RIEMANNIAN GEOMETRY
EXCERCISE 14

A Riemannian manifold (M, g) is called *homogeneous* if for any $p, q \in M$, there exists an isometry $\varphi : M \to M$ such that $\varphi(p) = q$. (M, g) is called *two-point homogeneous*, if for any two pairs of points p_1, p_2 and $q_1, q_2 \in M$ with

$$d(p_1, p_2) = d(q_1, q_2),$$

there exists an isometry $\varphi : M \to M$ such that

$$\varphi(p_i) = q_i, \quad i = 1, 2.$$

1. Prove that any simply-connected space forms are two-point homogeneous. (Hint: Using the Theorem 9 (lecture notes V, §8) we discussed.)