RIEMANNIAN GEOMETRY
EXCERCISE 10

1. Let $\mathbb{S}^2$ be the unit sphere in $\mathbb{R}^3$ with the metric
$$g = d\theta^2 + \cos^2\theta d\varphi^2.$$
Note that
$$X_1 = \frac{\partial}{\partial\theta},\ X_2 = \frac{1}{\cos\theta}\frac{\partial}{\partial\varphi}$$
satisfy $g(X_i, X_j) = \delta_{ij}$. Show that $R(X_1, X_2)X_1 = X_2$, and, hence, the sectional curvature $K(X_1, X_2) = 1$.

2. Let $\mathbb{B}^2$ be the unit disc in $\mathbb{R}^2$ with the metric
$$g = \frac{4}{(1-r^2)^2}(dr^2 + r^2 d\theta^2).$$
Note that
$$X_1 = \frac{1-r^2}{2}\frac{\partial}{\partial r},\ X_2 = \frac{1-r^2}{2r}\frac{\partial}{\partial\theta}$$
satisfy $g(X_i, X_j) = \delta_{ij}$. Show that $R(X_1, X_2)X_1 = -X_2$, and, hence, the sectional curvature $K(X_1, X_2) = -1$.