Real Analysis Problem Set 1, Part 1

02/26/2018

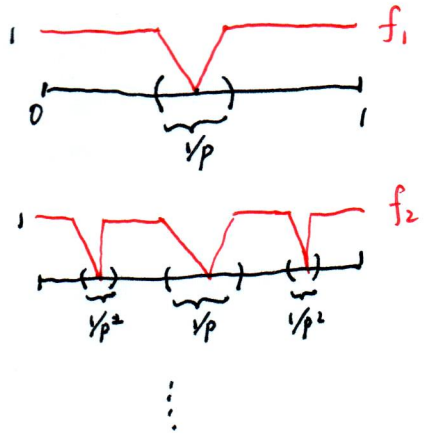
Due: 03/05, before class

1. Read: "周民强, 第三版", P15 - P24. (基数, 可数集, 不可数集)

2. A real number x is called algebraic if it is the root of a polynomial with integer coefficients.
Prove: The set of all algebraic numbers is countable.

3. Read "周民强, 第三版", P45 - 48. (Cantor集, 类Cantor集, Cantor函数)

4. Let C_δ be the Cantor-like set ~~of total length~~ constructed on Page 47.
It is a non-empty complete set without interior point, and it is not of measure zero.
Construct a sequence of continuous functions f_n as follows:



Prove. (1) $f_n \geq f_{n+1}$, and the limit function $f(x) = \lim_{n\to\infty} f_n(x)$ exists

(2) $f(x)$ is NOT Riemann integrable.