* proposed an “ $IS$ ” coupling scheme: the fine-structure interaction was treated as a perturbation on the states obtained by first coupling nuclear spin to the total electron spin [38]. This scheme explained well the observed abnormal suppression of two transitions in the  $2^3S_1$ – $3^3P_J$  manifold.



**Fig. 3** The  $1s2p$   $3^3P_J$  levels of  $^4\text{He}$  and  $^3\text{He}$ . The splittings are given in MHz.

### 2.3 The Lamb shift

The  $2^2S_{1/2}$ – $2^2P_{1/2}$  splitting in atomic hydrogen found by Lamb and Retherford 60 years ago [39] attracted great theoretical interest and resulted in the establishment of the QED theory. Precision measurements of the “Lamb shift” test the atomic theory including QED effects in the simple hydrogen- and helium-like systems. The largest Lamb shift in helium occurs at its ground level  $1^1S$ , where the electron wave function has the most overlap with the nucleus. However, the difficulty in the extreme ultraviolet 58 nm laser makes it a challenging experiment to measure the Lamb shift of  $1^1S$ . The  $1^1S$ – $2^1P$  transition of helium at 58 nm was measured by Eikema *et al.* to be 5 130 495 083(45) MHz [40]. The deduced ground state Lamb shift of 41 224(45) MHz agrees well with the theoretical value of 41 233(35) MHz from QED calculations to the order  $\alpha^5 Z^6$ .

Although the Lamb shift in the  $2^3S_1$  metastable level is one order of magnitude smaller than that in the ground level, the much more available near-infrared and visible lasers make the  $2^3S_1$  level easier to measure precisely.

The  $2^3S$ – $2^3P$  transition frequency was measured by Pastor *et al.* using an optical frequency comb [41]. The spin-averaged centroid value of the  $2^3S$ – $2^3P$  frequency was determined to be 276 736 495 624.6(2.4) kHz, based on which the Lamb-shift contributions to the  $2^3S$ – $2^3P$  transition and to the  $2^3P$  energies were derived to be 5 311.210 9(35) MHz and  $-1253.978(58)$  MHz, respectively. This is the most accurate Lamb shift measurement of helium determined so far. The values agree well with the QED calculations but the latter still has a much larger uncertainty. The Lamb shift of the  $2^3S_1$  level has also been investigated through other transitions. Pavone *et al.* measured the  $2^3S_1$ – $3^3P_0$  transition in  $^4\text{He}$  (389 nm) with respect to the previously calibrated  $^{87}\text{Rb}$  transition at 778 nm [42]. The absolute frequency of the helium transition was determined to be 770 732 839 058(190) kHz, and the Lamb shift of the  $2^3S_1$  level was determined to be 4 057.61(79) MHz. Dorner *et al.* measured the  $2^3S_1$ – $3^3D_1$  two-photon transition frequency at 762 nm, and determined the  $2^3S_1$ – $3^3D_1$  frequency to be 786 823 850.002(56) MHz with a relative uncertainty of  $7.1 \times 10^{-11}$ , which in turn gave a deduced  $2^3S_1$  Lamb shift of 4057.276(60) MHz [43].

## 3 Discussion and prospect

Overall, precision spectroscopy of the helium atom provides a favorable case for testing the QED theory and atomic physics in general. The significant discrepancy between theoretical and experimental values of the fine structure of  $2^3P_J$  requires further theoretical and experimental studies. A higher accuracy measurement of the ionization energy of the ground level  $1^1S_0$  in  $^4\text{He}$  will be an excellent test of the bound-state QED theory. There are additional exciting opportunities. Eyler *et al.* proposed to perform two photon spectroscopy from the  $2^3S$  level to higher Rydberg states [44]. Leeuwen *et al.* suggested the measurement of the “forbidden”  $2^3S$ – $2^1S$  magnetic dipole ( $M1$ ) transition of helium at 1.557  $\mu\text{m}$  [45]. The very small natural width (8 Hz) of the transition would allow very accurate testing of atomic structure calculations. We believe that helium spectroscopy is a fertile field for precision study on atomic structure, both theoretically and experimentally. A project dedicated to precision study of the helium atom is being planned by the authors at the Hefei National Laboratory for Physical Sciences at the Microscale (HFNL).

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