Strong Convergence Month in USTC

Strong convergence and its applications to random graphs and surfaces

Organizers: Wenbo Li, Shiping Liu, Joe Thomas

Venue: School of Mathematical Sciences, University of Science and Technology of China

Online Talks

Talk 1: Strong convergence of matrices: results and applications
Speaker: Charles Bordenave (CNRS & Aix-Marseille University, IAS)
Time: Oct 27, 8–9 AM

Abstract: Strong convergence is a notion of convergence for matrix algebras at the level of operator norms. Since the fundamental work of Haagerup-Thorbjørnsen in 2005 on the strong convergence of independent Gaussian unitary matrices, this notion has attracted a lot of attention. It plays a central role in recent advances in operator algebras, spectral graph theory or spectral geometry for example. In this lecture, we will explore the particularly intriguing case of strong convergence for unitary representations of finitely generated groups. Special emphasis will be placed on its applications.

Talk 2: A new approach to strong convergence (an overview)
Speaker: Jorge Garza Vargas (Princeton University)
Time: Oct 29, 8–9 AM

Abstract: In this talk I will give a high-level overview of the main techniques used in our new method for proving strong convergence. To make the exposition concrete, I will give examples of how each of these techniques can be used to prove statements that are of direct interest in the study of operator algebras. This is joint work with Chi-Fang Chen, Joel Tropp, and Ramon van Handel.

Talk 3: Strong convergence for random permutation representations of free groups
Speaker: Ewan Cassidy (Cambridge University)
Time: Nov 6, 4–5 PM

Abstract: One can obtain a random permutation representation of the free group $\mathbf{F}_r$ by composing a random $\varphi_n \in \mathrm{Hom}(\mathbf{F}_r, S_n)$ with a fixed complex representation ρ of S_n. Bordenave and Collins showed that, if we take ρ to be the $(n-1)$-dimensional standard representation, then $\rho \circ \varphi_n$ strongly converges (in probability) to the left regular representation λ of the free group. I will discuss a generalization of this idea in which we replace the standard representation with much higher dimensional representations. This also has some interesting consequences in spectral graph theory. The proof is based on the recent "polynomial method" for strong convergence of Chen, Garza-Vargas, Tropp and van Handel, combined with some new group theoretic inputs.

Mini-Courses

Part I: Random regular graphs: An introduction to Friedman's theorem
Speaker: Wenbo Li (University of Science and Technology of China)
Time: Oct 31, Nov 3, 5, 10–12 AM

Abstract: In these three talks of the mini-course we will discuss *Friedman's theorem*, which asserts that for random d-regular graphs all nontrivial eigenvalues are, with high probability, bounded by $2\sqrt{d-1}+o(1)$, matching the Ramanujan bound up to vanishing error. We will first introduce the notion of random regular graphs and explain the phenomenon of tangle behavior. Then we will present a proof of Friedman's theorem in the setting of random permutation models, following the recent approach of Chen, Garza-Vargas, Tropp, and van Handel. The lectures will serve as an introduction to this new method, which offers a conceptually simpler route to strong convergence.

Part II: Optimal spectral gap of random hyperbolic surfaces
Speaker: Joe Thomas (Durham University)
Time: Nov 7, 10, 12, 14, 10–12 AM

Abstract: The second part of the minicourse will be focussed on the size of the spectral gap of the Laplacian for random hyperbolic surfaces. We will begin with a crash course on hyperbolic geometry and surfaces with the main highlight being the Selberg trace formula. This formula gives a direct connection between the Laplacian spectrum and geometry of a surface. Next, we will give an overview of the construction of Weil-Petersson random hyperbolic surfaces. We will introduce the Mirzakhani integration formula which is a fundamental tool used in the computation of probabilities in this random surface model. We will then put all of these tools together along with the key ideas of the polynomial method to prove an optimal spectral gap result holding asymptotically almost surely.